

... for a brighter future







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#### **Astrophysics – Critical Nuclear Physics**

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NIC-X School, Argonne, July 2008





#### **Nuclei involved in Astrophysics**





## **Experiments with stable beams and targets:**

provide data for BB nucleo-synthesis and quiescent burning scenarios

**Need:** 

High beam intensities
thick targets, that can tolerate the beams
low backgrounds

Iong runs



See session 3: (C. Iliadis, G. Imbriani)

# Experiments with radioactive beams:

provide data for explosive burning scenarios:

## Need:

Beams of unstable nuclei (low intensities, contaminants)
thick targets (to compensate for the intensity)
long runs



#### **Radioactive beam production:**

### **Isotope Separation OnLine (ISOL)**





#### **Radioactive beam production:**

#### **Fragmentation Technique**





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#### Radioactive beam production: In-Flight technique



. . .



#### **The Nuclear Landscape**







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#### $B\rho - \Delta E - B\rho$ Separation Method







R. Schneider et al., Phys. Scr. T56, 67(1995)



# **Radioactive beam production at RIKEN**





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#### **Caveat: Existence/non-existence:** <sup>69</sup>Br



B. Blank et al. PRL 74,4611(1995)

M. F. Mohar et al, PRL66,1571(1991)











#### **Importance of nuclei with long half lives**



## **Principle of Half-life Measurements**



J. J. Prisciandaro et al. NIMA 505, 140(2003)



## **Example 2: n-rich Pd nuclei**



F. Montes et al. PRC73, 035801(2006)

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# Measurement of very long half lives: <sup>60</sup>Fe (T<sub>1/2</sub>~1.5 Ma)

### Principle:

- 1. Produce <sup>60</sup>Fe (e.g. in a beam stop of an accelerator) (N~10<sup>15</sup> atoms).
- 2. Measure the activity of the sample.
- 3. Calculate (Roy and Kohman, Can. J. Phys. 35, 649(1957) or
- 4. Measure (Kutschera et al., NIMB5, 430(1984)) the number of atoms produced.
- 5. Use the relation  $A(t) = \lambda N(t)$ .



# **Measuring the number of atoms:**

### **Technique:**







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# Measurement of intermediate half-lifes: <sup>44</sup>Ti (T<sub>1/2</sub>~60 years)







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### **Caveat: beware of the systematic errors!**



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#### The difference between half-life and mean life:

#### Half-Life of <sup>10</sup>Be: A Correction\*

Edwin M. McMillan

Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720 (Received 29 August 1972)

A mistake in computing the result of an earlier determination of the half-life of <sup>10</sup>Be is pointed out. The corrected value is  $(1.7 \pm 0.4) \times 10^6$  yr.

Yiou and Raisbeck<sup>1</sup> have published a redetermination of the half-life of <sup>10</sup>Be, which differs from the previous measurements of Hughes, Eggler, and Huddleston<sup>2</sup> and McMillan.<sup>3</sup> This discrepancy motivated me to check my orginal work sheets, and I discovered no mistakes except in the last step of the calculations, the conversion of the decay constant to the half-life, where I neglected to include the factor In2. Since both the decay constant and the half-life are given in the published paper, any reader can see where the mistake was made. I would therefore like to revise my 1947

result from  $(2.5 \pm 0.5) \times 10^6$  yr to  $(1.7 \pm 0.4) \times 10^6$ 

The result of Yiou and Raisbeck for the half-1 is  $(1.5\pm0.3)\times10^{6}$  yr, in agreement with my revised value. The Hughes, Eggler, and Huddless result of  $2.9\times10^{6}$  yr (no error given) has been vised to  $1.6\times10^{6}$  yr by Emery, Reynolds, and Wyatt,<sup>4</sup> using the ratios of new and old values for the relevant cross sections. These authors also give a new experimental determination,  $(1.6\pm0\times10^{6}$  yr. Thus there now seems to be general agreement that the half-life of <sup>10</sup>Be is close to  $1.6\times10^{6}$  yr.

\*Work performed under the auspices of the U.S. Atomic Energy Commission.

<sup>1</sup>F. Yiou and G. M. Raisbeck, Phys. Rev. Letters <u>29</u>,

Rev. 71, 269 (1947).

<sup>3</sup>E. M. McMillan, Phys. Rev. 72, 591 (1947).

<sup>4</sup>J. F. Emery, S. A. Reynolds, and E. I. Wyatt, Nuc











# Why do we need masses?

•Needed to determine the driplines

•Needed to determine the half-lives

•Needed to determine the path of the r-process



# **Techniques for mass measurements**

Reaction Q-values: A(a,b)B
 TOF + energy measurements: E=m\*s²/t²
 Cyclotron resonance: T<sub>cycl</sub>\*eB/2π=m/q
 Storage rings

For details see D. Lunney et al. RMP75, 1021(2003)




D. Lunney, Proc. Nuclei in the Cosmos IX, (2006)





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NAL LABORATORY

## How a Penning trap works -1



- constant axial magnetic field
- particle orbits in horizontal plane

$$\mathcal{W}_{c} = \frac{qB}{m}$$

• free to escape axially



## How a Penning trap works-2





## Add an axial harmonic potential to confine particles:





## Motion of ions in a Penning trap

## **Solve for equations of motion:**

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

## **Axial oscillations:**

$$\omega_z = \sqrt{\frac{eV}{md^2}}$$

## **Radial motion:**

$$\omega_{\pm} = \frac{\omega_c}{2} \pm \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$





## Penning trap mass spectrometry



## **Penning traps**













#### First Precision Mass Measurements of Refractory Fission Fragments

U. Hager,<sup>1</sup> T. Eronen,<sup>1</sup> J. Hakala,<sup>1</sup> A. Jokinen,<sup>1,\*</sup> V. S. Kolhinen,<sup>2</sup> S. Kopecky,<sup>1</sup> I. Moore,<sup>1</sup> A. Nieminen,<sup>1</sup> M. Oinonen,<sup>3</sup> S. Rinta-Antila,<sup>1</sup> J. Szerypo,<sup>2</sup> and J. Äystö<sup>1</sup>



n-rich nuclei are less bound than expected by mass formulae

 $\rightarrow$  neutron drip line moves closer to the valley of stability

# **Reactions in Nuclear Astrophysics**

Si in CasA



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## Critical reactions in nuclear astrophysics

**(**p,γ)  $\square(\alpha,\gamma)$ **α**(α,p)  $\square$ <sup>12</sup>C + <sup>12</sup>C fusion **(**n,γ) GT transitions **α**(α,n)  $(p,\alpha)$  $(\gamma, p), (\gamma, n), (\gamma, \alpha)$  (p-process), [session 11]

(novae, rp-process) (red giants) (rp-process) (supernovae) (r-process, s-process), [session 9,10] (supernovae), [session 5] (s-process, red giants), [session 10] (novae), [session 14]



## In Nature:



## v: Maxwellian distribution



## In the laboratory:



Reactions between Charged Particles (Astrophysical Reaction Rate)

Example:  ${}^{12}C(p,\gamma){}^{13}N$   $N_c$ :  ${}^{12}C$  particles/cm<sup>3</sup>  $N_p$ : protons/cm<sup>3</sup>  $v_o$ : relative velocity between C and p Rate:  $r=N_c \cdot N_p \cdot v_o \cdot \sigma_{p\gamma}(v_o) \{cm^{-3} s^{-1}\}$ 



Plasma: velocity distribution  $\phi(v)$  $v\sigma \rightarrow \langle v\sigma \rangle = \int \phi(v) \cdot v \cdot \sigma(v) dv$ ( $< v\sigma >$  reaction rate per particle pair) Particle densities N<sub>i</sub>:  $\rho = N_i \mu$   $\mu =$  weight of a particle  $\rho = N_i A / N_A N_A$ : Avogadro's Number  $N_i = \rho N_A / A$ 

Or, for a multi-particle gas with X<sub>i</sub> as a mass fraction:

 $N_i = \rho N_A / A X_i$ 



## In normal stellar matter (not in neutron stars)

$$\phi_{i}(v_{i}) = 4\pi v_{i}^{2} \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^{2}}{2kT}\right)$$
(Maxwellian)

$$<\sigma v > = \iint \phi(v_1)\phi(v_2)\sigma(v_{rel})v_{rel} dv_1 dv_2$$

 $v_1 = V + m_2/(m_1 + m_2)v$  V : center-of-mass velocity  $v_2 = V - m_1/(m_1 + m_2)v$  v : relative velocity  $(v_1 - v_2)$ 

$$<\sigma v > = \iint \Phi(V)\phi(v) v \sigma(v) dv dV$$



# Where: $\Phi(V) = 4\pi V^2 (M/(2\pi kT))^{3/2} \exp(-MV^2/(2kT))$ $M = m_1 + m_2$ $\phi(v) = 4\pi v^2 (\mu/(2\pi kT))^{3/2} \exp(-\mu v^2/(2kT))$ $\mu = m_1 m_2 / (m_1 + m_2)$ $\langle \sigma v \rangle = \int \phi(v) v \sigma(v) dv$ Because $\int \Phi(V) dV = 1$



$$\langle \sigma \mathbf{v} \rangle = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int \mathbf{v}^3 \,\sigma(\mathbf{v}) \,\exp\left(-\frac{\mu \,v^2}{2kT}\right) d\mathbf{v}$$

or

$$<\sigma v>=(\frac{8}{\pi \mu})^{1/2} \left(\frac{1}{kT}\right)^{3/2} \int \sigma(E) E \exp(-\frac{E}{kT}) dE$$



## Need $\sigma(E)$ :

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Non-resonant cross sections

resonant cross sections



To eliminate the strong energy dependence, one takes out the trivial factors :  $e^{-2\pi\eta}/E$  and defines a new parameter S (S-Factor) which contains the 'non-trivial' energy dependence:

 $\sigma = S(E)/E e^{(-2\pi\eta)}$ 

S(E)=σ E  $e^{(2\pi\eta)}$ 



With S(E) one can rewrite  $\langle \sigma v \rangle$ :

$$<\sigma v > = (\frac{8}{\pi\mu})^{1/2} (\frac{1}{kT})^{3/2} \int S(E) \exp(-E/kT - b/E^{1/2}) dE$$

argument of the exponent:





Maximum of the argument at  $E_0$ :

$$E_0 = (bkT/2)^{2/3}$$
 with  $b = (2\mu)^{1/2} \pi e^2 Z_1 Z_2 / \hbar$ 

or

E<sub>0</sub>=1.22(
$$Z_1^2 Z_2^2 \mu T_6^2$$
)<sup>1/3</sup> [keV]

## **Gamow peak**

T<sub>6</sub>: temperature in 10<sup>6</sup> K



## **Resonance Reactions**







TARGET A

FINAL STATE OF COMPOUND NUCLEUS B





## $\sigma_{resonance}$ : Breit-Wigner shape

$$\sigma_{i \to f} = \frac{\pi}{k^2} \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_i \Gamma_f}{(E-E_r)^2 + (\Gamma/2)^2}$$

## J: spin of the resonance

- J  $_{1,2}$ : spin of the particles in the entrance channel
- k: wave number

 $\Gamma_{i,f}$ : widths (decay probabilities) in the entrance or exit channel

E<sub>r</sub>: resonance energy

 $\Gamma$ : total width ( $\Gamma_i + \Gamma_f + ...$ )







$$\int \sigma_{\rm BW}(E) \, dE = \frac{\pi}{k^2} \, \omega \Gamma_{\rm i} \Gamma_{\rm f} \, \pi/(\Gamma/2)$$

$$= 2\pi^2/k^2 \frac{\omega \Gamma_i \Gamma_f}{\Gamma} = \frac{2\pi^2/k^2}{\omega \gamma}$$

## $\omega\gamma$ : resonance strength



$$\langle \sigma v \rangle = (\frac{2\pi}{\mu kT})^{3/2} \hbar^2 \omega \gamma \exp(-E_r/kT)$$

For several non-overlapping resonances:

$$\langle \sigma v \rangle = (\frac{2\pi}{\mu kT})^{3/2} \hbar^2 \sum \omega \gamma_i \exp(-E_i/kT)$$

High rates for:

1. Large  $\omega\gamma$ 





# (p,γ) reactions

# Center of activities with radioactive beams Mainly resonant Example <sup>21</sup>Na(p,γ)<sup>22</sup>Mg (TRIUMF)

S. Bishop et al. PRL90, 162501(2003) J. d'Auria et al. PRC69, 065803(2004)



Need 200-500 keV  $^{21}Na$  (T $_{1/2}$ =22.8 s) beams and hydrogen gas target Reaction studied as:  $p(^{21}Na,^{22}Mg)\gamma$ 



### "Gamow" windows







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Beam and Recoils



## **Other Recoil Separators for Astrophysics**



## DRAGON at TRIUMF ISAC Used to measure <sup>21</sup>Na(p,γ)<sup>22</sup>Mg



ARES at Louvain-Ia-Neuve Used to measure  ${}^{19}Ne(p,\gamma){}^{20}Na$ 



DRS at ORNL HRIBF Used to measure <sup>18</sup>F(d,p)<sup>19</sup>F



FMA at ANL ATLAS Used to measure <sup>18</sup>F(p,γ)<sup>19</sup>Ne



 (p,γ) reaction with stable nuclei: many examples, cross sections typically ~µb

• with radioactive beams studied so far: <sup>7</sup>Be(p, $\gamma$ ), <sup>13</sup>N(p, $\gamma$ ), <sup>17</sup>F(p, $\gamma$ ), <sup>21</sup>Na(p, $\gamma$ ), <sup>26</sup>Al(p, $\gamma$ )

• need beam intensities >  $10^8$  /s, which is difficult for radioactive beams

 $\rightarrow$ use indirect techniques



## Indirect techniques for $(p,\gamma)$ reactions:

$$\sigma_{p \to \gamma} \frac{\pi}{k^2} \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_p \Gamma_{\gamma}}{(E-E_r)^2 + (\Gamma/2)^2}$$

- 1. Determine E<sub>r</sub> (e.g. via transfer reactions)
- 2. Determine J (e.g. via angular distributions)
- 3. Determine  $\Gamma_{\gamma}$  (e.g. via a  $\gamma$  lifetime measurement)
- 4. Determine  $\Gamma_{p}$  (e.g. via elastic scattering)



## **Other Indirect Techniques:**

Coulomb dissociation: (<sup>8</sup>B(γ,p)<sup>7</sup>Be is the time-inverse reaction of <sup>7</sup>Be(p,γ)<sup>8</sup>B)




<sup>7</sup>Be(p,γ)<sup>8</sup>B

# Direct measurement $S_{17}(0)=22.1\pm0.6\pm0.6$ eVb

# Indirect measurement S<sub>17</sub>(0)=20.6±0.8±1.2 eVb



Junghans et al. PRC68, 065803(2003)

Schümann et al. PRC73, 015806(2006)



# **Other Indirect techniques:**

- 1. Transfer Reactions (Asymptotic Normalization Coefficients, ANC) (A. Mukhamedzanov et al. PRC56, 1302(1997)
- 2. γ-spectroscopy following fusion reactions (D. Jenkins et al. PRL 92, 031101 (2004)
- 3. γ-spectroscopy following knockout reactions (R. Clement et al., PRL 92 172502, (2004))
- 4. (<sup>3</sup>He,d) reactions (C. L. Jiang et al., subm. to PRC)



(α,γ) Reactions								
cross sections ~ $1/100 \sigma(p,\gamma)$								
Important examples:								
$\alpha + \alpha + \alpha \rightarrow ^{12}C$	bridging the mass 8 gap							
<sup>12</sup> C(α,γ) <sup>16</sup> O	'most important reaction in Nuclear Astrophysics							
<sup>15</sup> Ο(α,γ) <sup>19</sup> Ne	breakout from the hot CNO cycle							
<sup>40</sup> Ca(α,γ) <sup>44</sup> Ti	production of the gamma tracer <sup>44</sup> Ti							



## **Direct measurements of <sup>40</sup>Ca(α,γ)<sup>44</sup>Ti** (amount of <sup>44</sup>Ti in CasA SN remnant)

- 1977: high intensity <sup>4</sup>He beams + <sup>40</sup>Ca target, γ detection
- (E. Coopermann et al., Nucl. Phys. A284, 163 (1977))
- Target deterioration
- Detection efficiency
- Background

## New approaches (<sup>40</sup>Ca beam and <sup>4</sup>He target):

#### Accelerator mass spectrometry

(H. Nassar et al., PRL96, 041102(2006)

#### Measurements in inverse kinematics

(C. Vockenhuber et al., PRC76, 035801(2007))





#### **CASSIOPEIA A**



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# **Problems with <sup>44</sup>Ti signal**

Amount of <sup>44</sup>Ti measured in Cas A:

160±60 μM<sub>☉</sub> (3x10<sup>26</sup> kg)

Amount of <sup>44</sup>Ti <u>calculated</u>:

20 - 80  $\mu M_{\odot}$ 

(for comparison: mass of the earth ~  $6x10^{24}$  kg)



Mn <sup>1246°</sup> <sup>2061°</sup>	Mn44	Mn45	Mn46 41 ms	Mn47 100 ms	Mn48 158.1 ms	Mn49 382 ms	Mn50 283.88 ms	Mn51 46.2 m	Mn52 5.591 d	Mn53 3.74E+6 y
+2+3+4+7					4+	5/2-	0+	5/2-	6+	7/2-
0.000031%			ЕСр	ЕСр	ECp,ECα,	EC	EC	EC	EC	EC
Cr42	Cr43	Cr44	Cr45	Cr46	Cr47	Cr48	Cr49	Cr50	Cr51	Cr52
	(3/2+)	53 ms 0+	50 ms	0.26 s 0+	500 ms 3/2-	21.56 h 0+	42.3 m 5/2-	1.8E+17 y 0+	27.702 d 7/2-	0+
	ECp,ECα,	ЕСр	ЕСр	EC	EC	EC		ECEC 4.345	EC	83.789
V41	V42	V43	V44	V45		V47	V48	V49	V50	V51
		800 ms (7/2-)	90 ms (2+)	54/ms 7/2-	$(\alpha,p)$	32.6 m	15.9735 d 4+	530 d 7/2-	1.4E+1/y 6+	7/2-
		EC	* ECα	EC	C.C	EC	EC		EC,β-	99.750
Ti40	Ti41	Ti42	Ti43	Ti44	Ti45	Ti46	Ti47	Ti48	Ti49	<b>Ti50</b>
50 ms	80 ms	199 ms	509 ms	63 y	184.8 m	0+	5/2-	0+	7/2-	0+
EC.	EC.	EC	EC.	FC	FC III	0.0	5/2-			
E. 20	Se40	EC Sed1	C-42	8-42	EC Sadd	8.0	1.3	75.8	5.5	5.4
5039	182.3 ms	596.3 ms	681.3 as	3.891 h	3.927 h	5645	83.79 d	3.3492 d	5C48 43.67 h	57.2 m
(7/2-)	4-	7/2-	*	7/2-	2+ *	7/2- *	4+	7/2-	6+	7/2-
	ECp,ECα,	EC	Y.C.	EC	EC	100	β·	β-	β-	β-
Ca38	Ca39	Ca40	Ca41	Ca42	Ca43	Ca44	Ca45	Ca46	Ca47	Ca48
0+	3/2+	0+	7/2-	0+	7/2-	0+	7/2-	0+	4.330 u 7/2-	0L+18 y 0+
EC	EC	96.941	EC	0.647	0.135	2.086	β-	0.004	β-	β-,β-β- 0.187
K37	K38	K39	K40	K41	K42	K43	K44	K45	K46	K47
1.226 s 3/2+	7.636 m 3+	3/2+	1.277E+9 y 4-	3/2+	12.360 h 2-	22.3 h 3/2+	22.13 m 2-	17.3 m 3/2+	105 s (2-)	17.50 s 1/2+
EC	EC *	93.2581	EC,β·	6.7302	β-	β-	β-	β-	β-	β-
Ar36	Ar37	Ar38	Ar39	Ar40	Ar41	Ar42	Ar43	Ar44	Ar45	Ar46
0+	35.04 d	0+	269 y	0+	109.34 m	32.9 y	5.37 m (3/2 5/2)	11.87 m	21.48 s	8.4 s
0.007	FC	0.072	R-	00,000	R-	α.	(J) 2, J) 2)	8-	β	8.
0.337	EC	0.063	P	99.600	þ	p	p-	P	p.	p.



# Direct ( $\alpha, \gamma$ ) measurements (with stable beams)

Experimental setup for the  ${}^{40}Ca(\alpha,\gamma){}^{44}Ti$  experiment :





1. Bombard <sup>4</sup>He gas with a beam of <sup>40</sup>Ca particles (<sup>40</sup>Ca + <sup>4</sup>He  $\rightarrow$  <sup>44</sup>Ti) (done at Argonne)

<sup>4</sup>He

<sup>40</sup>Ca

2. Identify and count the number of <sup>44</sup>Ti particles implanted into the copper block (done at the Weizmann Institute in Israel)

























# Indirect ( $\alpha, \gamma$ ) measurements

$$\sigma(\alpha,\gamma) = \frac{\pi}{k^2} \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_a \Gamma_{\gamma}}{(E-E_r)^2 + (\Gamma/2)^2}$$

for  ${}^{15}O(\alpha, \gamma){}^{19}Ne$ :

Need:  $\Gamma_{\gamma}$  (from  $T_{\gamma}$ )

 $\Gamma_{lpha}/\Gamma_{\gamma}$ 

Er

J



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#### The (α,p) reaction in the (rp) process





# Direct measurement of <sup>44</sup>Ti(α,p)<sup>47</sup>V in inverse kinematics



A. Sonzogni et al., PRL84, 1651(2000)



# Beam contaminants at ATLAS(<sup>44</sup>Ti) (measure <sup>44</sup>Ti( $\alpha$ ,p)<sup>47</sup>V and <sup>44</sup>Ca( $\alpha$ ,p)<sup>47</sup>Sc)





<sup>4</sup>He(<sup>44</sup>Ti,<sup>47</sup>V)p or <sup>4</sup>He(<sup>44</sup>Ca,<sup>47</sup>Sc)p







## <u>The <sup>18</sup>Ne(α,p) reaction:</u> breakout from the hot CNO cycle





А



### Direct measurement of <sup>18</sup>Ne(α,p)<sup>21</sup>Na:

For recoil separator would need a large acceptance



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#### <u>Indirect methods (α,p):</u>

#### Inverse reactions:

<sup>21</sup>Na(p, $\alpha$ )<sup>18</sup>Ne, see: S. Sinha et al., BAPS 2004 2004 and to be publ.

#### Thick target technique:

C. B. Fu et al. PRC76, 0212603(2007)





# Fusion reactions in nuclear astrophysics Carbon burning





#### How to extrapolate towards lower energies Example: <sup>12</sup>C + <sup>12</sup>C fusion





#### **Problems:**

Cross sections are in the pb range
Data from various groups don't agree
There can be resonances at low E
How to extrapolate (fusion hindrance)



- C. L. Jiang et al., PRC 75, 015803(2007)
- L. R. Gasques et al., PRC76, 035802(2007)



#### **Need experimental data!**



### A new technique for particle detection in inverse kinematics





#### Simple kinematics





p(44Ti,p')44Ti kinematics






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## **Advantages of Solenoid Spectrometer**

Automatic particle identification

Excellent center-of-mass energy resolution

High detection efficient

Simple detector and electronics - few channels

Excellent center-of-mass angle resolution

