Topics in Nuclear Astrophysics

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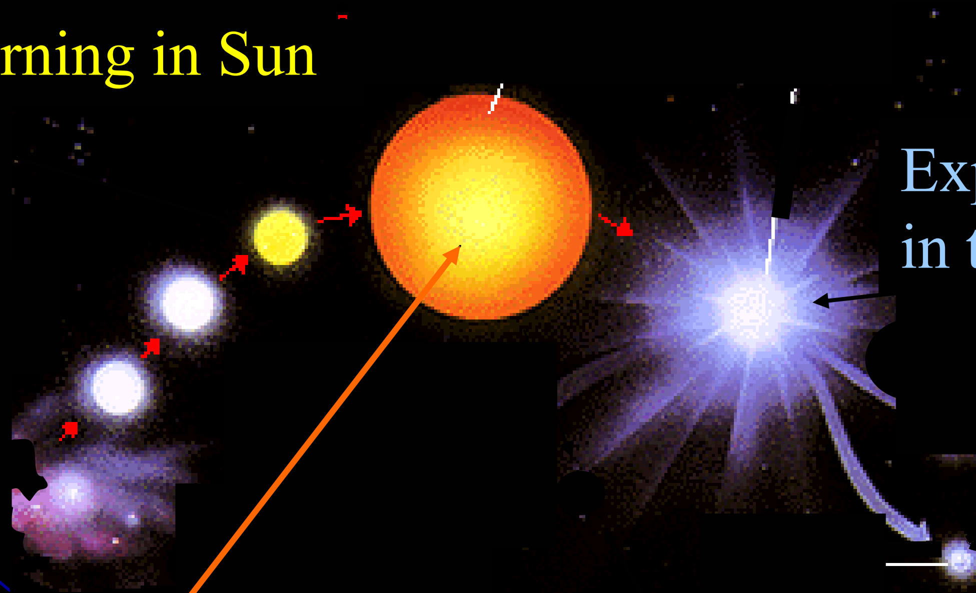
Hydrogen burning in Sun



Explosive reactions in the SN shock front

Helium burning in Red Giants

rp-process on accreting white dwarfs (Nova) or neutron stars (X-ray burst)

Out-Line

- General concept and definitions
- Hydrogen burning sequences
- Nucleosynthesis in late stellar evolution
- Nucleosynthesis in explosive burning in supernova
- Explosive burning in cataclysmic binary systems
- Nucleosynthesis in Big Bang and Cosmic Rays

Topics in Nuclear Astrophysics I

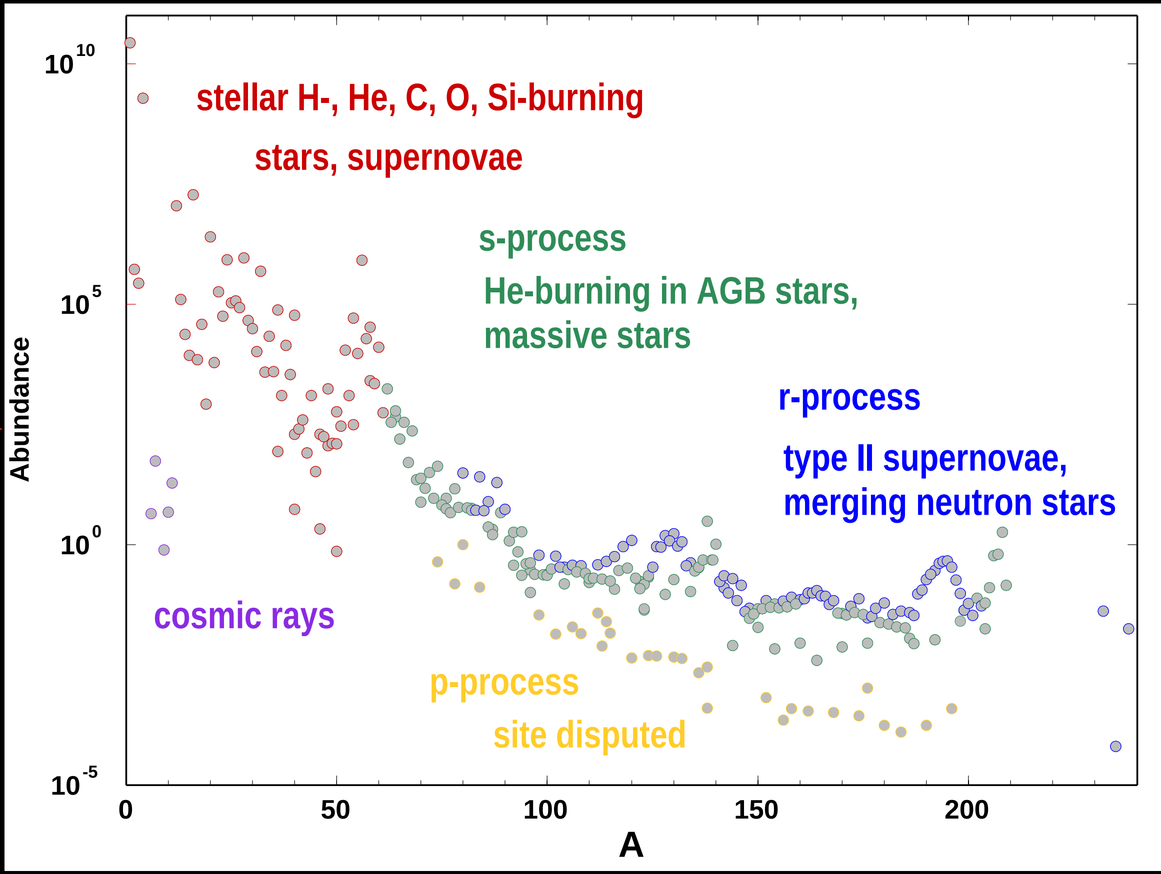
Nuclear Signatures & Nuclear Physics

- Observational Signatures
- Abundances
- Light-curves & radioactivity
- Nuclear energy production rate
- Nuclear reaction cross sections
- Nuclear reaction mechanisms
- Techniques for determining cross sections

Signatures of Nucleosynthesis

galactic abundance distribution

- nucleosynthesis processes
- nucleosynthesis history of our universe
- cosmic chemical evolution



References

Chemical Evolution of Galaxies
J Audouze, B M Tinsley
Annual Review of Astronomy and Astrophysics. Volume 14, Page 43-79, Sep 1976

Element Production in the Early Universe
D N Schramm, R V Wagoner
Annual Review of Nuclear Science. Volume 27, Page 37-74, Dec 1977

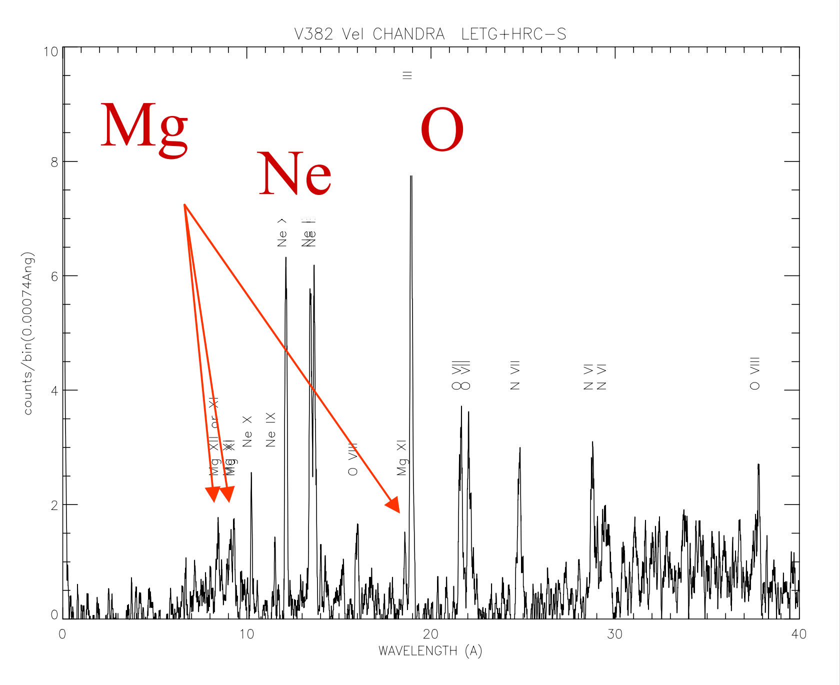
Elemental and Isotopic Composition of the Galactic Cosmic Rays
J A Simpson
Annual Review of Nuclear and Particle Science. Volume 33, Page 323-382, Dec 1983

Nucleosynthesis
J W Truran
Annual Review of Nuclear and Particle Science. Volume 34, Page 53-97, Dec 1984

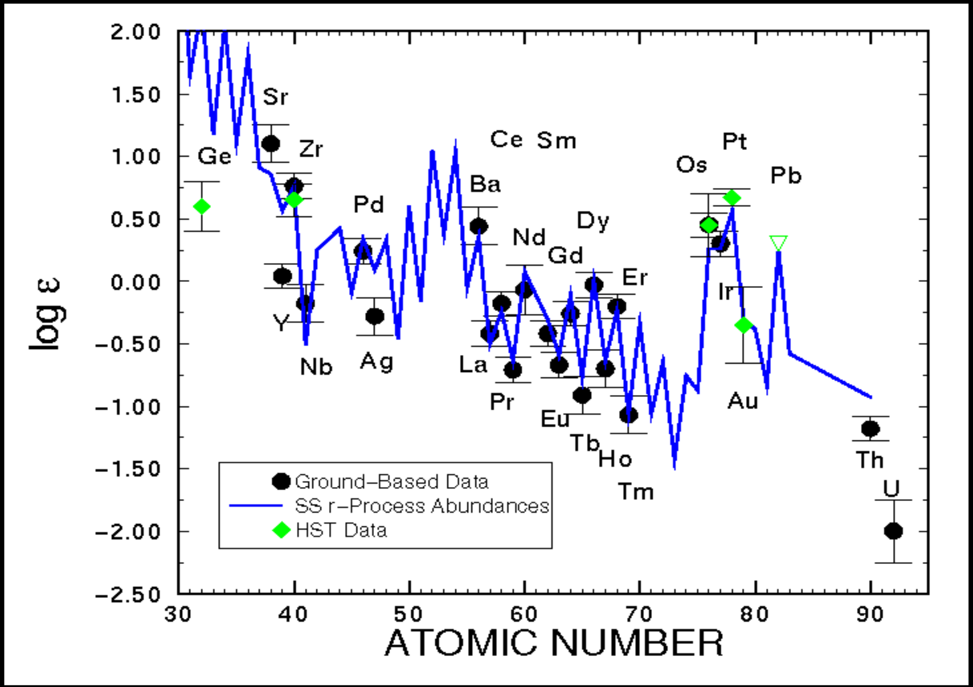
NUCLEOSYNTHESIS IN ASYMPTOTIC GIANT BRANCH STARS: Relevance for Galactic Enrichment and Solar System Formation
M. Busso, R. Gallino, G. J. Wasserburg
Annual Review of Astronomy and Astrophysics. Volume 37, Page 239-309, Sep 1999

Site-Specific Nucleosynthesis Pattern

Nova (Chandra)



Metal-Poor Halo Star (HST)



On-line observation of nucleosynthesis products!

Meteorites

Chondrites: Chondrules - small ~1mm size spherical inclusions in matrix

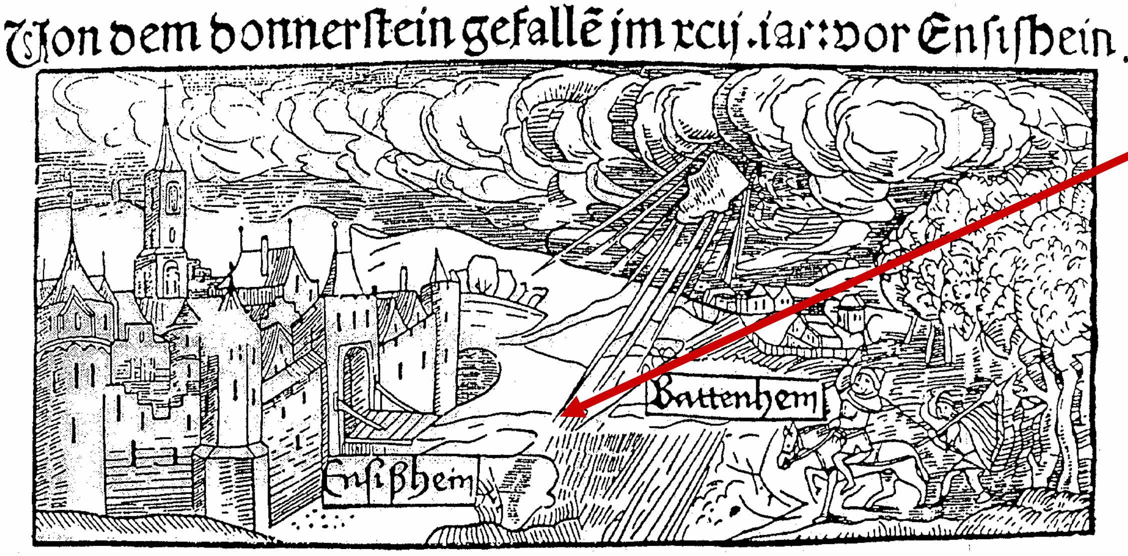


Figure I-2. Woodcut depicting the fall of the Ensisheim LL chondrite on 7 November 1492. A literal translation of the German caption (by Sebastian Brant) is "of the thunder-stone (that) fell in xcii (92) year outside of Ensisheim." This meteorite, which is preserved in the city hall of Ensisheim, Alsace, is the oldest recorded fall from which material is still available.

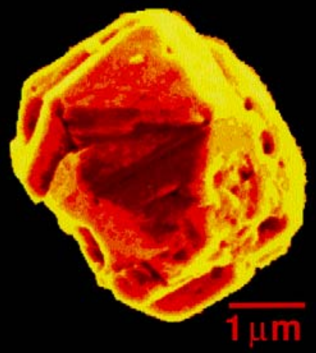
Meteoritic inclusions are early condensates of nucleosynthesis ejecta – supernovae, novae, stellar winds AGB stars

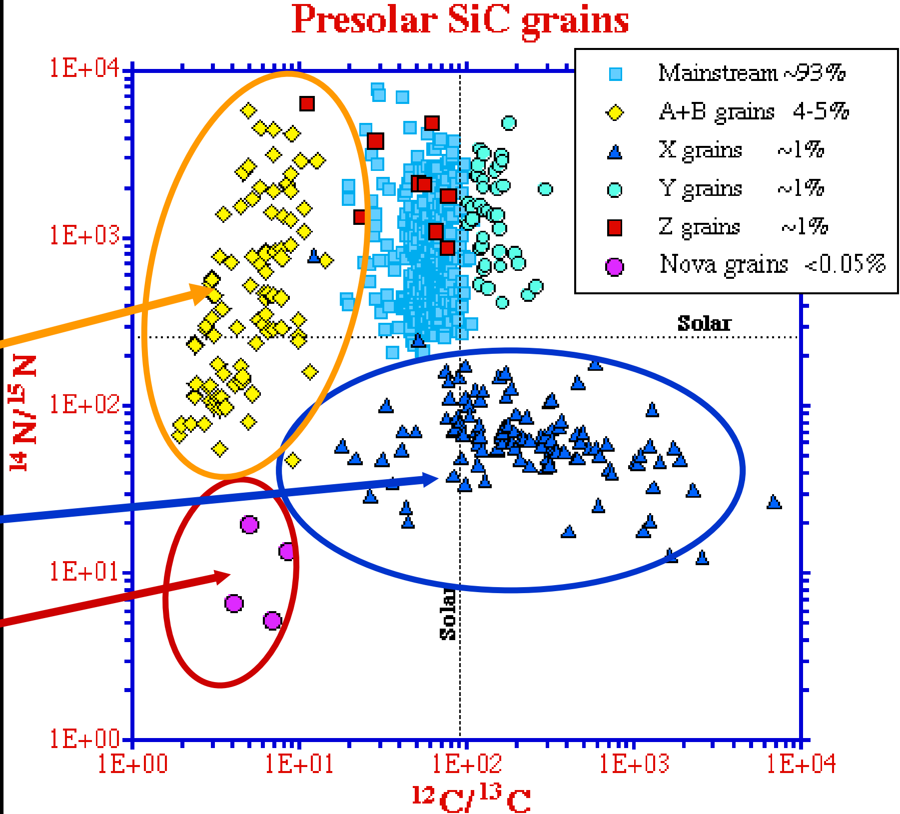
STELLAR NUCLEOSYNTHESIS AND THE ISOTOPIC COMPOSITION OF PRESOLAR GRAINS FROM PRIMITIVE METEORITES

Ernst Zinner, Annual Review of Earth and Planetary Sciences. Volume 26, Page 147-188, May 1998

Cosmo-Chemistry of Meteorites

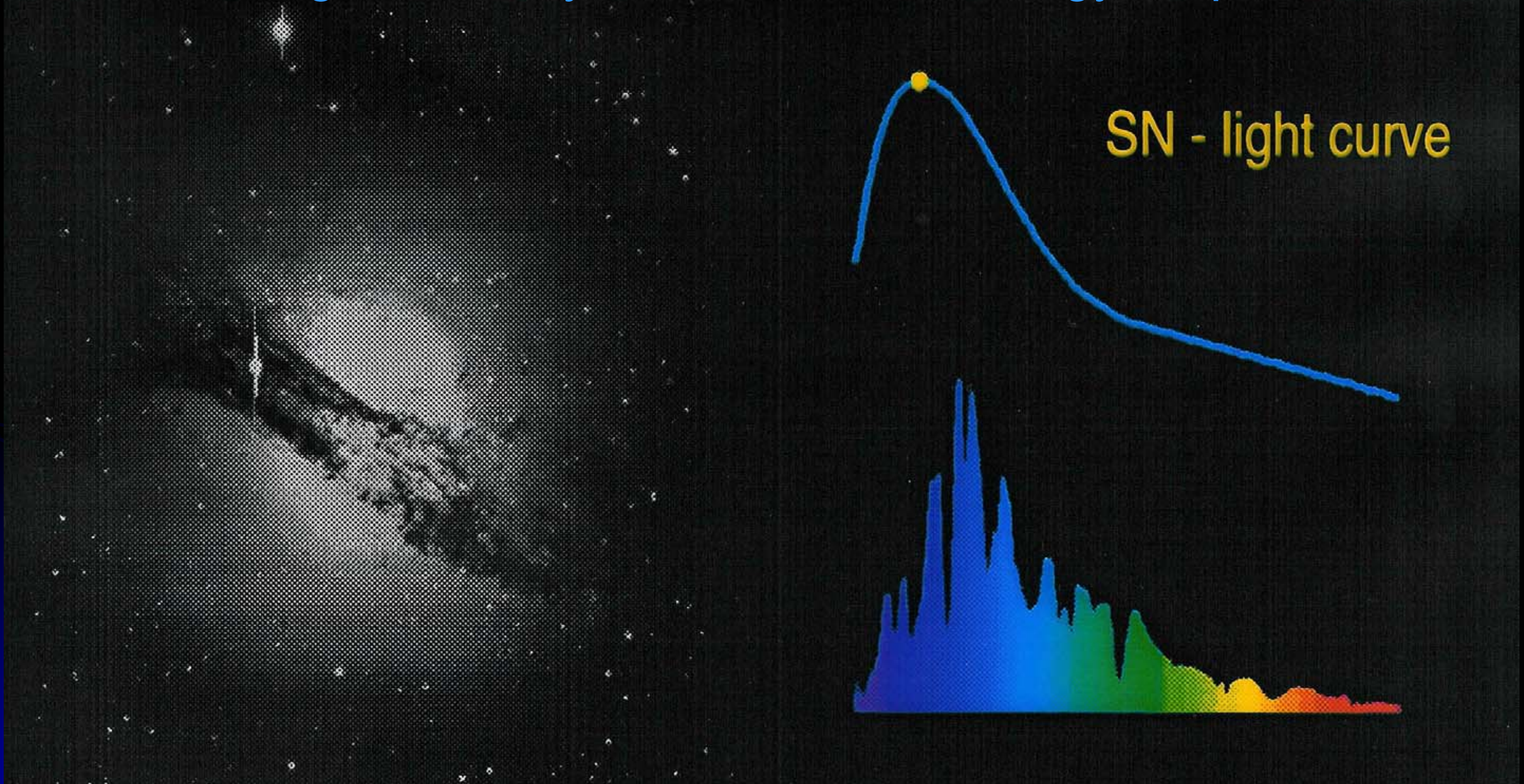
Chondrule





Light and Light-Curves

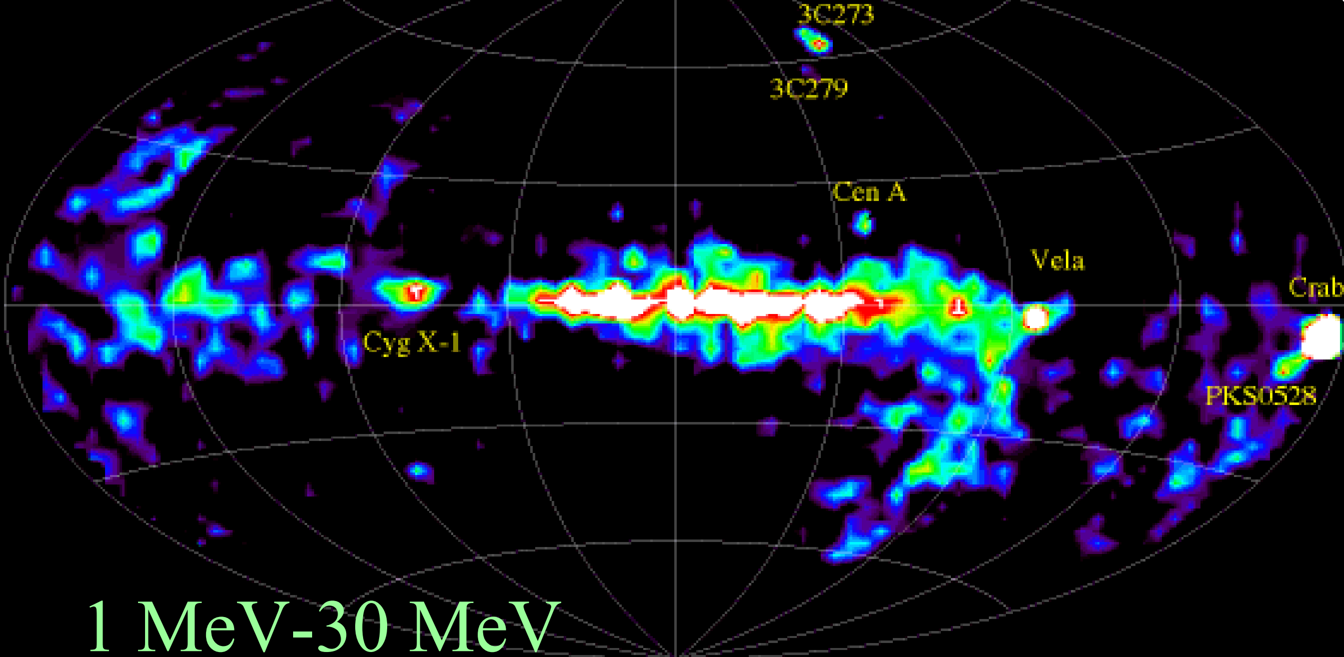
Light intensity correlates with energy-output



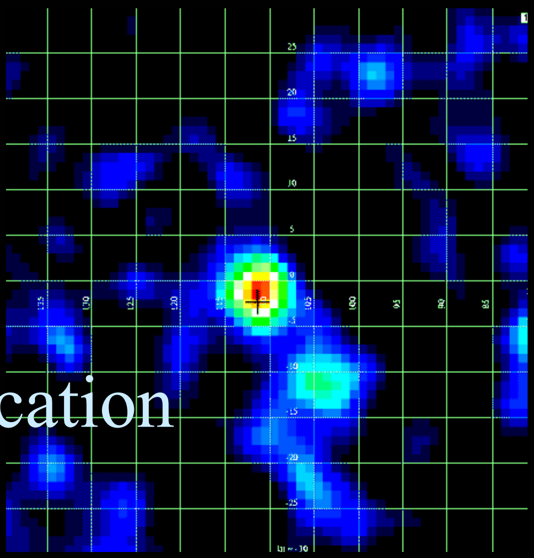
Light curve follows the radioactive decay law ^{56}Ni, ^{56}Co, ^{44}Ti

Galactic Radioactivity - γ-Radiation

Gamma-Ray Astronomy; *R Ramaty, R E Lingenfelter,*
Annual Review of Nuclear and Particle Science. Volume 32, Page 235-269, Dec 1982



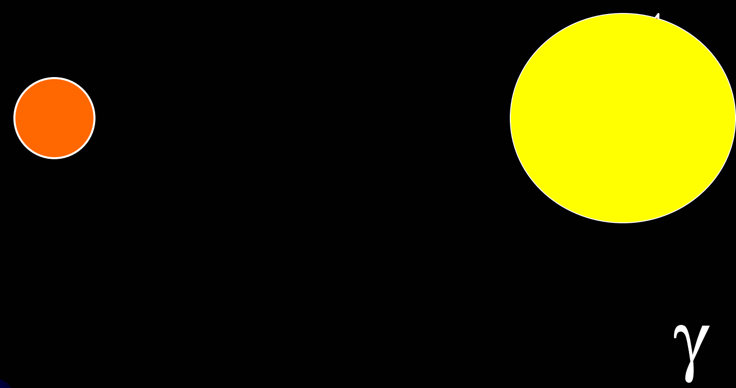
1 MeV-30 MeV
γ-Radiation in Galactic Survey



^{44}Ti in Supernova Cas-A Location
1.157 MeV γ-radiation

Nuclear Reactions in Stars

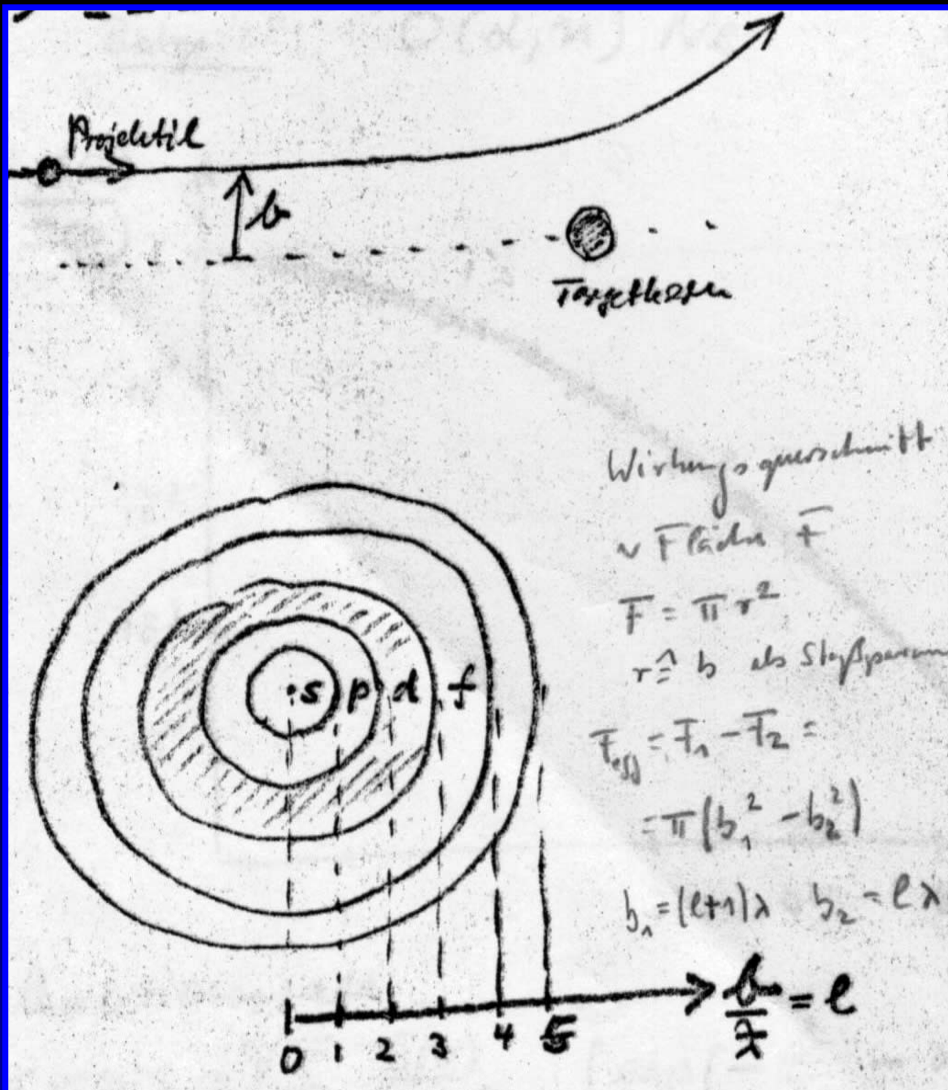
- generate energy
- create new isotopes and elements



$$^{12}C(p,\gamma)^{13}N$$

reaction probability ⇒ σ: reaction cross section

Cross Section – a reminder!



cross section =area!

$$\sigma \cong \pi \cdot r_{nucleus}^2 \approx 1.26 \cdot \pi \cdot A^{2/3} \quad [fm]$$

projectile orbital momentum ℓ

$$p = \hbar k = \frac{\hbar}{\lambda\!\!\!^{-}}; \quad L = \vec{b} \times \vec{p} \cong |b| \cdot |p| = b \cdot \hbar k = b \cdot \frac{\hbar}{\lambda\!\!\!^{-}}$$

$$L = \hbar \ell \quad \Rightarrow \quad b = \lambda\!\!\!^{-} \cdot \ell$$

orbital momentum ℓ in σ

$$\sigma_\ell \cong \pi \cdot F_\ell = \pi \cdot \left(r_{\ell+1}^2 - r_\ell^2\right) = \pi \cdot \left(b_{\ell+1}^2 - b_\ell^2\right)$$

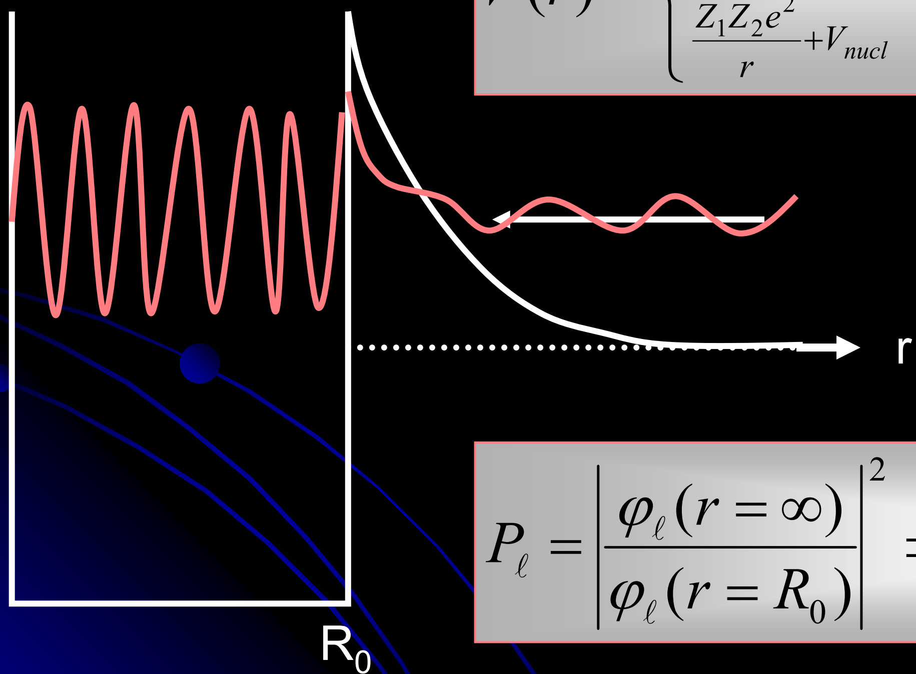
$$\sigma_\ell \cong \pi \cdot \left((\ell+1)^2 - \ell^2\right) \cdot \lambda\!\!\!^{-2} = \pi \cdot (2\ell+1) \cdot \lambda\!\!\!^{-2}$$

$$\sigma_\ell(E) \propto \frac{2\ell+1}{E}$$

Transmission through the barrier

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\ell(\ell+1)\hbar^2}{2\mu r^2}+V(r)-E\right]\varphi_\ell(r)=0$$

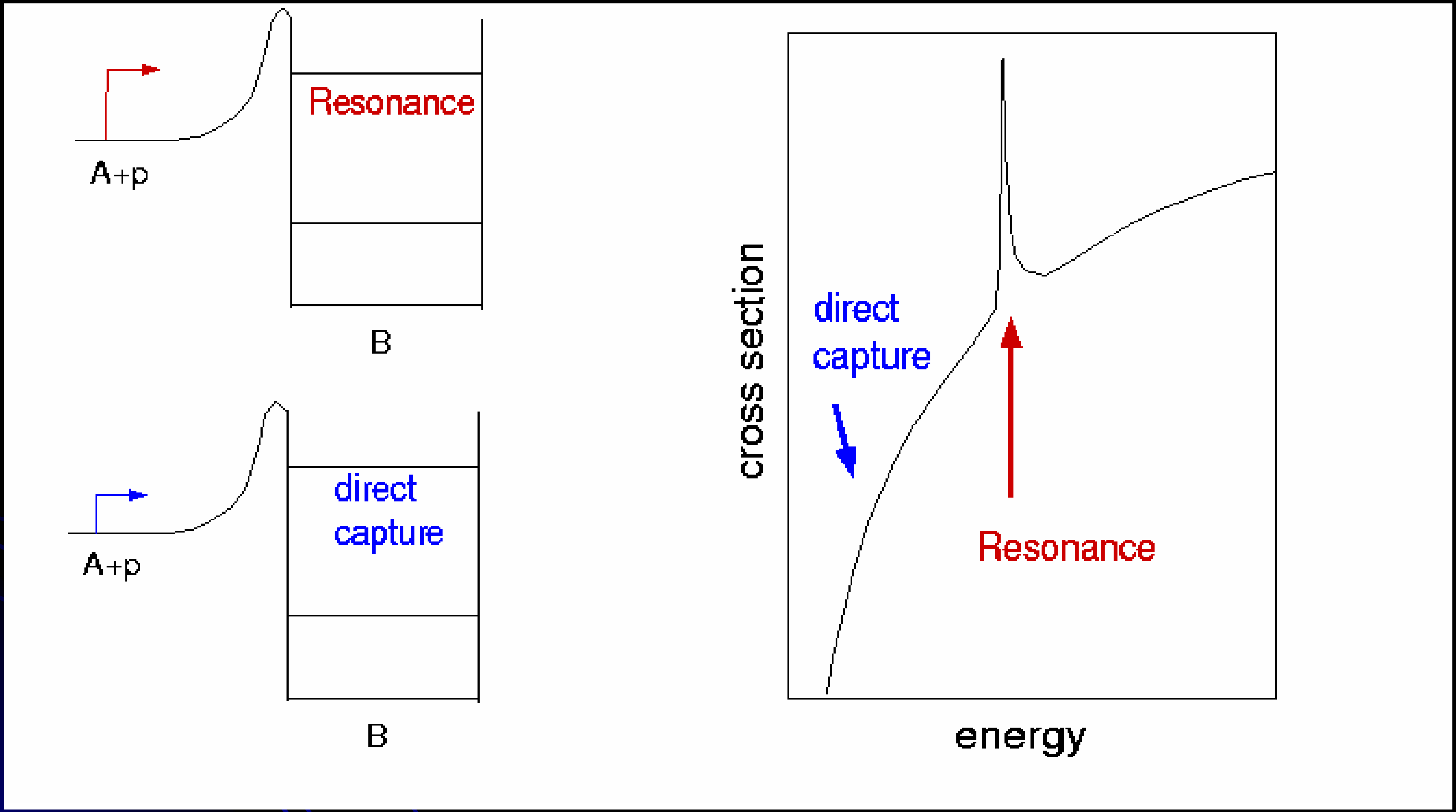
$$V(r)=\begin{cases}\frac{Z_1Z_2e^2}{r} & r\geq R_0\\ \frac{Z_1Z_2e^2}{r}+V_{nucl} & r<R_0\end{cases} \qquad \varphi_\ell(r)=A\cdot F_\ell(r)+B\cdot G_\ell(r)$$



$$P_\ell=\left|\frac{\varphi_\ell(r=\infty)}{\varphi_\ell(r=R_0)}\right|^2 \Rightarrow P_\ell(E,R_0)=\frac{1}{F_\ell^2(E,R_0)+G_\ell^2(E,R_0)}$$

F_ℓ , G_ℓ are the Coulomb wave functions

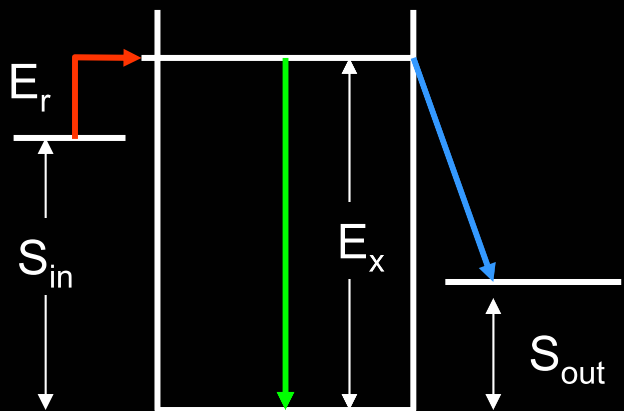
Cross section for charged particles

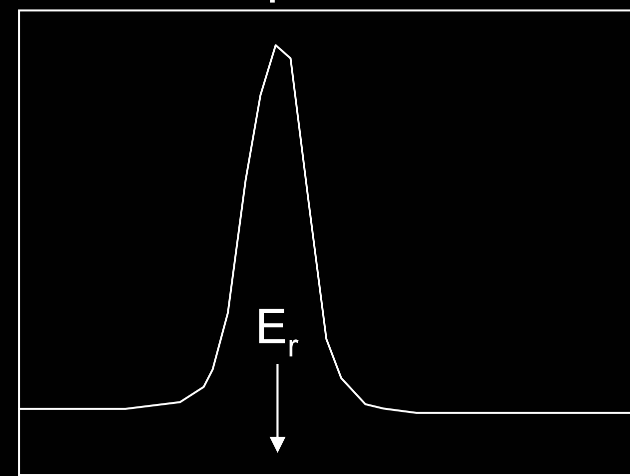


Exponential decline towards lower energy due to Coulomb barrier

Resonant Reaction Mechanism

resonance capture: population of unbound state in compound nucleus





$$\Gamma_i \cong P_\ell \cdot \left| \left\langle \psi_f \middle| H_{\text{int}} \middle| \psi_i \right\rangle \right|^2$$

$$\Gamma_{in} \cong P_\ell \cdot \left| \left\langle \psi_C \middle| H_s \middle| \psi_T + \psi_{in} \right\rangle \right|^2$$

$$\Gamma_{out} \cong P_\ell \cdot \left| \left\langle \psi_{out} + \psi_R \middle| H_s \middle| \psi_C \right\rangle \right|^2$$

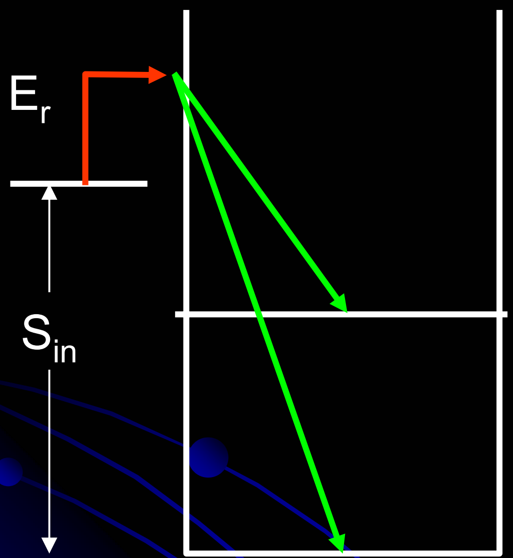
$$\Gamma_\gamma \cong E_\gamma^{2L+1} \cdot \left| \left\langle \psi_{gs} \middle| H_{em} \middle| \psi_C \right\rangle \right|^2$$

$$\sigma(E) = \pi \lambda\!\!\!^{-2}\ \omega \cdot \frac{\Gamma_{in} \cdot \Gamma_{out}}{(E - E_r)^2 + (\Gamma/2)^2}$$

$$\omega = \frac{2J+1}{(2j_p+1)\cdot(2j_T+1)}$$

Direct Capture Mechanism

one-step reaction mechanism through e-m interaction



$$\sigma(E) = P_\ell \cdot \left| \left\langle \psi_f \middle| H_{em} \middle| \psi_p + \psi_T \right\rangle \right|^2$$

P_ℓ is the penetrability through the Coulomb and orbital momentum barrier

γ energy changes with particle energy

cross section for neutron capture

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\ell(\ell+1)\hbar^2}{2\mu r^2}-E\right]\varphi_\ell(r)=0$$

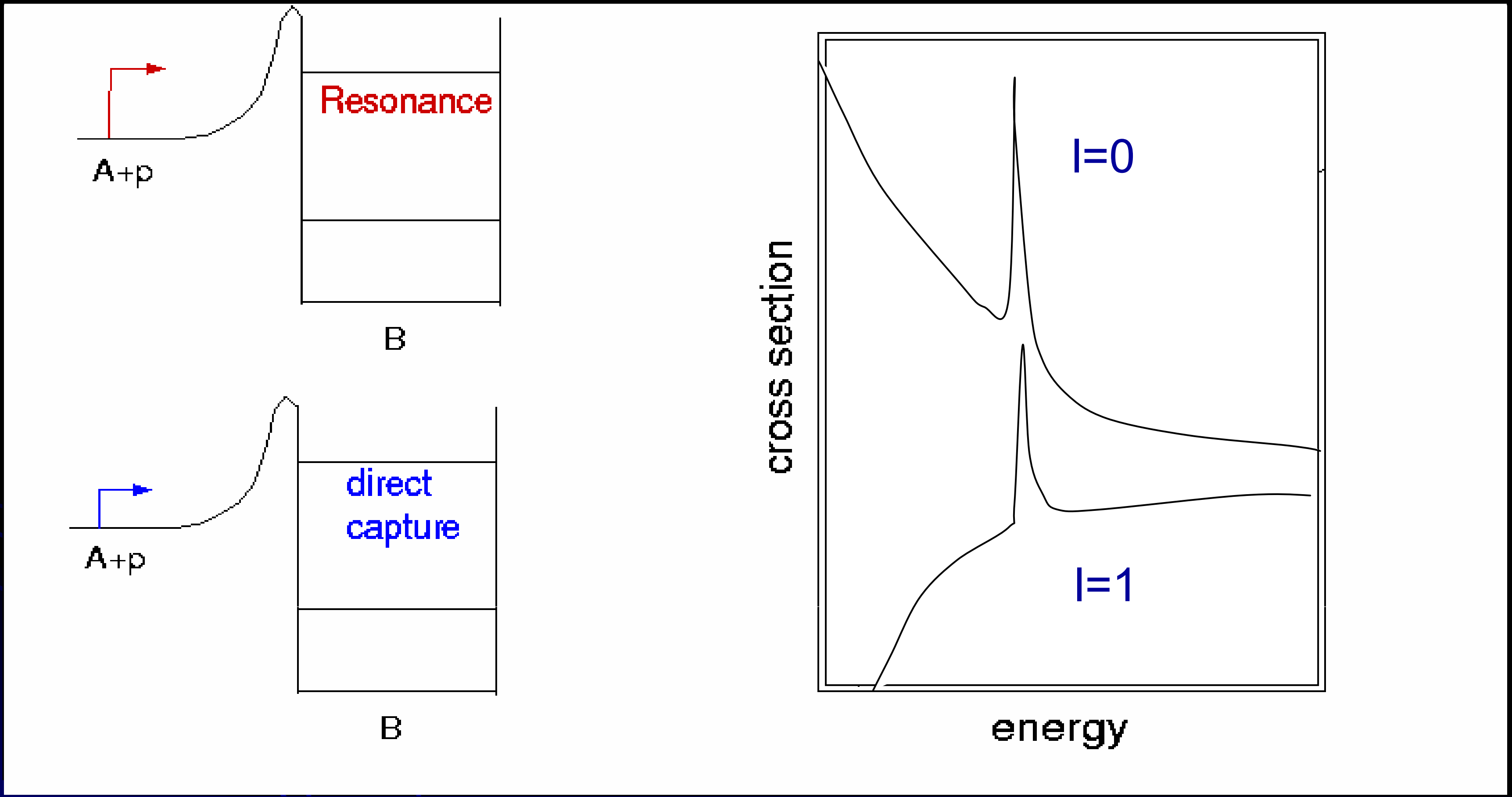
no Coulomb barrier, just orbital momentum term!

$$P_\ell=\left|\frac{\varphi_\ell(r=\infty)}{\varphi_\ell(r=R_0)}\right|^2 \Rightarrow P_{\ell=0}=\frac{r}{\lambda}=k\cdot r$$

$$P_{\ell=1}=\frac{(k\cdot r)^3}{\left(1+(k\cdot r)^2\right)}$$

$$P_{\ell=2}=\frac{(k\cdot r)^5}{\left(9+3(k\cdot r)^2+(k\cdot r)^4\right)}$$

Cross section for neutral particles



ℓ=0, s-wave follows 1/v law,

ℓ=1, p-wave experiences orbital momentum barrier

Hauser Feshbach Approach

assumption of many resonances!
(high level density ρ in particle unbound region of compound nucleus)

$$i + j \Rightarrow o + m$$

$$\sigma_i(E) = \pi \cdot \lambda\!\!\!^{-}{}_j^2 \cdot \frac{1+\delta_{i,j}}{(2j_i+1)\cdot(2j_j+1)} \sum_{J,\pi} (2J+1) \frac{T_j(E,J,\pi)\cdot T_o(E,J,\pi)}{T_{tot}(E,J,\pi)}$$

T : *transition probability*

Smooth Coulomb barrier determined energy dependence!

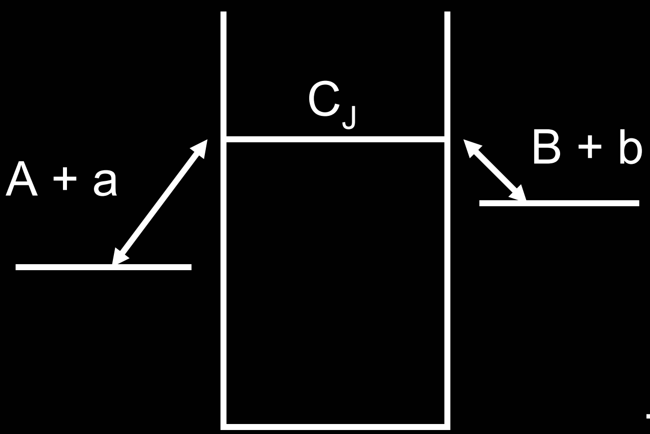
inverse reactions

$$A + a \Rightarrow B + b$$

or

$$B + b \Rightarrow A + a$$

detailed balance theorem I



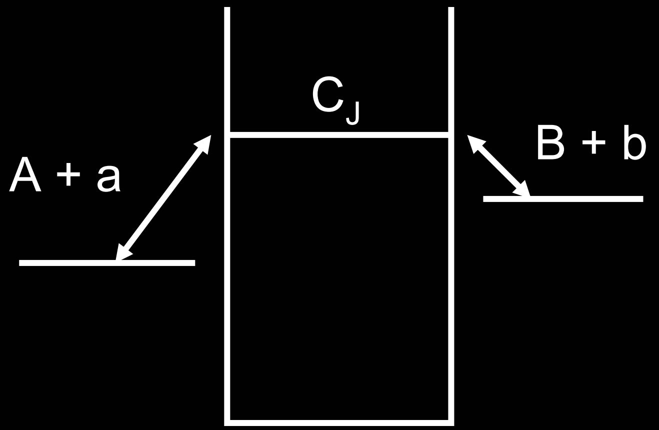
$$\left|\left\langle \varphi_{Bb}\right|H\left|\varphi_{Aa}\right\rangle\right|^2 = \left|\left\langle \varphi_{Aa}\right|H\left|\varphi_{Bb}\right\rangle\right|^2$$

identical matrix elements for transition through same compound configuration

$$\sigma_{Aa} = \pi \bar{\lambda}^2_{Aa} \cdot \frac{(2J+1)}{(2j_A+1)\cdot(2j_a+1)} \cdot \left|\left\langle \varphi_{Bb}\right|H\left|\varphi_{Aa}\right\rangle\right|^2$$

$$\sigma_{Bb} = \pi \bar{\lambda}^2_{Bb} \cdot \frac{(2J+1)}{(2j_B+1)\cdot(2j_b+1)} \cdot \left|\left\langle \varphi_{Aa}\right|H\left|\varphi_{Bb}\right\rangle\right|^2$$

detailed balance theorem II



theorem allows to calculate reaction cross section from known cross section of the inverse reaction process!

$$\lambda_{Aa}^2 \propto \frac{1}{\mu_{Aa} \cdot E_{Aa}}; \qquad \mu_{Aa} = \frac{m_A \cdot m_a}{m_A + m_a} \quad E_{Aa} = E_{cm}$$

$$\frac{\sigma_{Aa}}{\sigma_{Bb}} = \frac{(2j_B+1)(2j_b+1)}{(2j_A+1)(2j_a+1)} \cdot \frac{m_B \cdot m_b \cdot E_{Bb}}{m_A \cdot m_a \cdot E_{Ab}}$$

Conclusion

- observational indication for nuclear processes in the past and present universe!
- low energy reaction processes through resonant and non-resonant reaction channels
- cross section determination through
  - single resonance formalism
  - direct capture or transfer formalism
  - statistical model formalism
  - detailed balance approach