

NUCLEAR BURNING IN DENSE STELLAR MATTER

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- **Thermonuclear burning**
- **Physical conditions**
- **Screening in thermonuclear reactions**
- **Pycnonuclear burning**
- **General outlook and applications**
- **Summary**

JINA, September, 2004

Classical theory of thermonuclear reactions

$$Z_1 + Z_2 \rightarrow Z_c \rightarrow \dots \quad \rho \sim (1-10^{13}) \text{ g cm}^{-3} \quad T \leq 5 \times 10^9 \text{ K}$$

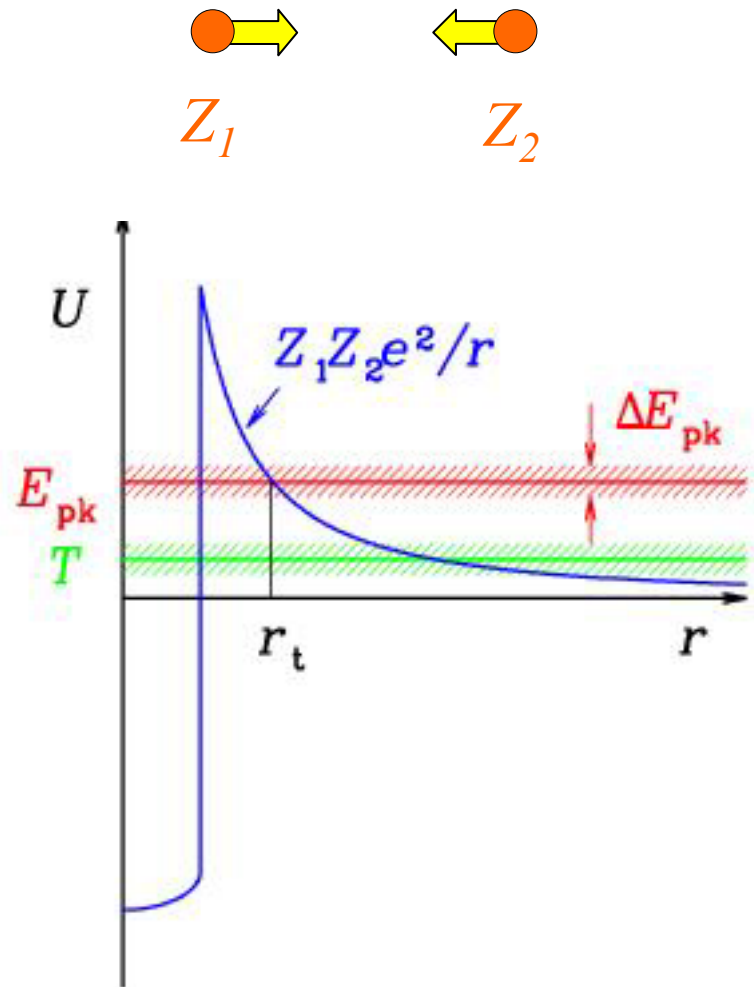
Reaction rate:

$$R = \frac{n_1 n_2}{1 + \delta_{12}} \langle v \sigma \rangle \quad \frac{\text{reactions}}{\text{cm}^3 \text{ s}}$$

$$\int_0^\infty dv \, v^3 e^{-E/T} \sigma(E)$$

$$\sigma(E) = \frac{S(E)P(E)}{E}, \quad P(E) = \exp(-\eta)$$

$$\eta(E) = \frac{2}{\hbar} \int_b^a dr \, |p(r)| = \frac{2\pi Z_1 Z_2 e^2}{\hbar v}$$



The Gamow Peak

$$E_{pk} = T \left(\frac{\pi^2 M Z_1^2 Z_2^2 e^4}{2 T \hbar^2} \right)^{1/3}$$

Example: center of the Sun:

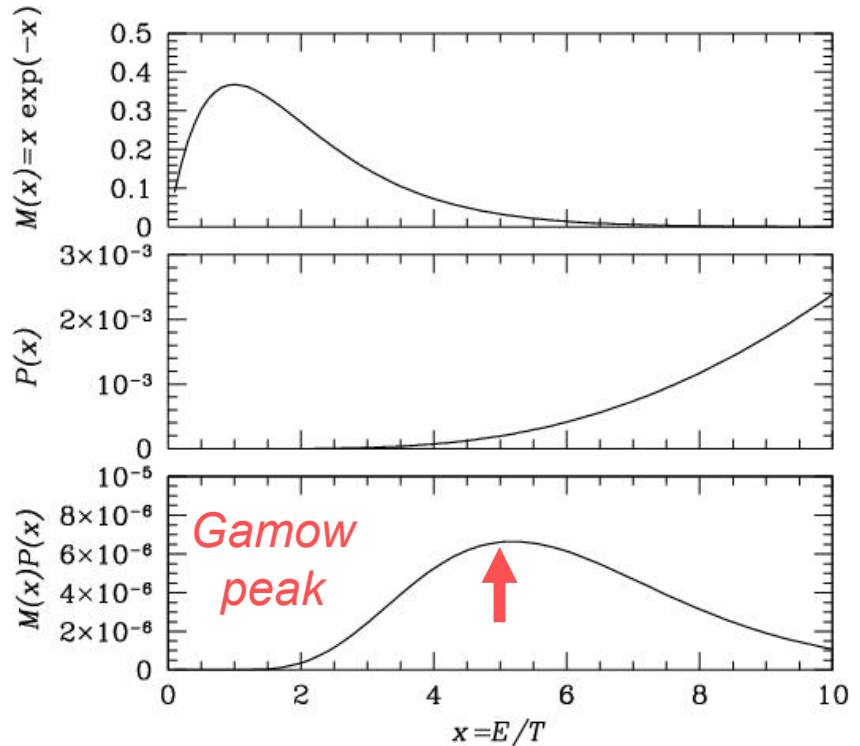
$$T = 1.57 \times 10^7 \text{ K}$$



$$E_{pk} \approx 4.5 T$$

$$\langle P(T) \rangle \sim 10^{-10}$$

$$\langle P(E_{pk}) \rangle \sim 10^{-6}$$



Classical theory of thermonuclear reactions

$$\langle v\sigma \rangle = 4 \sqrt{\frac{2E_{pk}}{3M}} \frac{S(E_{pk})}{T} \exp(-\tau)$$


$$\tau = \frac{3E_{pk}}{T} = \left(\frac{27\pi^2 M Z_1^2 Z_2^2 e^4}{2T\hbar^2} \right)^{1/3} \gg 1$$

$$\tau \propto T^{-1/3}, \quad \frac{E_{pk}}{T} = \frac{\tau}{3} \gg 1,$$

$$\frac{\Delta E_{pk}}{E_{pk}} \approx \frac{1}{\sqrt{\tau}} \ll 1$$

Reaction rate R depends mainly on T

Nobel Prize, 1967

● **History**  G. Gamow, H. Bethe, C. Critchfield, E. Salpeter, C. Von Weizsacker, A. Cameron, W. Fowler, G. Rivers

● **Example**  *Carbon burning:* $^{12}\text{C} + ^{12}\text{C} \Rightarrow ^{24}\text{Mg}^* \Rightarrow \dots$

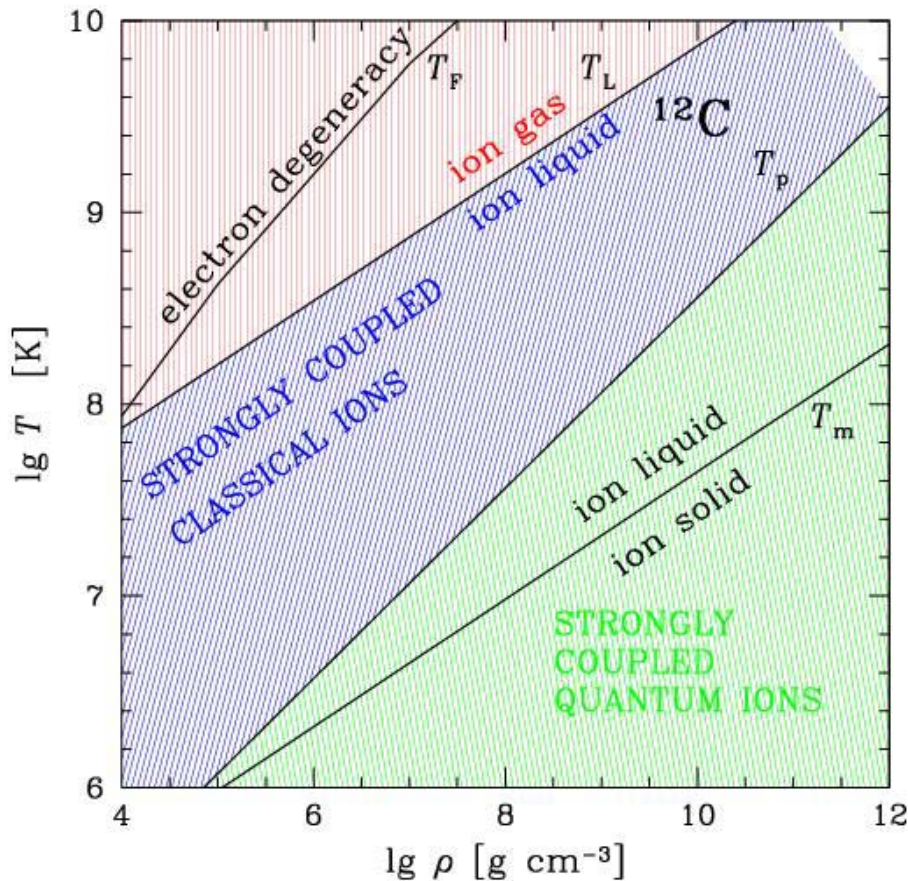
$$\rho = 10^{10} \text{ g cm}^{-3}, \quad t = n_i / R = \text{burning time}$$

$$T = 10^9 \text{ K} \quad \rightarrow \quad t = 6.9 \text{ s}$$

$$T = 10^8 \text{ K} \quad \rightarrow \quad t = 7.5 \times 10^{34} \text{ yr}$$

No burning at low T !

PHYSICAL CONDITIONS



$$T_F = m_e c^2 \left(\sqrt{1 + x^2} - 1 \right), \quad x = \frac{p_{Fe}}{m_e c}.$$

$$\Gamma = \frac{Z^2 e^2}{a T}, \quad a = \left(\frac{3}{4\pi n_i} \right)^{1/3}.$$

$$T_L = \frac{Z^2 e^2}{a} \Rightarrow \Gamma = 1; \quad T_m \Rightarrow \Gamma = 175$$

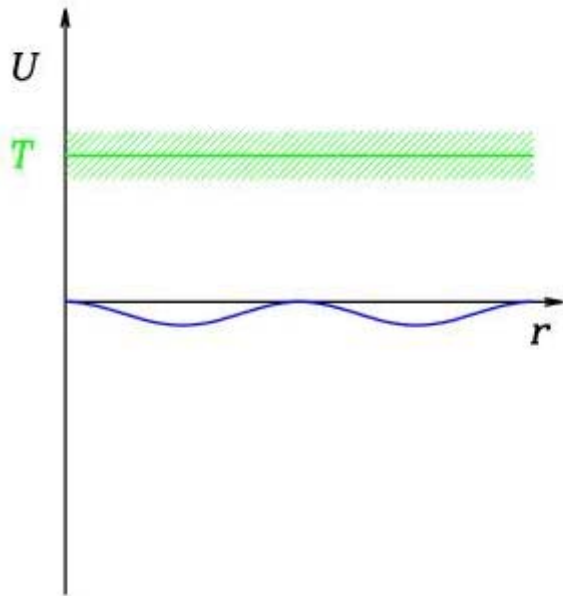
$$T_p = \hbar \omega_p, \quad \omega_p = \sqrt{\frac{4\pi Z^2 e^2 n_i}{m_i}}$$

PHYSICAL STATES OF IONS

Gas

$$T \gg \frac{Z^2 e^2}{a}$$

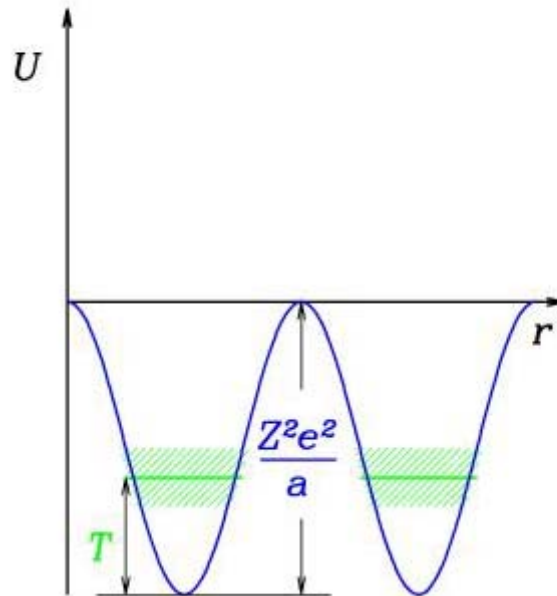
$$\Gamma \ll 1$$



Strongly coupled classical system

$$\hbar \omega_p \ll T \ll \frac{Z^2 e^2}{a}$$

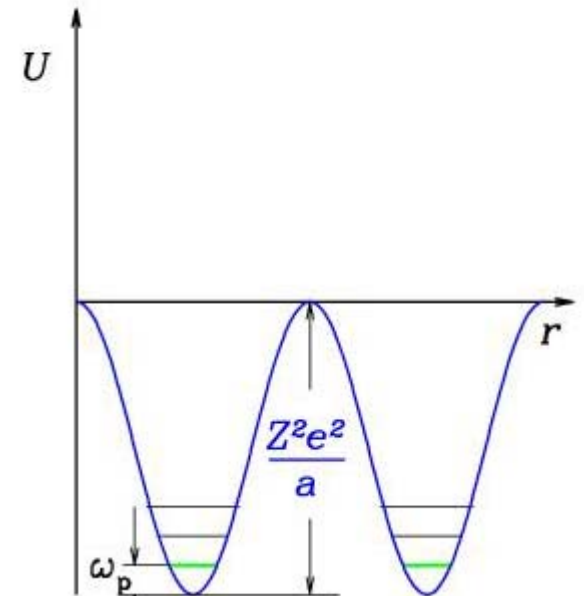
$$\Gamma \gg 1$$



Strongly coupled quantum system

$$T \ll \hbar \omega_p \ll \frac{Z^2 e^2}{a}$$

$$\Gamma \gg 1$$



PLASMA SCREENING IN THERMONUCLEAR REACTIONS

$$U(r) = \frac{Z_1 Z_2 e^2}{r} + \Phi(r) \quad \leftarrow \text{Created by neighbors}$$

$r \sim a$

Consider: $r_t \ll a \Rightarrow T \gg T_p$

Then $\Phi(r) \approx \text{const}$ for $r \leq r_t$

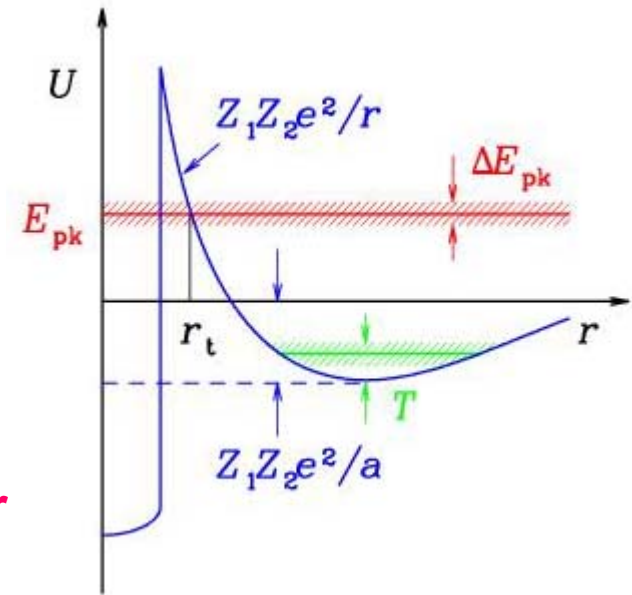
Two-stage process: (1) approaching the barrier
(2) penetrating the barrier

The reaction rate: $R = R_0 E$

$E > 1$ enhancement factor

$$E = \exp(f_1 + f_2 - f_{12}), \quad f_j = \frac{\mu_j}{T}$$

Thermodynamic function



Basic ideas:

Salpeter 1954, 1961

Salpeter & Van Horn 1969

DeWitt, Graboske & Cooper 1973

WEAK PLASMA SCREENING

Screening: **ion** + **electron**

$$T \gg \frac{Z^2 e^2}{a} \Leftrightarrow \Gamma \ll 1 \Rightarrow \text{Classical thermonuclear regime}$$

$$E = \exp\left(\frac{Z_1 Z_2 e^2}{r_D T}\right) \approx 1$$

pp cycle: $p + p \rightarrow D + e^+ + \nu_e$

in the Sun's center :

$$E \approx 1.055 \text{ for } T = 1.57 \times 10^7 \text{ K}$$

STRONG PLASMA SCREENING

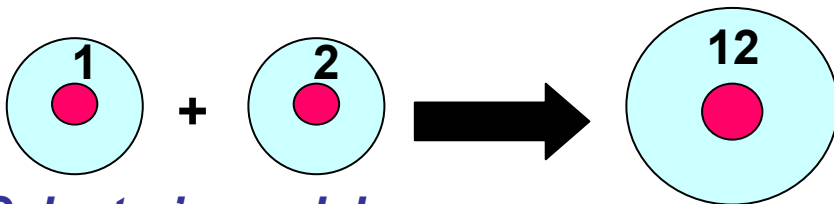
$$\frac{Z^2 e^2}{a} \ll T \ll T_p \Rightarrow \Gamma \gg 1 \Rightarrow \text{Thermonuclear regime with strong screening}$$

$$E = \exp(f_1 + f_2 - f_{12}) \gg 1$$

$$f_j = \frac{\mu_j}{T}; \quad \text{additive rule: } f_j = f(\Gamma_j)$$

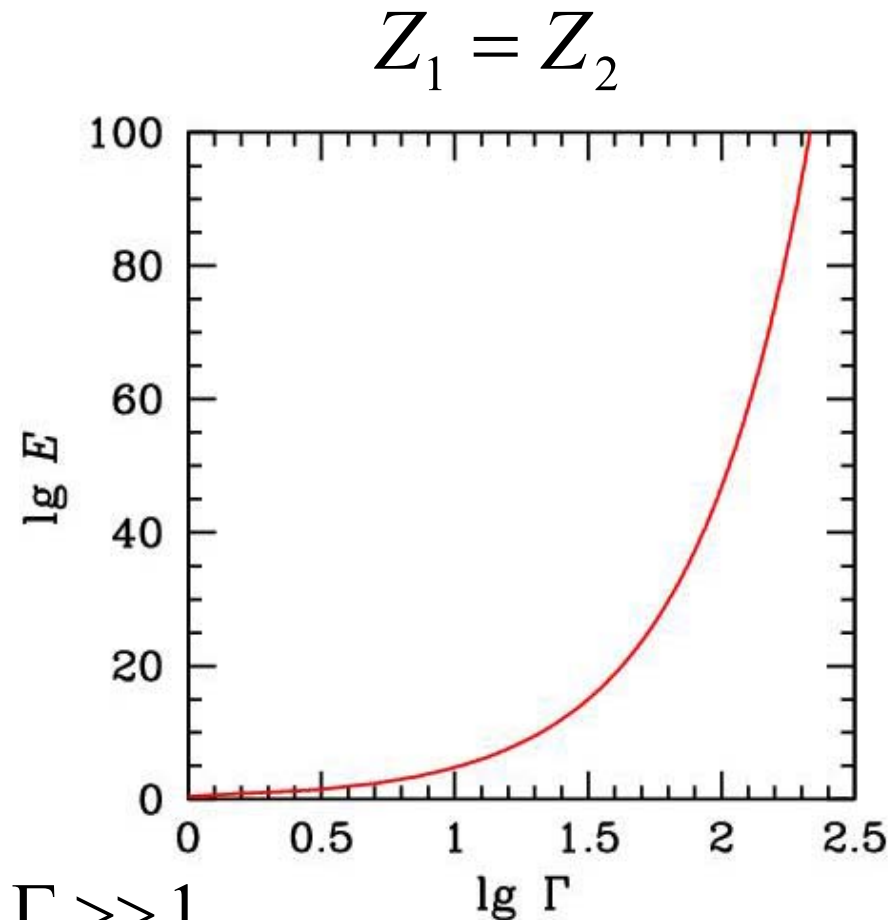
$$\Gamma_j = \frac{Z_j^2 e^2}{a_j T}, \quad a_j = a_e Z^{1/3}, \quad a_e = \left(\frac{3}{4\pi n_e} \right)^{1/3}$$

$f(\Gamma)$ - from OCP Monte Carlo,
with $f(\Gamma) \approx -0.9\Gamma$ for $\Gamma \gg 1$



Salpeter's model:

$$E \approx \exp(1.057\Gamma) \text{ for } Z_1 = Z_2 \text{ and } \Gamma \gg 1$$



Pycnonuclear reactions

Coulomb lattice of nuclei, $T=0$

Zero-point vibrations, $E \sim \hbar\omega$, $r_0 \sim \sqrt{\frac{\hbar}{m\omega}}$

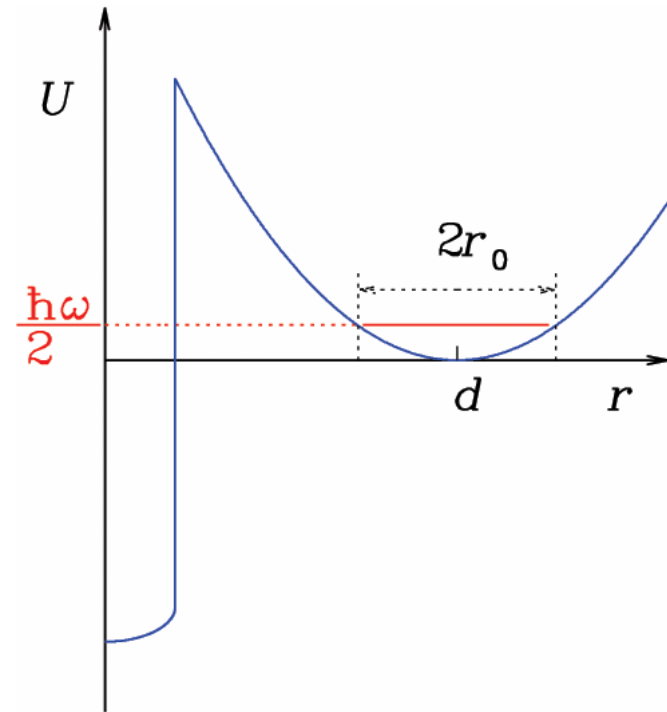
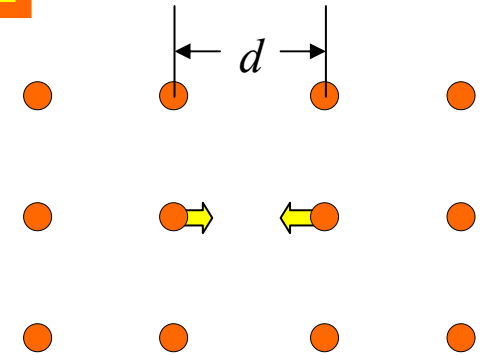
$$\omega \sim \omega_p = \sqrt{\frac{4\pi Z^2 e^2 n_i}{m}} \quad = \text{ion plasma frequency}$$

WKB tunneling:

$$\sigma(E) = \frac{S(E)P(E)}{E}, \quad P(E) = \exp(-\eta)$$

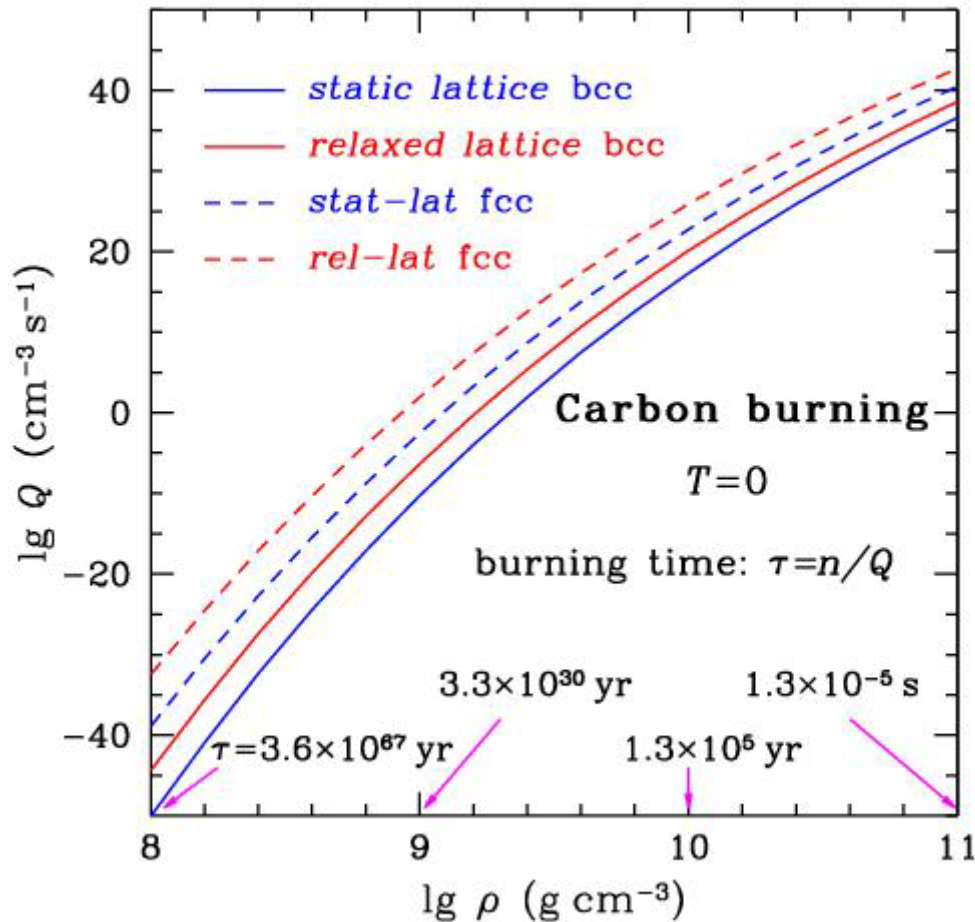
$$\eta(E) = \frac{2}{\hbar} \int_b^a dr \quad |p(r)| = \alpha \left(\frac{d}{r_0} \right)^2 \propto \frac{1}{\rho^{1/6}}$$

$$\alpha \sim 1 = ?$$



The reaction rate exponentially increases with growing density!

PYCNONUCLEAR BURNING OF DENSE STELLAR MATTER



After Schramm & Koonin (1990)

History:

Gamow (1938)

Wildhack (1940)

Zel'dovich (1957)

Cameron (1959)

Kirzhnits (1960)

Kopyshev (1964)

Wolf (1965)

Van Horn (1966)

Salpeter & Van Horn (1969)

Schramm & Koonin (1990)

Problems:

Dynamics of lattice response

Ionic mixtures, lattice imperfections

Cold Fusion Experiments (March 1989)

Cold Fusion Research

November 1989

A Report of the Energy Research Advisory Board to the United
States Department of Energy

Washington, DC 20585

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NCAS Introduction

CONTENTS

The recent interest in cold fusion was stimulated by reports from Utah scientists in March 1989 that fusion had occurred in experiments on the electrolysis of heavy water (D₂O). Dr. Stanley Pons and Dr. Martin Fleischmann at the University of Utah claimed to measure a production of heat that could only be explained by a nuclear process. Dr. Steven Jones at Brigham Young University did not observe heat but claimed to observe neutron emission that would also indicate a nuclear process. The claims were particularly astounding given the simplicity of the equipment, just a pair of electrodes connected to a battery and immersed in a jar of D₂O--equipment easily available in many laboratories.

This was not the first time fusion had been claimed to occur in electrolysis experiments, the earliest dating to the late 1920's in experiments that were later retracted, as discussed below. Nonetheless the implications of the Utah claims, if they were correct, and the ready availability of the required equipment, led scientists around the world to attempt to repeat the experiments within hours of the announcement. The Panel estimates that several tens of millions of dollars have been spent in the United States on cold fusion experiments. These experiments are discussed in the following sections.

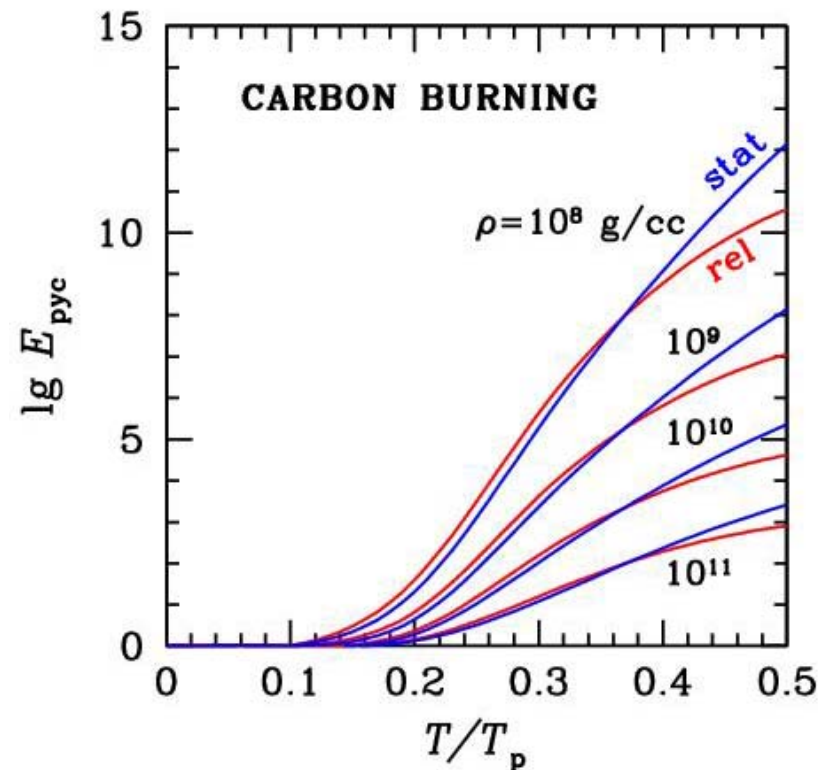
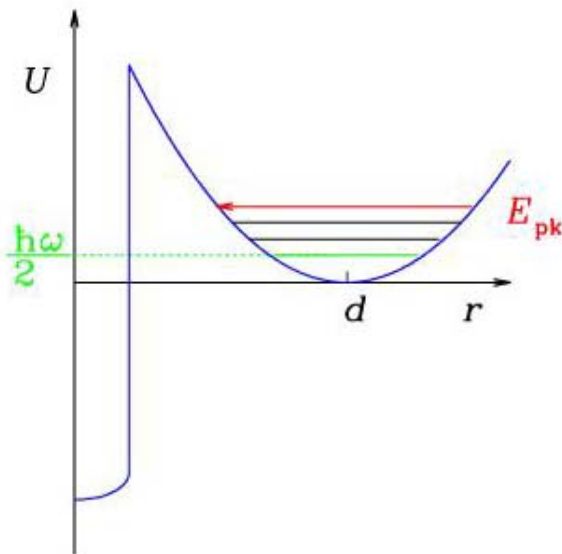
THERMALLY ENHANCED PYCNONUCLEAR BURNING

Increase T from $T=0$: $E_{pyc} = R(T) / R(0) = \text{enhancement factor}$

$$E_n \approx \hbar \omega_0 \left(n + \frac{1}{2} \right); \quad f_n \approx \exp \left(-\frac{E_n}{T} \right) \ll 1$$

$$E_{pk} \approx \frac{Z^2 e^2}{a} \exp \left(-\frac{\hbar \omega_0}{T} \right) \ll \frac{Z^2 e^2}{a}$$

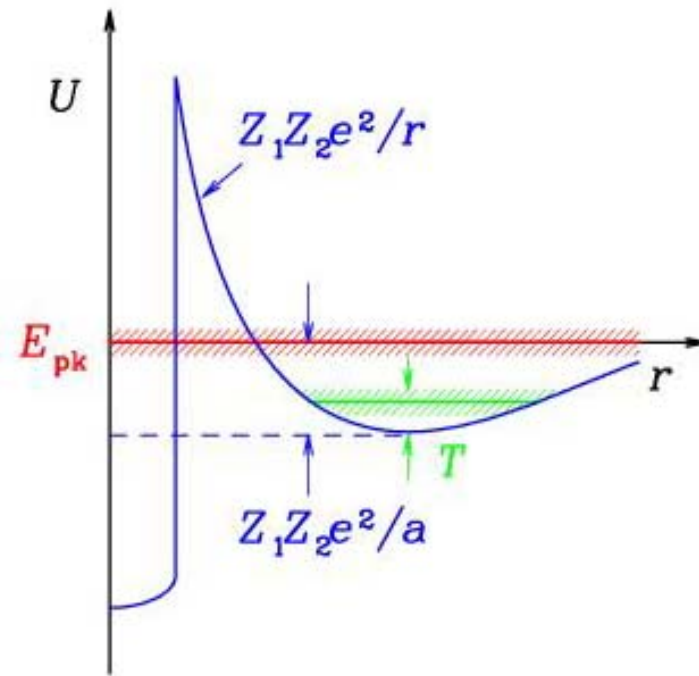
Salpeter and Van Horn 1969



INTERMEDIATE THERMO-PYCNONUCLEAR BURNING

$$T \sim T_p; \quad r_t \sim a; \quad E_{pk} \sim \frac{Z^2 e^2}{a}; \quad \Phi(r) \neq \text{const}$$

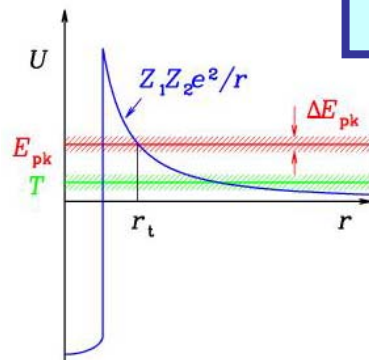
Most difficult for the theory!
Many attempts!



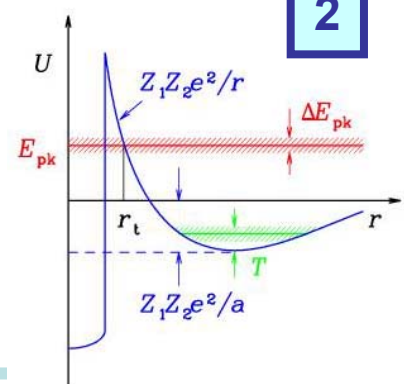
FIVE REGIMES OF BURNING IN DENSE MATTER

Thermonuclear →

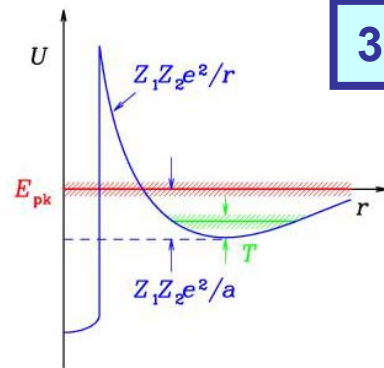
Classical



Strong screening

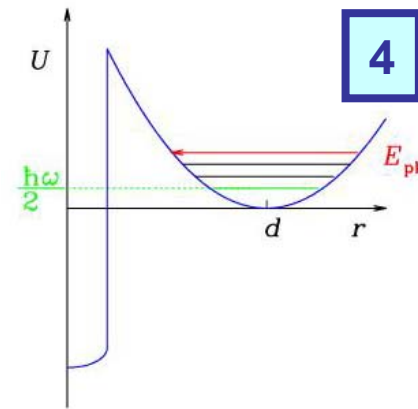


Intermediate →

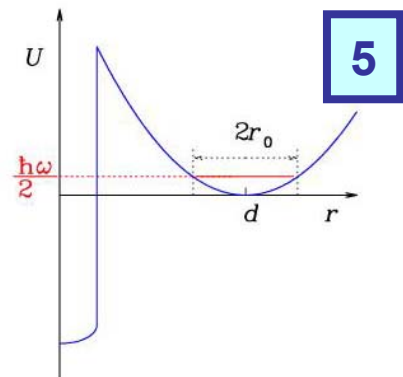


Pycnonuclear →

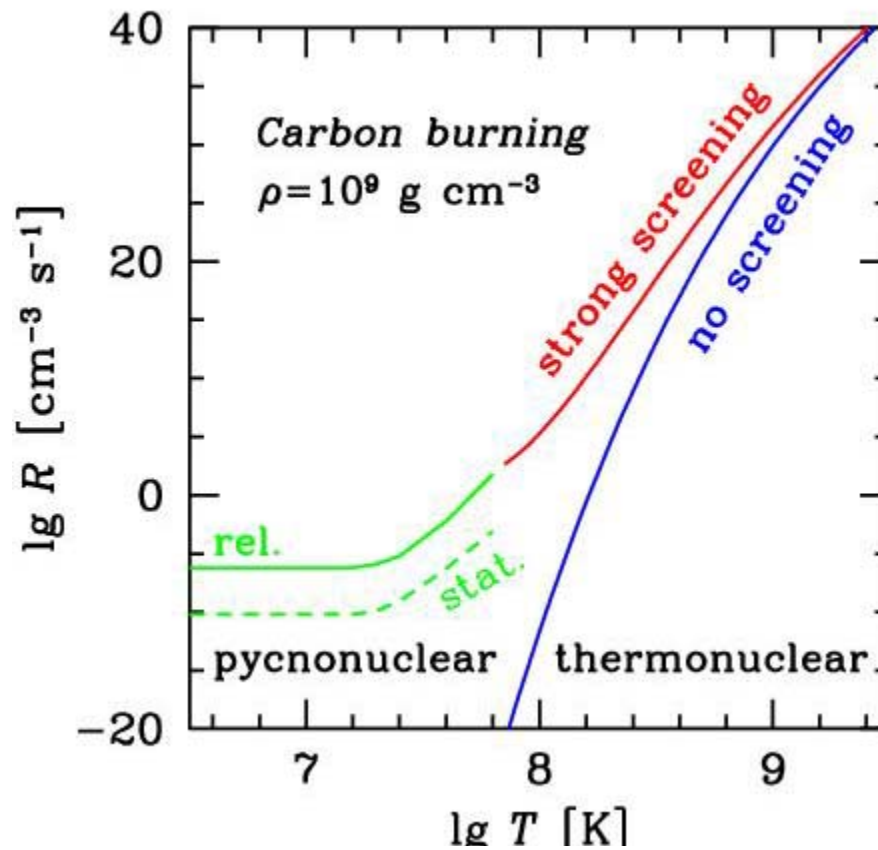
T-enhanced



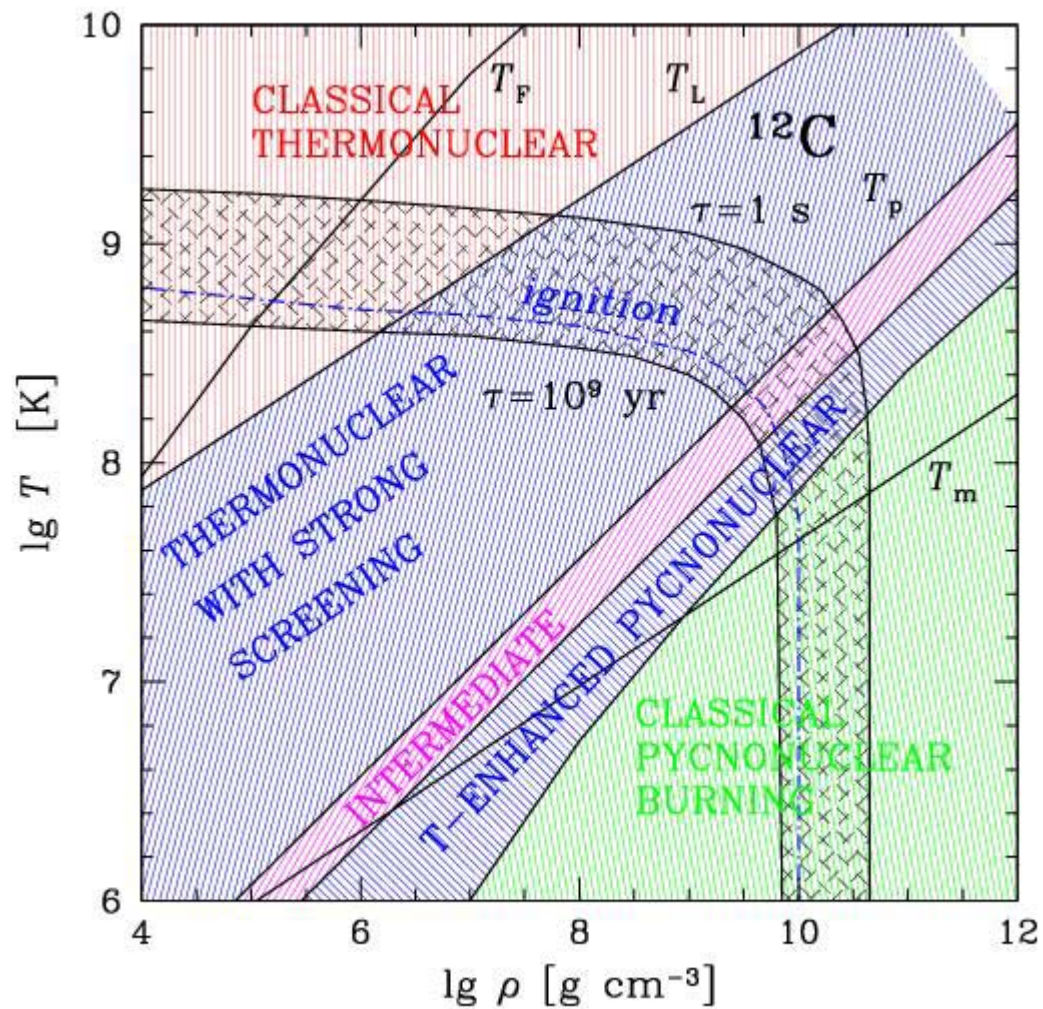
$T=0$



TEMPERATURE DEPENDENCE



GENERAL OUTLOOK



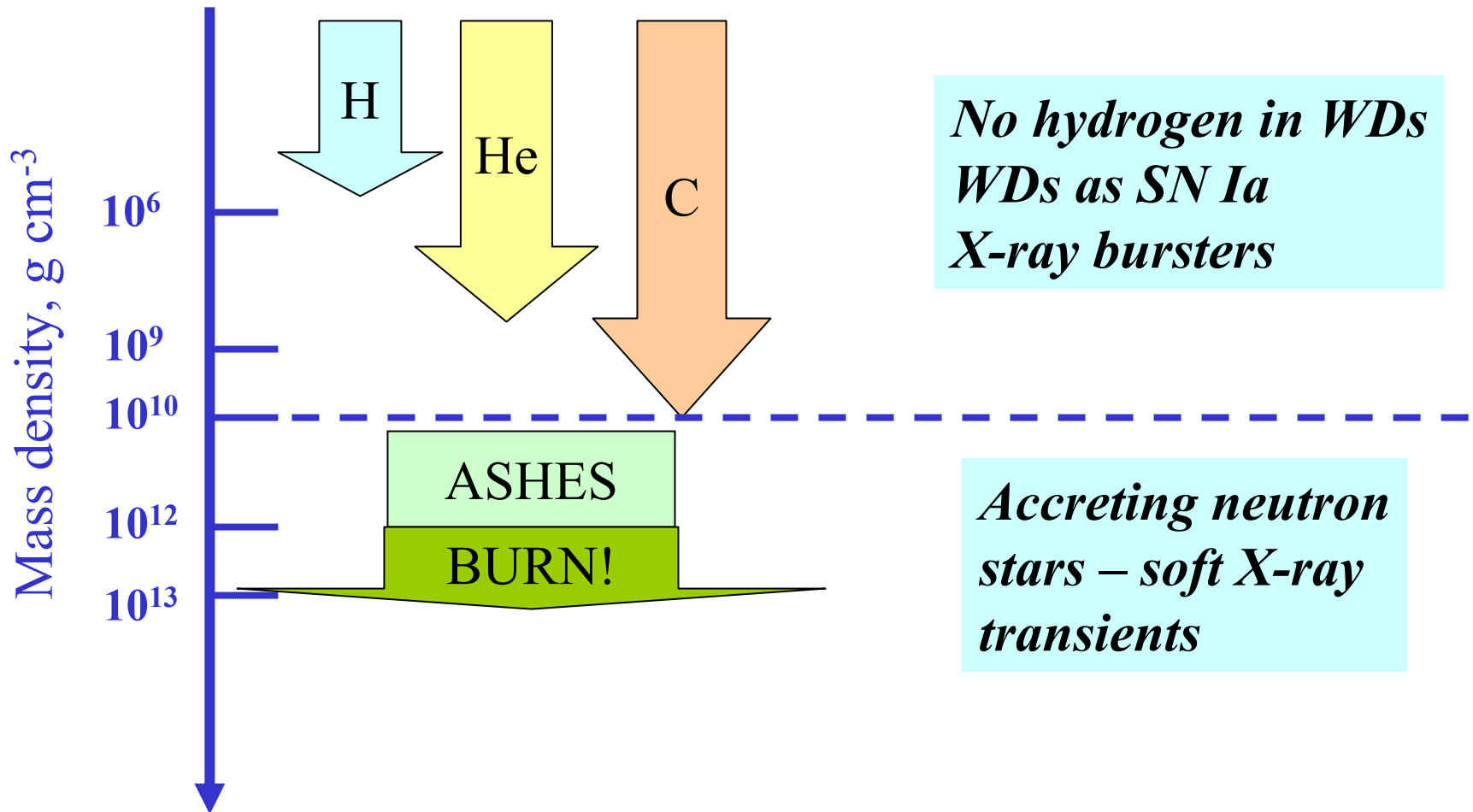
Applications of thermonuclear burning



**Any star from main
sequence to giants,
supergiants,
presupernovae ...**

Applications of pycnonuclear burning

White dwarfs and neutron stars (particularly, accreting)



SUMMARY

1. *There are five regimes of nuclear burning*
2. *Classical thermonuclear regime is well elaborated; other regimes – much less (especially the intermediate thermo-pycnonuclear regime)*
3. *The problem belongs to the physics of strongly coupled Coulomb plasma*
4. *The main problem is to calculate quantum tunneling in fluctuating plasma potential (with dynamical effects, etc.)*
5. *The result should depend on two parameters: Gamma and T/T_p*
6. *Theory goes back in time!*