The Joint Institute for Nuclear Astrophysics

Strong and Weak Nuclear Evolution of Material Behind a Deflagration Front in a Type Ia Supernova

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Nuclear Evolution in a Type Ia Supernova

Near the core of the exploding white dwarf (WD), a nuclear flame burns ^{12}C and ^{16}O to a hot soup made up of elements near ^{56}Ni and some 20% ^{4}He by mass. This material is in an *active* equilibrium state, termed nuclear statistical equilibrium (NSE), in which continuously occurring particle fusion is balanced by photo-disintegration. This NSE state, notably the relative fraction of ^{4}He to heavy elements, then evolves as the ashes undergo hydrodynamic evolution. As much as 40% of the available nuclear energy, $\simeq 0.4$ MeV per nucleon, is released during this post-flame evolution, as the ^{4}He is used to build elements near ^{56}Ni. Proper treatment of this energy release is necessary for accurate hydrodynamical simulation of rising bubbles and plumes of this NSE ash, making it essential to simulation of the deflagration and early expansion stages of the supernova. (See also Calder et al., this poster session.)

Material which remains at high density for more than several tenths of a second can convert a significant number of protons into neutrons via e^- capture and e^+ emission. Though some energy is lost due to neutrino emission, as the ash becomes more neutron rich, more tightly bound species closer to the valley of stability such as ^{56}Fe are favored over ^{56}Ni. This adjustment of NSE can release an additional ~ 0.15 MeV per nucleon, raising the temperature. (Compare Fig. 1 & 2.)

Motivation

- **Non-spherical Effects:** The non-spherical nature of the Ia supernova event, the thermonuclear explosion of a carbon oxygen white dwarf, is still under active study and remains poorly understood. Accurate energy release is an essential component of ongoing simulation efforts.
- **Cosmology:** The apparent brightness of distant Type Ia supernovae provided important evidence for the revelation that the expansion of the universe is accelerating (Riess et al. 1998; Perlmutter et al. 1999). These measurements continue in earnest and promise to constrain the dark energy equation of state.
- **Nucleosynthesis:** The nuclear species produced in the supernova and visible in the spectrum depend sensitively on the temperature and density history of the ejected material, providing an important probe of the explosion. These hydrodynamic trajectories are affected by energy release and weak interactions.
- **Energy Release:** Energy stored in light nuclei at high temperatures immediately following the flame is released later when the hydrodynamic evolution lowers the density and temperature and the NSE abundances shift toward heavier and more tightly bound nuclei.

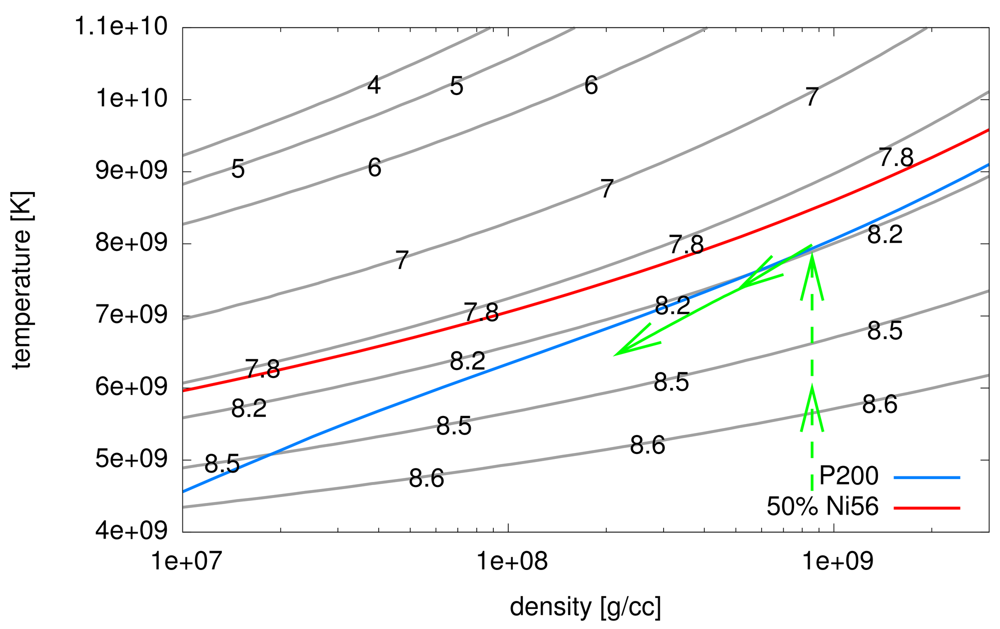

Figure 1: Contours of average binding energy per nucleon, $\bar{q}$, (MeV) for matter in nuclear statistical equilibrium (NSE). Higher values indicate more energy release. The electron fraction $Y_e = \bar{Z}/\bar{A} = 0.5$.

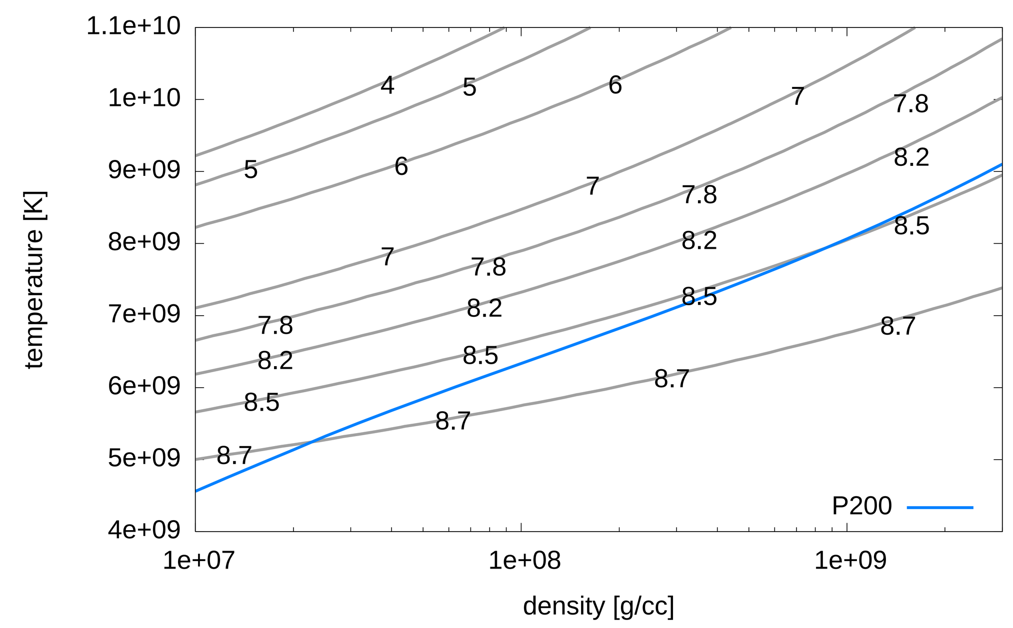

Figure 2: Contours of average binding energy per nucleon, $\bar{q}$, (MeV) for matter in NSE at an evolved electron fraction, $Y_e = 0.475$.

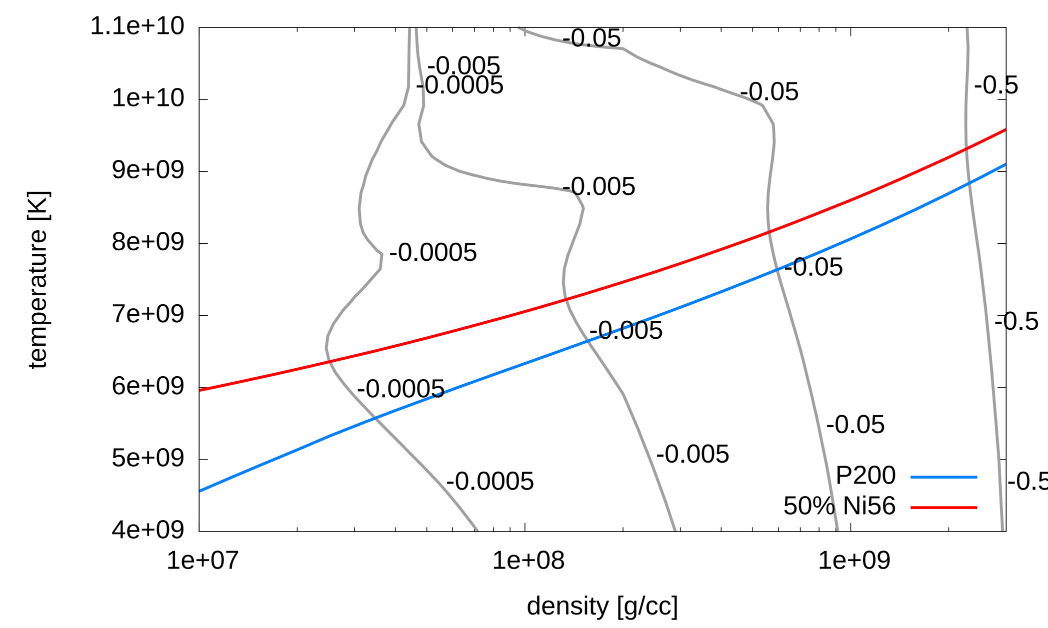

Figure 3: Contours of instantaneous bulk neutronization rate $\dot{Y}_e$ (s^{-1}) for matter in NSE with $Y_e = 0.5$. Note the large negative rates at high densities.

Also shown in each figure are the temperature at each density reached when burning a 50/50 mix of ^{12}C and ^{16}O to NSE (blue curve), calculated using a 200-isotope network, and a curve of 50% Ni abundance (red curve) in a two-component (^{4}He and ^{56}Ni) NSE, representing the line of separation between heavy and light element dominance in the mixture. In Fig. 1, green arrows indicate a possible path of a fluid element, the dashed section is very fast (milliseconds) upon passing through the flame, while the solid section might occur in the first few tenths of a second of the WD or bubble expansion.

Calculation and Application of NSE

- Solve equations of nuclear statistical equilibrium (NSE) for 238 nuclear species with good coverage down to $Y_e = \bar{Z}/\bar{A} \simeq 0.42$

$$\mu_i^{\rm id} + \mu_i^{\rm C} + m_i c^2 = Z_i(\mu_p^{\rm id} + \mu_p^{\rm C} + m_p c^2) + (A_i - Z_i)(\mu_n + m_n c^2)$$

  where $\mu_i^{\rm C}$ is the plasma Coulomb correction evaluated via the linear mixing rule for multicomponent plasma. We use the fitting form of Chabrier & Potekhin 1998, good for $\Gamma = Z_i^{5/3} e^2 (\frac{4}{3}\pi n_e)^{1/3}/kT \leq 160$. The rest mass difference can be written conveniently as the nuclear binding energy $Q = [Z_i m_p + (A_i - Z_i) m_n - m_i]c^2$.
  - Excited state partition functions included (Rauscher & Thielemann 2000)
- Average over equilibrium abundance distribution to calculate
  - mean atomic mass, $\bar{A}$; sets ionic heat capacity
  - mean binding energy per nucleon, $\bar{q} = \bar{Q}/\bar{A}$; determines energy release

  Bar denotes number-weighted average over species, e.g. $\bar{A} = \sum_i A_i n_i / \sum_i n_i$ where n_i is the number density of species i.
- Convolve e^- and e^+ capture and emission rates with abundance distribution of the full set of 238 nuclei using up to date rate table for pf-shell nuclei (Langanke & Martínez-Pinedo 2000). Determines neutrino loss rate, ϵ_ν, and neutronization rate, $\dot{Y}_e$.

Nuclear and Energetic Evolution

- In regions where the flame has passed, and reaction rates are fast compared to the hydrodynamic timescale, we can assume material is in NSE with a given temperature, density and electron fraction (ρ, T, Y_e).
  - Neutronization rate, $\dot{Y}_e$, neutrino energy loss rate ϵ_ν, and average nuclear binding energy, $\bar{q}$ are tabulated as functions of ρ, T, and Y_e.
  - Hydrodynamic evolution of Y_e and $\bar{q}$ is calculated using a reduced set of representative species.
  - A total nuclear energy deposition/loss term $\epsilon_{\rm nuc} = (E_{n+1} - E_n)/\Delta t$, where E is the internal thermal energy, is found by solving

$$E_{n+1} - \bar{q}_{n+1} = E_n - \bar{q}_n + \Delta t \left[-\dot{Y}_e N_A (m_e + m_p - m_n)c^2 - \epsilon_\nu \right]$$

    between each hydrodynamic timestep. This accounts for both losses due to neutronization and conversion of internal energy to and from nuclear binding (rest mass) energy.
- In regions where the flame has passed and reaction rates are comparable or slower than hydrodynamic timescale, we assume freeze-out of ash abundance.

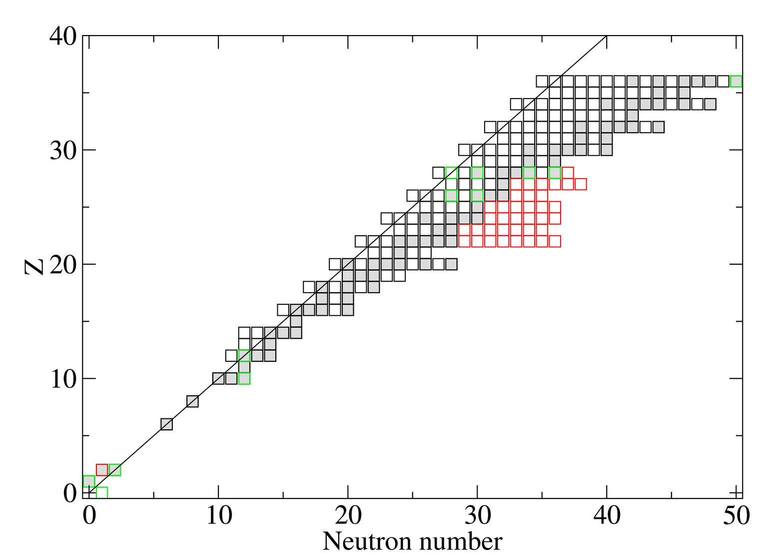

Figure 4: Nuclides used in calculation. Black indicates nuclei in the 200 species dynamic network calculation. Red nuclides were added to extend the Y_e range of the NSE calculation. Green nuclides are the reduced set used in supernova simulations. Grey indicates stable nuclides.

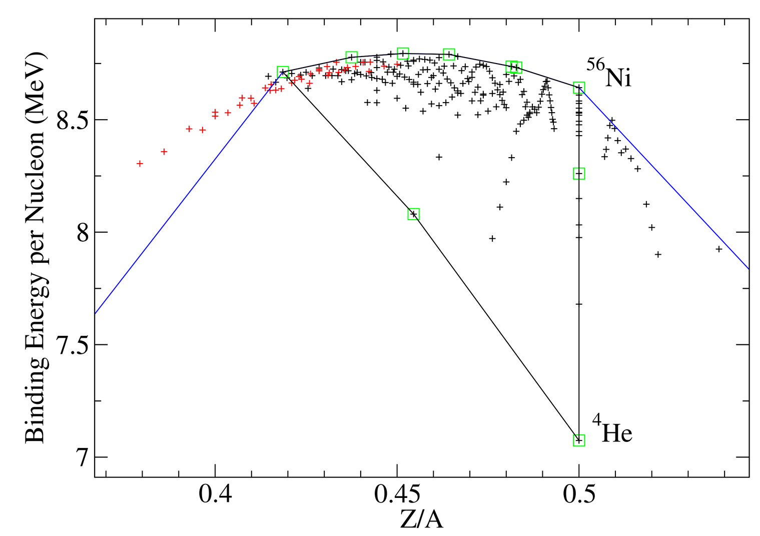

Figure 5: Binding energy per nucleon Q/A vs. Z/A (respectively $\bar{q}$ and Y_e of pure sample) for nuclides used in calculation. (^{3}He, n, and p are off the plot). Color scheme is as in Fig 4. The set of heavy nuclides is chosen to allow spanning of the entire possible area of q and Y_e (outlined in black and blue) and also allows accurate representation of $\bar{A}$, precise matching of $\bar{A}$ is achieved in most cases in all but the lowest Y_e portion of the black-outlined area.

Screening in Network and NSE Calculations

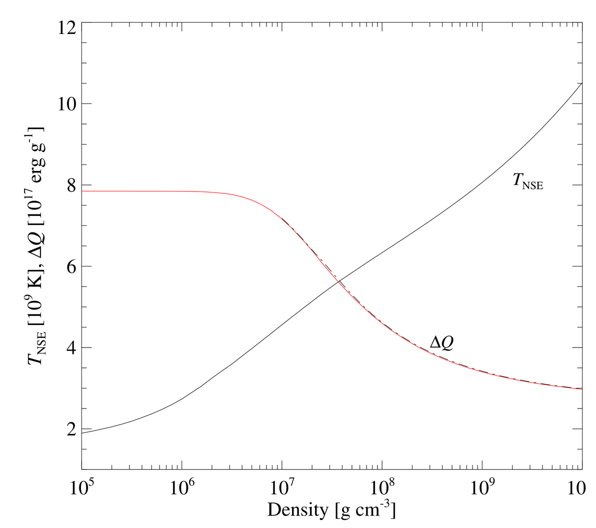

Figure 6: Network ignition calculation and NSE agree very well in energy release $\Delta Q = \bar{q} - \bar{q}_{\rm CO}$. 200-isotope network is used to burn 50/50 mix of CO to NSE at constant density, resulting T is used in NSE calculation of $\bar{q}$ at same density.

Dynamic network calculations of the burning and NSE calculations of the post-flame stage were reconciled by screening the forward rates and then calculating the reverse rates for for $i(j,k)l$ and $i(j,\gamma)l$ reactions using

$$\frac{\langle\sigma v\rangle_r}{\langle\sigma v\rangle_f} = \frac{g_i g_j}{g_k g_l}\left(\frac{m_i m_j}{m_k m_l}\right)^{3/2} \exp\left(\frac{Q_i + Q_j - Q_k - Q_l}{kT}\right) \exp\left(\frac{-\mu_i^{\rm C} - \mu_j^{\rm C} + \mu_k^{\rm C} + \mu_l^{\rm C}}{kT}\right)$$

$$\frac{\lambda_\gamma}{\langle\sigma v\rangle_f} = \frac{g_i g_j}{g_l}\left(\frac{m_i m_j}{m_k}\right)^{3/2}\left(\frac{2\pi kT}{h^2}\right)^{3/2} \exp\left(\frac{Q_i + Q_j - Q_l}{kT}\right) \exp\left(\frac{-\mu_i^{\rm C} - \mu_j^{\rm C} + \mu_l^{\rm C}}{kT}\right)$$

where $\mu_i^{\rm C}$ is the plasma Coulomb correction to the chemical potential for species i, Q_i is the nuclear binding energy, g_i is the temperature dependent nuclear partition function (Rauscher & Thielemann 2000), and m_i is the nuclear mass. The same fitting form is used for $\mu_i^{\rm C}$ as in the NSE calculation (Chabrier & Potekhin 1998).

Screening of forward rates was evaluated as described in Wallace, Woosley, & Weaver (1982) and rates are from REACLIB (Thielemann et al. 1986; Rauscher & Thielemann 2000).

Reduced Isotope set for Supernova Simulation

For several reasons it is convenient to implement proper conservation of $\bar{q}$ and Y_e by using a reduced set of nuclei which can exactly reproduce both these parameters with only a modest amount of extra storage required. These nuclides are chosen from those in the 238 used in the NSE calculation in such a way that a mixture of them will fully span the requisite $(\bar{q}, Y_e)$ space. The reduced set chosen for this application are shown in green in Fig. 4 and 5, and include neutrons, protons, ^{4}He, ^{22}Ne, ^{24}Mg, ^{54}Fe, ^{56}Fe, ^{56}Ni, ^{62}Ni, ^{64}Ni, ^{86}Kr. The heavy elements listed form the outer boundary of the convex surface in $\bar{q}$ as shown in Fig. 5. In most cases (generally within the black borders) it is possible to simultaneously match $\bar{q}$, Y_e and $\bar{A}$ with the reduced set. Near the Low Y_e border of this area and in the area delineated by the blue lines an error of a few percent in $\bar{A}$ is possible.

References

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Summary

We have described a treatment of the NSE material that, in addition to modeling energy release behind the flame, accurately accounts for electron capture, neutrino loss, and screening of charged particle reactions. The screening utilized is, for the first time, fully consistent between the NSE calculation and the dynamic burning (nuclear network) calculation which models the flame itself. This treatment has been implemented in the FLASH code, and is being used to simulate the early stages of a Type Ia supernova (see Calder et al., this poster session). Precise calculation of the energy release is necessary for calculation of nucleosynthetic information for the ejecta. Comparison of such calculations with observed spectral characteristics of the ejecta has consistently proven one of the most powerful observational tools for understanding the supernova process.

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