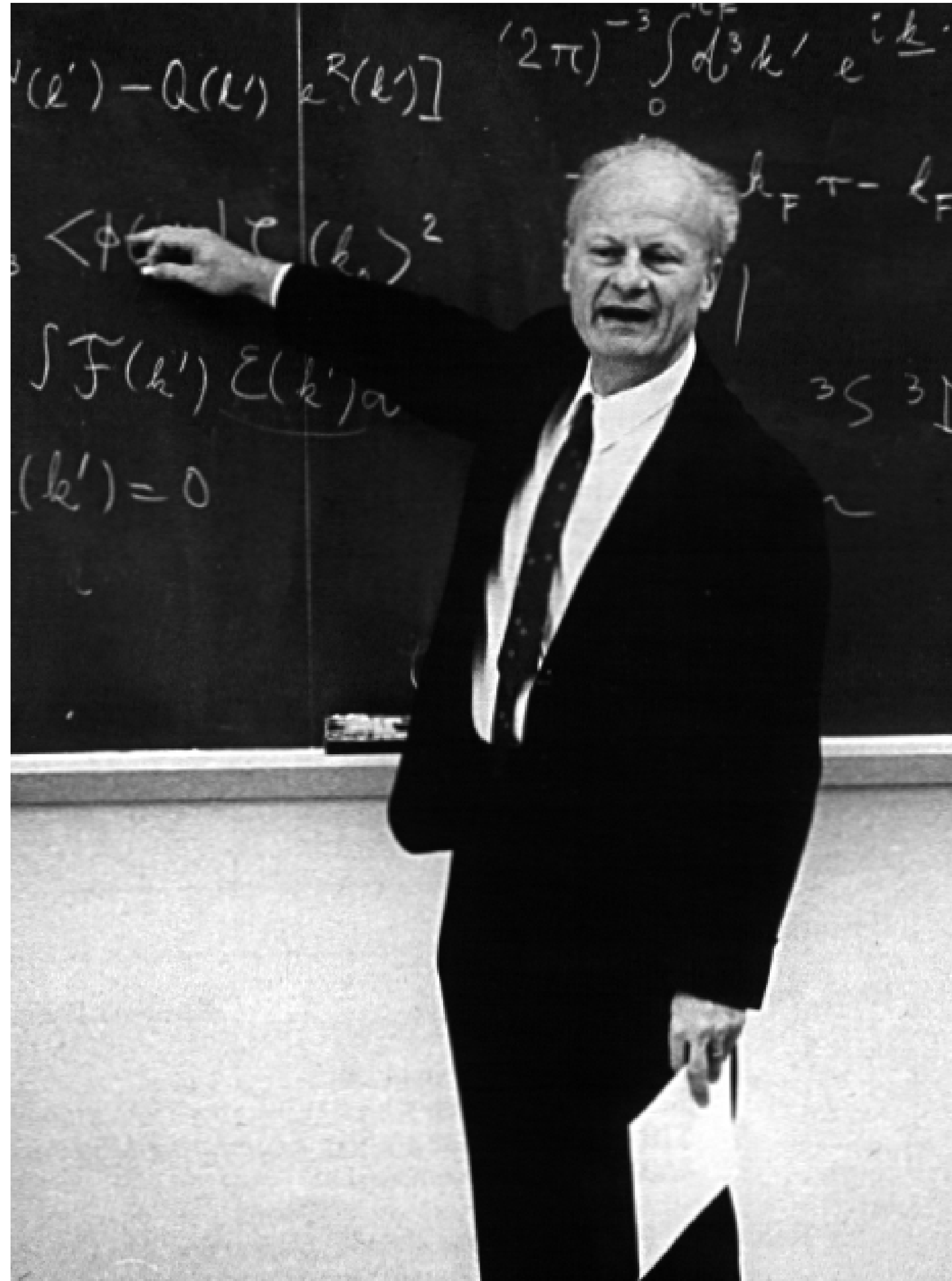
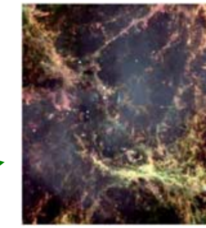
Hans Bethe, 1906 -

"Energy Production in Stars"
Phys. Rev. 55, 434 (1939)



Fusion Reactions in Stars: Challenges and Solutions

C.A. Bertulani NSCL, Michigan State University

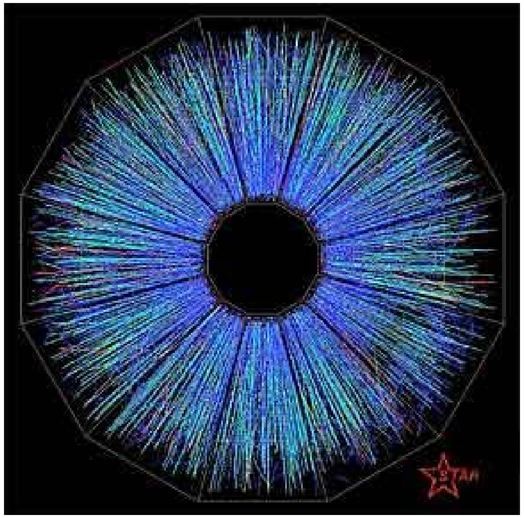


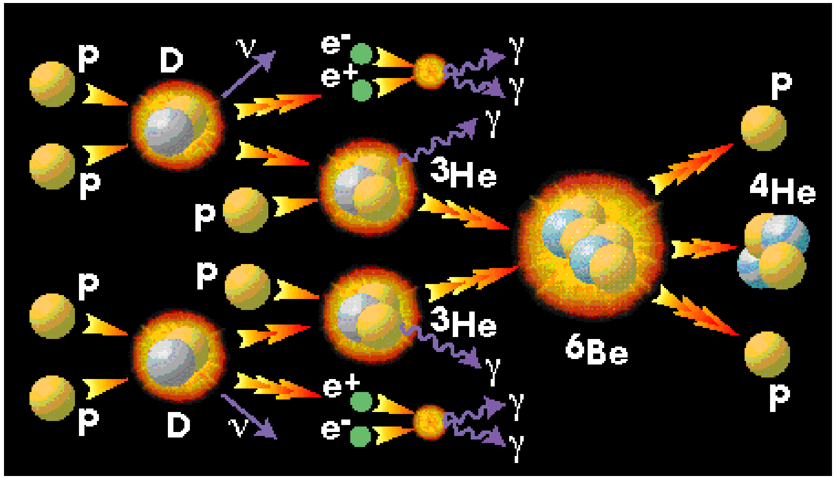
- Very high vs. very low energies
- <u>Problems with low energy experiments</u>
- NSCL/MSU's role: indirect methods
- Applications and status
- Perspectives

Jina Pizza Talk, MSU, March 8, 2004

Challenges in Nuclear Astrophysics

Very high vs. very low nuclear energies





100 GeV/nucleon

keV/nucleon

???

Exotic stellar site

Quark matter in compact stars, Big Bang

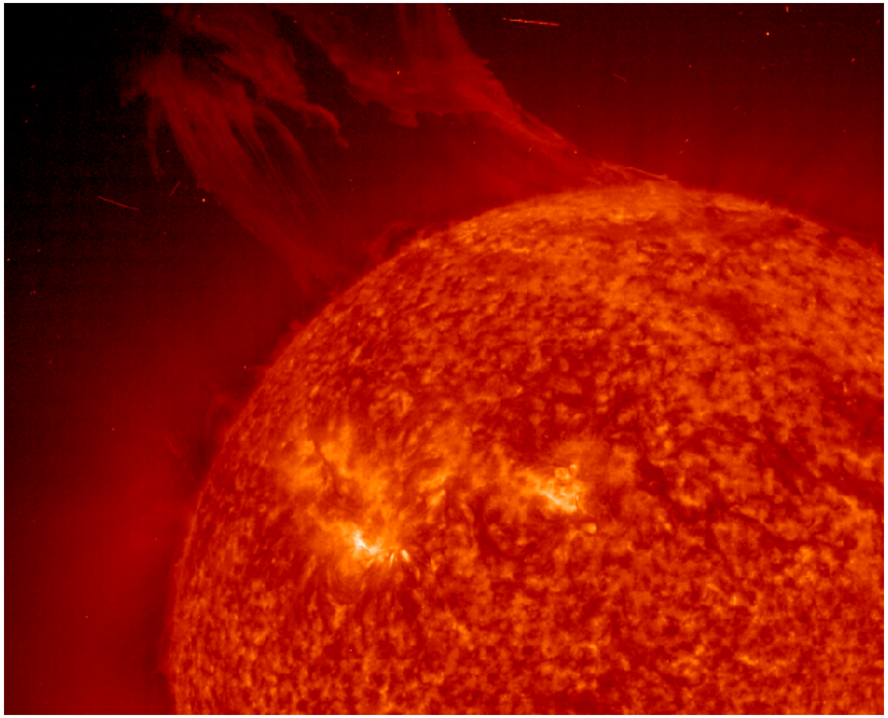
???

Typical stellar site

Stellar evolution

Why are low energies interesting?

(a) Understand our Sun



p-p chain

$p(p,e^+\nu)d$

$d(p,\gamma)^3He$

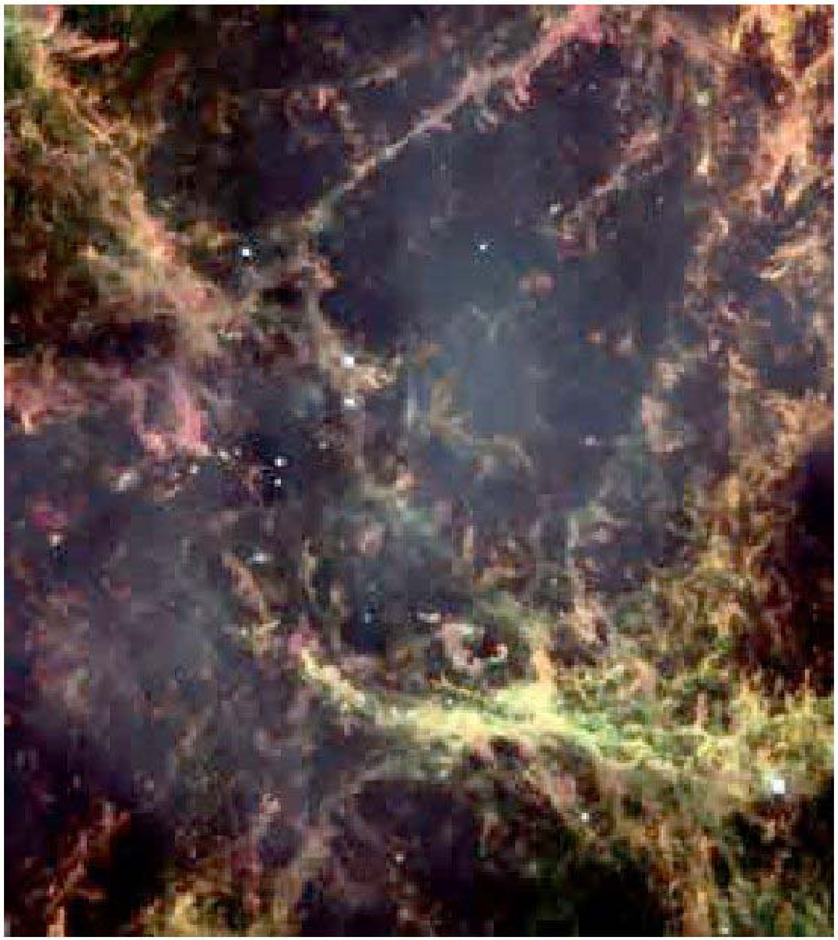
86% | 14%

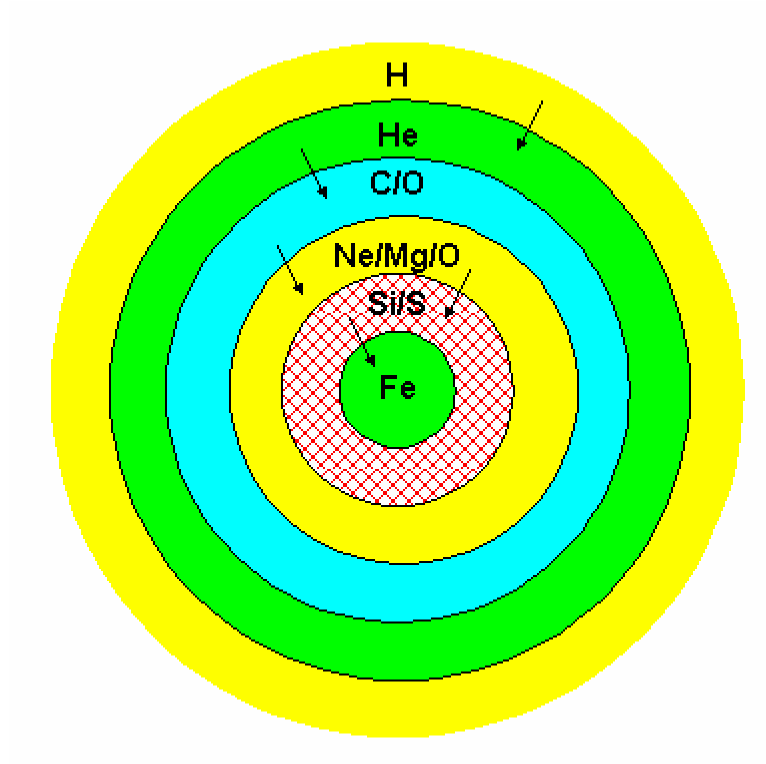
- 86%: $^3He(^3He,2p)^4He$
- 14%: $^3He(\alpha,\gamma)^7Be$
  - 14%: $Be(e^-,\nu)^7Li$, $^7Li(p,\alpha)^4He$
  - 0.02%: $^7Be(p,\gamma)^8B$, $^8B(e^+,\nu)^8Be$, $^8Be(\alpha)^4He$

Solar neutrinos

E_ν < 15 MeV

(b) Fate of massive stars

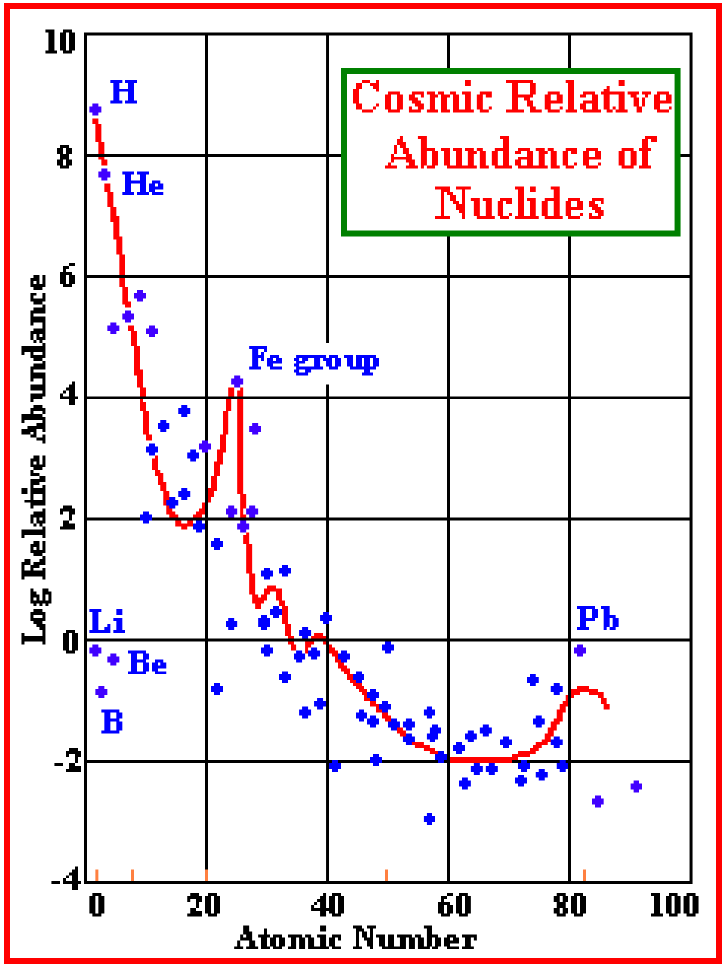


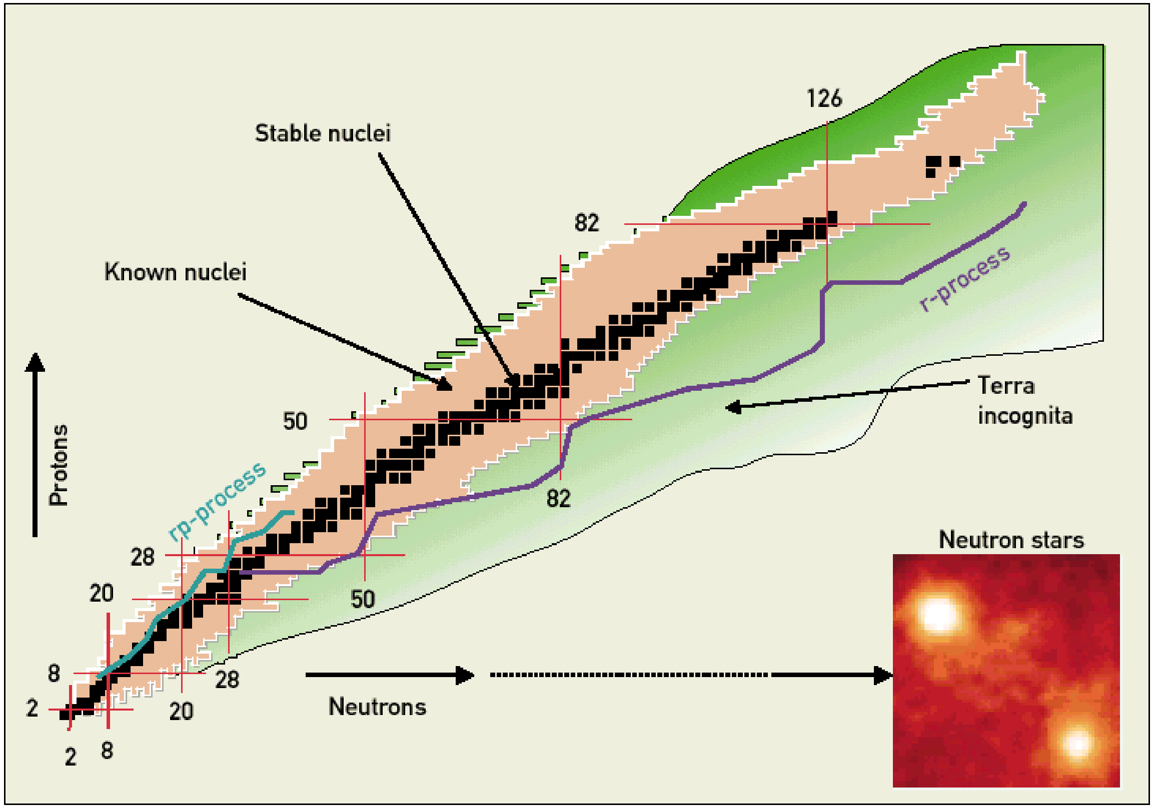


Helium burning: $^{12}C(\alpha,\gamma)^{16}O$

Supernovae remnants: black holes or neutron stars?

(c) Nucleosynthesis





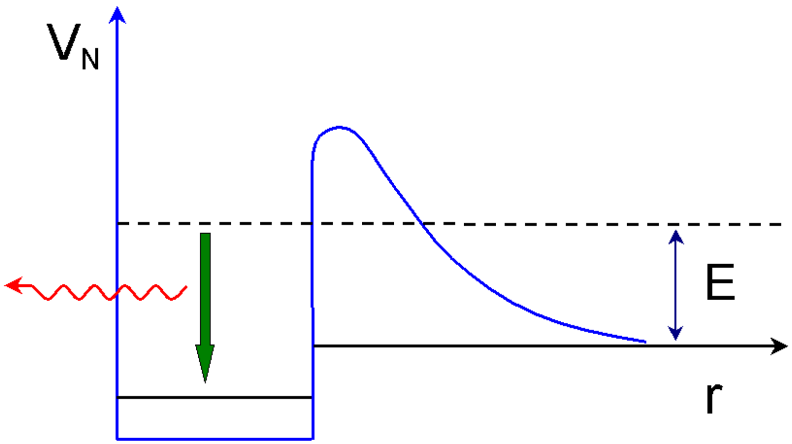
Rapid n-capture:

(n,γ) faster than β-decay

Numerous σ(n,γ) needed

Experimental problems

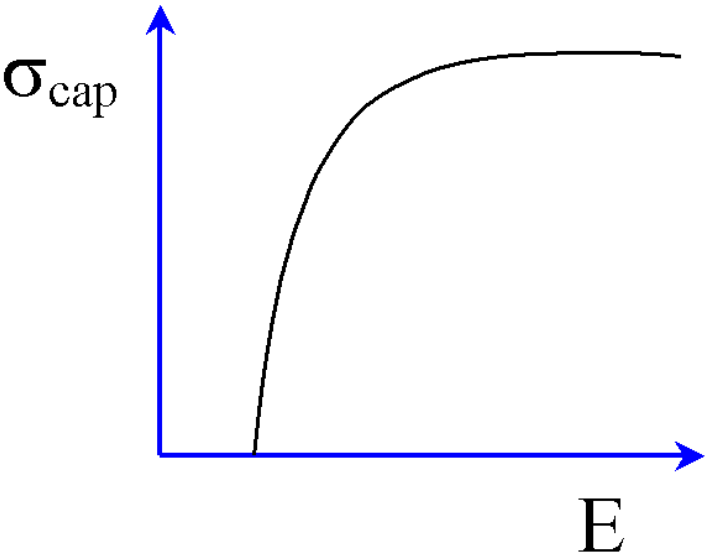
Charged particles



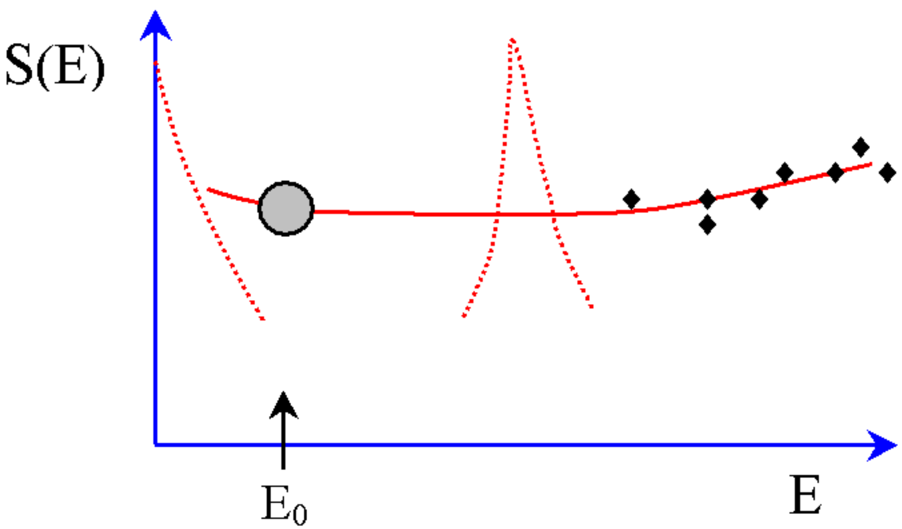
Astrophysical S-factor

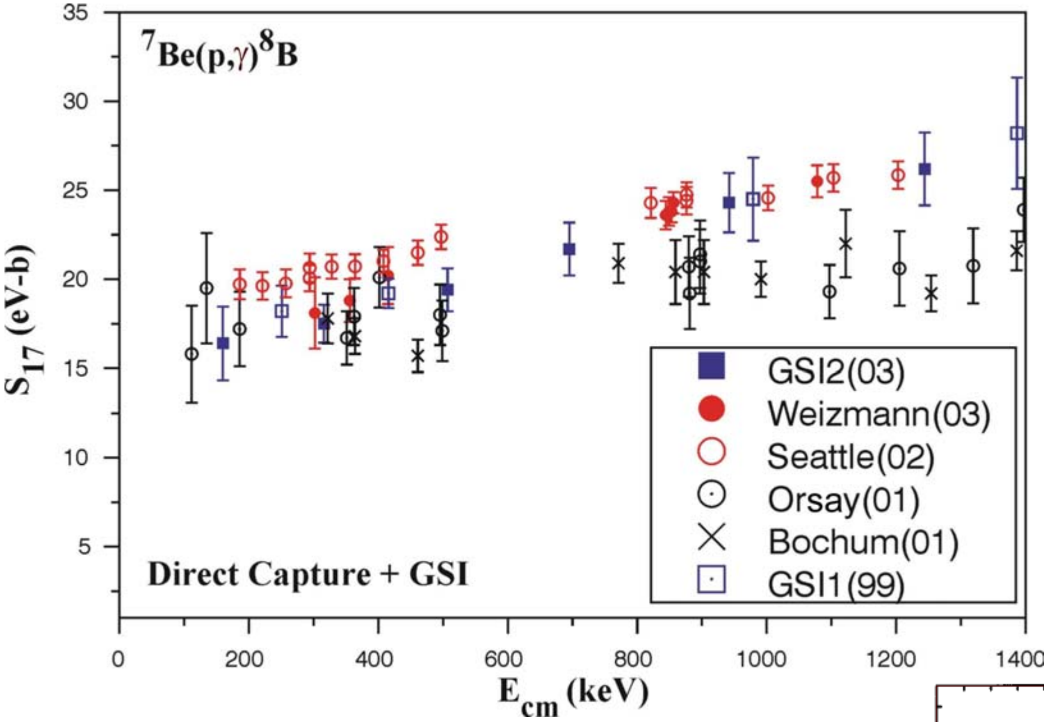
$$\sigma(E) = \frac{1}{E} S(E) \exp\left[-2\pi \frac{Z_1 Z_2 e^2}{\hbar v} \right]$$

Steep energy dependence



Unreliable extrapolations



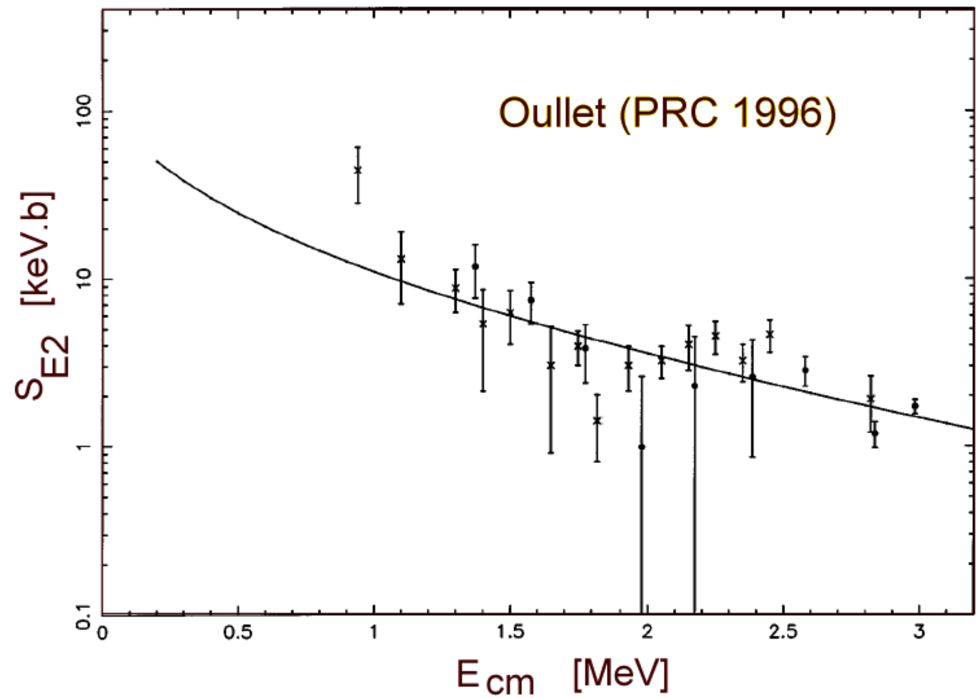


$^{7}Be(p,\gamma)^{8}B$

Solar neutrino problem might be due to ν-oscillations

But this reaction needs to be known more accurately

- Bahcall

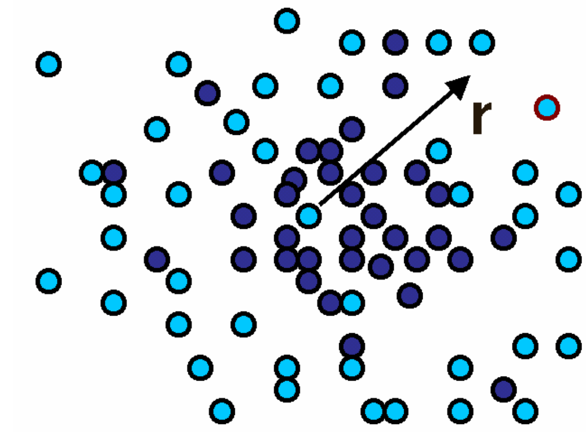


$^{12}C(\alpha,\gamma)^{16}O$

Has to be known better than 20% - Woosley 1993

Other (serious) problems
(a) electron screening (in stars)

Debye-Hueckel screening (Salpeter 1959)



$$< \sigma v >_{plasma} = f(E) < \sigma v >_{bare}$$

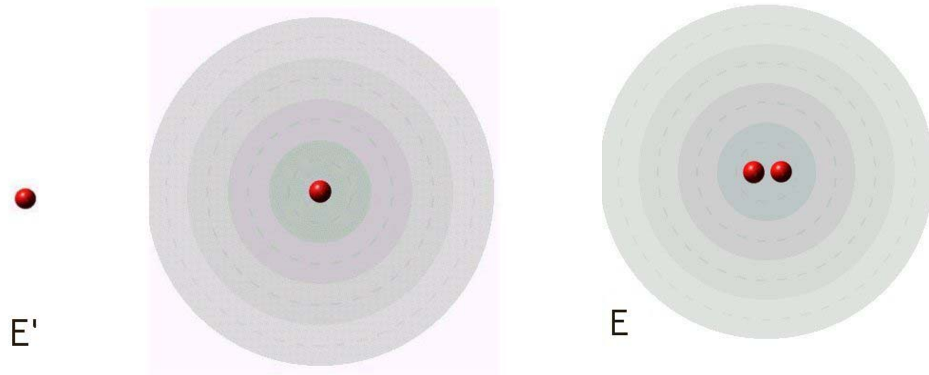
$$V_{eff} = \frac{Z_1 Z_2 e^2}{r} e^{-r/R_D} \sim \frac{Z_1 Z_2 e^2}{r} - U_e$$

$$f(E) = e^{U_e/kT}, \qquad R_D \sim \sqrt{kT/\rho} \sim 0.218\ \mathring{A}\ \text{(Sun)}$$

$$^{7}\text{Be}(p,\gamma)^{8}\text{B} \ (T = 20\,\text{keV}): \qquad f(E) \simeq 1.2$$ (20 % effect)

- Dynamics of continuum electrons 1 – 10 % effect
- Fluctuations in ion number in Debye-Hueckel sphere

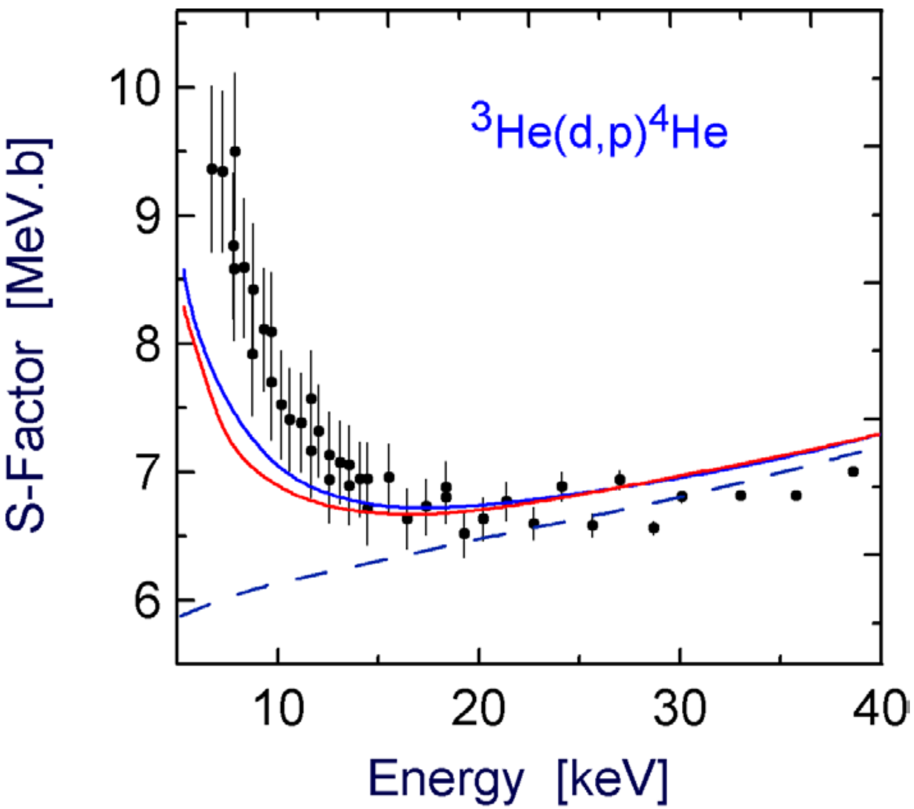
(b) Electron screening (in the laboratory)



Adiabatic model: $\Delta E = E' - E$

$$\sigma_{lab} \sim \sigma_{bare}(E + \Delta E)$$

$$\sim \exp\left[\pi\,\eta(E)\frac{\Delta E}{E}\right]\sigma_{bare}(E)$$



S_{bare} (dashed line)

Dynamic (red line)

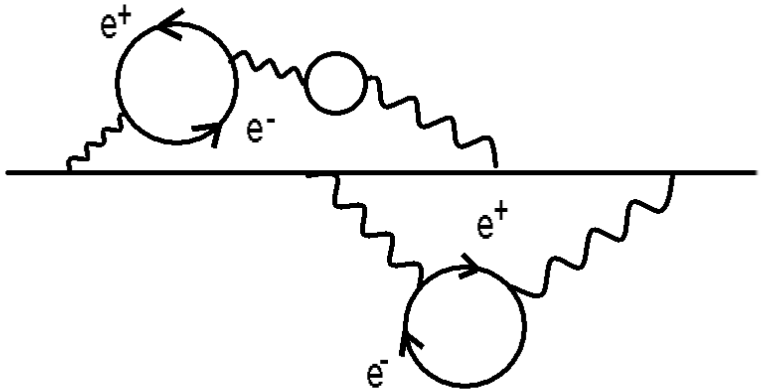
Adiabatic (blue line)

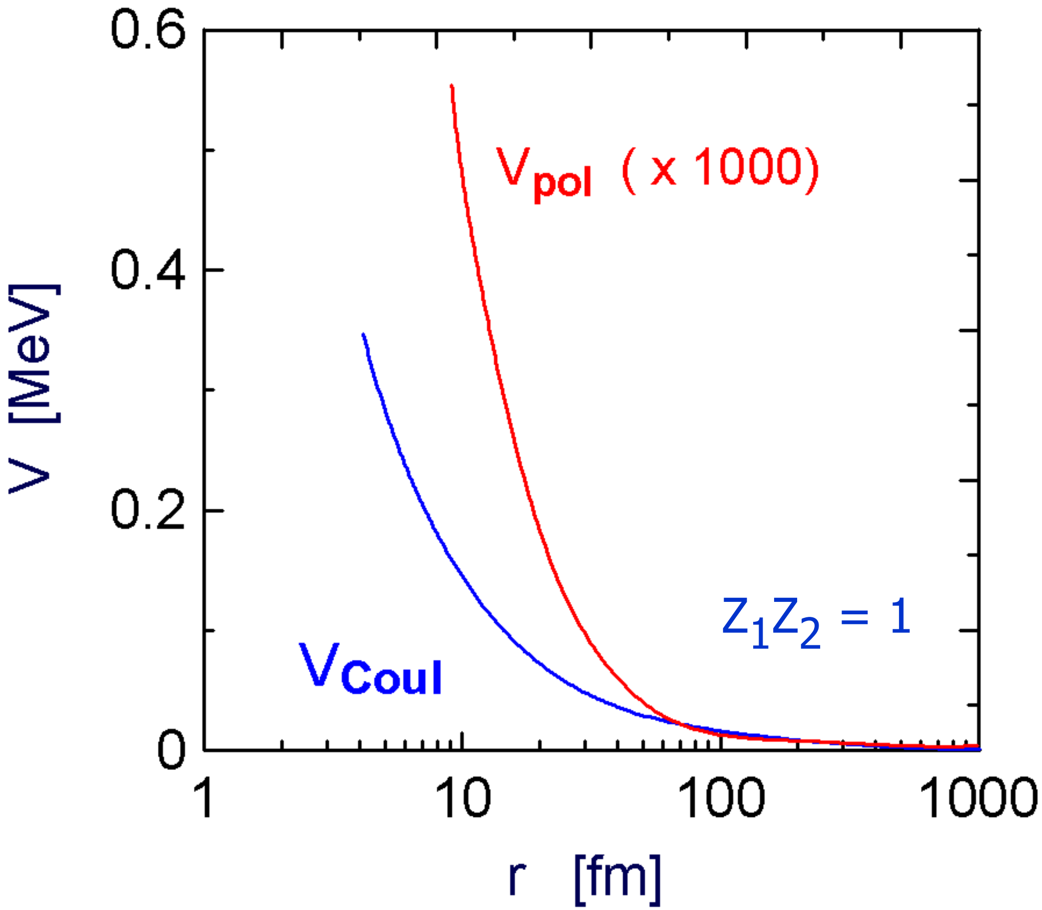
Rolfs, 1995

Reaction	ΔE [eV] experiment	ΔE [eV] adiabatic limit
$d(^3He, p)^4He$	180 ± 30	119
$^6Li(p, \alpha)^3He$	470 ± 150	186
$^6Li(d, \alpha)^4He$	380 ± 250	186
$^7Li(p, \alpha)^4He$	300 ± 280	186
$^{11}B(p, \alpha)2^4He$	620 ± 65	348

Possible solutions
(a) Amplificaton of small effects

Vacuum polarization





- Relativistic effects
- Bremsstrahlung
- Atomic polarizabilities

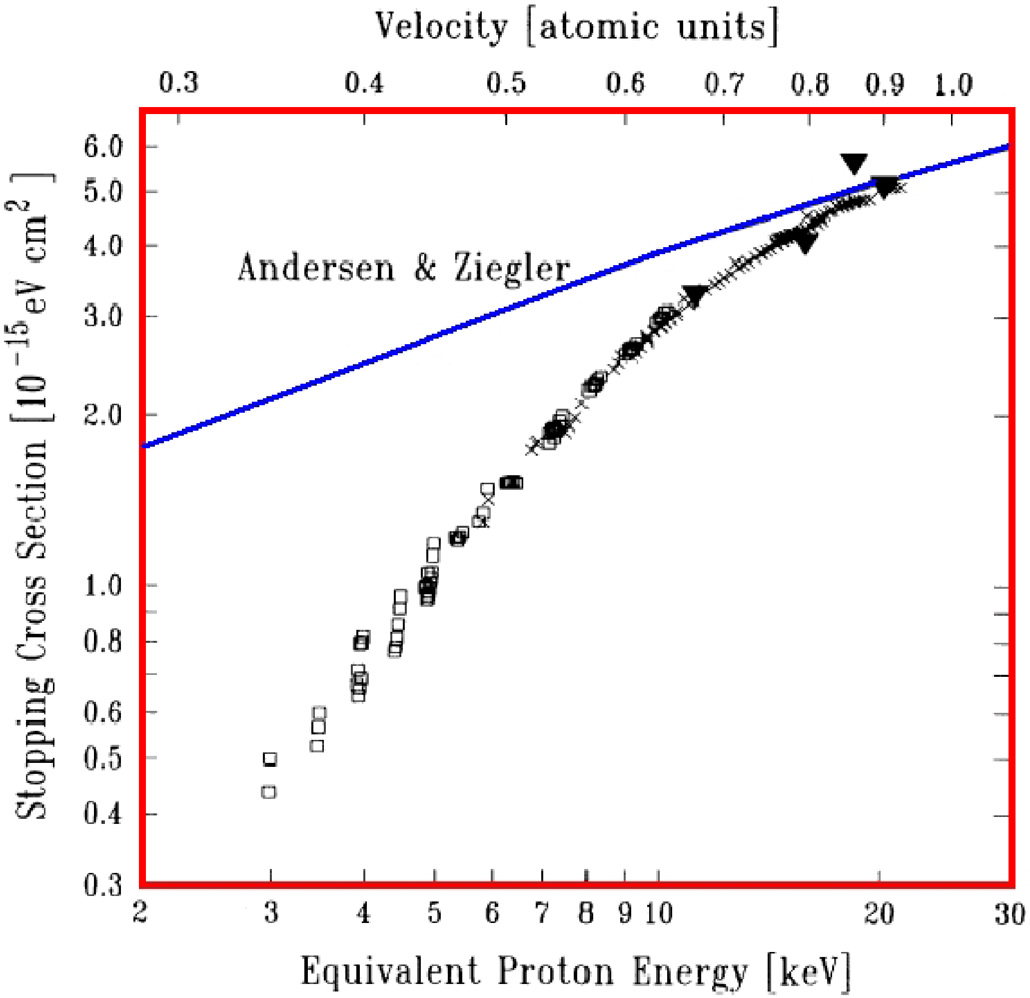
C.B., Balantekin, Hussein, NPA 1997

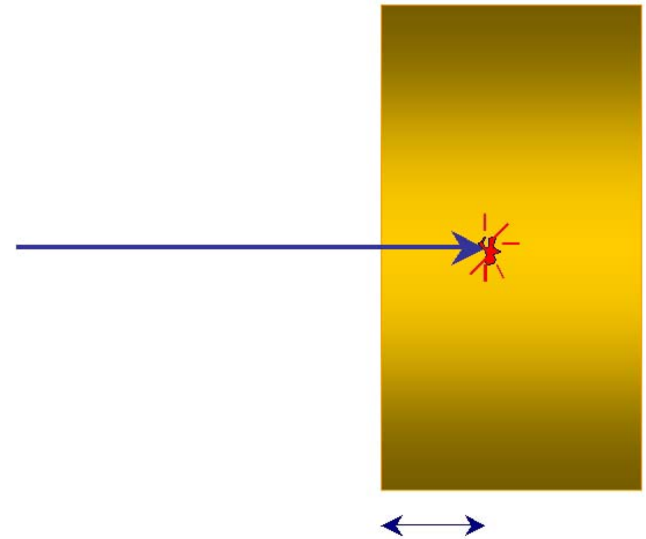
Corrections	
Vaccuum Polarization	$\sim 1\%$
Relativity	10^{-3}
Bremsstrahlung	10^{-3}
Atomic porarization	10^{-5}
Nuclear polarization	$<10^{-10}$

Not a solution!

(b) Wrong extrapolation of stopping power?

Bang, PRC 1996; Langanke, PLB 1996





$$S = -\frac{dE}{dx}$$

E' = E – S.Δx

H + He

Golser and Semrad, PRL 1991

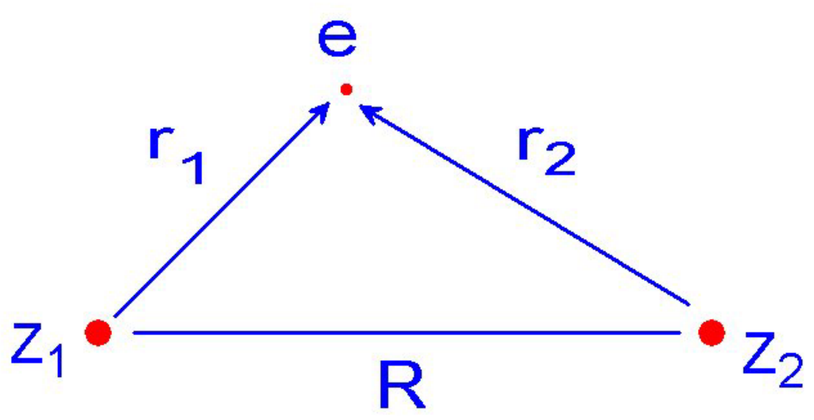
Mainly charge-exchange

Stopping power at very low energies
(a) Test with the simplest system

p + H

P + D

C.B. and de Paula, PRC 2000



c.c. t.d. Schroedinger eq. in a two-center basis |m>

$$\xi = \frac{r_1 + r_2}{R}; \qquad \eta = \frac{r_1 - r_2}{R}; \quad \phi$$

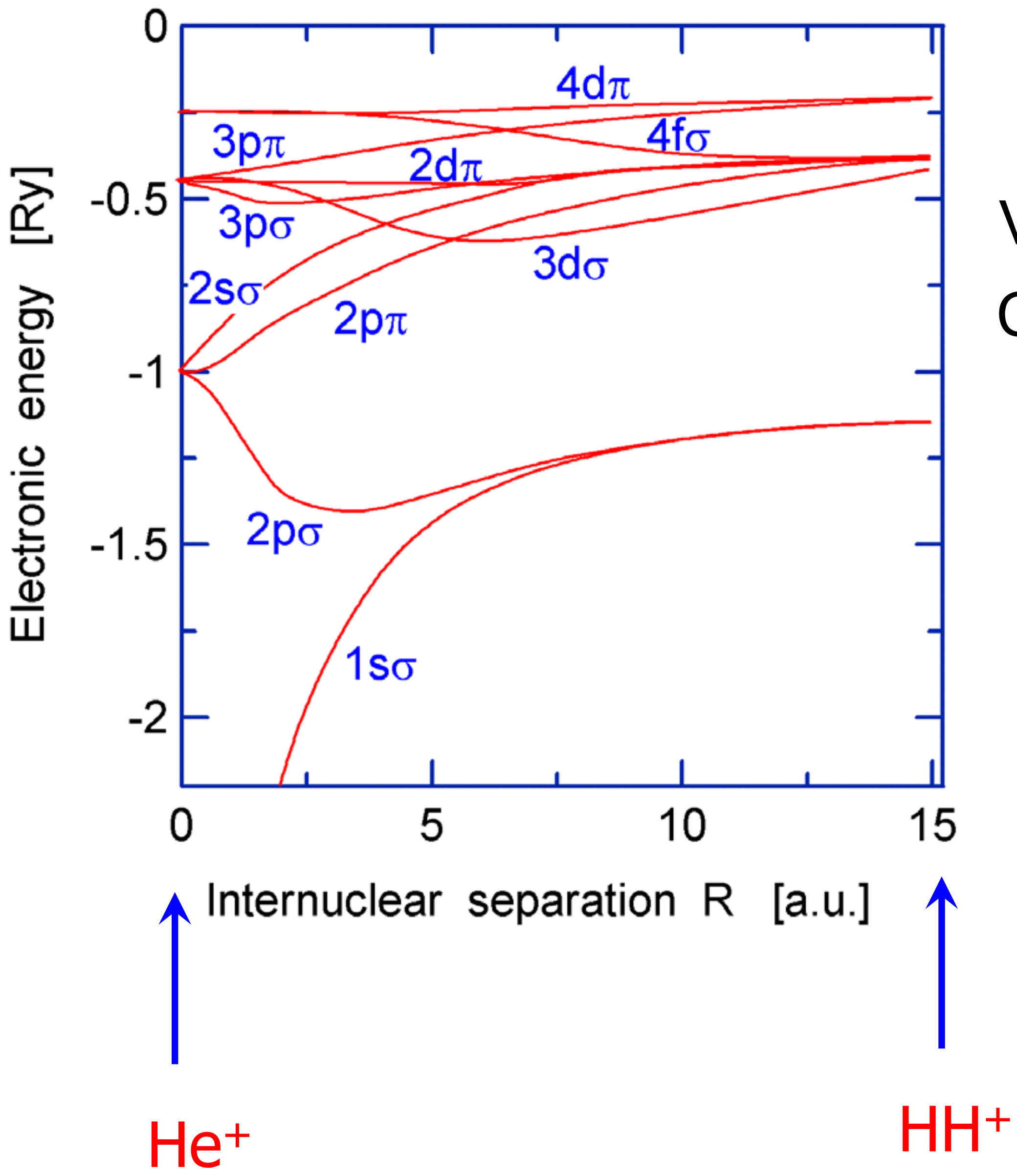
$$\Psi = F(\xi)\, G(\eta)\, e^{im\phi}$$

Elliptic coordinates

$$\frac{d}{d\xi}\left[\left(\xi^2 - 1\right)\frac{dF}{d\xi}\right] + \left[\frac{R^2\xi^2}{2}E + 2R\xi - \frac{m^2}{\xi^2 - 1}\right]F(\xi) = 0$$

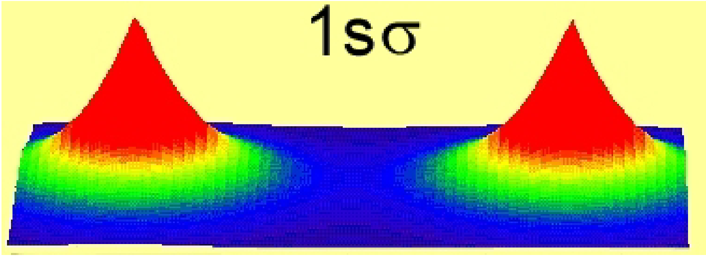
$$\frac{d}{d\eta}\left[\left(1 - \eta^2\right)\frac{dG}{d\eta}\right] - \left[\frac{R^2\xi^2}{2}E + 2R\xi + \frac{m^2}{\eta^2 - 1}\right]G(\eta) = 0$$

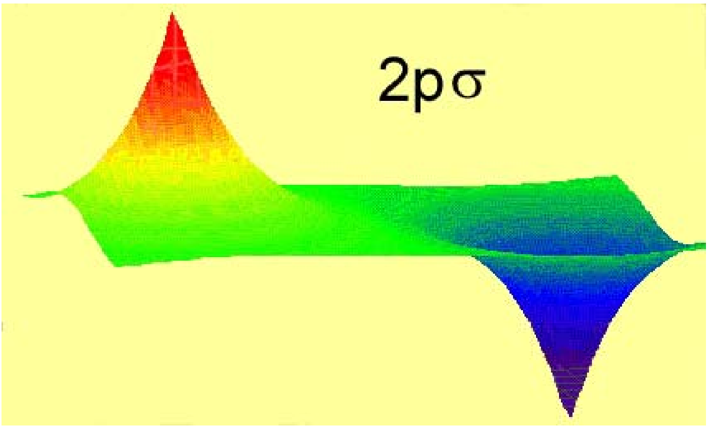
Expansion basis: molecular orbitals for p+H



$$l_z \Phi_s = \pm \lambda \Phi_s$$

Value of λ	0	1	2	3
Code letter	σ	π	δ	$\phi, \cdots$





Dynamical calculations

$$i\hbar \frac{d}{dt} a_m(t) = E_m(t) a_m(t) - i\hbar \sum_n a_n(t) \left\langle m \left| \frac{d}{dt} \right| n \right\rangle$$

$$\left\langle m \left| \frac{d}{dt} \right| n \right\rangle = \frac{\left\langle m \left| dV_p / dt \right| n \right\rangle}{E_n(t) - E_m(t)}, \qquad 0 \ \text{otherwise}$$

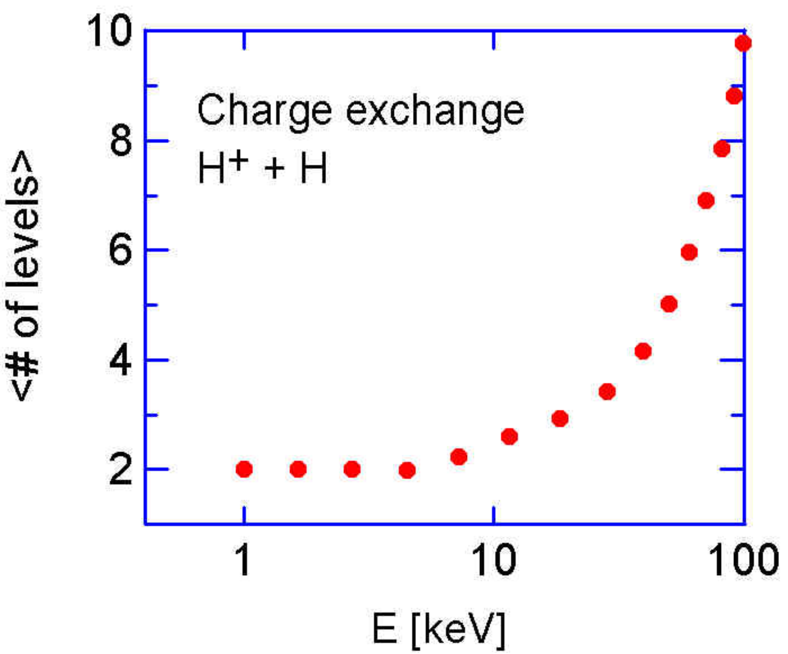
Hellman, Feynmann relation

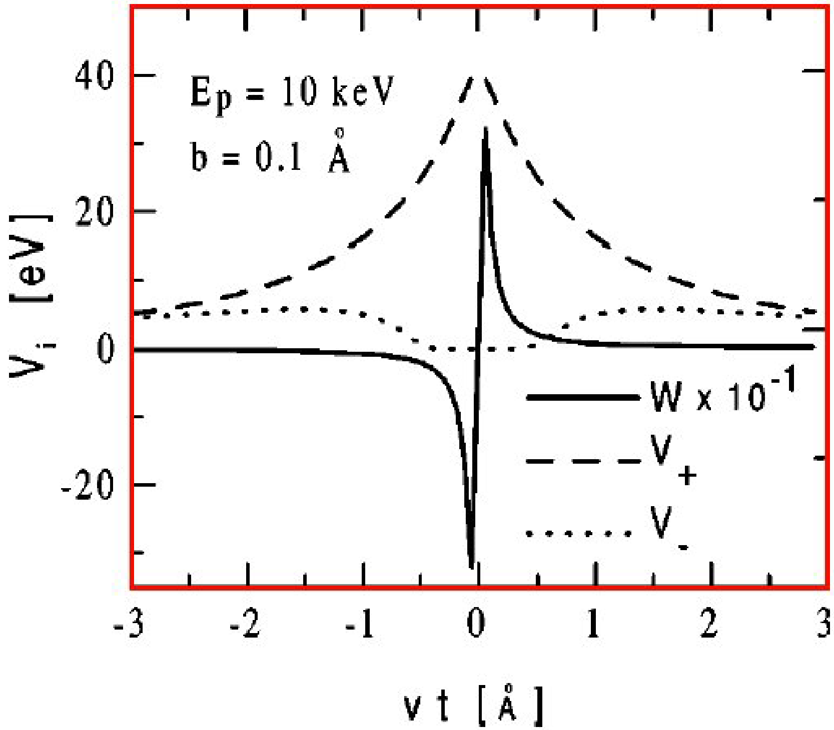
For E_p < 30 keV, only 1sσ and 2pσ

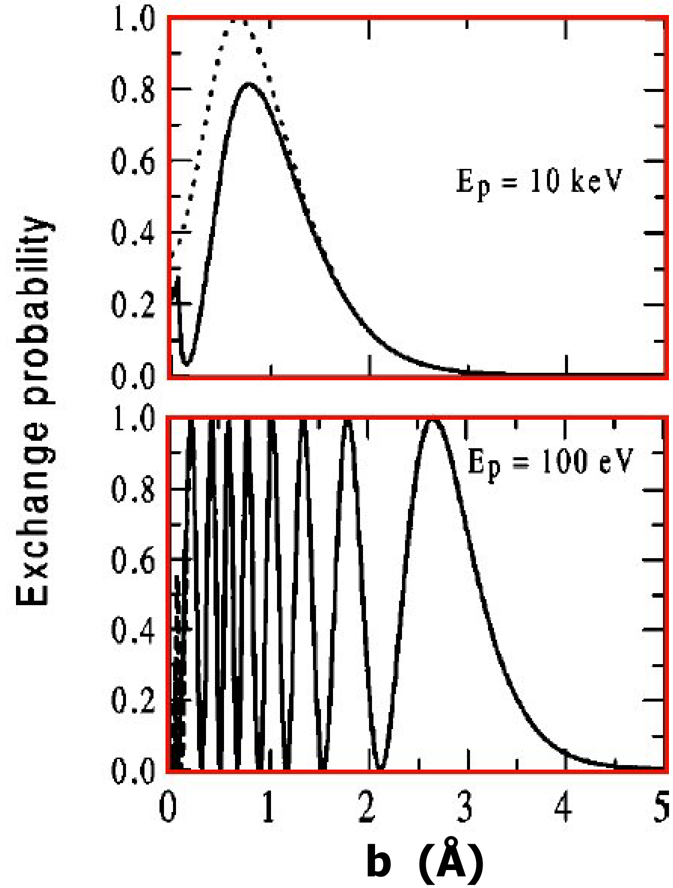
2-level problem

$$i\hbar \frac{d}{dt} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = \begin{pmatrix} V_+ + E_0 & iW \\ iW & V_- + E_0 \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

$$W(t) = \hbar \frac{\left\langle \Psi_{1s\sigma} \left| dV_p / dt \right| \Psi_{2p\sigma} \right\rangle}{E_{1s\sigma}(t) - E_{2p\sigma}(t)}$$







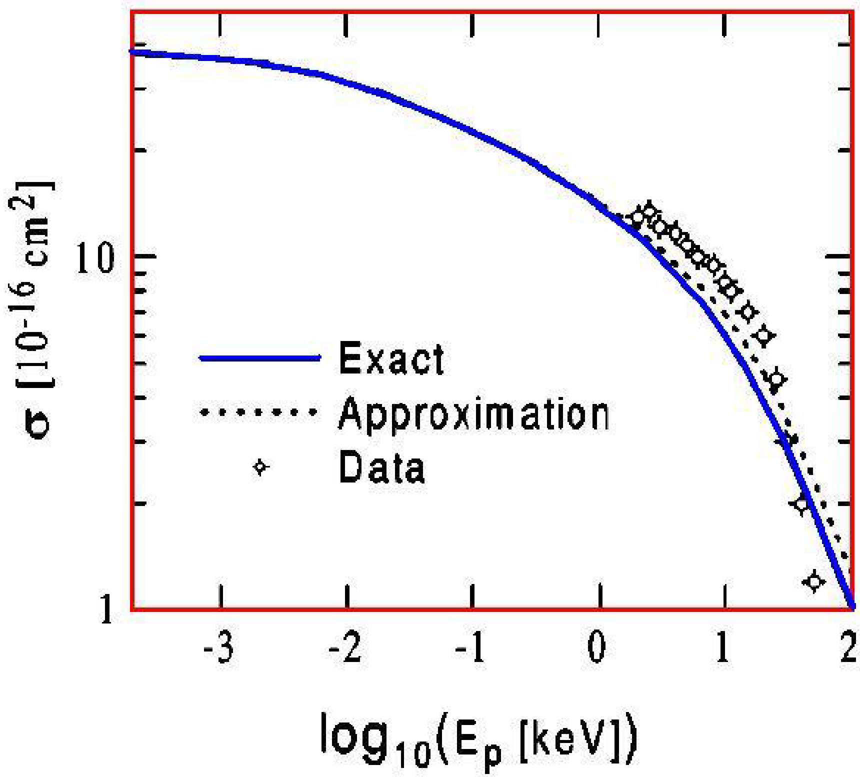
$$P_{exch} = \frac{1}{2} + \frac{1}{2}\cos\left\{\frac{1}{\hbar}\int_{-\infty}^{\infty}\left[E_-(t) - E_+(t)\right]dt\right\}$$

Resonant exchange (b < 4 Å)

$$\int_{-\infty}^{\infty}\left[E_-(t) - E_+(t)\right]dt = 2\pi\hbar(n + 1/2), \qquad n = 0, 1, 2, \cdots, N$$

state	MO	United Atom	Energy [a.u.]	LCAO
Even	$1\sigma_g$	1s	-4	$\frac{1s_1 + 1s_2}{\sqrt{2}}$
Odd	$1\sigma_u$	2p	-1	$\frac{1s_1 - 1s_2}{\sqrt{2}}$

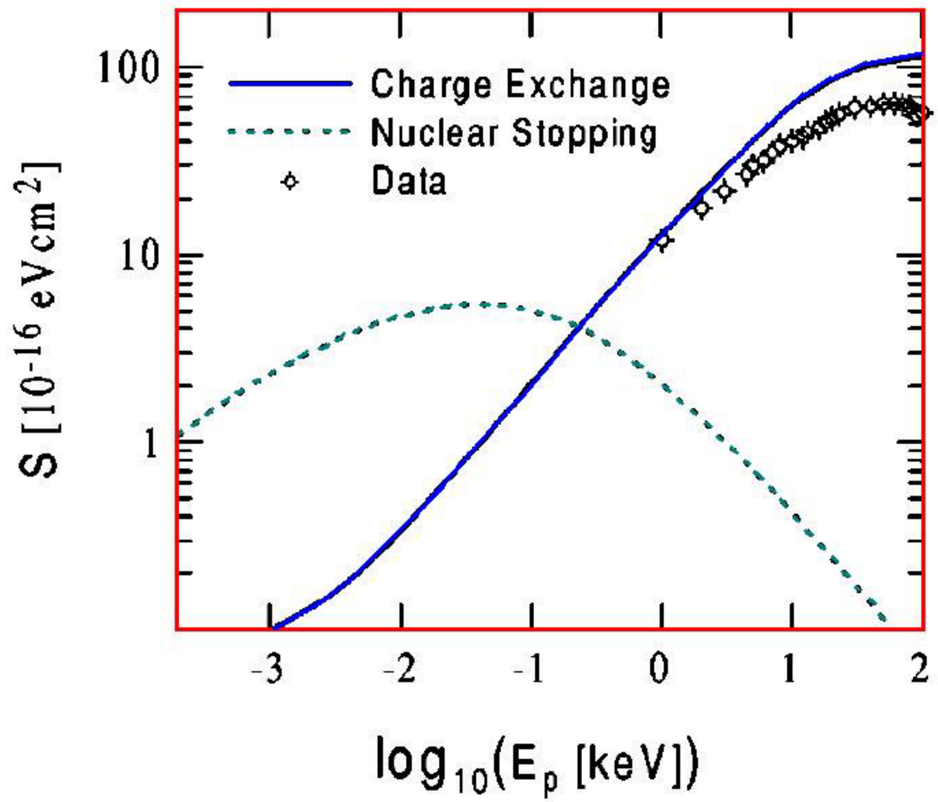
Charge-exchange x nuclear stopping



$$\sigma = 2\pi \int P_{exch}(b)\,db$$

$$S = n\,S_p$$

Data: McClure, PR 1966



Data Andersen-Ziegler, 1977:

S ~ v (E_p = 100 eV – 1 keV)

S ~ $v^{1.35}$ (Golser & Semrad: S ~ $v^{3.34}$: He^+ + He)

Stopping in H^+ + He collisions

C.B., PLB 2004

Slater-type orbitals

$$\phi = N\, r^{n-1}\, e^{-\xi r}\, Y_{lm}(\theta,\phi)$$

$$\Phi_i = \sum_{i=1}^{n}\left[c_{ji}^{A}\phi_i^{A} + c_{ji}^{B}\phi_i^{B}\right]$$

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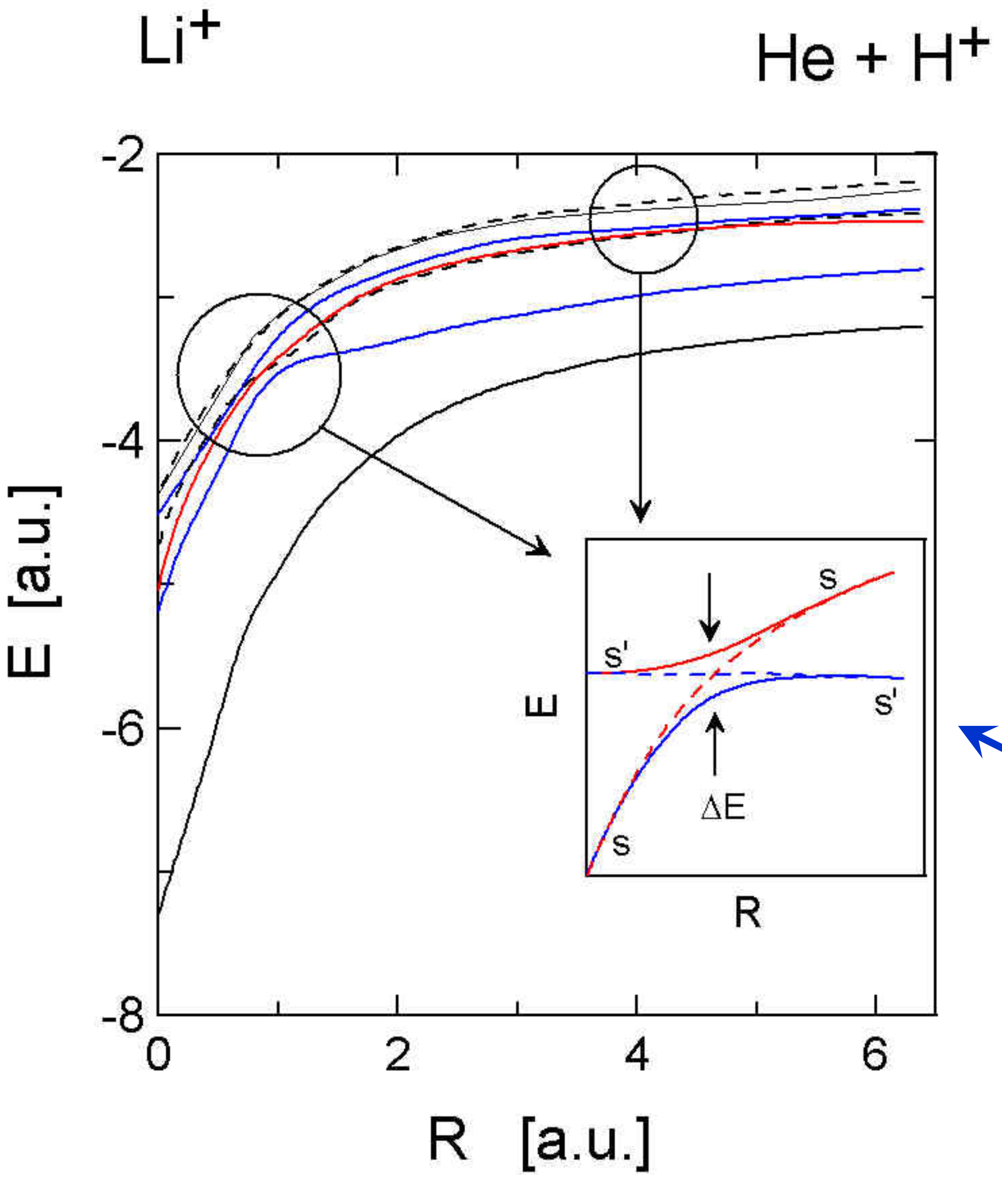
$$\mathrm{F}_{\mu\nu} = H_{\mu\nu} + \sum_{\lambda\rho} P_{\lambda\rho}\left[(\mu\nu\,|\,\lambda\rho) - \frac{1}{2}(\mu\rho\,|\,\lambda\nu)\right]$$

$$H_{\mu\nu} = \iint \phi_\mu^*(1)\left[-\frac{1}{2}\nabla_1^2 - \sum_A \frac{1}{r_{1A}}\right]\phi_\nu^*(1)\, d\tau_1\,, \qquad P_{\lambda\rho} = 2\sum_{i=1}^{occ} c_{\lambda i} c_{\rho i}$$

$$(\mu\nu\,|\,\lambda\rho) = \iint \phi_\mu(1)\,\phi_\nu(1)\frac{1}{r_{12}}\phi_\lambda(2)\,\phi_\rho(2)\, d\tau_1 d\tau_2\,, \qquad S_{\mu\nu} = \int \phi_\mu(1)\,\phi_\nu(1)\, d\tau_1$$

$$\mathrm{E(R)} = \sum_{\mu\nu} P_{\mu\nu} H_{\mu\nu} + \frac{1}{2}\sum_{\mu\nu\lambda\rho} P_{\mu\nu} P_{\lambda\rho}\left[(\mu\nu\,|\,\lambda\rho) - \frac{1}{2}(\mu\rho\,|\,\lambda\nu)\right]$$

8 lowest levels in H^+ + He molecule



Separated atom	United atom
$H^+ + He(1s^2)$	0Σ
$H(1s) + He^+(1s)$	1Σ
$H^+(1s) + He(1s2s)$	2Σ
$H(n=2) + He^+(1s)$	1Π
$H(n=2) + He^+(1s)$	3Σ
$H(n=2) + He^+(1s)$	4Σ
$H^+ + He(1s1p)$	5Σ
$H^+ + He(1s1p)$	2Π

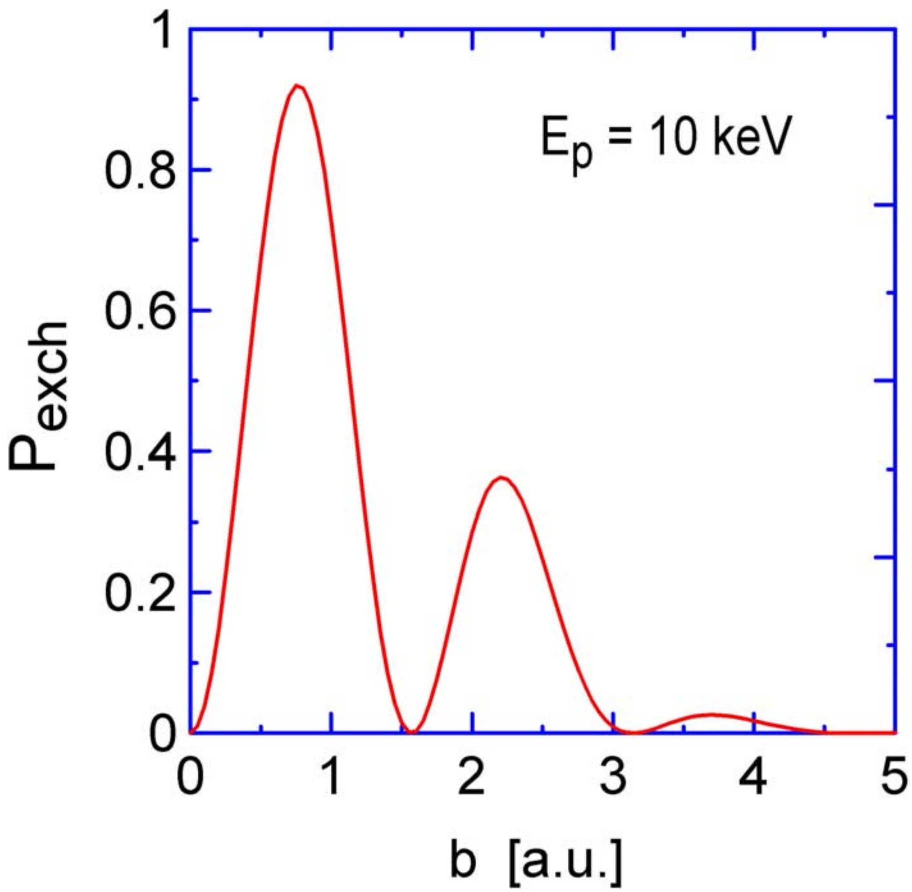
Von Neumann – Wigner

non-crossing rule

Dynamics of H^+ + He collisions

Damping of resonant exchange

H(1s) ⇔ He(1s2s)



$$P = e^{-y}$$ Landau-Zener

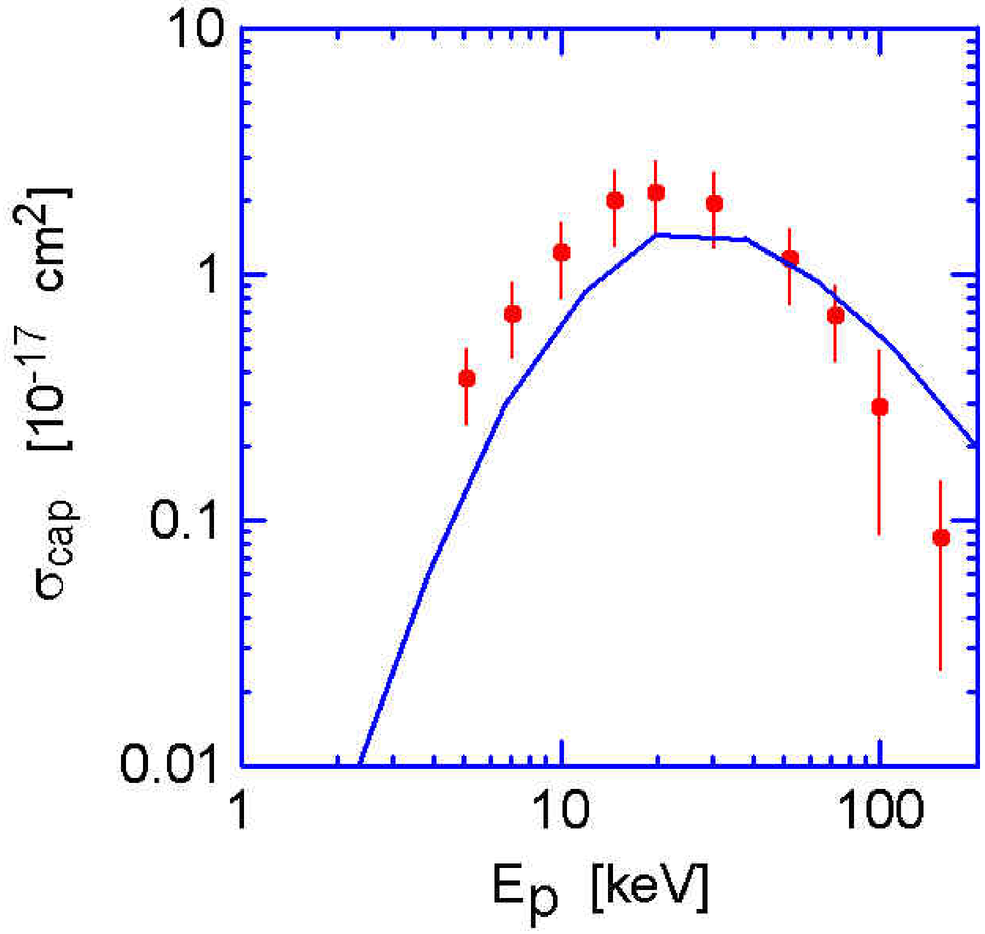
$$y = \frac{2\pi H_{12}^{\,2}}{\hbar\,|\dot{E}_1 - \dot{E}_2|} \sim \frac{2\pi^2\Gamma^2\,\Delta t_{coll}}{E_1 - E_2}$$

$$\frac{2\Gamma}{E_1 - E_2} \sim 0.1$$

$$P = e^{-\Gamma\,\Delta t_{coll}} \cos\left[\frac{H_{12}\,a}{2\mathrm{v}}\right]$$

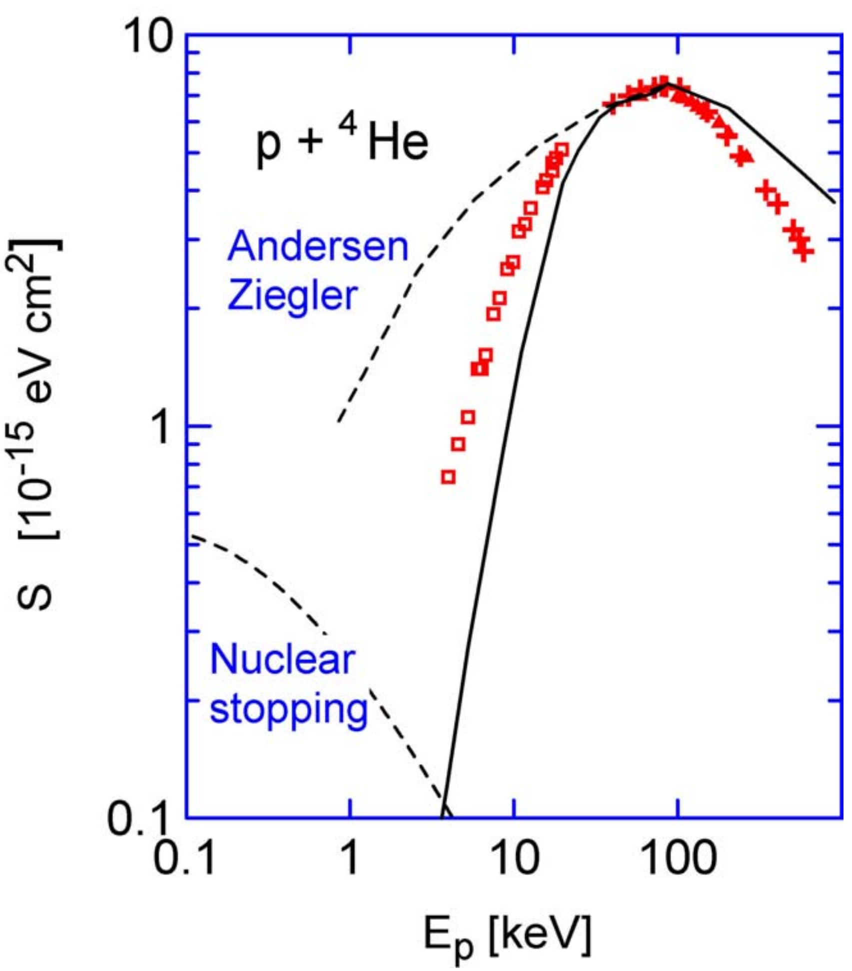
H^+ + He exchange cross sections

Data: Rudd et al, PRA 1983



Threshold effect

Data: Golser & Semrad, NIM 1992



Minimum momentum transfer:

$$q_{\min} = \frac{\mu v}{\hbar}\left[1 - \sqrt{1 - \frac{2\Delta E}{\mu v^2}}\right]$$

$$\frac{\hbar^2 q_{\min}^2}{2m_e} \geq \Delta E$$

$$\Rightarrow \quad E_P \geq \frac{\mu^2}{4M_P m_e}\Delta E$$

He: $1s^2\,(^1S_0) \rightarrow 1s2s\,(^1S)$: 19.8 eV

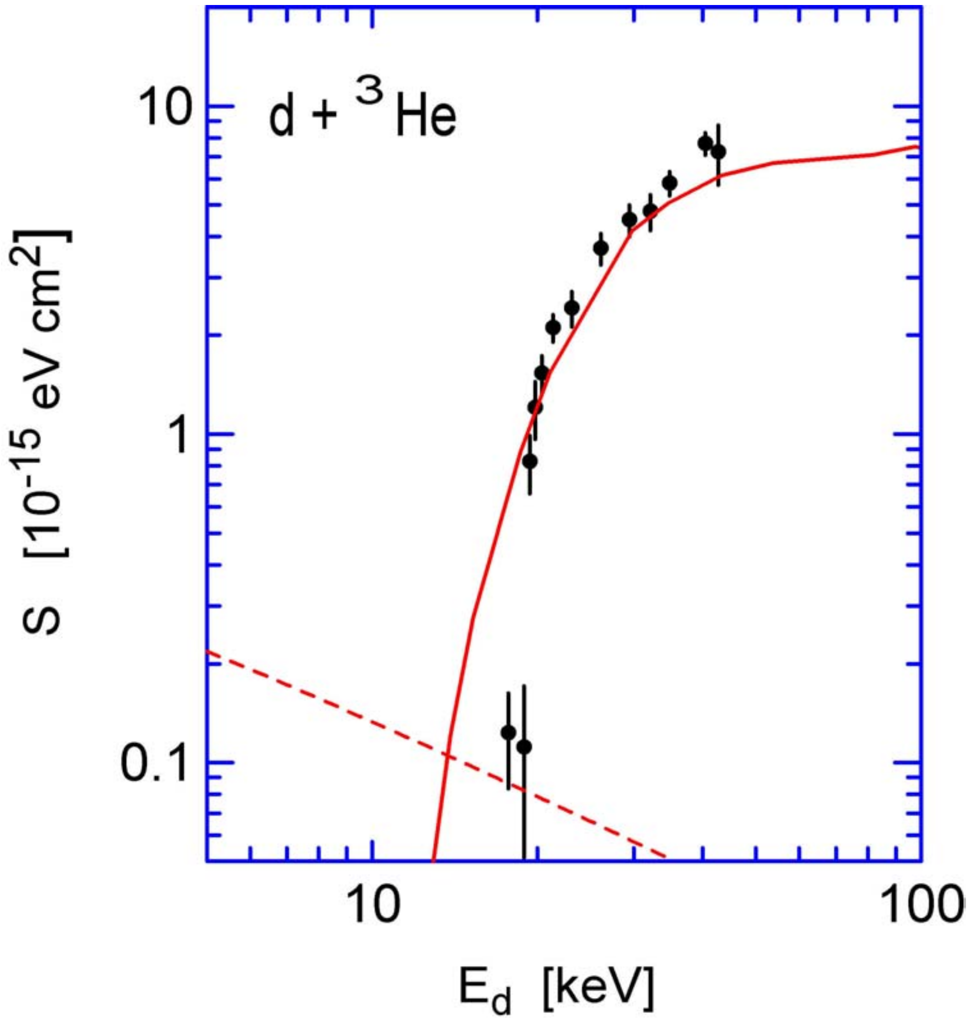
$$\Rightarrow \quad E_P \geq 8\,keV$$

Experimental Proof

Formicola et al, Eur. Phys. J. A 2000

d → ^{3}He

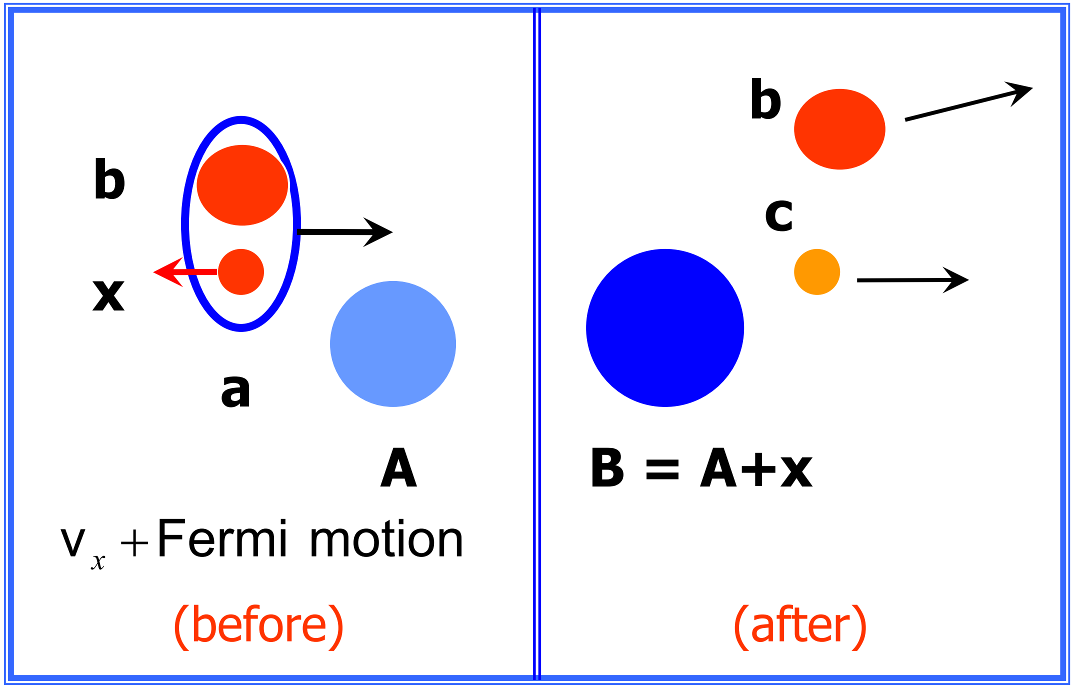
How can we understand the plasma screening in the stars if we can not understand it in the laboratory?



The role of NSCL/MSU: Indirect methods

(A) Trojan horse method:

Baur, 1986



Measuring $\mathrm{A} + \mathrm{a} \rightarrow \mathrm{b} + \mathrm{c} + \mathrm{B}$

with $\mathrm{a} = \mathrm{b} + \mathrm{x} \quad \Rightarrow$

$\mathrm{A} + \mathrm{x} \rightarrow \mathrm{B} + \mathrm{c}$ (astrophysics)

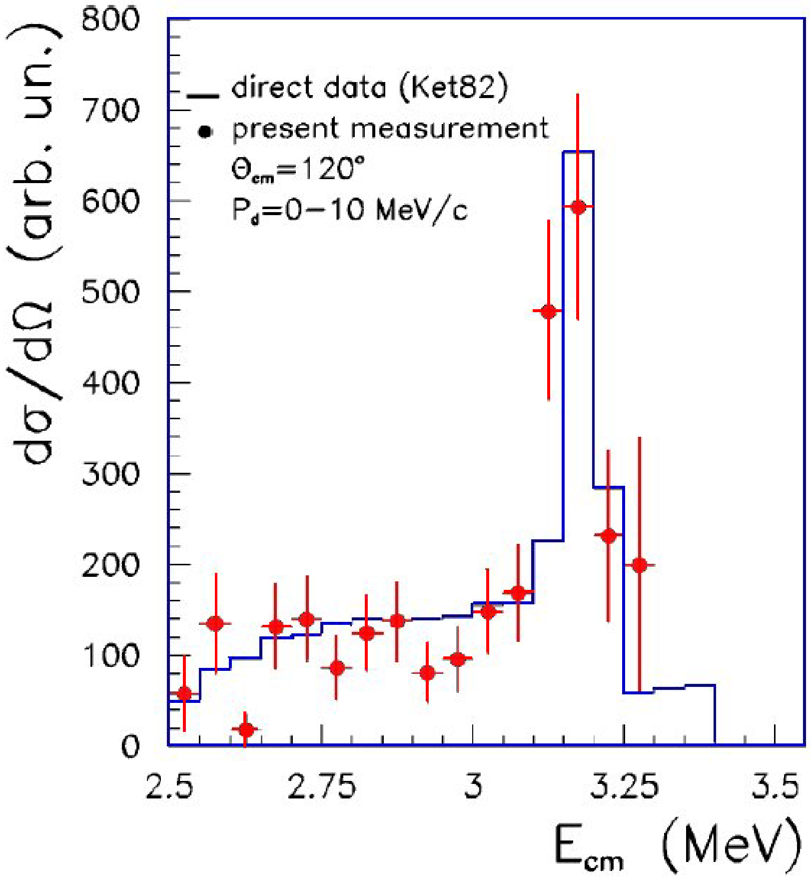
$$d\sigma/d\Omega_c d\Omega_b dE_b \sim \left| \sum_{lm} T_{lm}\left(\mathbf{k}_a, \mathbf{k}_b, \mathbf{k}_c\right) S_{lx} Y_{lm}(\mathbf{k}_c) \right|^2$$

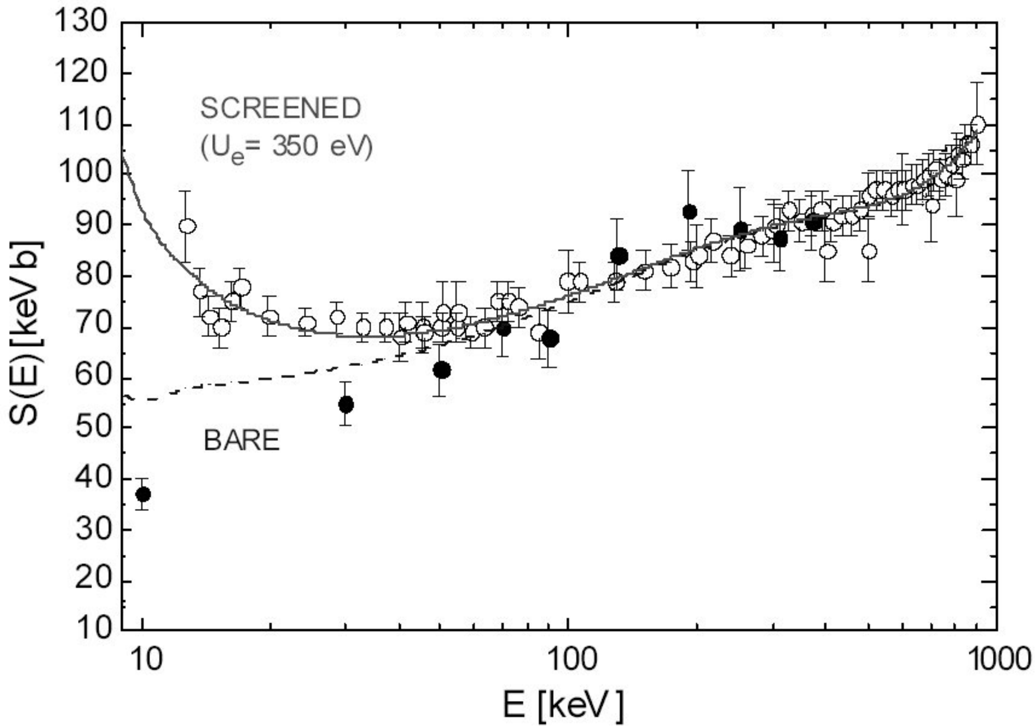
where $T_{lm} = < \chi_b^{(-)} Y_{lm} f_l \,|\, V_{bx} \,|\, \chi_b^{(+)} \phi_{bx} >$

But, $S_{lx} \sim e^{-2\pi\eta} \quad \Rightarrow \quad \sigma_{\mathrm{A+x} \,\rightarrow\, \mathrm{B+}c} \sim (1/k_x^2)\, e^{-2\pi\eta}$

$$f_l(r) \sim (k_x r)^{1/2} e^{2\pi\eta}\; K_{2l+1}(0.53 Z_A Z_B \sqrt{r}) \rightarrow T_{lm} f_l \sim \text{const.}$$

Thus, $d\sigma/d\Omega_a d\Omega_b dE_b \rightarrow \text{const.}$





Alliota et al, Eur. Phys. J. A 2000

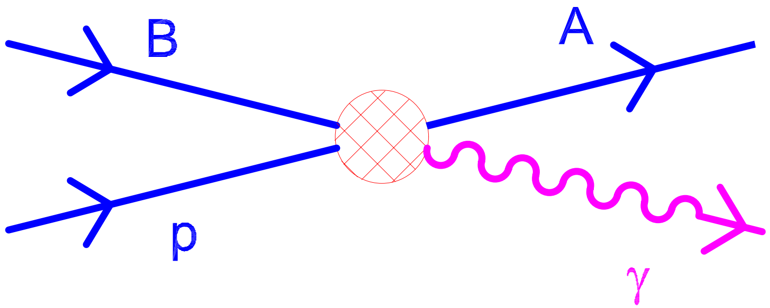
Spitaleri et al, Eur. Phys. J. A 2000

^{7}Li(p,α)α with ^{2}He(^{7}Li,$\alpha\alpha$)n

^{12}C(α,γ)^{16}O with ^{6}Li(^{12}C,α^{12}C)^{2}H

(B) Asymptotic normalization coefficients

$$\sigma \propto |M|^2 \qquad [S(E) = Ee^{2\pi\eta}\sigma]$$



$$\sigma_{capture} \propto (C^{A}_{Bp})^2$$

$$M = \left\langle \phi_A(\xi_B, \xi_p, \xi_{Bp}) \middle| \hat{O}(r_{Bp}) \middle| \phi_B(\xi_B)\phi_p(\xi_p)\psi_i^{(+)}(r_{Bp}) \right\rangle$$

Integrate over ξ:

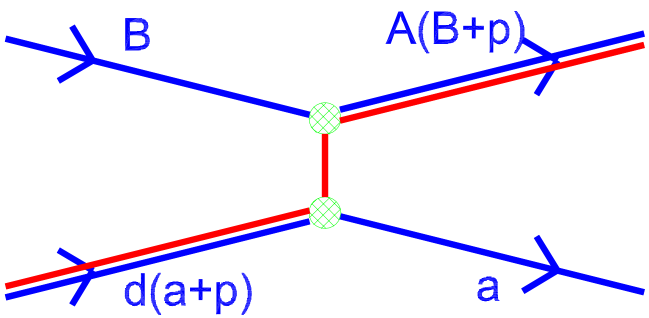
$$M = \left\langle I^{A}_{Bp}(r_{Bp}) \middle| \hat{O}(r_{Bp}) \middle| \psi_i^{(+)}(r_{Bp}) \right\rangle$$

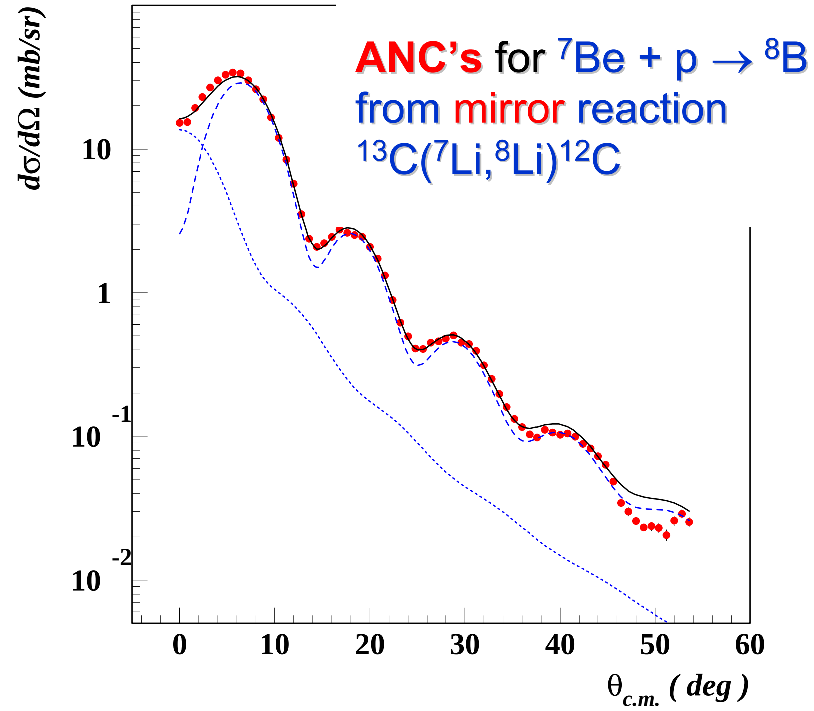
Low Energy:

$$I^{A}_{Bp}(r_{Bp}) \overset{r_B > R_N}{\approx} C^{A}_{Bp} \frac{W_{-\eta_A, l+\frac{1}{2}}(2\kappa_{Bp} r_{Bp})}{r_{Bp}}$$

Mukhamedzanov, 1993: Transfer reactions

Texas A&M





Peripherality:

$$M = \sum \left\langle \chi_f^{(-)} I_{Bp}^{A} \middle| \Delta V \middle| I_{ap}^{d} \chi_i^{(+)} \right\rangle$$

$$\frac{d\sigma}{d\Omega} = \frac{(C_{Bpl_A j_A}^{A})^2 (C_{apl_d j_d}^{d})^2}{b_{Bpl_A j_A}^{2} \, b_{apl_d j_d}^{2}} \sigma_{l_A j_A l_d j_d}^{DW}$$

- separate $p_{1/2}$ and $p_{3/2}$
- Fits ⇒ **ANC's**

$^7Li + n \rightarrow {}^8Li$:

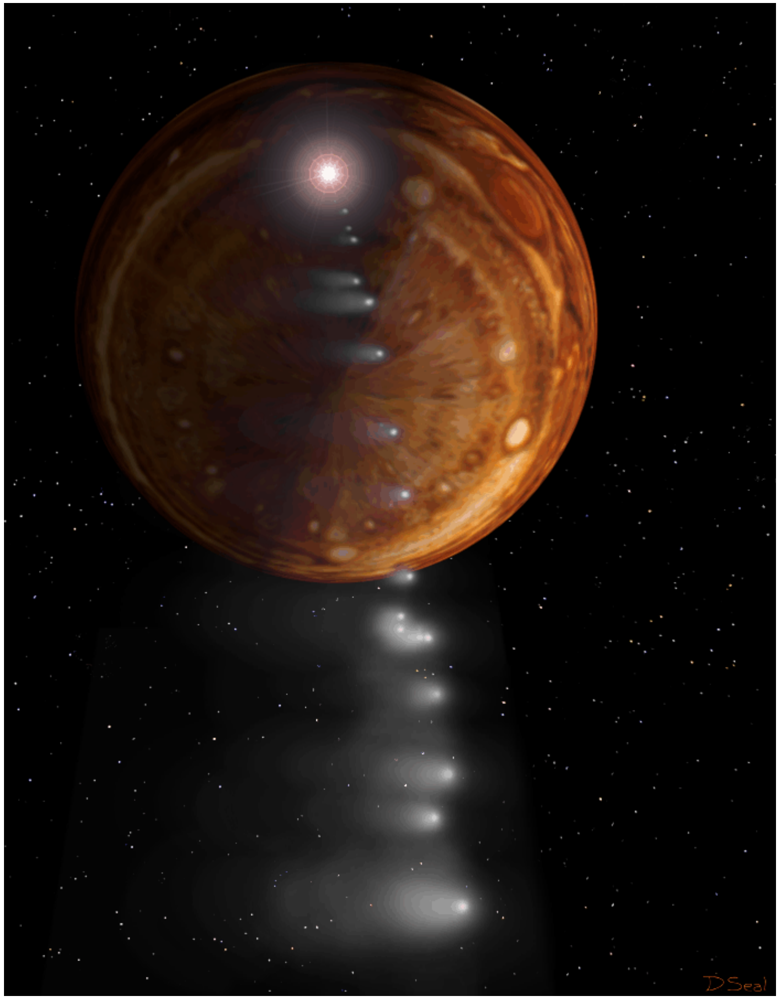
- $C^2(p_{3/2}) = .384 \pm .038\ fm^{-1}$
- $C^2(p_{1/2}) = .048 \pm .006\ fm^{-1}$

⇒ **$^7Be + p \rightarrow {}^8B$:**

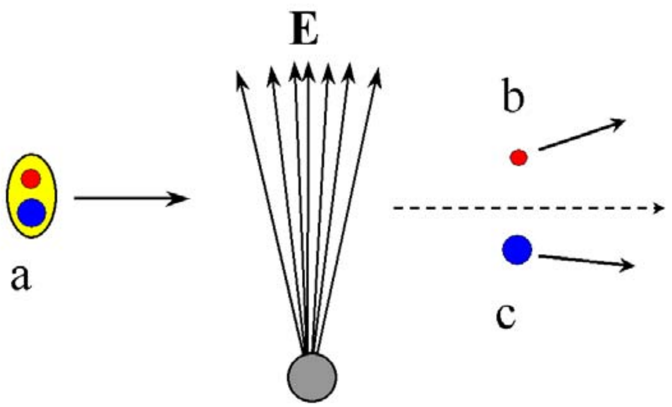
- $C^2(p_{3/2}) = .405 \pm .041\ fm^{-1}$
- $C^2(p_{1/2}) = .050 \pm .006\ fm^{-1}$

$S_{17}(0) = 17.6 \pm 1.7$ eV.b

(C) Coulomb dissociation method



Shoemaker-Levy comet + Jupiter



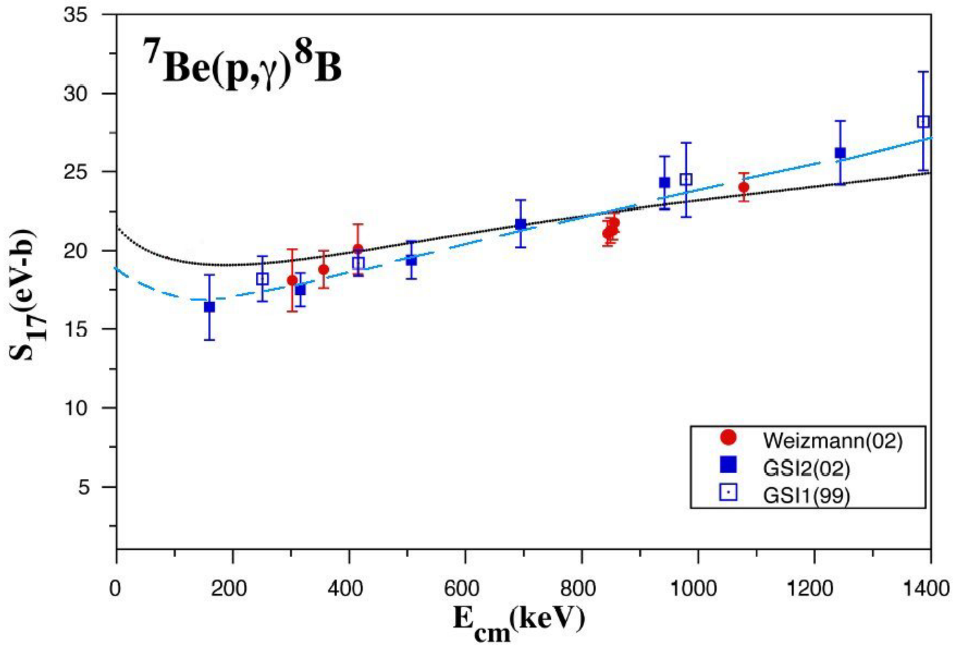
$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma} \sum_l \frac{dn_l(E_\gamma, \Omega)}{dE_\gamma d\Omega} \sigma_{\gamma + \mathrm{a} \to \mathrm{b} + \mathrm{c}} (E_\gamma)$$

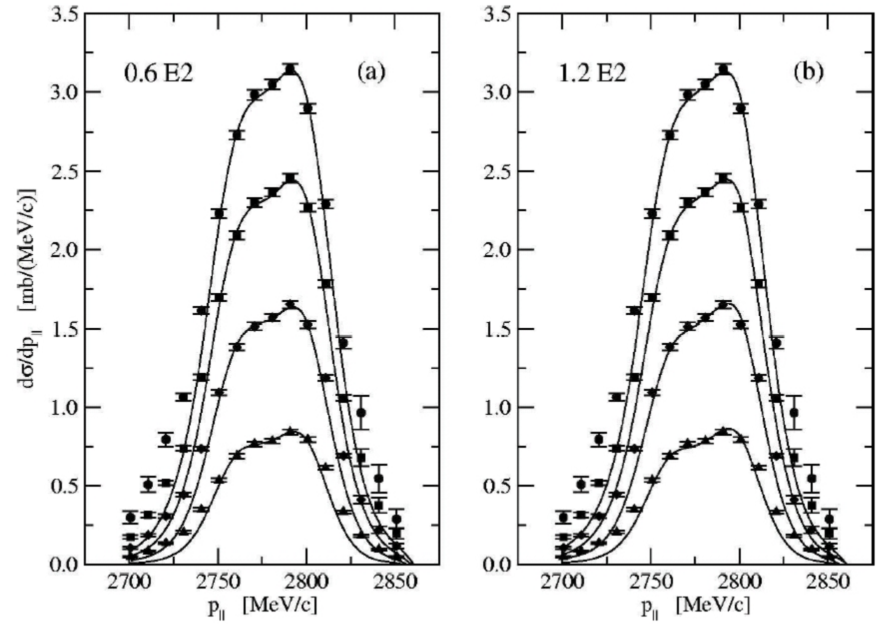
Theory

Detailed balance:

$$\sigma_{\gamma+a} = \frac{k_{bc}^2}{k_\gamma^2} \sigma_{b+c}$$

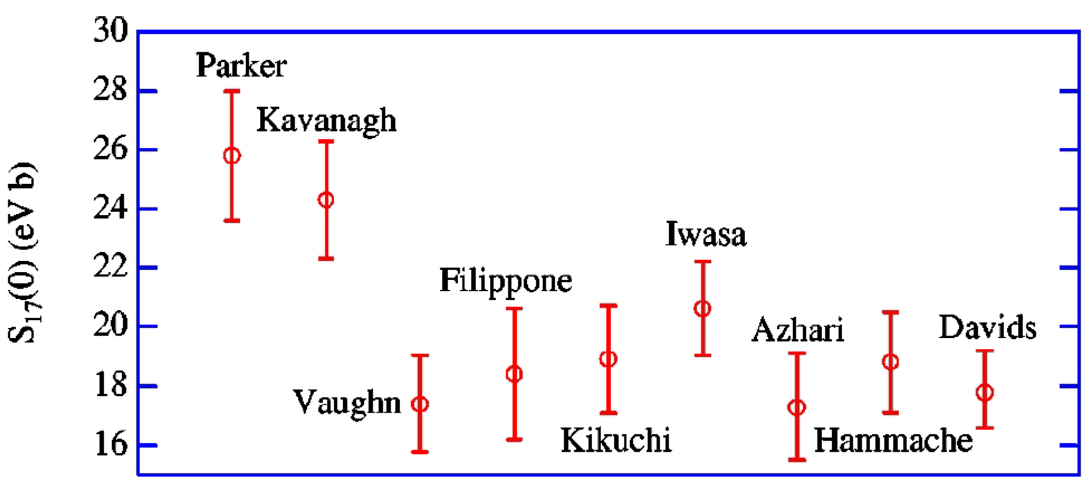
Baur, C.B. and Rebel, 1986





GSI, RIKEN: Invariant mass of fragments

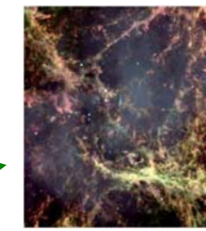
Sam Austin: Longitudinal momentum distribution



S_{17} (0) = 18 ± 1.1 eV.b

Conclusions

- Problems in nuclear astrophysics
  - Atomic physics effects
  - poor statistics due to small cross sections
  - Some needed reactions will never be measured directly



- Indirect methods solve part of these problems
  - Transfer of nucleons: Trojan horse, ANC's
  - Coulomb dissociation method
  - Charge-exchange experiments (Zegers, JINA Talk February 2003)