Carbon Fusion Rates and the Ignition Curve

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Outlook:

(i) Nuclear Physics:

- Nuclear and Coulomb potentials;
- The barrier penetration model (BPM);
- Fusion cross section results.

(ii) Nuclear Astrophysics:

- Astrophysical S-factor;
- Reaction rate equation;
- $^{12}C + {}^{12}C$ ignition curve.

The Nuclear Potential (Folding Potential)

$$V_F(R) = \int \rho_1(\vec{r_1})\, \rho_2(\vec{r_2})\, v_{NN}(\vec{R} - \vec{r_1} + \vec{r_2})\, d\vec{r_1}\, d\vec{r_2}$$

↳ density dependent

Two ways to solve this problem:

(i) Theoretical calculations (RMF, for example).

→ M. Stoitsov et al., Phys. Rev. C58 (1998) 2086.
A. V. Afanasjev et al., Phys. Rev. C60 (1999) 051303.

(ii) Systematization of the nuclear densities (Global description);

- Two-parameter Fermi distribution (2pF)
- Appropriate for stable nuclei

→ L. C. Chamon et al., Phys. Rev. C66 (2002) 014610.

$$V_{LE}(R) \approx V_{Fold}(R)\, e^{-4v^2/c^2}$$

← Phys. Rev. Lett. 78 (1997) 3270
Phys. Rev. Lett. 79 (1997) 5218
Phys. Rev. C58 (1998) 576
Phys. Rev C66 (2002) 014610

Coulomb Potential

$$V_C(R) = \int \int e^2 \, \frac{\rho_{ch1}(r_1) \, \rho_{ch2}(r_2)}{|\vec{R} - \vec{r_1} + \vec{r_2}|} \, dV_1 \, dV_2$$

$$\frac{e^2}{|\vec{R} - \vec{r_1} + \vec{r_2}|} \longrightarrow$$ interaction between two elements of charge

$$V_C(R) = \int \int \frac{dq_1 \, dq_2}{|\vec{R} - \vec{r_1} + \vec{r_2}|}$$

Barrier Penetration Model (BPM)

(Fusion Cross Section Calculations)

$$V_{\text{eff}}(R,E) = V_C(R) + V_N(R,E) + \frac{\ell(\ell+1)\hbar^2}{2\mu R^2}$$

$$E \geq V_{B\ell} \;\rightarrow\; T_\ell = \left\{1 + \exp\left[\frac{2\pi(V_{B\ell}-E)}{\hbar\omega_\ell}\right]\right\}^{-1} \quad \hbar\omega_\ell = \left|\frac{\hbar^2}{\mu}\,\frac{d^2V_{\text{eff}}}{dr^2}\right|^{1/2}_{R_{B\ell}}$$

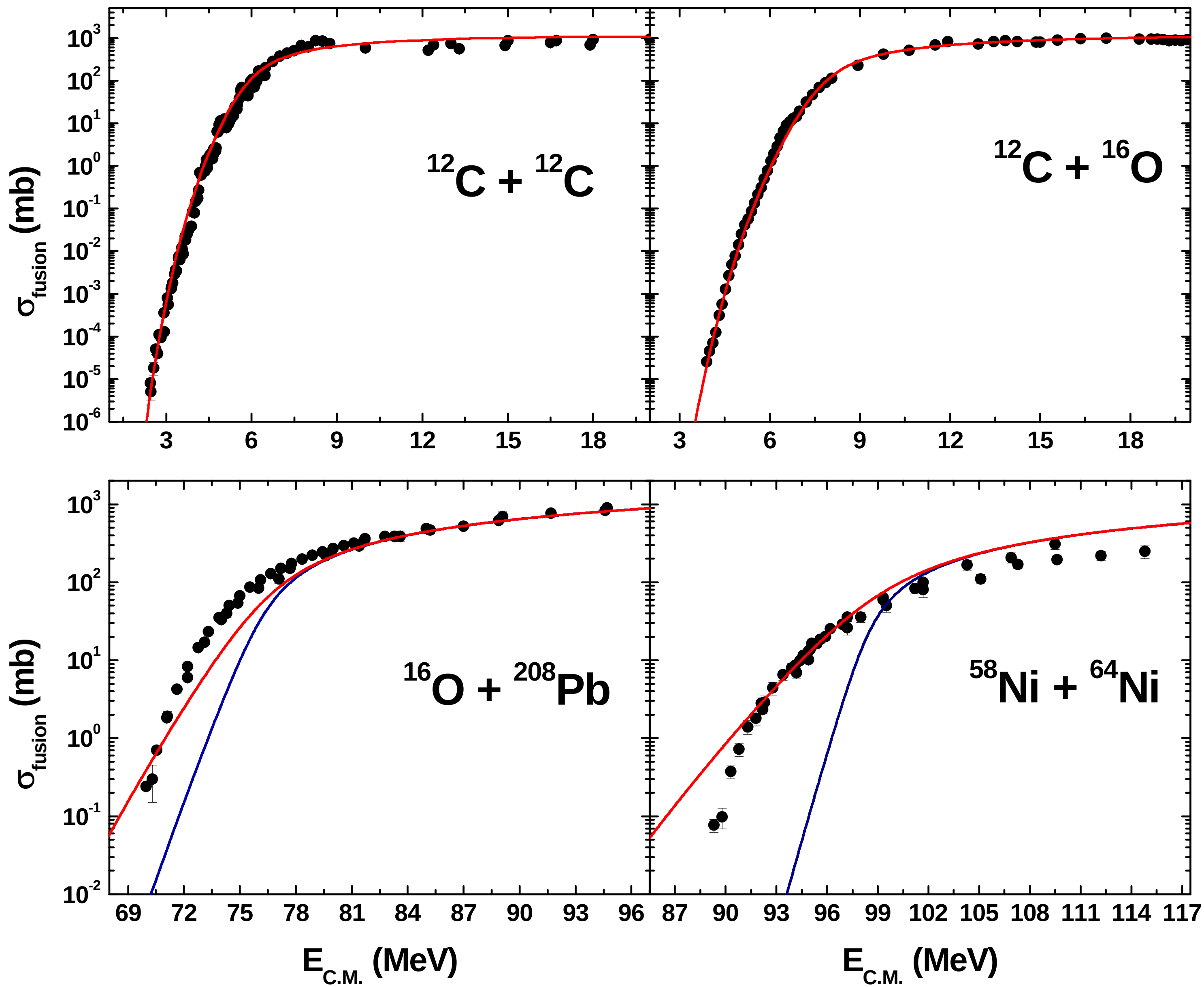
(Hill-Wheeler formula)

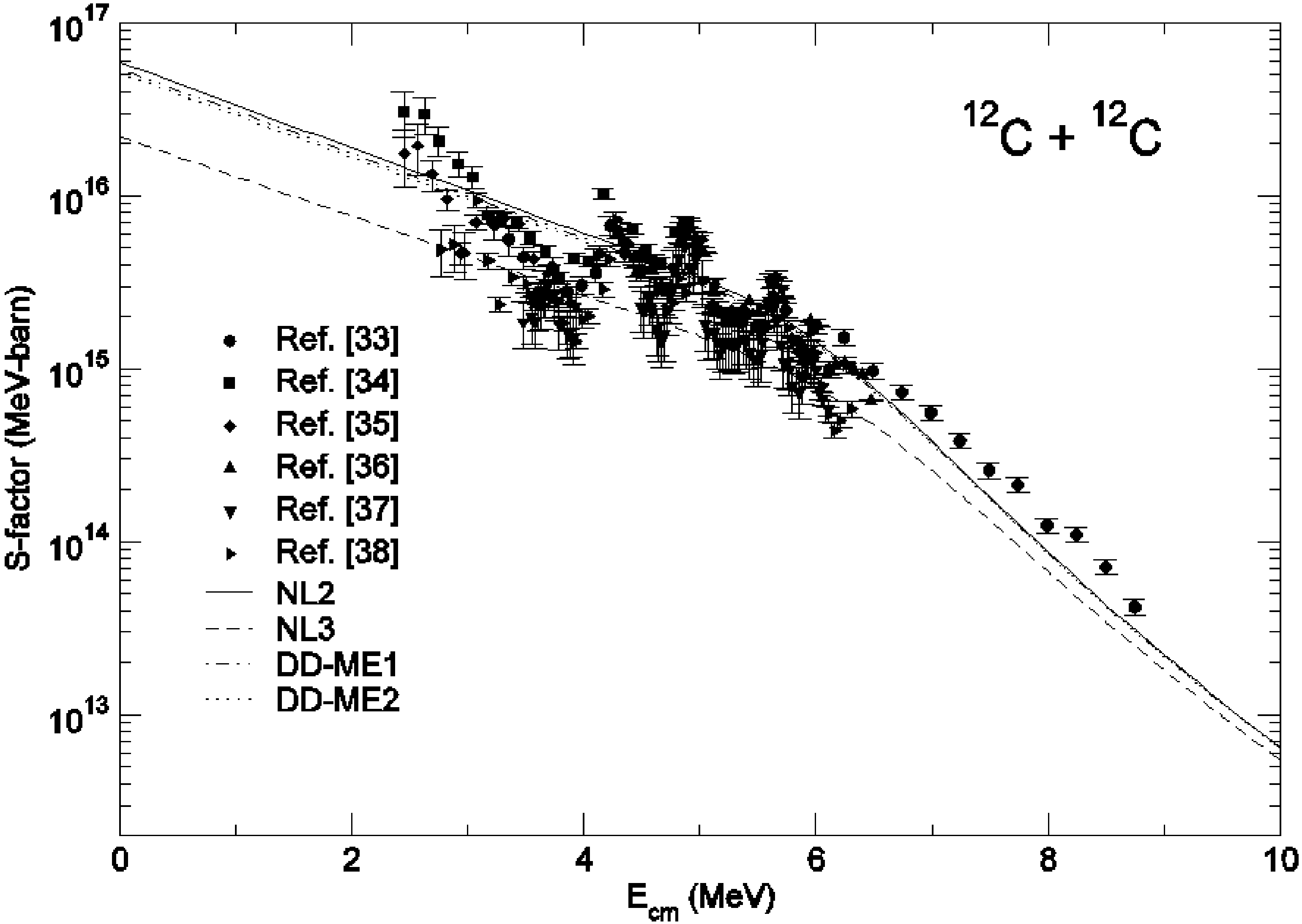
$$E \leq V_{B\ell} \;\rightarrow\; T_\ell = [1+\exp(S_\ell)]^{-1} \quad \begin{cases} \hbar\omega_\ell = \dfrac{2\pi(V_{B\ell}-E)}{S_\ell} \\ S_\ell = \displaystyle\int_{r_1}^{r_2} \sqrt{\frac{8\mu}{\hbar^2}[V_{\text{eff}}(r,E)-E]}\,dr \end{cases}$$

(WKB approximation)

$$\sigma_{BPM}(E) = \frac{\pi}{k^2}\sum_{\ell=0}^{\ell_{cr}}(2\ell+1)\,T_\ell$$

$$\hbar\omega_{\ell eff} = \begin{cases} \hbar\omega_{\ell} & \text{for } \mu \leq 8 \text{ a.m.u.} \\ \hbar\omega_{\ell}\,[1+\lambda(\mu-8)] & \text{for } \mu > 8 \text{ a.m.u.} \end{cases}$$





Extrapolations around E_{cm} = 3.5 MeV !!

Single analytical approximation in all regimes

$$R = R_{\mathrm{pyc}}(T=0) + \Delta R(T)$$

$$R_{\mathrm{pyc}} = \rho\, X_i\, A Z^4\, S(E_{\mathrm{pk}})\, C_{\mathrm{pyc}}\, 10^{46}\, \lambda^{3-C_{\mathrm{pl}}}\, \exp\left(-C_{\mathrm{exp}}/\sqrt{\lambda}\right)$$

$$\Delta R(T) = \frac{n_i^2}{2}\, S(E_{\mathrm{pk}})\, \frac{\hbar}{m Z^2 e^2}\, P\, F$$

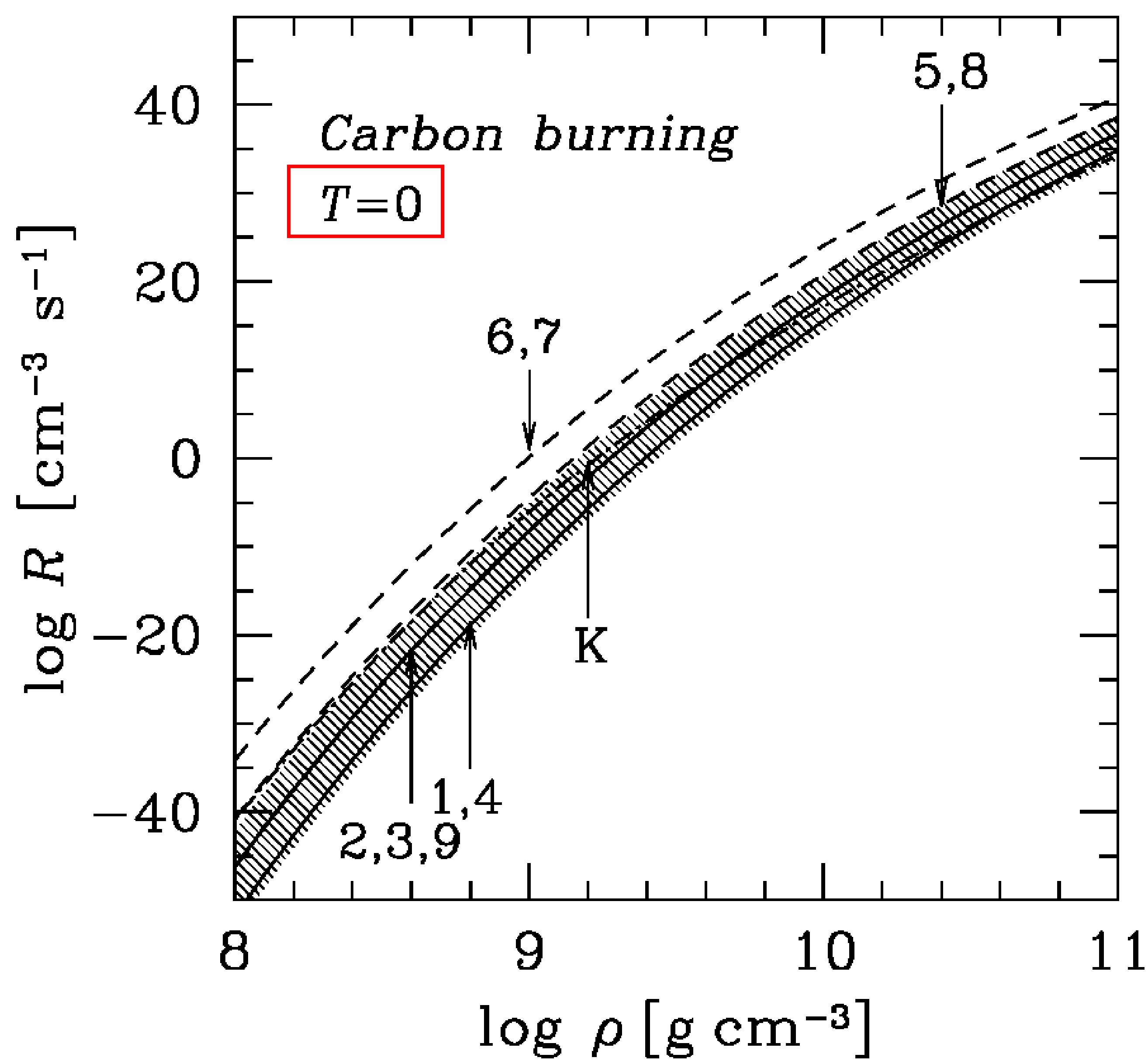
$$F = \exp\left(-\tilde{\tau} + C_{\mathrm{sc}}\tilde{\Gamma}\,\varphi\, \mathrm{e}^{-\Lambda T_p/T} - \Lambda\frac{T_p}{T}\right)$$

$$P = \frac{8\,\pi^{1/3}}{\sqrt{3}\,2^{1/3}}\left(\frac{E_{\mathrm{a}}}{\tilde{T}}\right)^{\gamma}$$

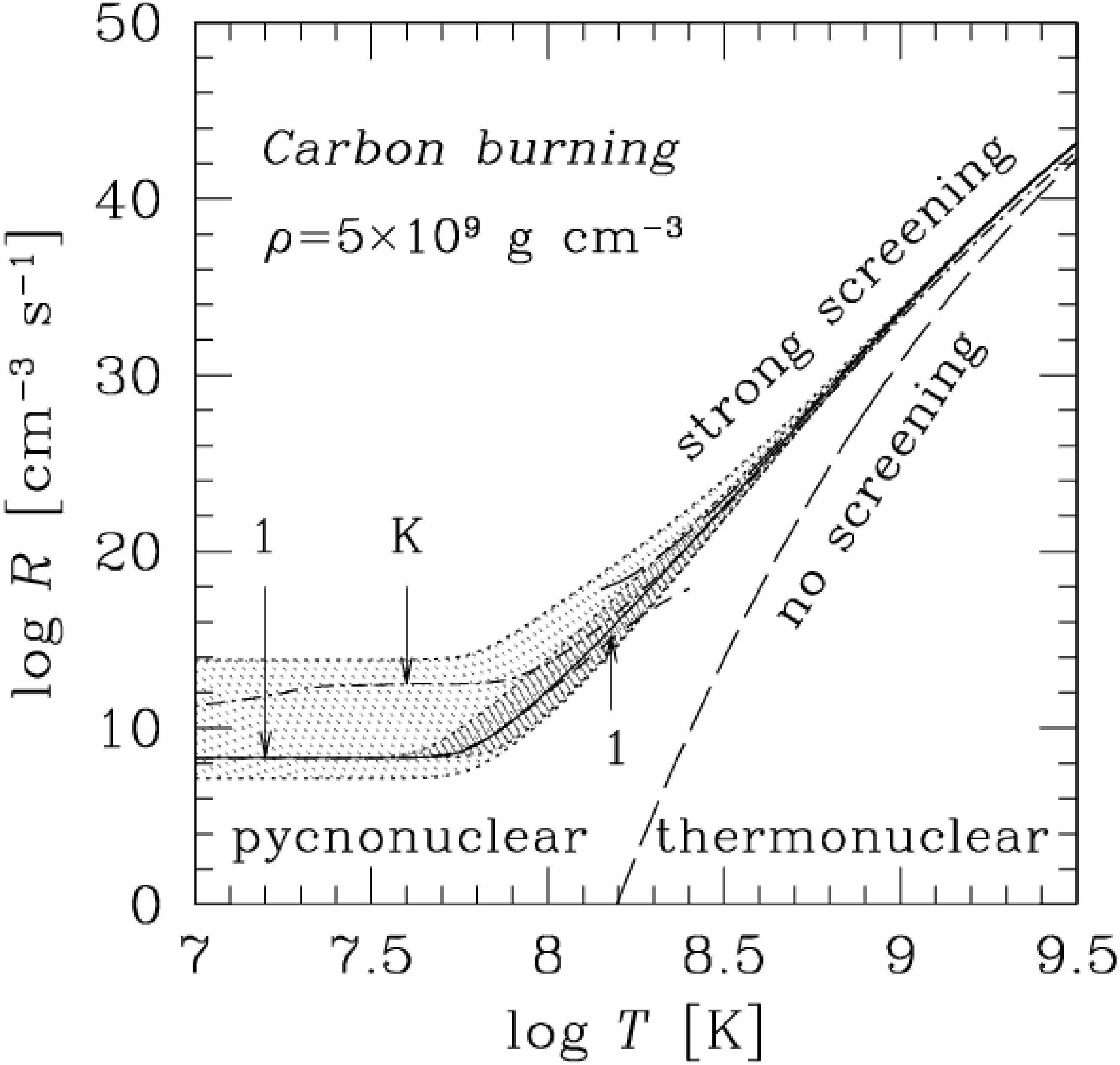
Table 1: The table presents the coefficients $C_{\rm exp}$, $C_{\rm pyc}$ and $C_{\rm pl}$ of the pycnonuclear reaction rate. The dimensionless parameter C_T, which is related to the "renormalized" temperature, is also included in the table.

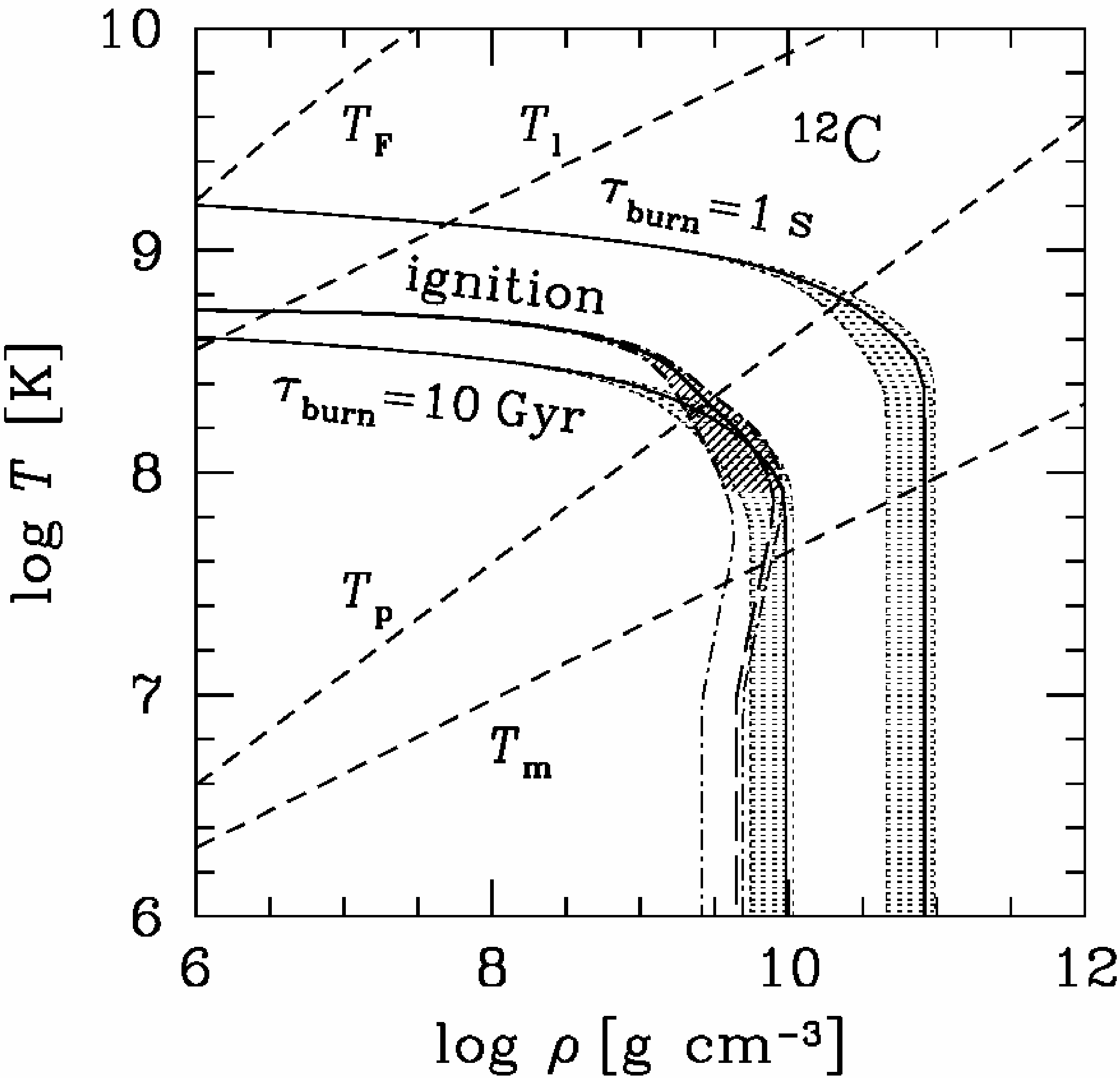
No.	$C_{\rm exp}$	$C_{\rm pyc}$	$C_{\rm pl}$	C_T	Model	Refs.
1	2.638	3.90	1.25	0.724	bcc; static lattice	SVH
2	2.516	4.76	1.25	0.834	bcc; relaxed lattice – WS	SVH
3	2.517	4.58[a)]	1.25	0.834	bcc; relaxed lattice	SK
4	2.659	5.13[a)]	1.25	0.707	bcc; effective mass approx.	SK
5	2.401	7.43[a)]	1.25	0.960	fcc; static lattice	SK
6	2.265	13.5[a)]	1.25	1.144	fcc; relaxed lattice – WS	SK
7	2.260	12.6[a)]	1.25	1.151	fcc; relaxed lattice	SK
8	2.407	13.7[a)]	1.25	0.953	fcc; effective mass approx.	SK
9	2.460	0.00181	1.809	0.893	bcc; MC calculations	SK
10	2.450	50	1.25	0.904	maximum rate	present paper
11	2.650	0.5	1.25	0.711	minimum rate	present paper

[a)] Corrected for the curvature factor as explained in the text.



Carbon burning
T=0
5,8
6,7
K
1,4
2,3,9
40
20
0
−20
−40
8
9
10
11
log R [cm−3 s−1]
log ρ [g cm−3]


Carbon burning
ρ=5×10⁹ g cm⁻³
strong screening
no screening
1
K
1
pycnonuclear
thermonuclear
log R [cm⁻³ s⁻¹]
log T [K]
0
10
20
30
40
50
7
7.5
8
8.5
9
9.5



10
9
8
7
6
6
8
10
12
log T [K]
log ρ [g cm⁻³]
T_F
T_l
¹²C
τ_burn = 1 s
ignition
τ_burn = 10 Gyr
T_p
T_m

Summary

Nuclear Physics:

The Sao Paulo potential seems appropriate to describe the fusion process.

The only parameter (λ) introduced in the BPM model is energy- and -system independent.

Nuclear Astrophysics:

We are proposing a single analytical expression for the fusion rate, which is valid in all regimes. The parameters reflect theoretical uncertanties of the reaction rates.

We show that carbon burning is actually important in a narrow (ρ-T) strip, which is mainly determined by the temperature (T ~ 4 - 15 x 10^8 K) as long as $\rho \leq 10^9$ g cm^{-3}, and by the density (ρ ~ 5 - 50 x 10^9 g cm^{-3}) as long as T ≤ 10^8 K.

An exact calculation should take into account many effects as lattice impurities and imperfections, classical motion of plasma ions, related structure of Coulomb plasma fields, etc.

The next step is extend the treatment presented for one-component-plasma case towards a general formalism for the fusion rate between different isotopes.