The Joint Institute for Nuclear Astrophysics

Wavelet Analysis of Giant Resonances

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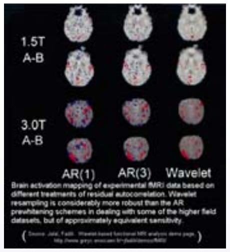
Motivation



Continuous Wavelet Transforms (CWT) and Discrete Wavelet Transforms (DWT) are novel techniques used for example in applications such as MRI scans (left) and JPEG image compression. Here, CWT and DWT are used to probe the fine-structure of giant resonances excited via nuclear charge-exchange reactions [(^{3}He,t) at 140 MeV/nucleon]. The goals are:

- detailed tests of theoretical descriptions of giant resonances of importance for astrophysics
- model-independent strength extraction of giant resonances
- extraction of level densities via autocorrelation analysis after model-independent background subtraction

Different Wavelets (Ψ) used in the CWT analysis; best results are obtained with the Morlet Wavelet

CWT

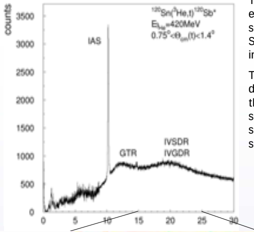
^{120}Sn(^{3}He,t)^{120}Sb* energy spectrum taken at RCNP, Osaka University.

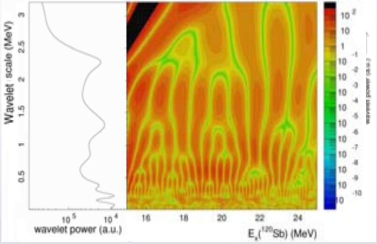
The Morlet wavelet (below) was used in the CWT and the coefficients were calculated for energy scales up to 3.2 MeV. The result is shown below the spectrum. The peaks at specific energies and scales characterize the spectrum and indicate dominant energy scales which can be compared with theory.

Continuous Wavelet Transform:

$$C(E_x,\delta E)=\frac{1}{\sqrt{\delta E}}\int\sigma(E)\,\Psi\left(\frac{E_x-E}{\delta E}\right)dE$$

$$\Psi(x)=\frac{1}{\sqrt[4]{\pi}}\cos\,(kx)\exp\left(-\frac{x^2}{2}\right)$$

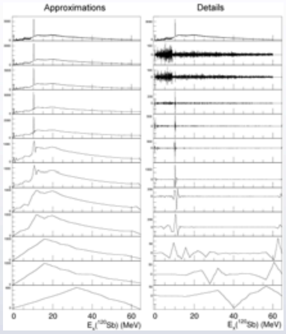




DWT

The discrete wavelet transform (DWT) is very similar to the CWT, except that transformations are only performed at specific energy scales. These levels of transformations are orthogonal to each other. Subsequent levels of “detail” are removed in the DTW, leaving behind increasingly coarse grained approximations of the data.

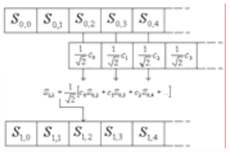
The spectrum can be reconstructed using the different levels of details and approximations, thus emphasizing certain features at those energy scales. This allows us to remove certain scales, and to see which scales are important for a specific resonance. The power spectrum is used to identify the dominant levels of detail (energy scales).



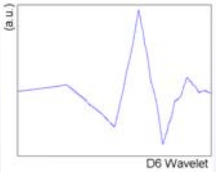
A Simple Example

In this example, S(m,n) refers to the nth bin in the mth level approximation. ‘m=0’ refers to the original spectrum. T(m,n) represents the details. From the deconstruction diagram, the wavelet coefficients are multiplied by the data to the right of the target bin in order to calculate the next level approximation. The details are calculated as the difference between the two. The reconstruction scheme essentially does the process backwards, creating the lower level approximation from the higher level approximation and corresponding details.

Deconstruction



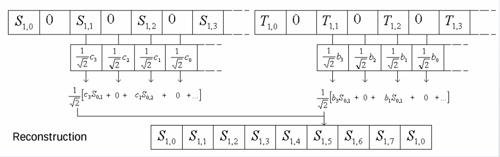
In the case of the DWT, we use the Daubechies family of wavelets, which are very compact. Wavelet D6 of this family is used (see wavelet coefficients below). It allows for removal of 2nd order background to the data in a model-independent way.



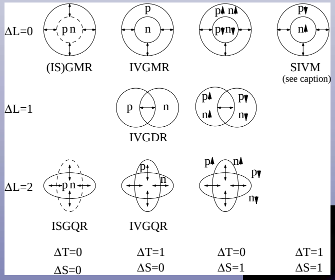
$$S_{1,n}=\frac{1}{\sqrt{2}}\sum_k c_k S_{0,2n+k}$$

$$T_{1,n}=\frac{1}{\sqrt{2}}\sum_k b_k S_{0,2n+k}$$

c_1=0.47046721
c_2=1.14111692
c_3=0.650365
c_4=-0.19093442
c_5 =-0.12083221
c_6 =0.0498175

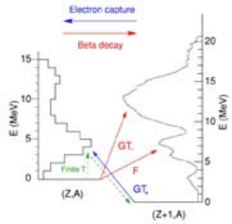


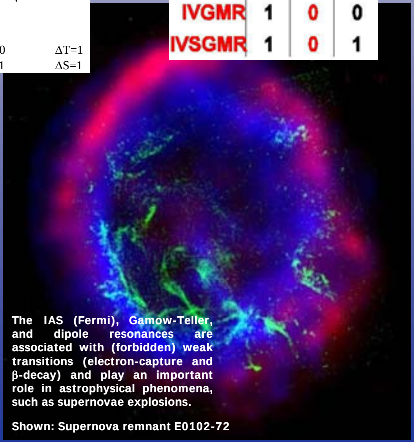
Giant Resonances



	ΔT	ΔL	ΔS
IAS	1	0	0
GTR	1	0	1
GDR	1	1	0
SDR	1	1	1
IVGQR	1	2	0
IVGMR	1	0	0
IVSGMR	1	0	1

Macroscopically, Giant Resonances can be described as oscillations/vibrations of the nuclear liquid drop around its equilibrium shape. Microscopically, they are coherent superpositions of 1-particle-1-hole transitions in the nucleus associated with certain quantum numbers.

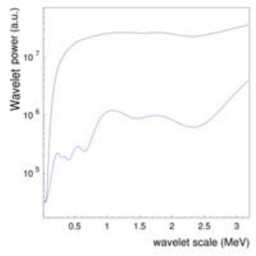




The IAS (Fermi), Gamow-Teller, and dipole resonances are associated with (forbidden) weak transitions (electron-capture and β-decay) and play an important role in astrophysical phenomena, such as supernovae explosions.

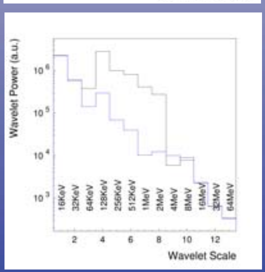
Shown: Supernova remnant E0102-72

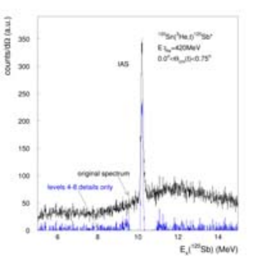
The Power Spectrum



Single narrow states in the experimental data dominate the power spectrum. The black line shows the wavelet power for the full data set. The blue line shows the wavelet power for which the IAS (see below) has been removed from the spectrum. Energy scales corresponding to the (wide) giant resonances are revealed.

Since in the DWT only certain energy scales are probed, the power spectrum is very coarse. However, it can be used to identify the important energy scales for the reconstruction of a certain excitation. Below left, the example is given for the IAS. The black line is the power spectrum from DWT of the full spectrum; the blue line refers to the DWT of the spectrum without the IAS. Scales 4-8 are important for the IAS.

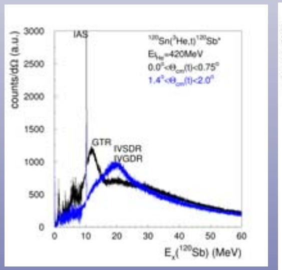


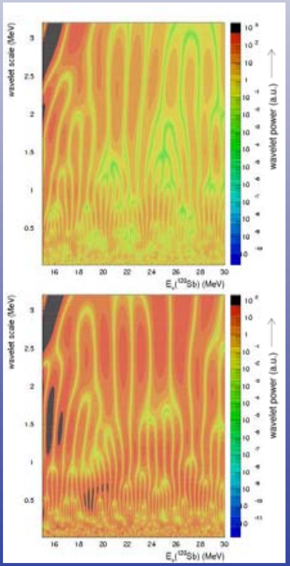


$$P_m=\sum_E\frac{\det(E)^2}{\sqrt{2}^{\,m}}$$

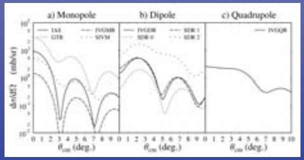
Since scales 4-8 are important for the IAS state, we can perform a reconstruction using only those scales. The original spectrum (black) is plotted together with such a reconstruction (blue). Notice that the IAS was almost perfectly reconstructed, and the “background” under the state removed.

Selecting Resonances





Resonances of different multi-polarity are prominent at different triton angles in the experimental spectra. At the lowest angles (0.0º-0.75º; black) monopole transitions (GTR and IAS) are strong, whereas at larger angles (1.4º-2.0º; blue) dipole resonances dominate. By comparing CWT and DWT analyses at different angles, the features of each type of resonance can be studied individually.



Measurements of angular distributions provide a clean tool to identify giant resonances with different multi-polarity.

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