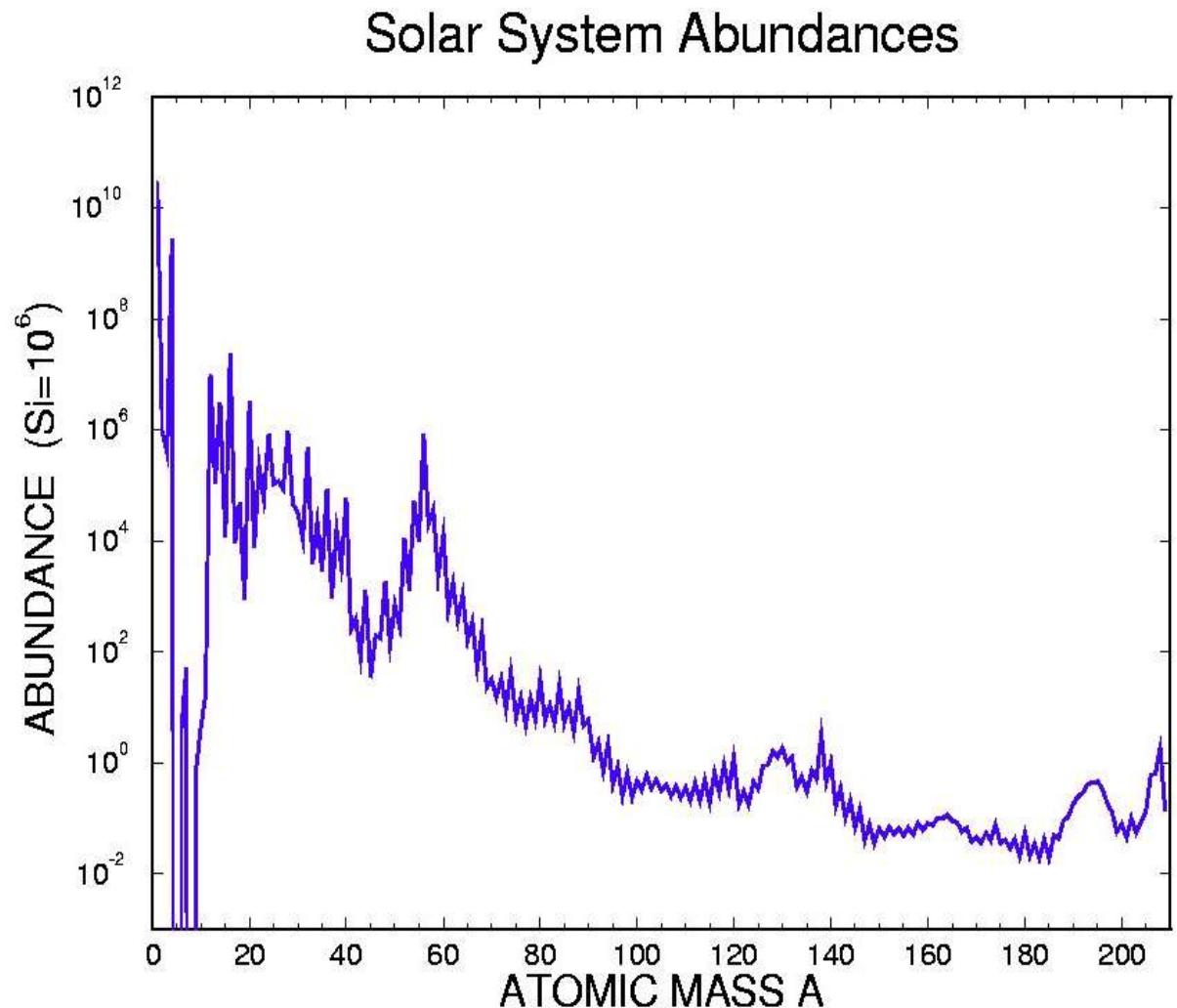
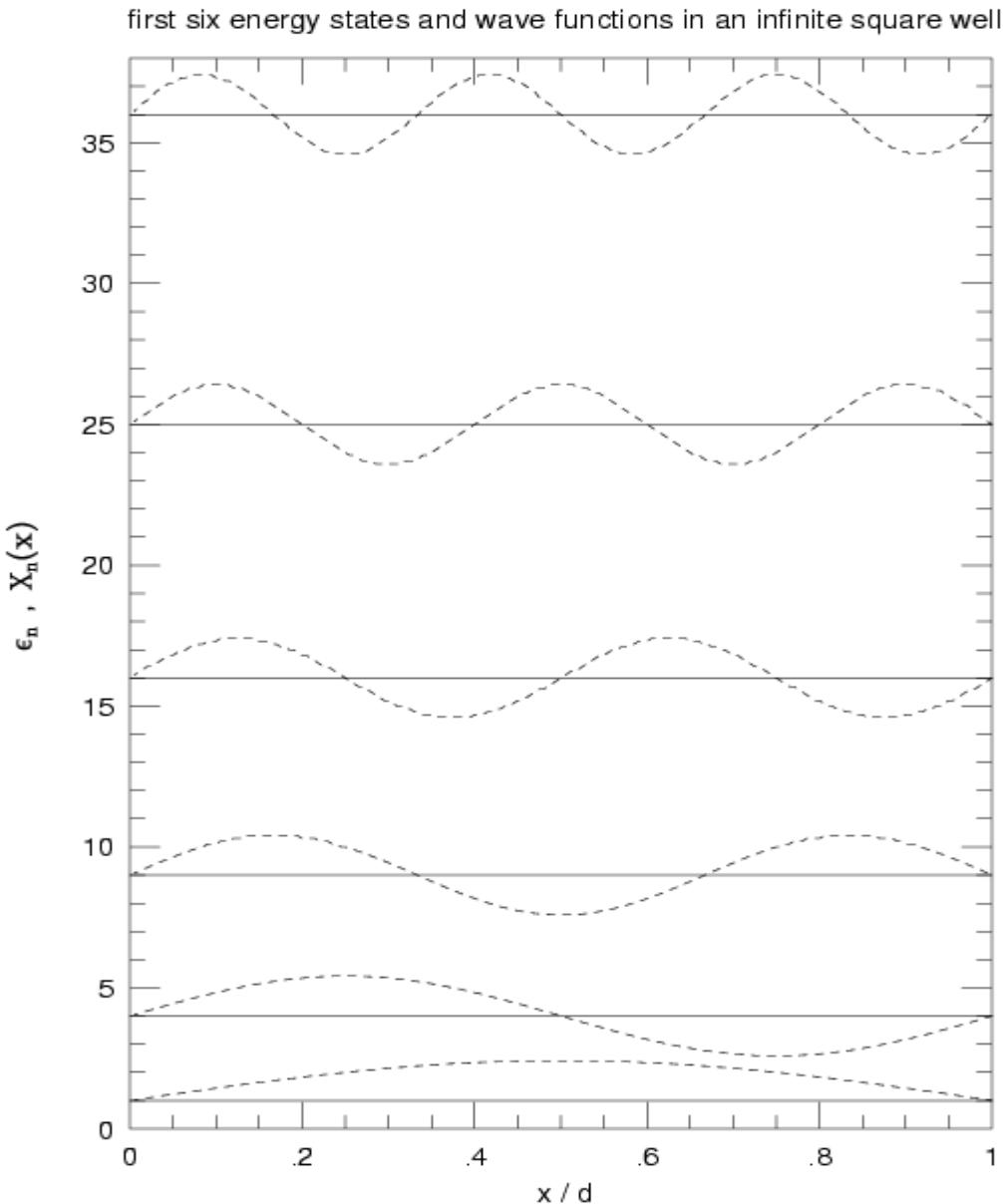


Nuclear Masses and their Relevance for (Nuclear) Burning in Astrophysical Plasmas

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Statistical Mechanics of astrophysical plasmas/gases



a non-interacting gas can be represented by a 3D box in which it is contained (with impenetrable walls)

energy eigenvalues

$$\epsilon_{x,n_x} = \frac{\pi^2 \hbar^2}{2md^2} n_x^2$$

$$\epsilon_{n_x, n_y, n_z} = \frac{\pi^2 \hbar^2}{2md^2} (n_x^2 + n_y^2 + n_z^2)$$

all these states can be occupied!

Total number of states and state density

$$\tilde{\Phi}(E) = g \frac{4\pi}{3} \frac{V}{h^3} (2m)^{3/2} E^{3/2}$$

$$\tilde{\omega}(E) = \frac{d\tilde{\Phi}(E)}{dE} = 2\pi g \frac{V}{h^3} (2m)^{3/2} E^{1/2}$$

total number of states in a given volume $V=d^3$ up to energy E , and state density at that energy

g measures degeneracy of state

$$\Phi(E) = \frac{4\pi}{3} \frac{g}{h^3} (2m)^{3/2} E^{3/2}$$

$$\omega(E) = 2\pi \frac{g}{h^3} (2m)^{3/2} E^{1/2}.$$

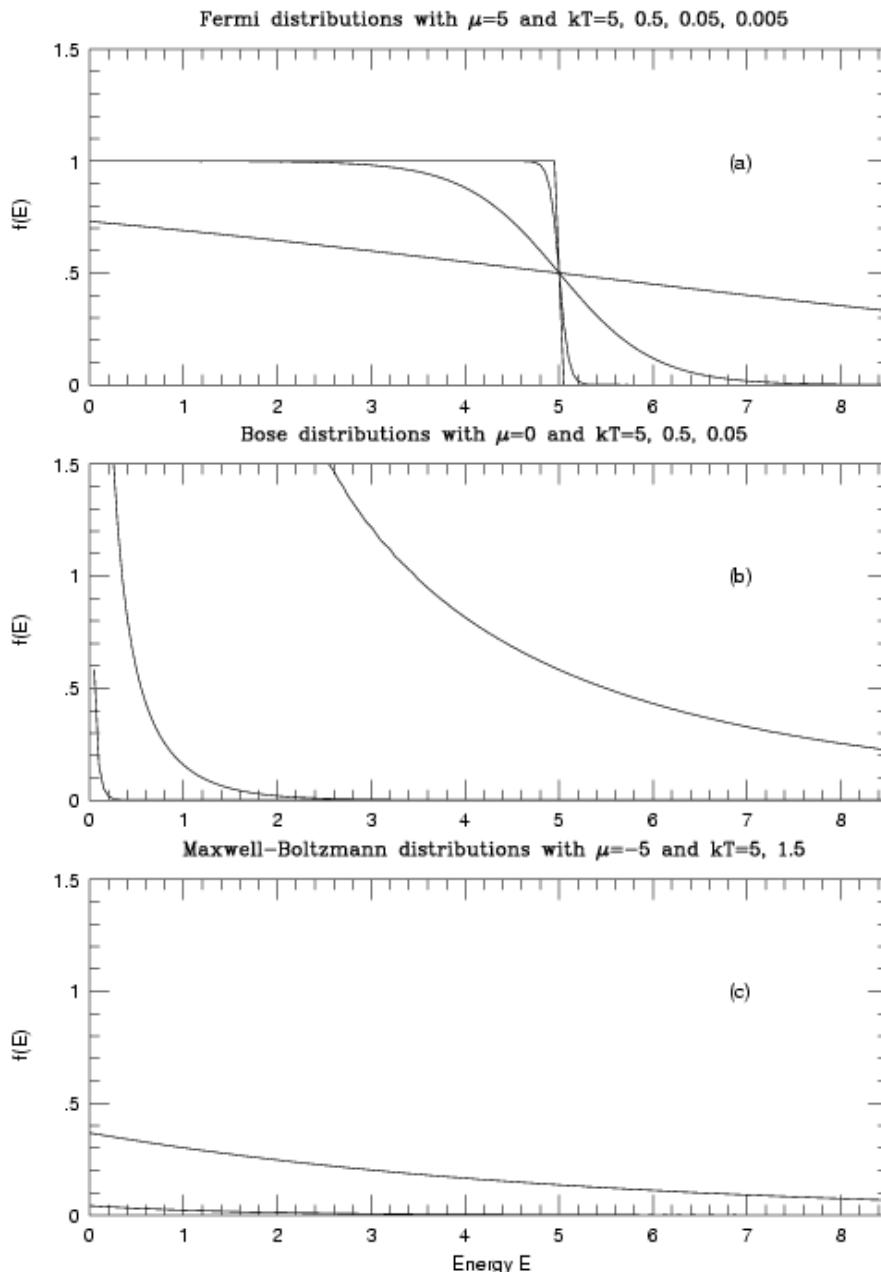
the same per volume

$$\Phi(p) = \frac{4\pi}{3} \frac{g}{h^3} p^3$$

$$\omega(p) = \frac{d\Phi(p)}{dp} = 4\pi \frac{g}{h^3} p^2$$

the same for momentum p with $E=p^2/2m$

Occupation probability for different statistics



$$f(p) = \begin{cases} [e^{(E(p)-\mu)/kT} + 1]^{-1} & \text{Fermions} \\ [e^{(E(p)-\mu)/kT} - 1]^{-1} & \text{Bosons} \\ e^{-(E(p)-\mu)/kT} & \text{Maxwell-Boltzmann} \end{cases}$$

$f(p)$ measures the probability that a state at energy E or momentum p is occupied.

How is the chemical potential determined?

Thermodynamic Properties

$$n = \frac{N}{V} = \int_0^\infty \omega(p) f(p) dp$$

$$u = \frac{U}{V} = \int_0^\infty E \omega(p) f(p) dp$$

$$P = \frac{1}{3} \int_0^\infty p v \omega(p) f(p) dp.$$

if not already known (0 for photons), the chemical potential can be determined from the first equation, as we know the number density n of gas particles.

$$\bar{\mu} = \mu + mc^2 = kT \ln \left(\frac{nh^3}{g} \frac{1}{(2\pi m k T)^{3/2}} \right) + mc^2$$

chemical potential and dn for a Maxwell-Boltzmann gas

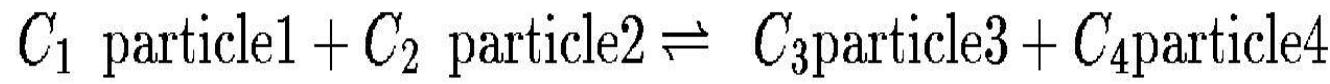
$$dn_j = n_j \frac{4\pi p_j^2}{(2\pi m_j k T)^{3/2}} \exp\left(-\frac{p_j^2}{2m_j k T}\right) dp_j$$

dn for a photon (Planck) gas

$$dn_\gamma = \frac{8\pi}{c^3} \frac{\nu^2 d\nu}{\exp(h\nu/kT) - 1} = \frac{1}{\pi^2 (c\hbar)^3} \frac{E_\gamma^2 dE_\gamma}{\exp(E_\gamma/kT) - 1}$$

Preview on chemical equilibria

a reaction involving particles 1 through 4 (with the C's being integer numbers) is in equilibrium, i.e. the forward and backward reactions occur on timescales shorter than the observing time. Then the following relation holds between the chemical potentials.



$$C_1 \bar{\mu}_1 + C_2 \bar{\mu}_2 = C_3 \bar{\mu}_3 + C_4 \bar{\mu}_4$$

$$\bar{\mu} = \mu + mc^2.$$

The chemical potential obtained from the total number density n provides information on energy/momentum distributions of particles. It is only determined up to a constant. If energy generation due to mass differences in reactions is involved, the above equation is correct, if the rest mass energy is added.

The above equation leads to solutions for the relative concentrations as a function of total (mass) density and temperature.

A sketch on nuclear reactions

$$\sigma = \frac{\text{number of reactions target}^{-1}\text{sec}^{-1}}{\text{flux of incoming projectiles}} = \frac{r/n_i}{n_j v}$$

$$\sigma = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) T_l$$

if one neglects spins of participating particles, the fusion cross section can be determined just by the sum of partial waves with transmission coefficients T_l for angular momentum l

$$\sigma \approx \frac{\pi}{k^2} T_{l=0}$$

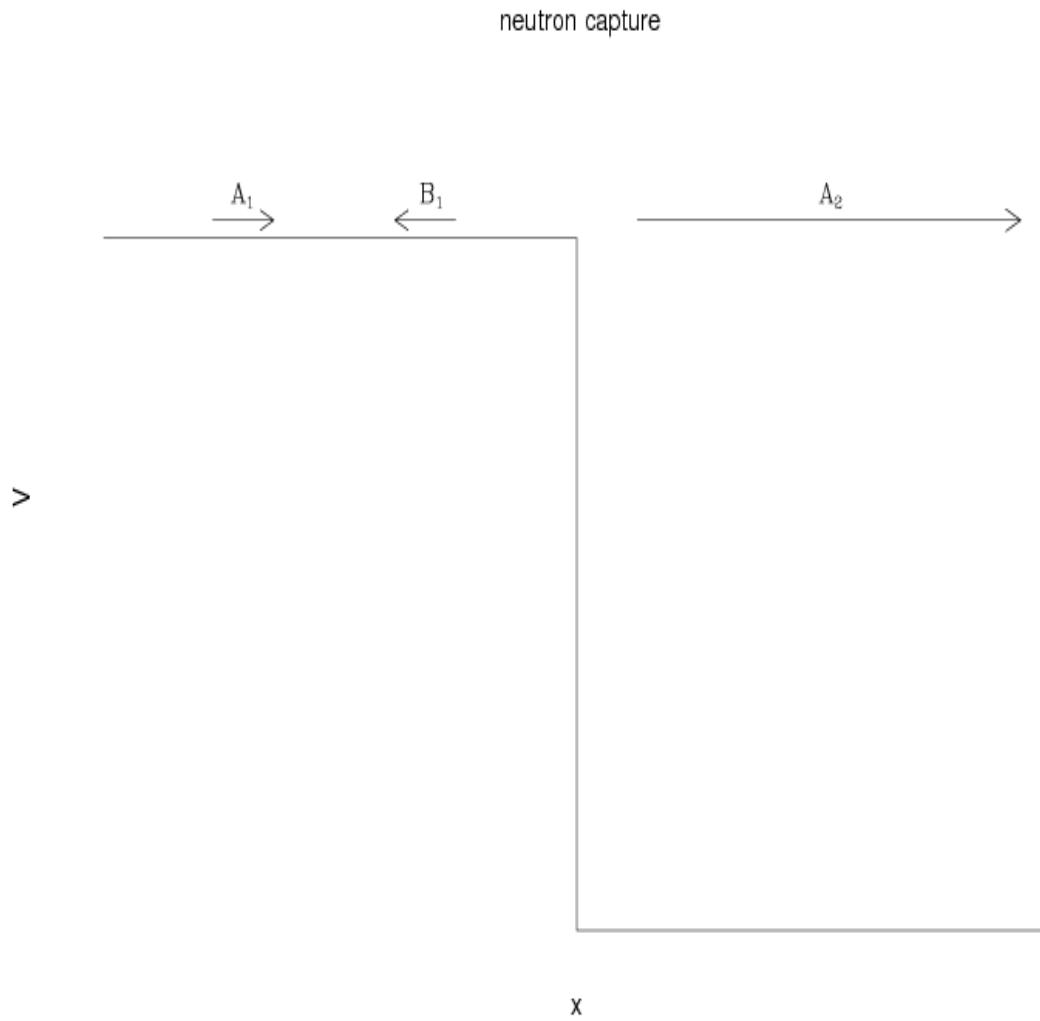
for low energies the fusion cross section is dominated by s-waves ($l=0$)

$$T = \frac{j_{fin}}{j_{in}} = \frac{k_{fin} |\phi_{fin}|^2}{k_{in} |\phi_{in}|^2}$$

transmission coefficient determined by ratio of penetrating to incoming flux.

Reactions with neutrons

“central collision”, $l=0$



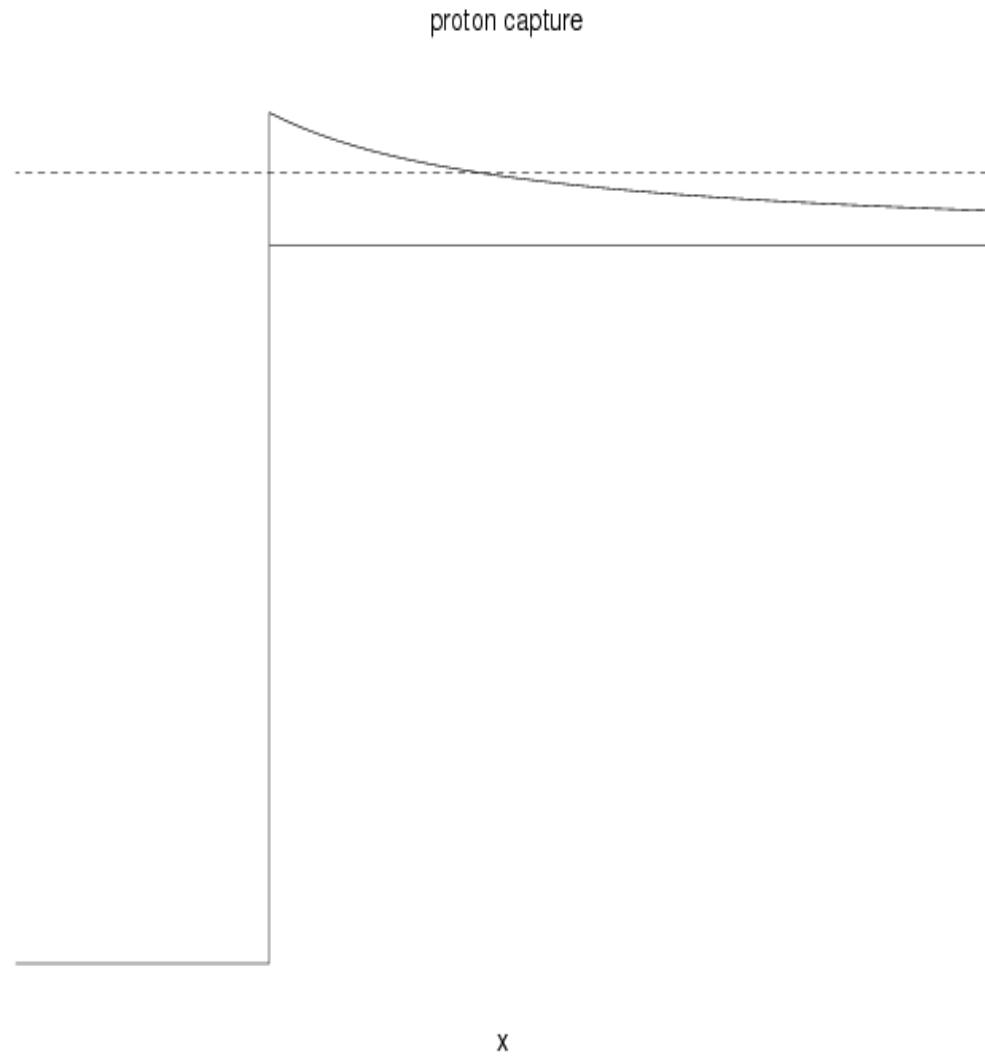
plane wave with momentum k_1 approaches nucleus, partially reflected and partially entering nucleus (inside nucleus momentum k_2).

$$T = \frac{k_2 |A_2|^2}{k_1 |A_1|^2} = \frac{4k_1 k_2}{|k_1 + k_2|^2}.$$

k_2 dominates over k_1 due to potential depth

$$T \approx \frac{4k_1}{k_2}$$

Cross sections with charged particles



$$T \approx e^{-\frac{2}{\hbar} \int_{x_1}^{x_2} \sqrt{2m(V(x)-E)} dx}$$

transmission coefficient
from WKB approximation

>

$$T = e^{-2\pi\eta}$$

$$\eta = \sqrt{\frac{m}{2E}} \frac{Z_1 Z_2 e^2}{\hbar},$$

Sommerfeld parameter.

for Coulomb barrier
penetration

cross sections for neutrons and charged particles

(i) neutrons $T_{n,0} \approx \frac{4k_1}{k_2}$ []

$$k_1 = \frac{\sqrt{2\mu E}}{\hbar} \quad k_2 = \frac{\sqrt{2\mu(E+Q)}}{\hbar} \approx \text{const} \quad \text{for } E \ll Q$$

$$\Rightarrow \sigma = \frac{\pi}{k_1^2} \cdot 4 \frac{k_1}{k_2} \propto \frac{1}{k_1}$$
$$\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

declining as function of
bombarding energy

(ii) charged particle captures $T_{c,0} = e^{-2\pi\eta}$ []

$$\sigma = \frac{\pi}{k^2} e^{-2\pi\eta} = \frac{\hbar^2 \pi}{2\mu E} e^{-2\pi\eta}$$

$$\eta = \sqrt{\frac{\mu}{2E}} \frac{Z_i Z_j e^2}{\hbar}$$

increasing as function of energy
by orders of magnitude due to
Coulomb penetration

Introduction to reaction rates

$$\sigma = \frac{\text{number of reactions target}^{-1}\text{sec}^{-1}}{\text{flux of incoming projectiles}} = \frac{r/n_i}{n_j v} \quad r = \sigma v n_i n_j$$

reaction rate r (per volume and sec) for a fixed bombarding velocity/energy (like in an accelerator)

$$r_{i;j} = \int \sigma \cdot |\vec{v}_i - \vec{v}_j| dn_i dn_j \quad \text{for thermal distributions in a hot plasma}$$

e.g. Maxwell-Boltzmann (nuclei/nucleons) or Planck (photons)

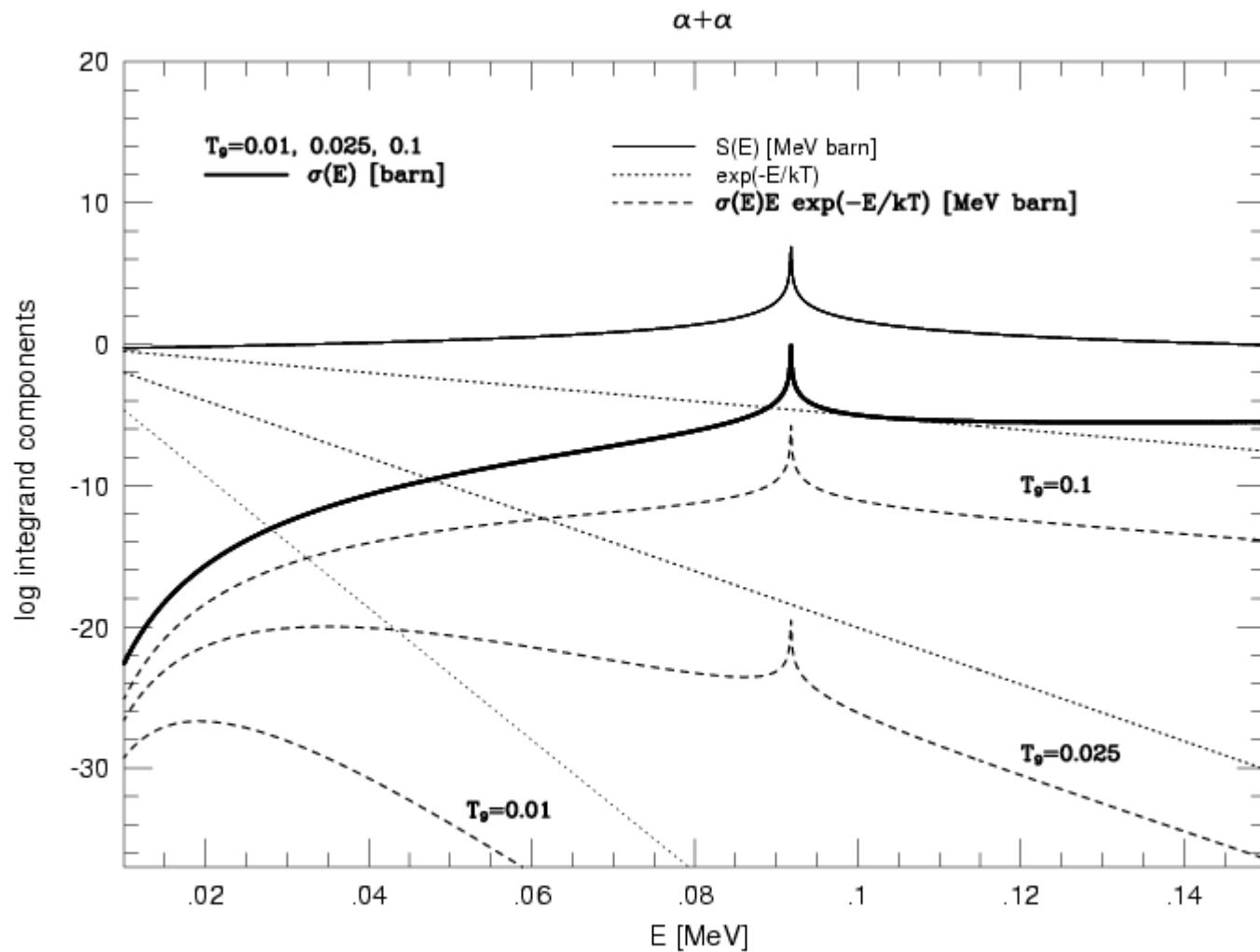
$$dn_j = n_j \frac{4\pi p_j^2}{(2\pi m_j kT)^{3/2}} \exp\left(-\frac{p_j^2}{2m_j kT}\right) dp_j$$

$$dn_\gamma = \frac{8\pi}{c^3} \frac{\nu^2 d\nu}{\exp(h\nu/kT) - 1} = \frac{1}{\pi^2 (c\hbar)^3} \frac{E_\gamma^2 dE_\gamma}{\exp(E_\gamma/kT) - 1}$$

for two MB-distributions for i and j one obtains after variable transformations

$$r_{i;j} = n_i n_j \langle \sigma v \rangle_{i;j} \quad \langle \sigma v \rangle(T) = \left(\frac{8}{\mu\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) \exp(-E/kT) dE$$

Temperature dependence of rates



for neutron captures close to constant (at higher temperatures, i.e. higher velocities, multiplied with $1/v$ dependence of cross section)

for charged particles the contribution to the integral is strongly rising with temperature

Reaction networks

reaction $i+j \rightarrow m+o$ $i(j,o)m$ with reaction rate

$$r_{i;j} = \frac{1}{1 + \delta_{ij}} n_i n_j \langle \sigma v \rangle$$

(avoiding double counting for reactions of identical particles)

resulting changes in number densities of participating nuclei (for constant mass densities!)

$$\left(\frac{\partial n_i}{\partial t} \right)_\rho = \left(\frac{\partial n_j}{\partial t} \right)_\rho = -r_{i;j}$$

$$\left(\frac{\partial n_o}{\partial t} \right)_\rho = \left(\frac{\partial n_m}{\partial t} \right)_\rho = +r_{i;j}$$

Introducing abundances Y and mass fractions X

$$Y_i = \frac{n_i}{\rho N_A}$$

$$\rho = \frac{1}{V} = \sum_i n_i m_i = \sum_i \frac{n_i}{N_A} m_i N_A$$

$$1 = \frac{\rho}{\rho} = \sum_i \frac{n_i}{\rho N_A} m_i N_A = \sum_i Y_i A_i = \sum_i X_i$$

Reaction networks

i(j,o)m

decay i->m

$$\dot{Y}_i = \frac{1}{\rho N_A} \left(\frac{\partial n_i}{\partial t} \right)_\rho = -\frac{r_{i;j}}{\rho N_A} = -\frac{1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_j = \frac{-1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_o = \frac{1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_m = \frac{1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j.$$

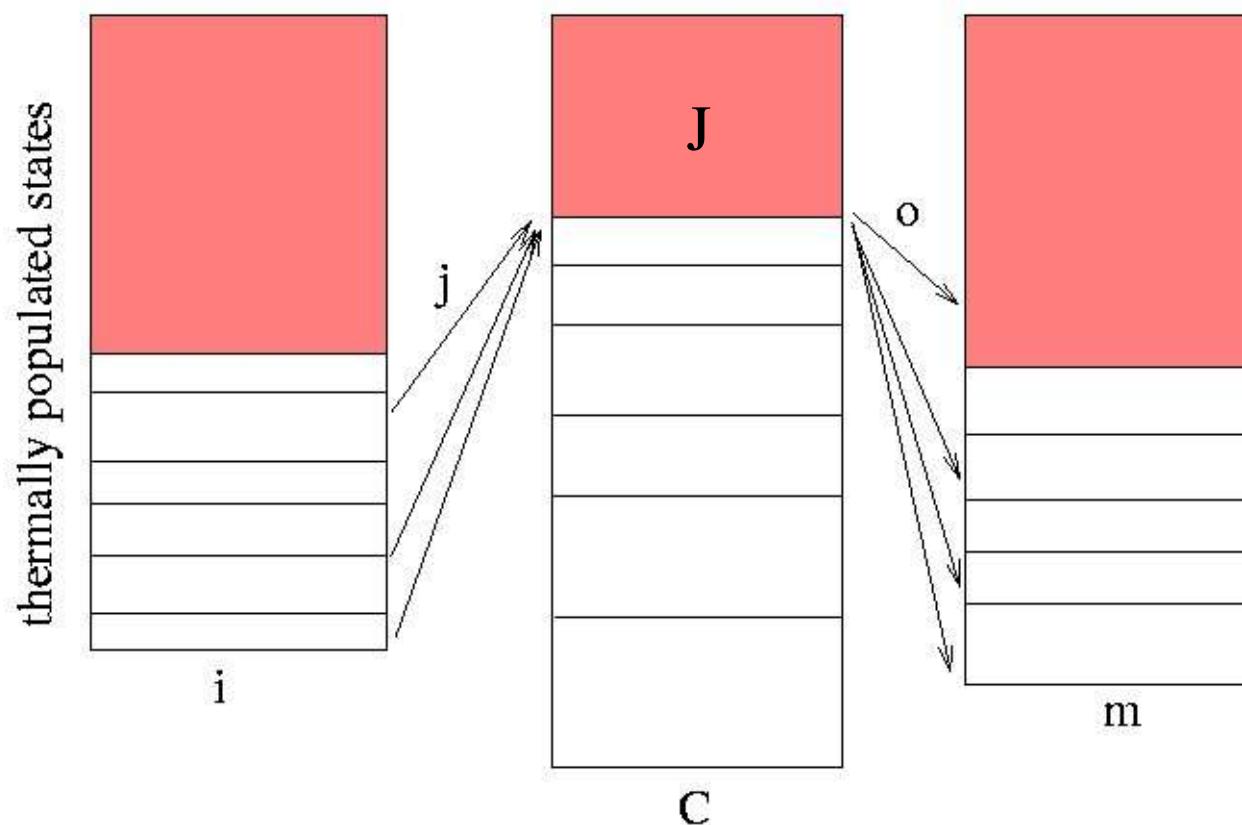
$$\dot{Y}_i = \left(\frac{\dot{n}_i}{\rho N_A} \right)_\rho = -\frac{r_i}{\rho N_A}$$

$$\Rightarrow \dot{Y}_i = -\lambda_i Y_i \quad \dot{Y}_m = \lambda_i Y_i$$

general: N's count number of particles produced/distroyed in the reaction (positive/negative)

$$\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} \frac{N_{j,k}^i}{1 + \delta_{jk}} \rho N_A \langle \sigma v \rangle_{j;k} Y_j Y_k.$$

General compound cross section



$$\sigma_i(j, o) = \frac{\pi}{k_j^2} \frac{(1 + \delta_{ij})}{(2I_i + 1)(2I_j + 1)} \sum_{J,\pi} (2J+1) \frac{T_j(E, J, \pi) T_o(E, J, \pi)}{T_{tot}(E, J, \pi)}$$

including spin and parity dependence

www.nuastro.org
for statist. model
cross sections

Reverse rates

$$\sigma_m(o, j)_J = \frac{\pi}{k_o^2} \frac{(1 + \delta_{om})(2J + 1)}{(2I_m + 1)(2I_o + 1)} \frac{T_o T_j}{T_{tot}}$$

$$\sigma_i(j, o)_J = \frac{\pi}{k_j^2} \frac{(1 + \delta_{ij})(2J + 1)}{(2I_i + 1)(2I_j + 1)} \frac{T_j T_o}{T_{tot}}$$

$$k_o = \frac{p_o}{\hbar} = \frac{\sqrt{2\mu_{om}E_{om}}}{\hbar} \quad k_j = \frac{p_j}{\hbar} = \frac{\sqrt{2\mu_{ij}E_{ij}}}{\hbar}$$

$$g_x = (2I_x + 1) \quad E_{ij} = E_{om} + Q_{o,j}.$$

$$\frac{\sigma_i(j, o)_J}{\sigma_m(o, j)_J} = \frac{1 + \delta_{ij}}{1 + \delta_{om}} \frac{g_o g_m}{g_i g_j} \frac{k_o^2}{k_j^2}$$

going through a
specific state J in the
compound nucleus

but true for any state at that
energy

$$\sigma_i(j, o; E_{ij}) = \frac{1 + \delta_{ij}}{1 + \delta_{om}} \frac{g_o g_m}{g_i g_j} \frac{k_o^2}{k_j^2} \sigma_m(o, j; E_{om})$$

Reverse rates

$$\langle \sigma v \rangle_{i;j,o} = \frac{1 + \delta_{ij}}{1 + \delta_{om}} \left(\frac{8}{\mu_{ij}\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E_{ij} \frac{g_o g_m}{g_i g_j} \frac{k_o^2}{k_j^2} \sigma_m(o, j; E_{om})$$

$$\times \exp(-E_{ij}/kT) dE_{ij}$$

$$= \frac{1 + \delta_{ij}}{1 + \delta_{om}} \frac{g_o g_m}{g_i g_j} \left(\frac{\mu_{om}}{\mu_{ij}} \right)^{3/2} \exp(-Q_{o,j}/kT)$$

$$\times \left(\frac{8}{\mu_{om}\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E_{om} \sigma_m(o, j; E_{om}) \exp(-E_{om}/kT) dE_{om}$$

$$\langle \sigma v \rangle_{i;j,o} = \frac{1 + \delta_{ij}}{1 + \delta_{om}} \frac{G_m g_o}{G_i g_j} \left(\frac{\mu_{om}}{\mu_{ij}} \right)^{3/2} \exp(-Q_{o,j}/kT) \langle \sigma v \rangle_{m;o,j} \boxed{\quad}$$

containing the Q-value of the reaction (nuclear mass differences)

Reverse photodisintegrations

$$r_{i\gamma} = n_i \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{\sigma_i(\gamma, o; E_\gamma) E_\gamma^2}{\exp(E_\gamma/kT) - 1} dE_\gamma \\ = n_i \lambda_{i;\gamma,o}(T)$$

$$\lambda_{i;\gamma,o}(T) = \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{\sigma_i(\gamma, o; E_\gamma) E_\gamma^2}{\exp(E_\gamma/kT) - 1} dE_\gamma.$$

photodisintegration rates only T-dependent!

$$k_\gamma = \frac{w}{c} = \frac{\hbar w}{\hbar c} = \frac{E_\gamma}{\hbar c} \quad g_\gamma = 2$$

$$k_o = \frac{p}{\hbar} = \frac{\sqrt{2\mu_{om}E_{om}}}{\hbar} \quad E_\gamma = E_{om} + Q_{o,\gamma}$$

$$\sigma_i(\gamma, o; E_\gamma) = \frac{g_o g_m}{(1 + \delta_{om}) g_i} c^2 \frac{\mu_{om} E_{om}}{E_\gamma^2} \sigma_m(o, \gamma; E_{om})$$

$$\lambda_{i;\gamma,o} = \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{g_o g_m}{(1 + \delta_{om}) g_i} c^2 \frac{\mu_{om} E_{om}}{E_\gamma^2} \sigma_m(o, \gamma; E_{om}) E_\gamma^2 \\ \times \exp(-E_\gamma/kT) dE_\gamma \\ = \frac{1}{\pi^2 \hbar^3} \frac{g_o g_m}{g_i} \mu_{om} \exp(-Q_{o,\gamma}/kT)$$

$$\times \int_0^\infty E_{om} \sigma_m(o, \gamma; E_{om}) \exp(-E_{om}/kT) dE_{om}$$

$$\lambda_{i;\gamma,o}(T) = \frac{g_o G_m}{(1 + \delta_{om}) G_i} \left(\frac{\mu_{om} kT}{2\pi \hbar^2} \right)^{3/2} \exp(-Q_{o,\gamma}/kT) \langle \sigma v \rangle_{m;o} \boxed{\quad}$$

relation between photodisintegration rate and reverse capture rate

Reaction equilibria

reaction network for i(j,o)m

$$\dot{Y}_i = \dot{Y}_j = -\rho N_A \langle \sigma v \rangle_{i;j,o} Y_i Y_j + \rho N_A \langle \sigma v \rangle_{m;o,j} Y_o Y_m$$

$$\dot{Y}_m = \dot{Y}_o = -\dot{Y}_i$$

$$\dot{Y}_i = \dot{Y}_j = -\rho N_A \langle \sigma v \rangle_{i;j,\gamma} Y_i Y_j + \lambda_{m;\gamma,j} Y_m$$

$$\dot{Y}_m = -\dot{Y}_i \quad \text{in this case o is a photon}$$

if forward and backward reaction are in equilibrium, we have for all indices

$$\dot{Y} = 0$$

this leads to the following abundance relations

$$\begin{aligned} \frac{Y_m}{Y_i} &= \frac{Y_j}{Y_o} \frac{\langle \sigma v \rangle_{i;j,o}}{\langle \sigma v \rangle_{m;o,j}} \\ &= \frac{Y_j}{Y_o} \frac{g_o G_m}{g_j G_i} \left(\frac{m_o m_m}{m_i m_j} \right)^{3/2} \exp(Q_{j,o}/kT) \end{aligned}$$

$$\begin{aligned} \frac{Y_m}{Y_i} &= \frac{\rho N_A \langle \sigma v \rangle_{i;j,\gamma}}{\lambda_{m;\gamma,j}} Y_j \\ &= \rho N_A Y_j \frac{G_m}{g_j G_i} \left(\frac{m_m}{m_i m_j} \right)^{3/2} \left(\frac{2\pi\hbar^2}{kT} \right)^{3/2} \exp(Q_{j,\gamma}/kT) \end{aligned}$$

The same results would have been obtained, if the equations for chemical equilibria would have been utilized which include the chemical potentials!!

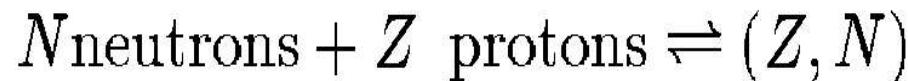
Nuclear Statistical Equilibrium (NSE)

$$\bar{\mu}(Z, N) + \bar{\mu}_n = \bar{\mu}(Z, N + 1)$$

$$\bar{\mu}(Z, N) + \bar{\mu}_p = \bar{\mu}(Z + 1, N)$$

i.e. neutron or proton captures on nucleus (Z, N) are in chemical equilibrium with the reverse photodisintegrations.

If this is the case for all neutron and proton captures on all nuclei (hot enough to overcome all Coulomb barriers as well as having high energy photons...) this leads to



$$N\bar{\mu}_n + Z\bar{\mu}_p = \bar{\mu}_{Z,N}.$$

with

$$\bar{\mu}_i = kT \ln \left(\frac{\rho N_A Y_i}{G_i} \left(\frac{2\pi\hbar^2}{m_i kT} \right)^{3/2} \right) + m_i c^2$$

Solving NSE

$$\begin{aligned}
& kT \ln \left(\frac{\rho N_A Y(Z, N)}{G_{Z,N}} \left(\frac{2\pi\hbar^2}{m_{Z,N}kT} \right)^{3/2} \right) + m_{Z,N}c^2 \\
&= N \left[kT \ln \left(\frac{\rho N_A Y_n}{g_n} \left(\frac{2\pi\hbar^2}{m_n kT} \right)^{3/2} \right) + m_n c^2 \right] \\
&+ Z \left[kT \ln \left(\frac{\rho N_A Y_p}{g_p} \left(\frac{2\pi\hbar^2}{m_p kT} \right)^{3/2} \right) + m_p c^2 \right]
\end{aligned}$$

$$\begin{aligned}
& \times \ln \left(\frac{\rho N_A Y(Z, N)}{G_{Z,N}} \left(\frac{2\pi\hbar^2}{m_{Z,N}kT} \right)^{3/2} \right) \\
& - N \ln \left(\frac{\rho N_A Y_n}{g_n} \left(\frac{2\pi\hbar^2}{m_n kT} \right)^{3/2} \right) - Z \ln \left(\frac{\rho N_A Y_p}{g_p} \left(\frac{2\pi\hbar^2}{m_p kT} \right)^{3/2} \right) \\
& = \frac{1}{kT} (N m_n c^2 + Z m_p c^2 - m_{Z,N} c^2) = B_{Z,N}/kT.
\end{aligned}$$

Solving NSE

with $A = N + Z$ $m_n \approx m_u$ $m_p \approx m_u$ $m_{Z,N} \approx A m_u$

this leads to

$$Y(Z, N) = G_{Z,N} (\rho N_A)^{A-1} \frac{A^{3/2}}{2^A} \left(\frac{2\pi\hbar^2}{m_u kT} \right)^{\frac{3}{2}(A-1)} \exp(B_{Z,N}/kT) Y_n^N Y_p^Z$$

and can be solved via two equations
(mass conservation and total proton
to nucleon ratio Y_e) for neutron and
proton abundances

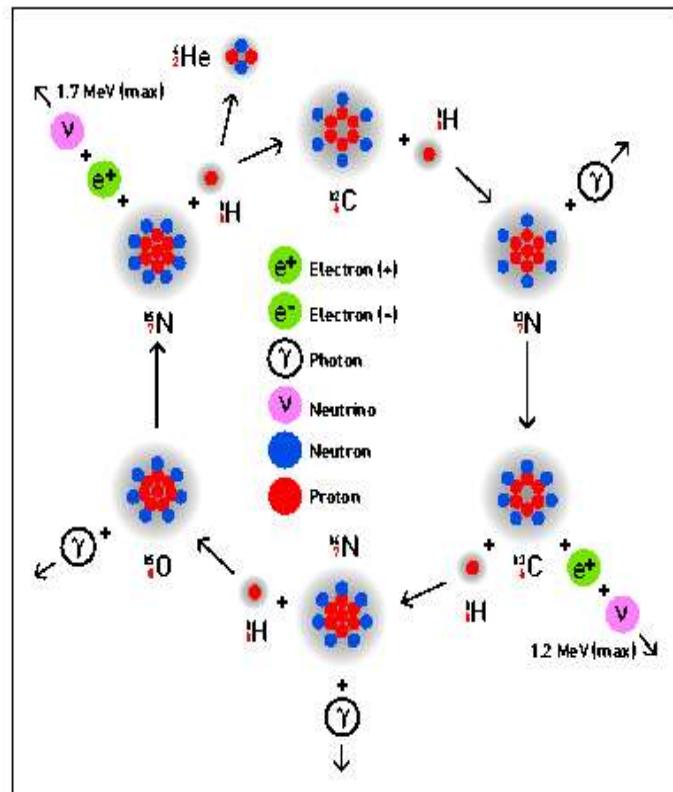
$$\sum_i A_i Y_i = 1$$

$$\sum_i Z_i Y_i = Y_e$$

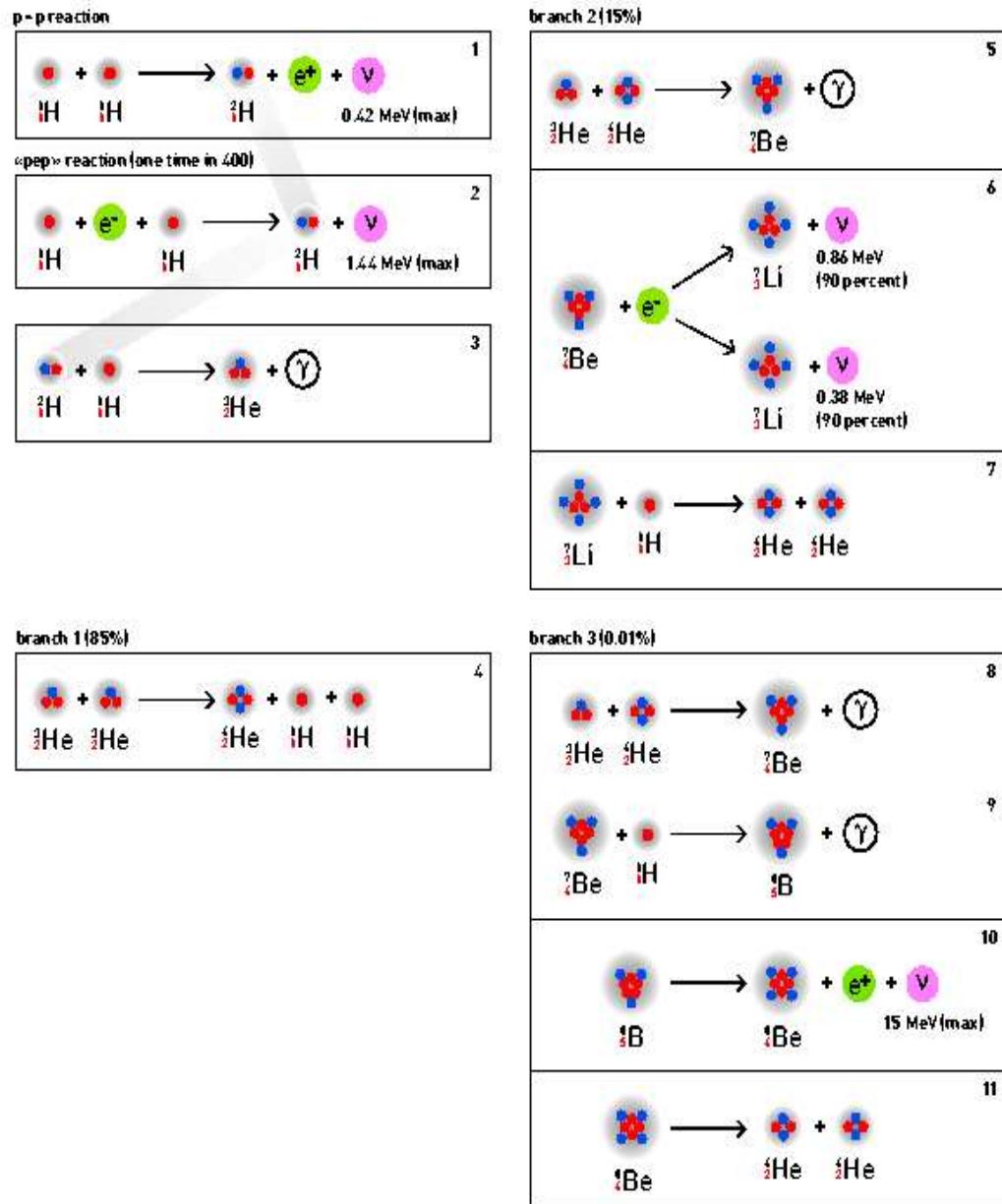
Stellar Burning Stages

Hydrogen Burning

The CNO Cycle



P-P Cycles



Stellar Burning Stages

- | | |
|--|---|
| 1. Hydrogen Burning | $T = (1-4) \times 10^7 K$ |
| pp-cycles | ${}^1H(p, e^+ \nu) {}^2H$ |
| CNO-cycle | ${}^{14}N(p, \gamma) {}^{15}O$ |
| 2. Helium Burning | $T = (1-2) \times 10^8 K$ |
| ${}^4He + {}^4He \rightleftharpoons {}^8Be$ | ${}^8Be(\alpha, \gamma) {}^{12}C [(\alpha, \gamma) {}^{16}O]$ |
| ${}^{14}N(\alpha, \gamma) {}^{18}F(\beta^+) {}^{18}O(\alpha, \gamma) {}^{22}Ne(\alpha, n) {}^{25}Mg$ | |
| 3. Carbon Burning | $T = (6-8) \times 10^8 K$ |
| ${}^{12}C({}^{12}C, \alpha) {}^{20}Ne$ | ${}^{23}Na(p, \alpha) {}^{20}Ne$ |
| ${}^{12}C({}^{12}C, p) {}^{23}Na$ | ${}^{23}Na(p, \gamma) {}^{24}Mg$ |

Stellar Burning Stages

4. Neon Burning

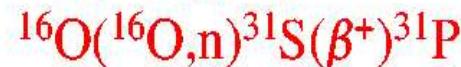
$T = (1.2-1.4) \times 10^9 K$



$$30kT = 4 \text{ MeV}$$

5. Oxygen Burning

$T = (1.5-2.2) \times 10^9 K$

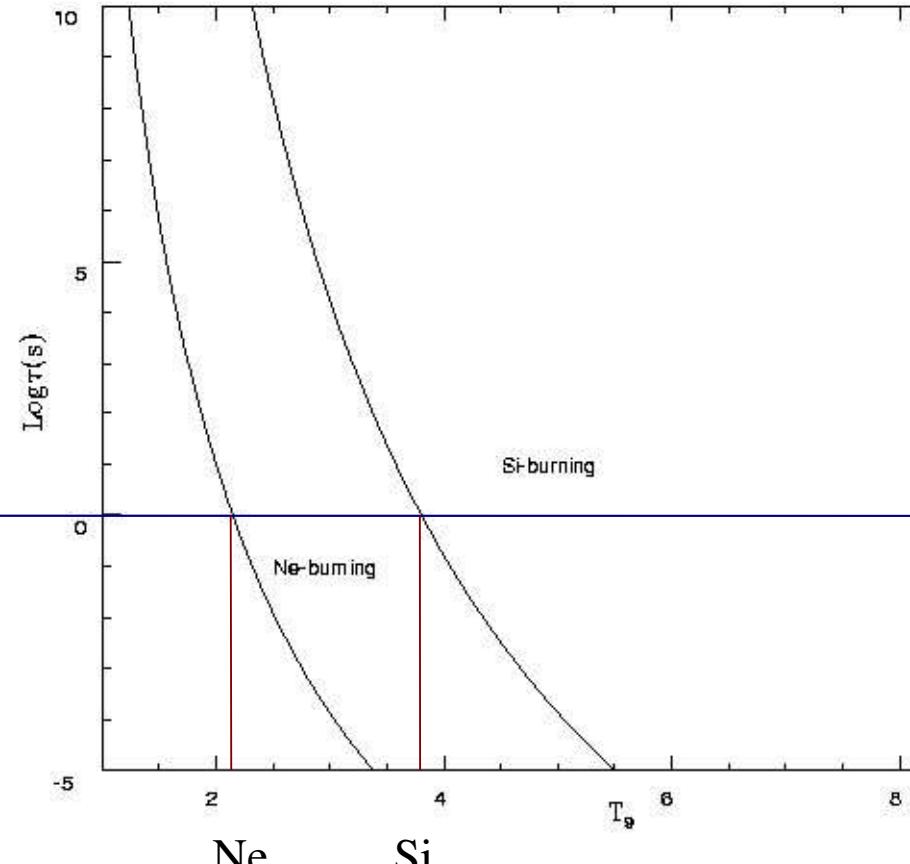
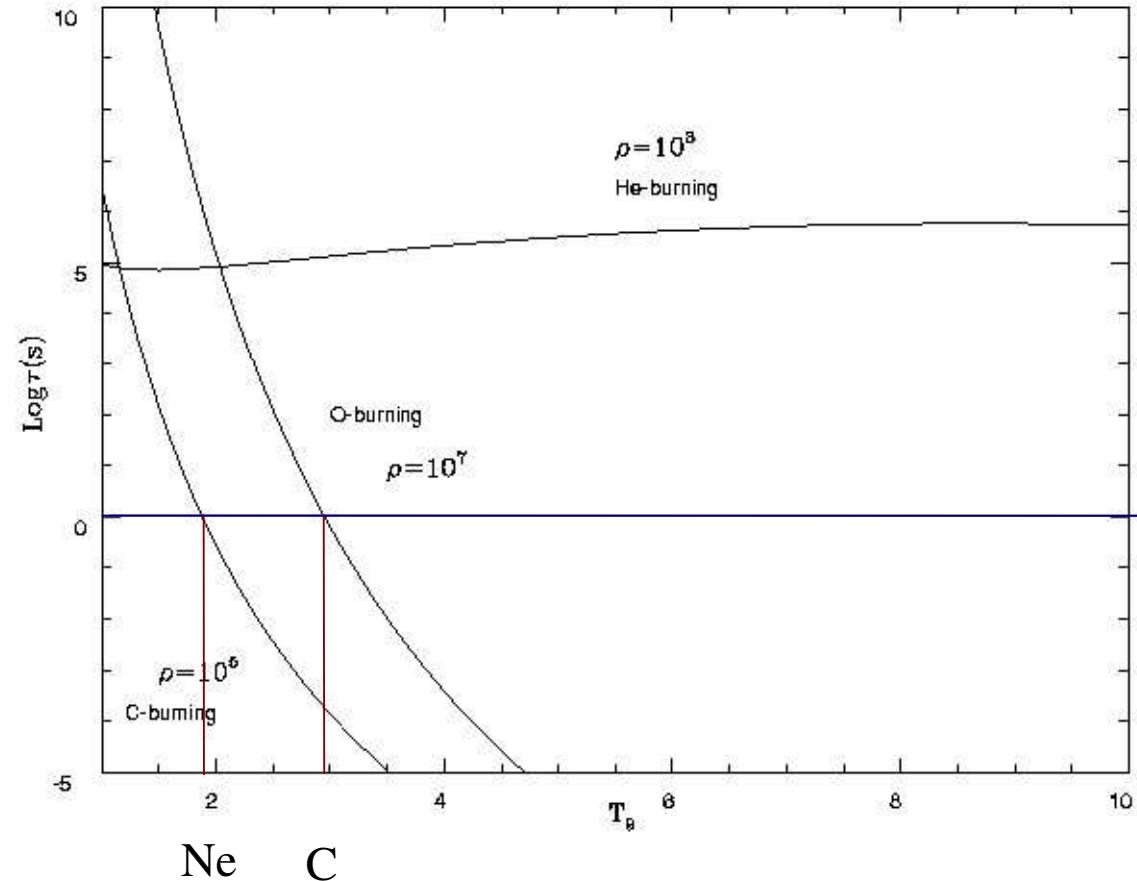


6. “Silicon” Burning

$T = (3-4) \times 10^9 K$

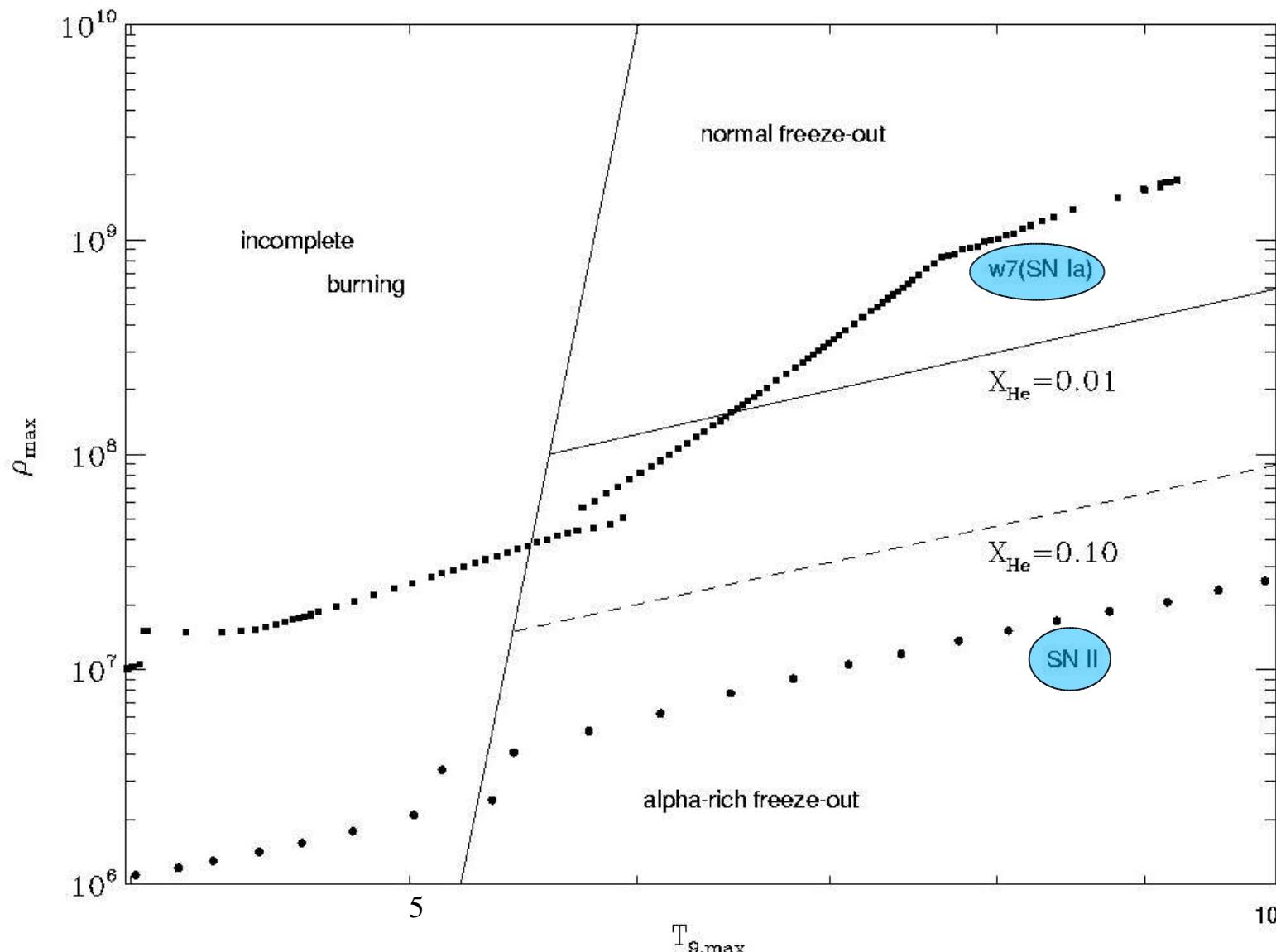
(all) photodisintegrations and capture reactions possible
⇒ thermal (chemical) equilibrium

Explosive Burning



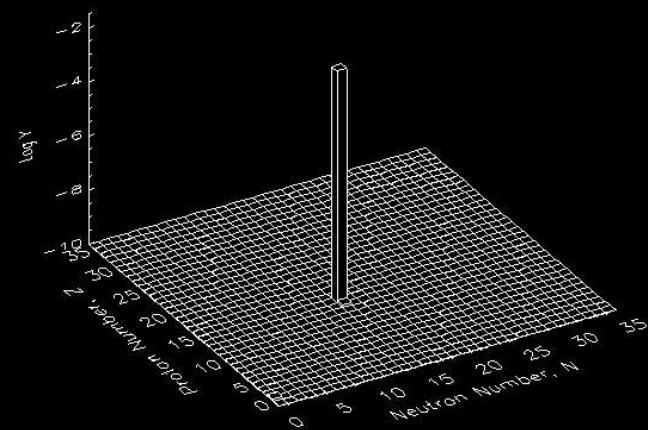
typical explosive burning process timescale order of seconds: fusion reactions (He, C, O) density dependent (He quadratic, C,O linear) photodisintegrations (Ne, Si) not density dependent

Explosive Si-Burning



Explosive Burning above a critical temperature destroys (photodisintegrates) all nuclei and (re-)builds them up during the expansion. Dependent on density, the full NSE is maintained and leads to only Fe-group nuclei (normal freeze-out) or the reactions linking ${}^4\text{He}$ to C and beyond freeze out earlier (alpha-rich freeze-out).

t (s) = 4.78200e-15 T_9 = 5.50 ρ (g/cc) = 1.00000e+07

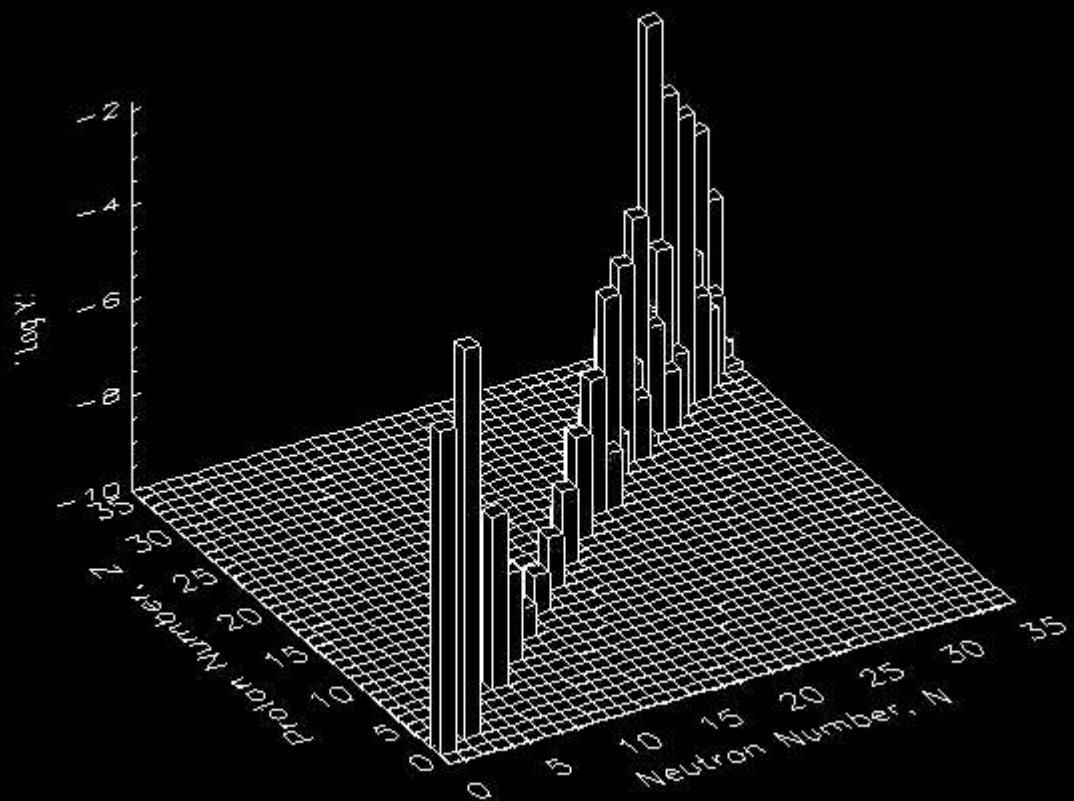
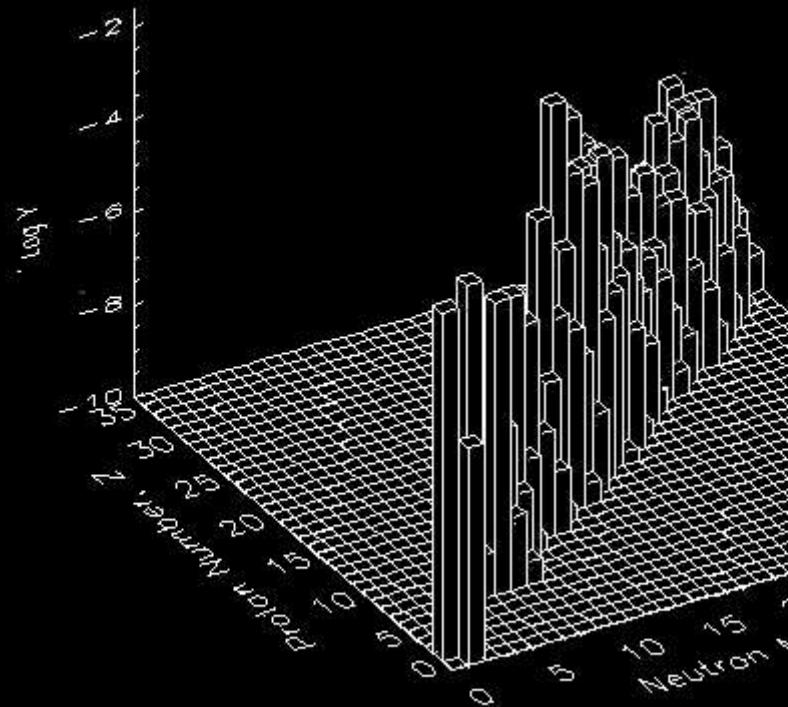


Explosive Si-burning

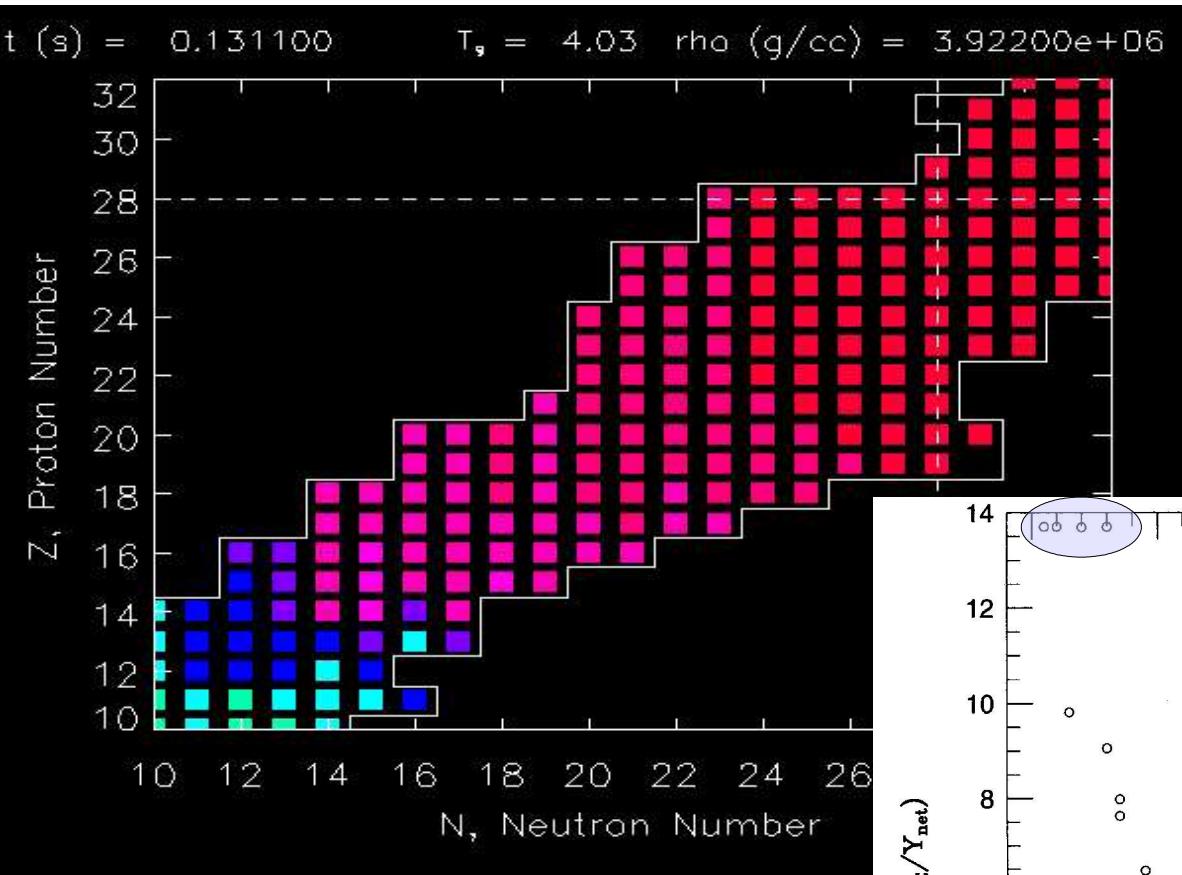
initially only ^{28}Si , fully burned, finally alpha-rich freeze-out

visualization: B.S. Meyer

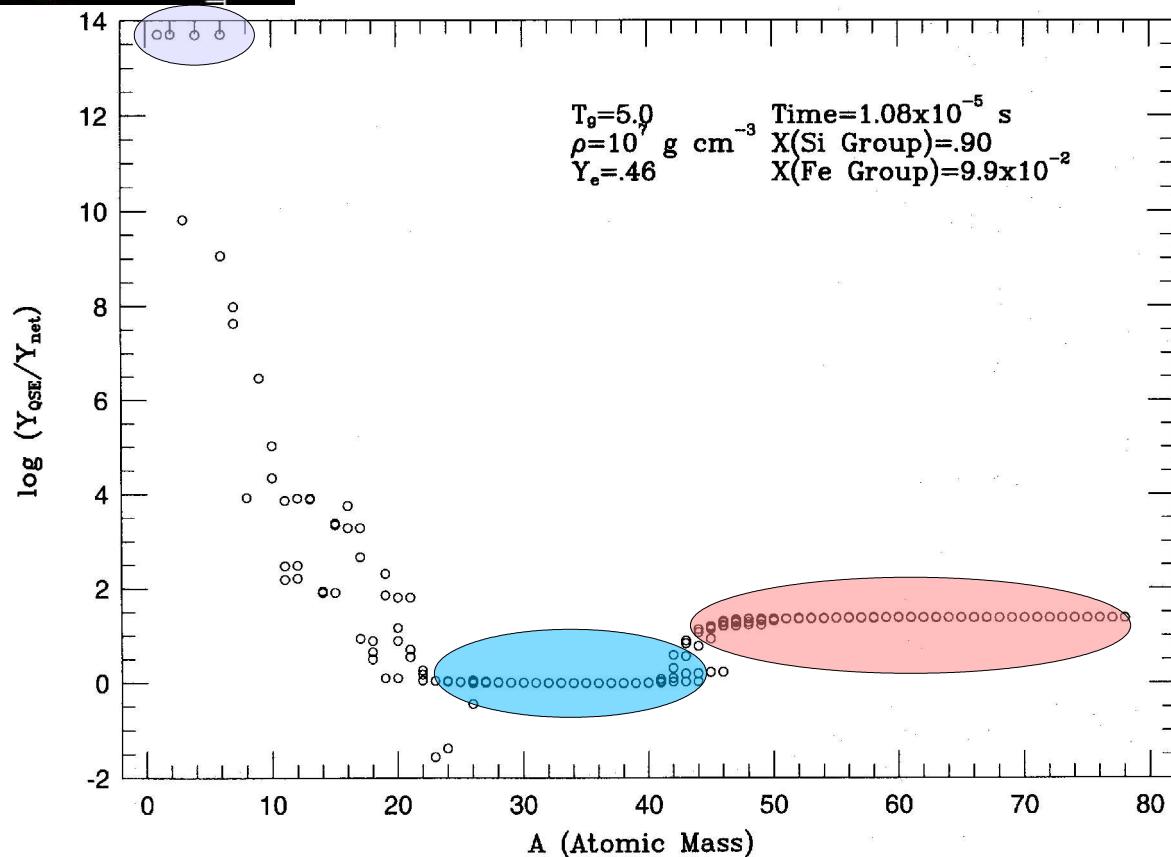
t (s) = 5.90400e-07 T_9 = 5.50 ρ (g/cc) = 1.00000e+07 t (s) = 0.303400 T_9 = 2.67 ρ (g/cc) = 1.14500e+06



Quasi-Equilibrium (QSE)

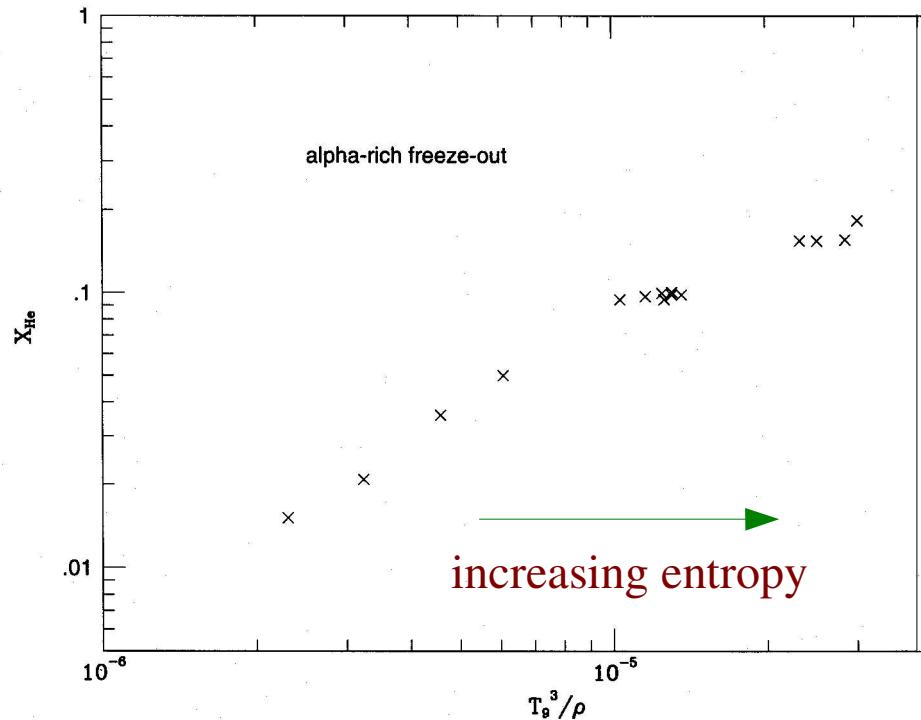


full NSE is not attained, but there exist equilibrium groups around ^{28}Si , ^{56}Ni and n,p, ^4He , which are separated by slow reactions



Sample Calculations from
• B.S. Meyer and
• Hix and Thielemann

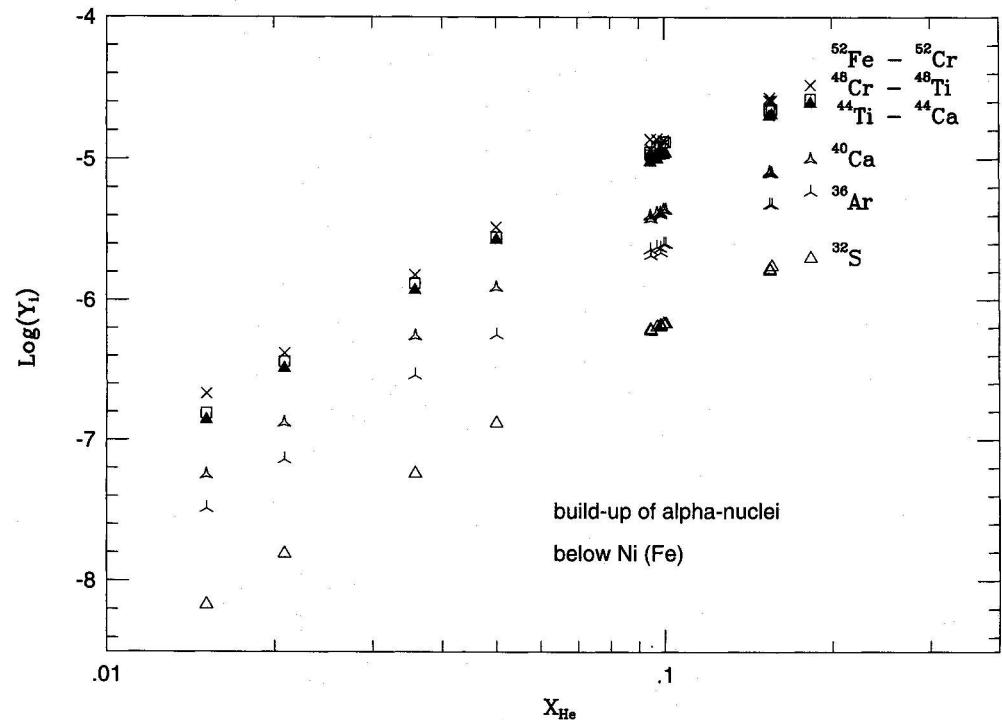
alpha-rich freeze-out



Thielemann et al. (1996)

alpha-rich freeze-out occurs at high temperatures and/or low densities and is a function of entropy S in radiation-dominated matter

- it leads to the enhancement of “alpha-elements”
- and also to the extension of the Fe-group to higher masses (^{56}Ni to ^{64}Ge and for very high entropies up to $A=80$)



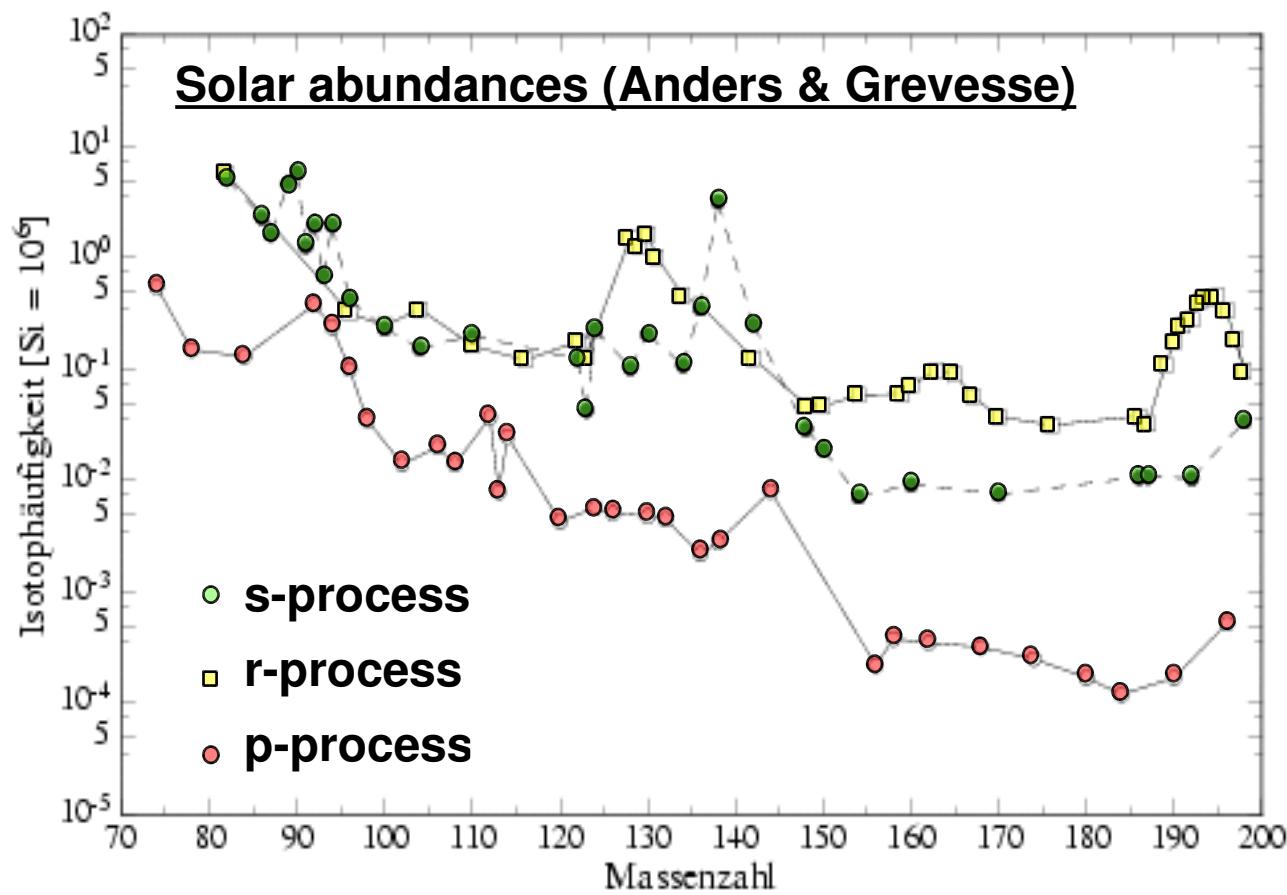
increasing entropy

“Historical” Burning Processes (B²FH)

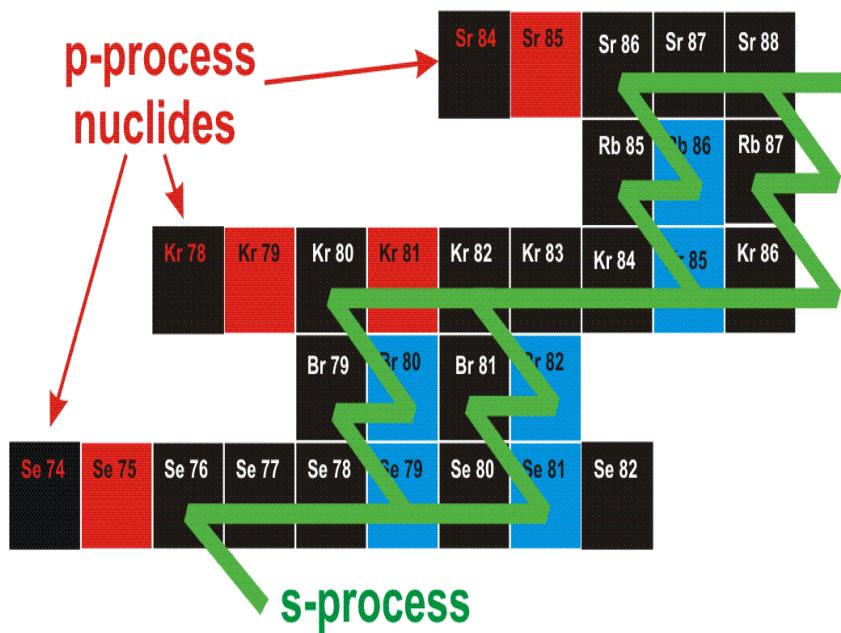
- H-Burning
- He-Burning
- alpha-Process
- e-Process
- s-Process
- r-Process
- p-Process
- x-Process

- ## Present Understanding
- H-Burning
 - He-Burning
 - expl. C, Ne, O-Burning, incomplete Si-Burning
 - explosive Si-Burning
 - about 70% normal freeze-out with $Y_e=0.42-47$, about 30% alpha-rich freeze-out with $Y_e=0.5$
 - s-Process (core and shell He-burning, neutrons from alpha-induced reactions on ^{22}Ne and ^{13}C)
 - r-Process (see below)
 - p-Process (see below)
 - x-Process (light elements D, Li, Be, B [big bang, cosmic ray spallation and neutrino nucleosynthesis])

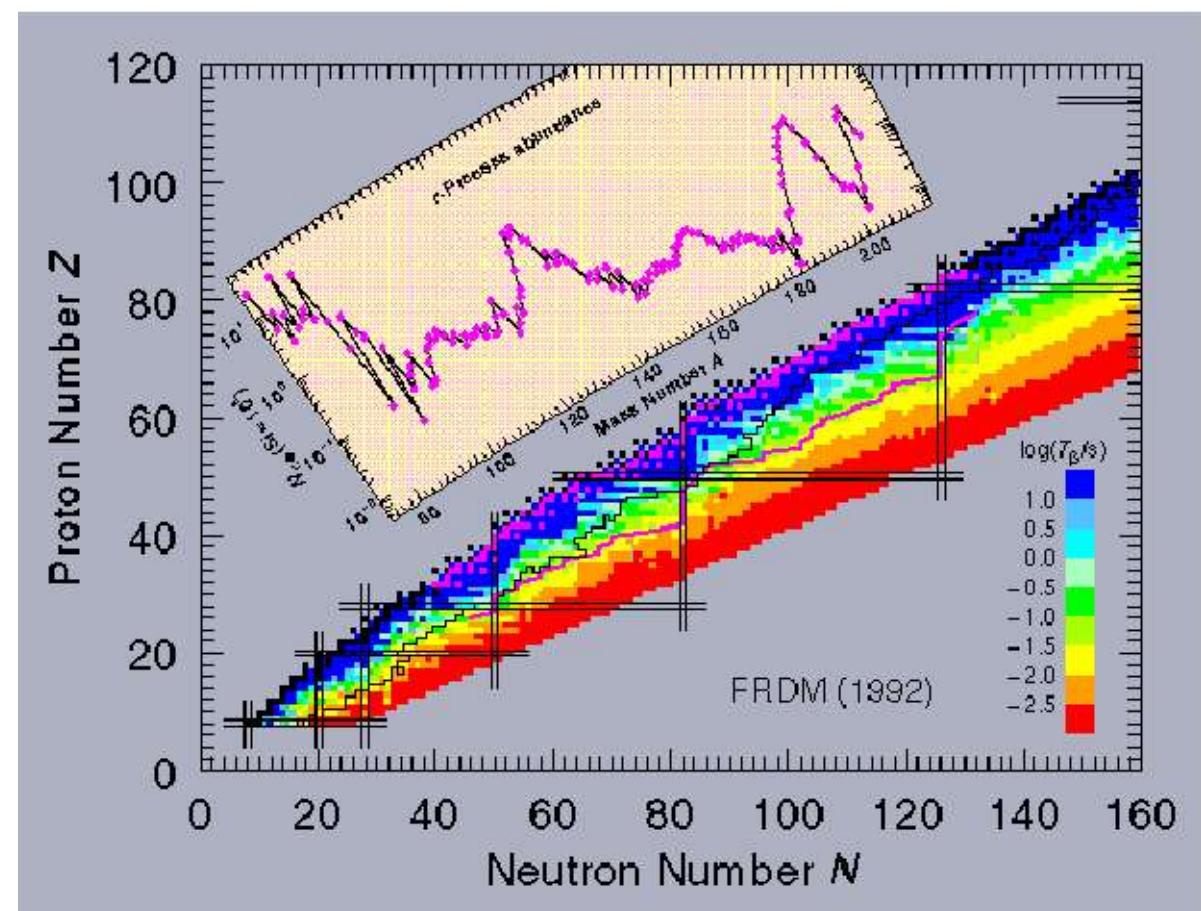
The Heavy Elements



S-, r- and p-Process



F. Käppeler



Processes in the Nuclear Chart

