Nucleosynthesis of Heavy and Neutron Capture Elements:

A collection of problems at the interface of nuclear structure/reactions, reaction equilibria, stellar evolution/explosions and the origin of the elements

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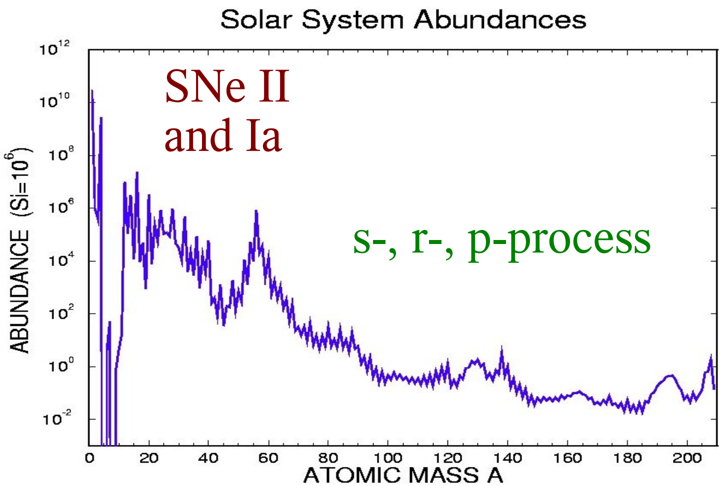
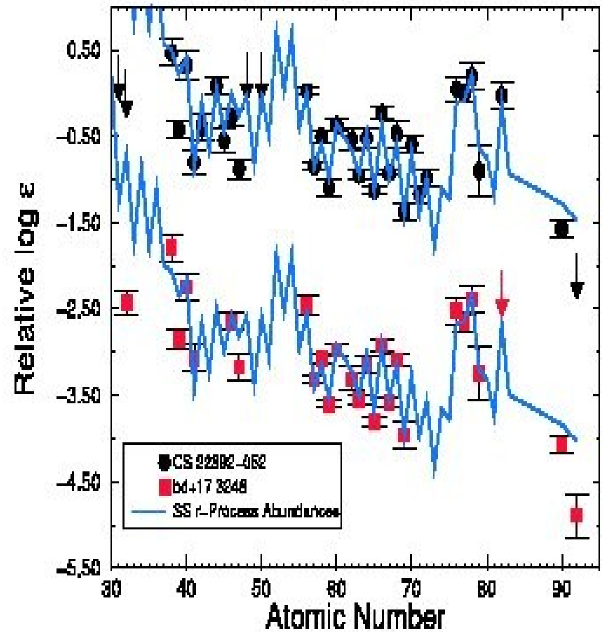
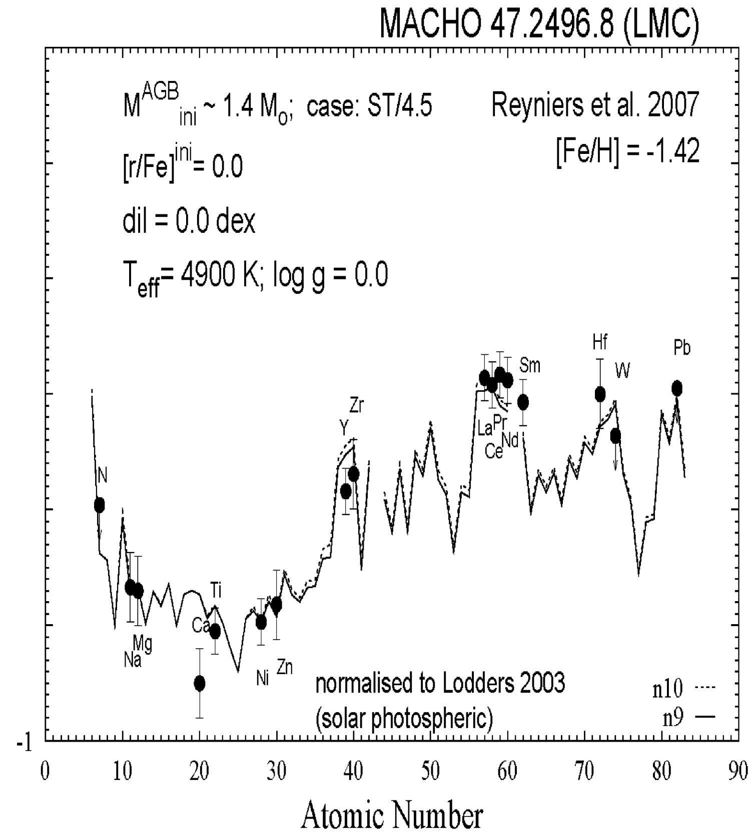
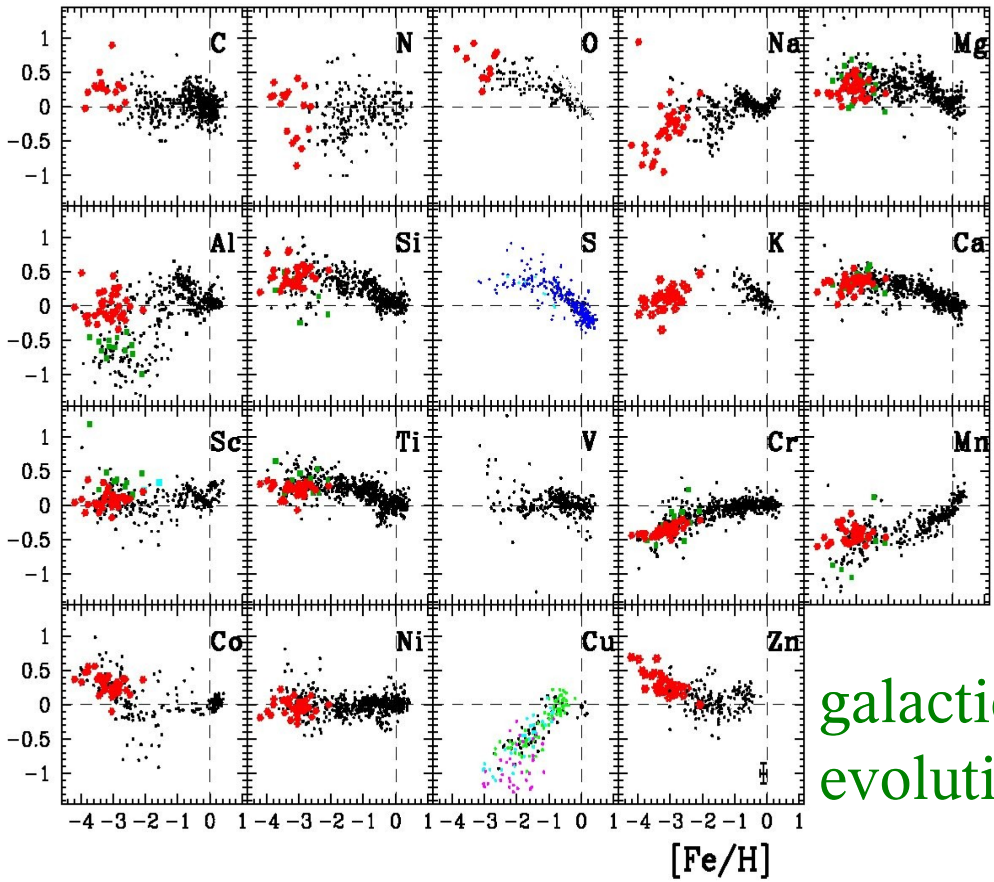




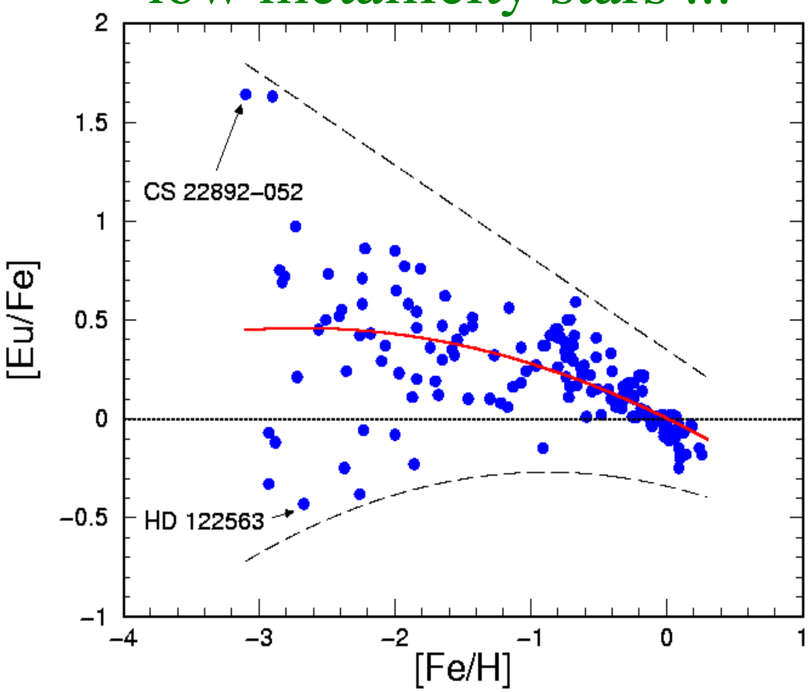


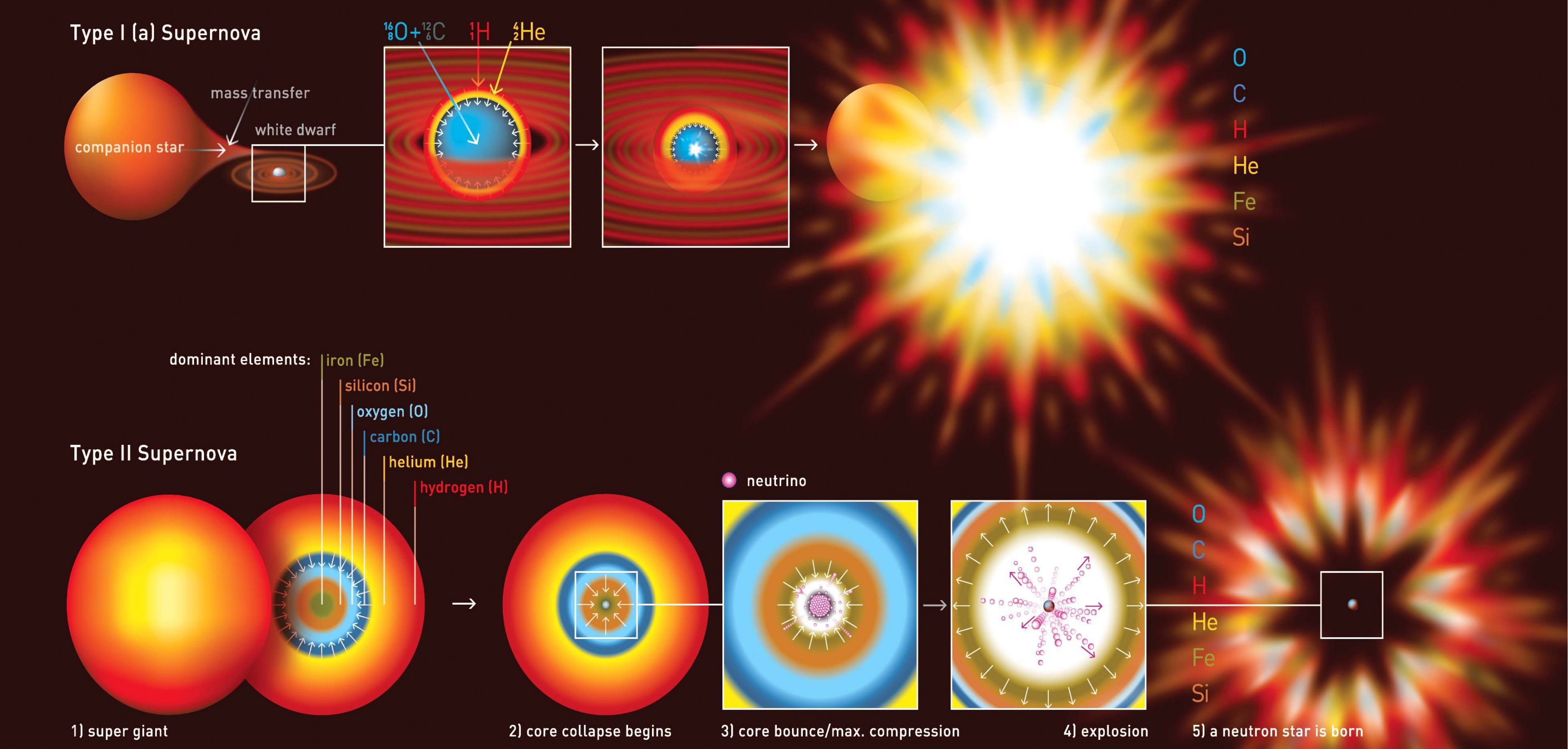
How do we understand:
solar system abundances..

low metallicity stars ...

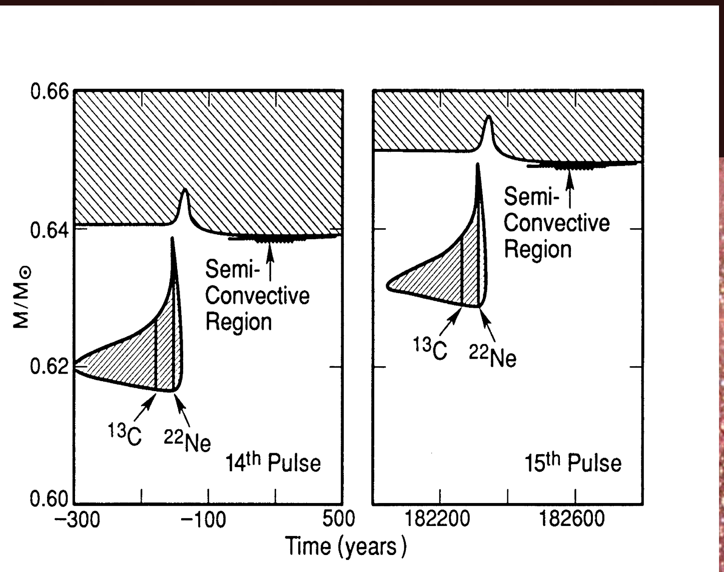


galactic
evolution?



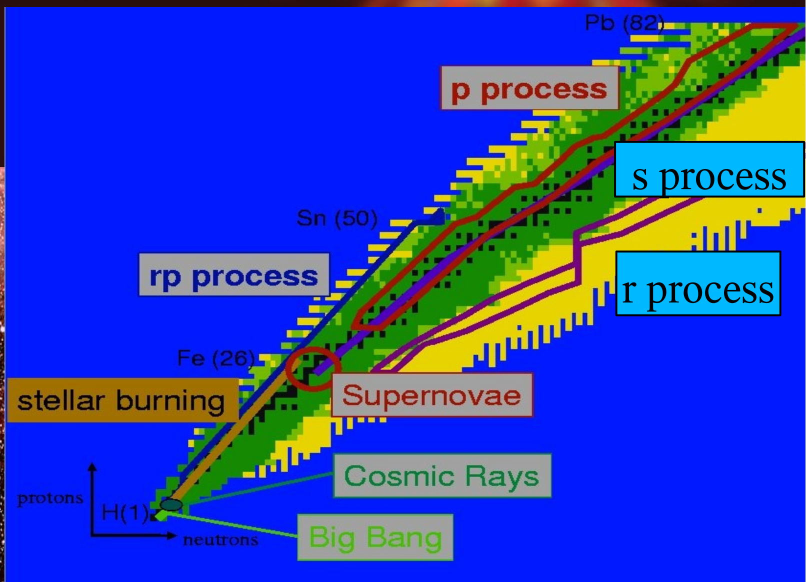


Type I (a) Supernova
$^{16}_{8}O+^{12}_{6}C$
$^{1}_{1}H$
$^{4}_{2}He$
mass transfer
white dwarf
companion star
O
C
H
He
Fe
Si
dominant elements:
iron (Fe)
silicon (Si)
oxygen (O)
carbon (C)
helium (He)
hydrogen (H)
Type II Supernova
neutrino
O
C
H
He
Fe
Si
1) super giant
2) core collapse begins
3) core bounce/max. compression
4) explosion
5) a neutron star is born



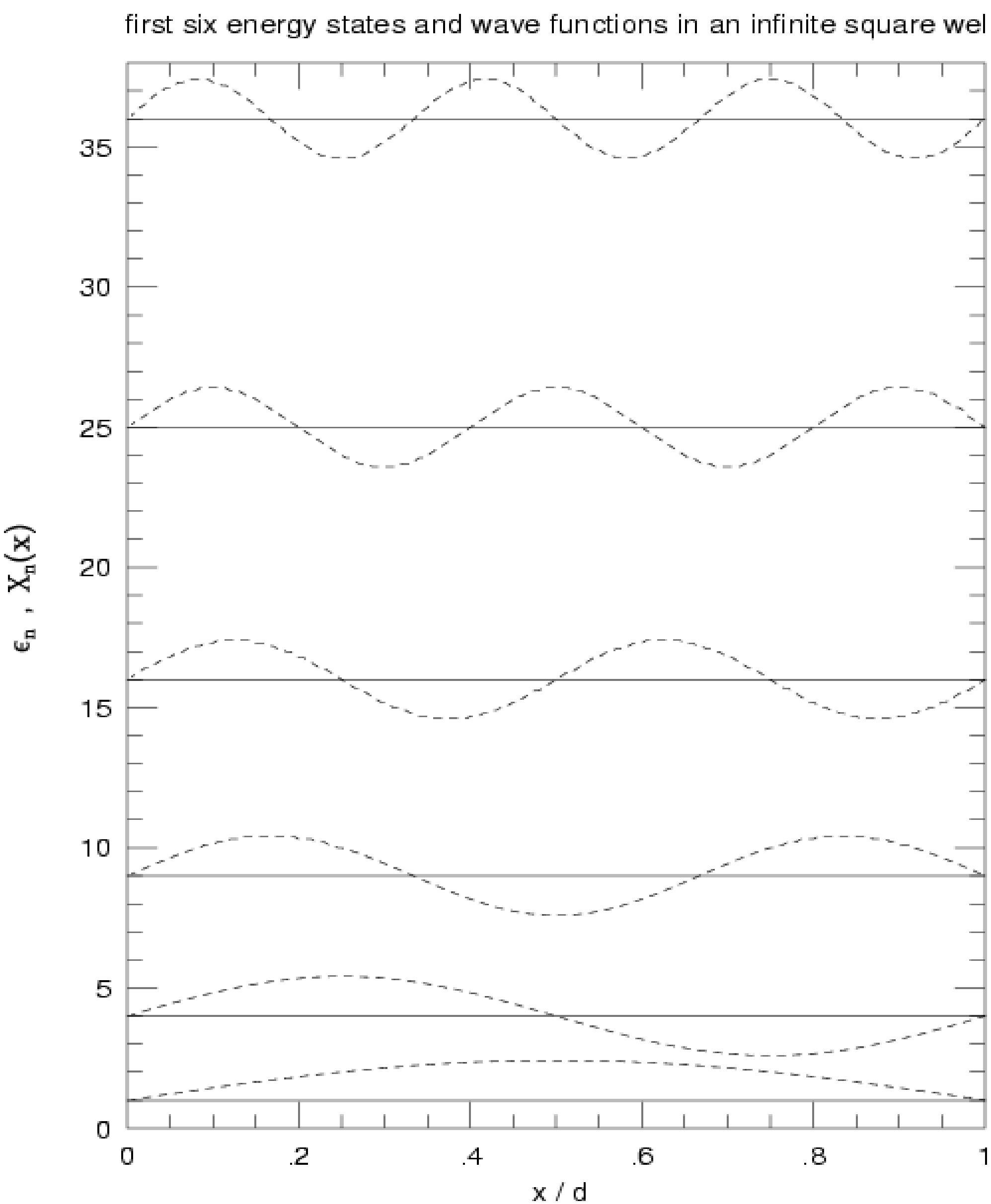
0.66
0.64
0.62
0.60
M/M⊙
Semi-
Convective
Region
^{13}C
^{22}Ne
14th Pulse
15th Pulse
−300
−100
500
182200
182600
Time (years)

SN 1987A



Pb (82)
p process
s process
Sn (50)
rp process
r process
Fe (26)
stellar burning
Supernovae
Cosmic Rays
protons
H(1)
neutrons
Big Bang

The Basics: statistical mechanics of astrophysical plasmas/gases



a non-interacting gas can be represented by a 3D box in which it is contained (with inpenetrable walls)

energy eigenvalues

$$\varepsilon_{x,n_x} = \frac{\pi^2 \hbar^2}{2md^2} n_x^2$$

$$\varepsilon_{n_x,n_y,n_z} = \frac{\pi^2 \hbar^2}{2md^2} (n_x^2 + n_y^2 + n_z^2)$$

all these states can be occupied!

Total number of states and state density

$$\tilde{\Phi}(E) = g\frac{4\pi}{3}\frac{V}{h^3}(2m)^{3/2}E^{3/2}$$

$$\tilde{\omega}(E) = \frac{d\tilde{\Phi}(E)}{dE} = 2\pi g\frac{V}{h^3}(2m)^{3/2}E^{1/2}$$

total number of states in a given volume $V=d^3$ up to energy E, and state density at that energy

g measures degeneracy of state

$$\Phi(E) = \frac{4\pi}{3}\frac{g}{h^3}(2m)^{3/2}E^{3/2}$$

$$\omega(E) = 2\pi\frac{g}{h^3}(2m)^{3/2}E^{1/2}.$$

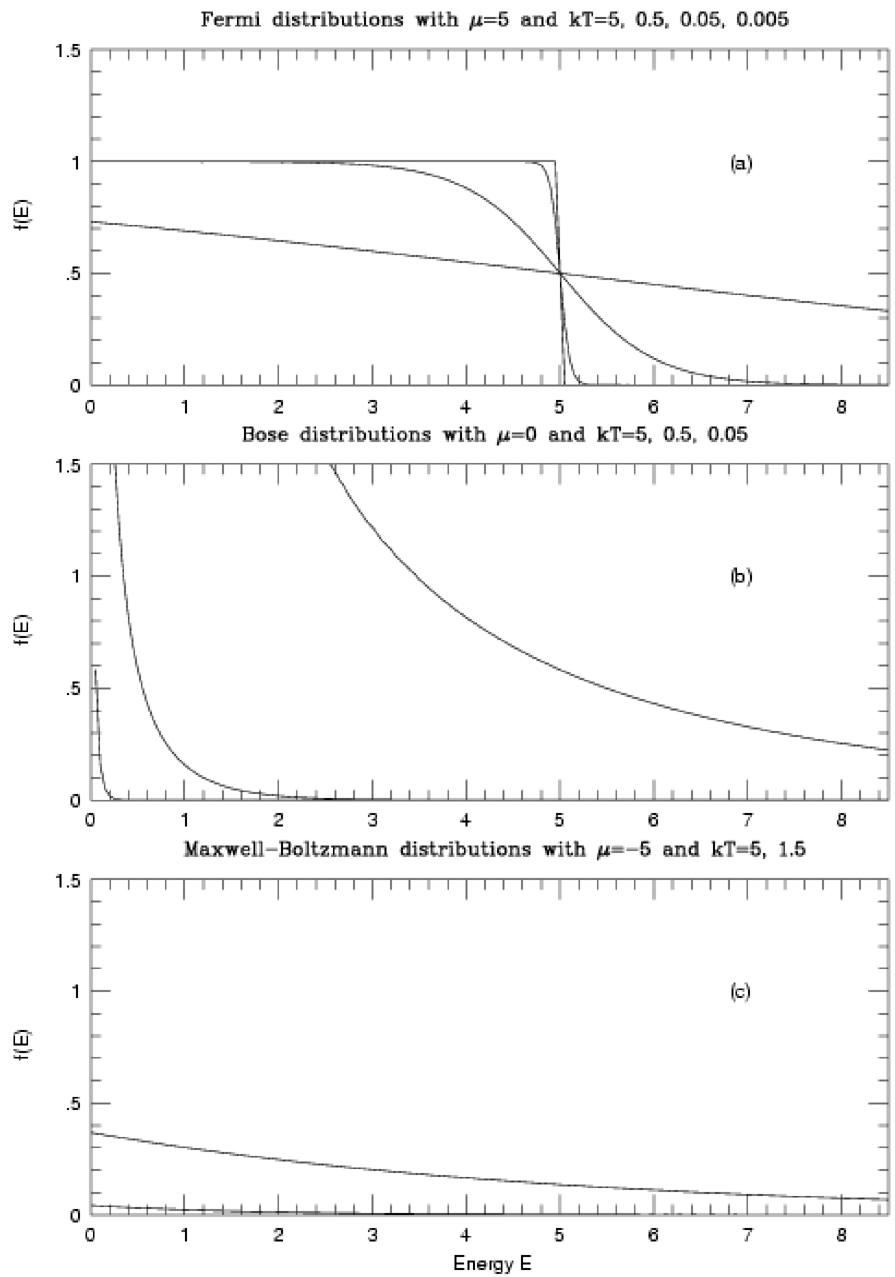
the same per volume

$$\Phi(p) = \frac{4\pi}{3}\frac{g}{h^3}p^3$$

$$\omega(p) = \frac{d\Phi(p)}{dp} = 4\pi\frac{g}{h^3}p^2$$

the same for momentum p with $E=p^2/2m$

Occupation probability for different statistics



$$f(p) = \begin{cases} \left[e^{(E(p)-\mu)/kT} + 1\right]^{-1} & \text{Fermions} \\ \left[e^{(E(p)-\mu)/kT} - 1\right]^{-1} & \text{Bosons} \\ e^{-(E(p)-\mu)/kT} & \text{Maxwell-Boltzmann.} \end{cases}$$

f(p) measures the probability that a state at energy E or momentum p is occupied.

How is the chemical potential determined?

Thermodynamic Properties

$$n = \frac{N}{V} = \int_0^\infty \omega(p) f(p) dp$$

$$u = \frac{U}{V} = \int_0^\infty E\omega(p) f(p) dp$$

$$P = \frac{1}{3} \int_0^\infty p v \omega(p) f(p) dp.$$

if not already known (0 for photons), the chemical potential can be determined from the first equation, as we know the number density n of gas particles.

$$\bar{\mu} = \mu + mc^2 = kT \ln\left(\frac{nh^3}{g} \frac{1}{(2\pi mkT)^{3/2}}\right) + mc^2$$

chemical potential and dn for a Maxwell-Boltzmann gas

$$dn_j = n_j \frac{4\pi p_j^2}{(2\pi m_j kT)^{3/2}} \exp\left(-\frac{p_j^2}{2m_j kT}\right) dp_j$$

dn for a photon (Planck) gas

$$dn_\gamma = \frac{8\pi}{c^3} \frac{\nu^2 d\nu}{\exp(h\nu/kT) - 1} = \frac{1}{\pi^2 (c\hbar)^3} \frac{E_\gamma^2 dE_\gamma}{\exp(E_\gamma/kT) - 1}$$

Preview on chemical equilibria

a reaction involving particles 1 through 4 (with the C's being integer numbers) is in equilibrium, i.e. the forward and backward reactions occur on timescales shorter than the observing time. Then the following relation holds between the chemical potentials.

$$C_1 \text{ particle1} + C_2 \text{ particle2} \rightleftharpoons C_3\text{particle3} + C_4\text{particle4}$$

$$C_1\bar{\mu}_1 + C_2\bar{\mu}_2 = C_3\bar{\mu}_3 + C_4\bar{\mu}_4$$

$$\bar{\mu} = \mu + mc^2.$$

The chemical potential obtained from the total number density n provides information on energy/momentum distributions of particles. It is only determined up to a constant. If energy generation due to mass differences in reactions is involved, the above equation is correct, if the rest mass energy is added.

The above equation leads to solutions for the relative concentrations as a function of total (mass) density and temperature.

A sketch on nuclear reactions

$$\sigma = \frac{\text{number of reactions target}^{-1}\text{sec}^{-1}}{\text{flux of incoming projectiles}} = \frac{r/n_i}{n_j v}$$

$$\sigma = \frac{\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)T_l$$

if one neglects spins of participating particles, the fusion cross section can be determined just by the sum of partial waves with transmission coefficients T_l for angular momentum l

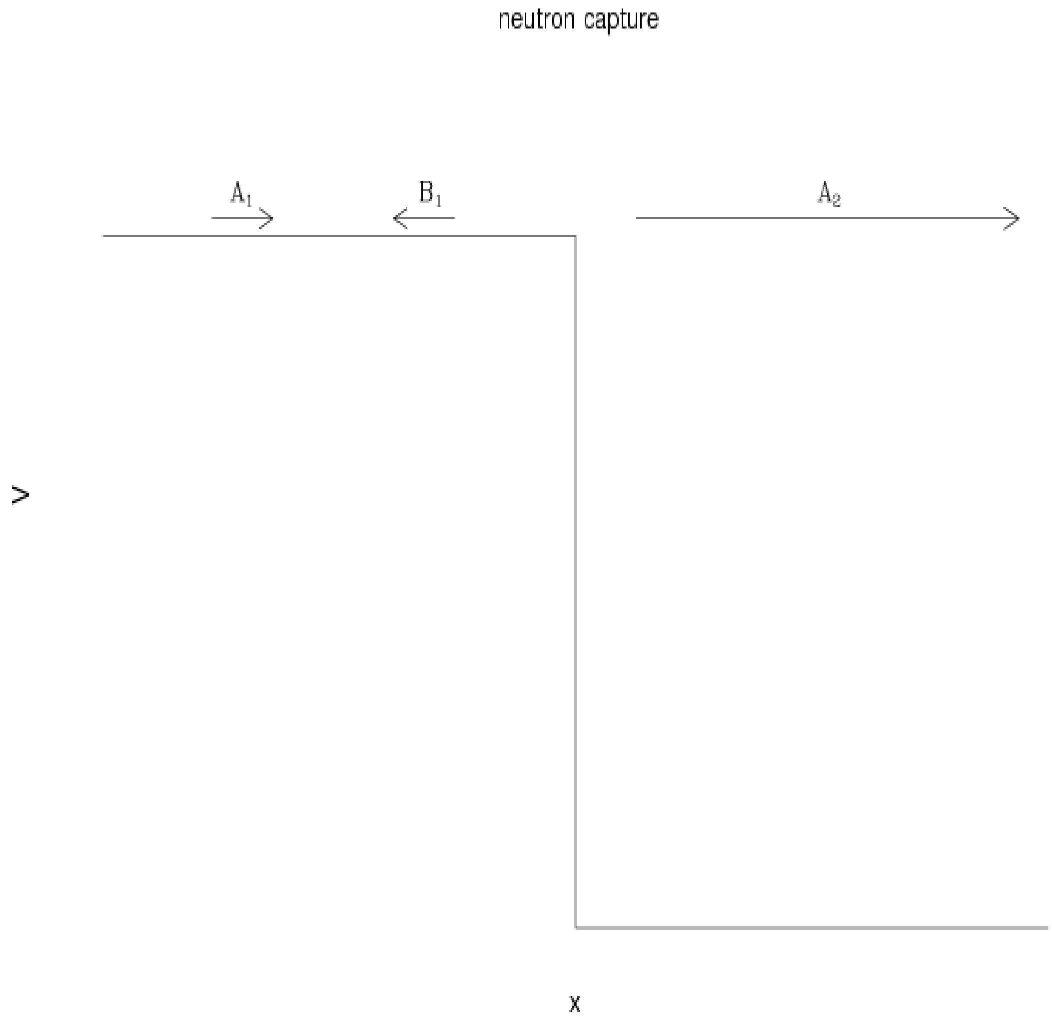
$$\sigma \approx \frac{\pi}{k^2}T_{l=0}$$

for low energies the fusion cross section is dominated by s-waves (l=0)

$$T = \frac{j_{fin}}{j_{in}} = \frac{k_{fin}|\phi_{fin}|^2}{k_{in}|\phi_{in}|^2}$$

transmission coefficient determined by ratio of penetrating to incoming flux.

Reactions with neutrons



"central collision", l=0

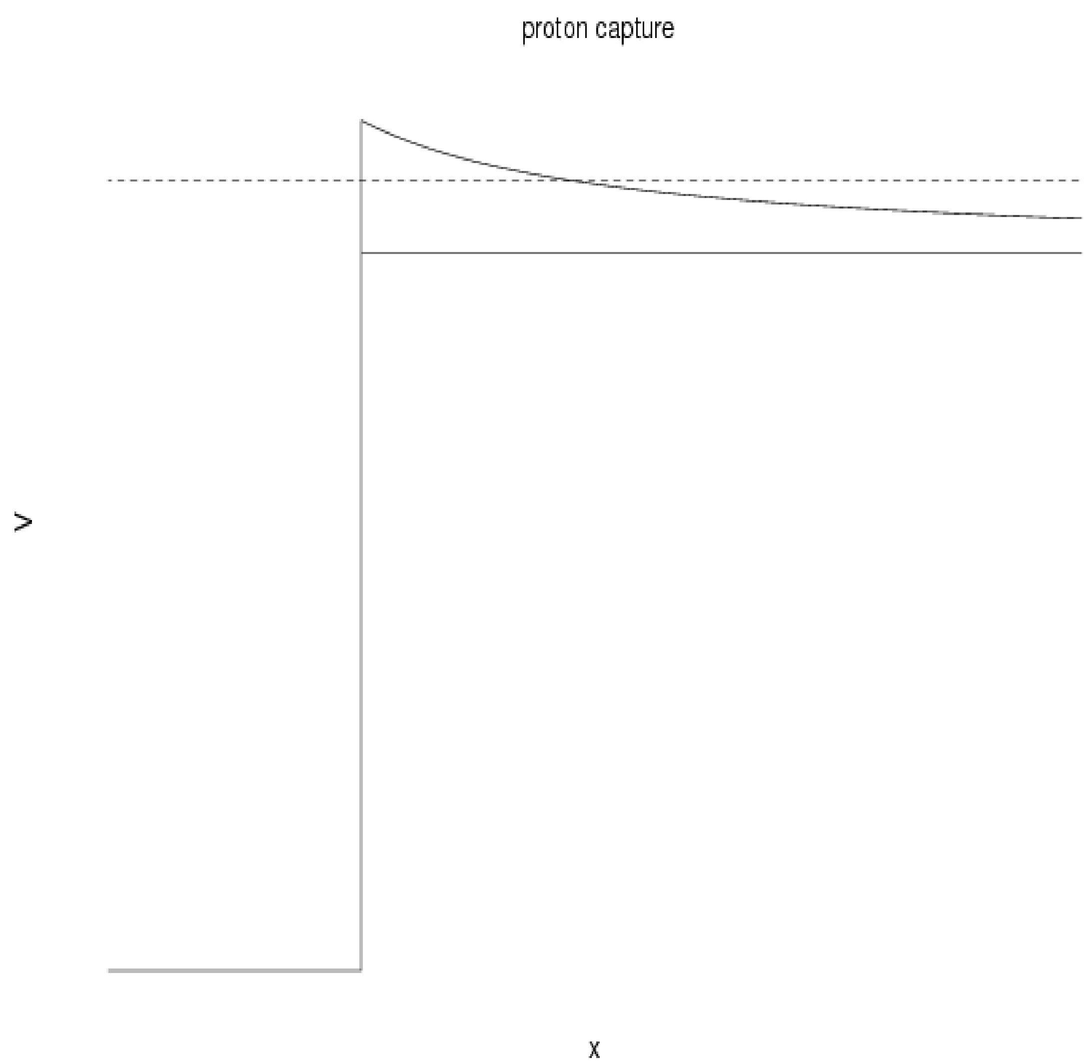
plane wave with momentum k_1 approaches nucleus, partially reflected and partially entering nucleus (inside nucleus momentum k_2).

$$T = \frac{k_2|A_2|^2}{k_1|A_1|^2} = \frac{4k_1k_2}{|k_1+k_2|^2}.$$

k_2 dominates over k_1 due to potential depth

$$T \approx \frac{4k_1}{k_2}$$

Cross sections with charged particles



$$T \approx e^{-\frac{2}{\hbar}\int_{x_1}^{x_2}\sqrt{2m(V(x)-E)}dx}$$

transmission coefficient from WKB approximation

$$T = e^{-2\pi\eta}$$

$$\eta = \sqrt{\frac{m}{2E}}\frac{Z_1 Z_2 e^2}{\hbar},$$

Sommerfeld parameter.

for Coulomb barrier penetration

cross sections for neutrons and charged particles

(i) neutrons $T_{n,0} \approx \frac{4k_1}{k_2}$

$$k_1 = \frac{\sqrt{2\mu E}}{\hbar} \qquad k_2 = \frac{\sqrt{2\mu(E+Q)}}{\hbar} \approx \text{const} \quad \text{for } E \ll Q$$

$$\Rightarrow \sigma = \frac{\pi}{k_1^2} \cdot 4\frac{k_1}{k_2} \propto \frac{1}{k_1}$$

$$\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

declining as function of bombarding energy

(ii) charged particle captures $T_{c,0} = e^{-2\pi\eta}$

$$\sigma = \frac{\pi}{k^2} e^{-2\pi\eta} = \frac{\hbar^2 \pi}{2\mu E} e^{-2\pi\eta}$$

$$\eta = \sqrt{\frac{\mu}{2E}} \frac{Z_i Z_j e^2}{\hbar.}$$

increasing as function of energy by orders of magnitude due to Coulomb penetration

Introduction to reaction rates

$$\sigma = \frac{\text{number of reactions target}^{-1}\text{sec}^{-1}}{\text{flux of incoming projectiles}} = \frac{r/n_i}{n_j v}$$

$$r = \sigma v n_i n_j$$

reaction rate r (per volume and sec) for a fixed bombarding velocity/energy (like in an accelerator)

$$r_{i;j} = \int \sigma \cdot |\vec{v}_i - \vec{v}_j| dn_i dn_j$$

for thermal distributions in a hot plasma

e.g. Maxwell-Boltzmann (nuclei/nucleons) or Planck (photons)

$$dn_j = n_j \frac{4\pi p_j^2}{(2\pi m_j kT)^{3/2}} \exp(-\frac{p_j^2}{2m_j kT}) dp_j$$

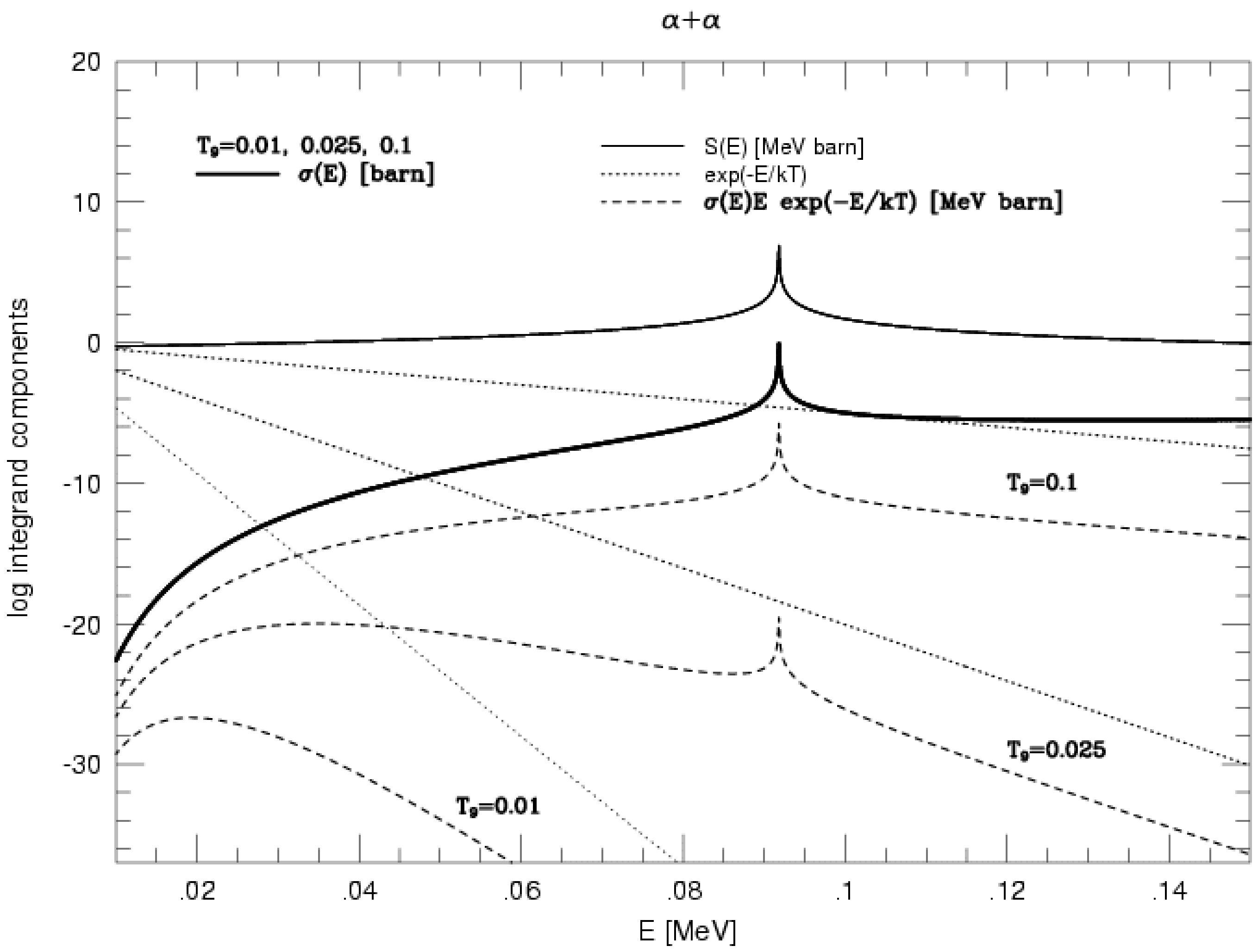
$$dn_\gamma = \frac{8\pi}{c^3} \frac{\nu^2 d\nu}{\exp(h\nu/kT) - 1} = \frac{1}{\pi^2 (c\hbar)^3} \frac{E_\gamma^2 dE_\gamma}{\exp(E_\gamma/kT) - 1}$$

for two MB-distributions for i and j one obtains after variable transformations

$$r_{i;j} = n_i n_j \langle \sigma v \rangle_{i;j}$$

$$\langle \sigma v \rangle (T) = \left(\frac{8}{\mu\pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\sigma(E)\ \exp(-E/kT) dE$$

Temperature dependence of rates



for neutron captures close to constant (at higher temperatures, i.e. higher velocities, multiplied with 1/v dependence of cross section)

for charged particles the contribution to the integral is strongly rising with temperature

Reaction networks

reaction i+j->m+o i(j,o)m with reaction rate

$$r_{i;j} = \frac{1}{1+\delta_{ij}} n_i n_j \langle \sigma v \rangle$$

(avoiding double counting for reactions of identical particles)

$$\left(\frac{\partial n_i}{\partial t}\right)_\rho = \left(\frac{\partial n_j}{\partial t}\right)_\rho = -r_{i;j}$$

$$\left(\frac{\partial n_o}{\partial t}\right)_\rho = \left(\frac{\partial n_m}{\partial t}\right)_\rho = +r_{i;j}$$

resulting changes in number densities of participating nuclei (for constant mass densities!)

Introducing abundances Y and mass fractions X

$$\rho = \frac{1}{V} = \sum_i n_i m_i = \sum_i \frac{n_i}{N_A} m_i N_A$$

$$Y_i = \frac{n_i}{\rho N_A}$$

$$1 = \frac{\rho}{\rho} = \sum_i \frac{n_i}{\rho N_A} m_i N_A = \sum_i Y_i A_i = \sum_i X_i$$

Reaction networks

i(j,o)m

$$\dot{Y}_i = \frac{1}{\rho N_A}\left(\frac{\partial n_i}{\partial t}\right)_\rho = -\frac{r_{i;j}}{\rho N_A} = -\frac{1}{1+\delta_{ij}}\rho N_A \langle\sigma v\rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_j = \frac{-1}{1+\delta_{ij}}\rho N_A \langle\sigma v\rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_o = \frac{1}{1+\delta_{ij}}\rho N_A \langle\sigma v\rangle_{i;j} Y_i Y_j$$

$$\dot{Y}_m = \frac{1}{1+\delta_{ij}}\rho N_A \langle\sigma v\rangle_{i;j} Y_i Y_j.$$

decay i->m

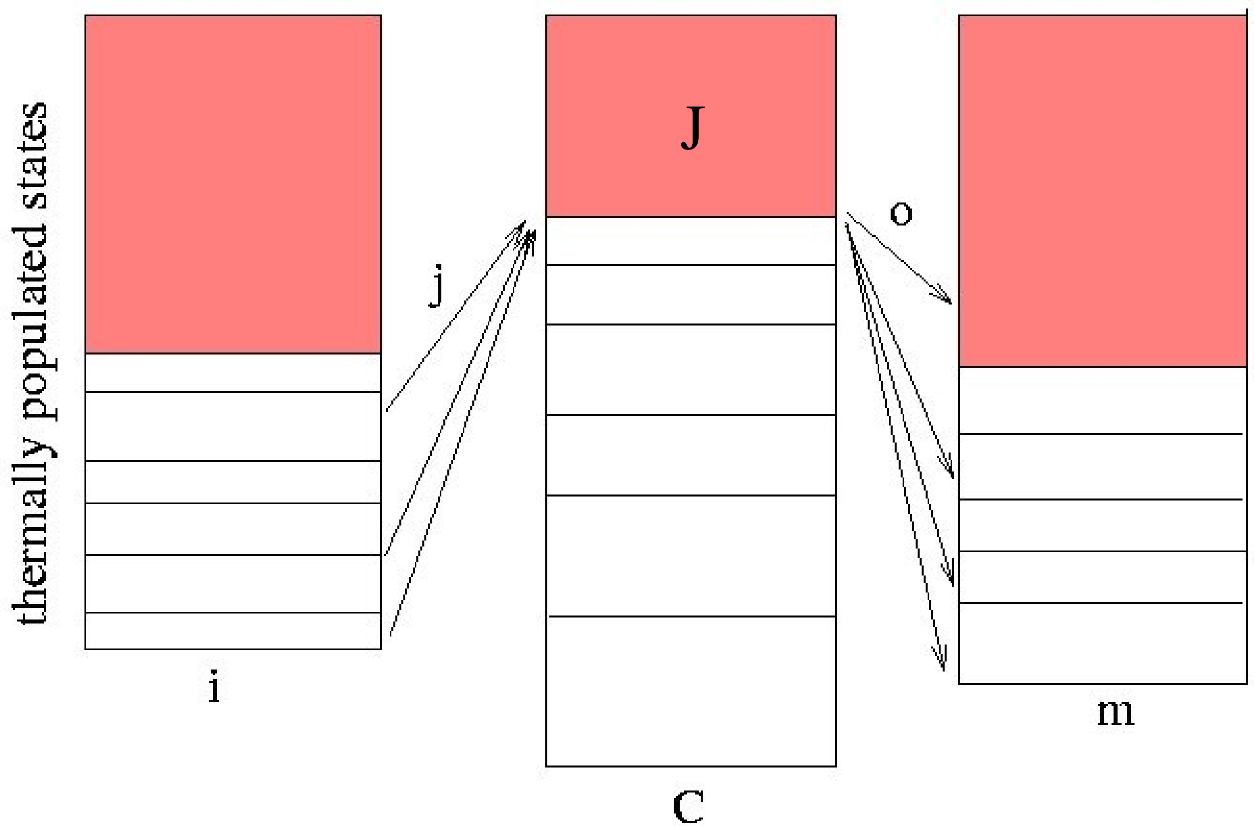
$$\dot{Y}_i = \left(\frac{\dot{n}_i}{\rho N_A}\right)_\rho = -\frac{r_i}{\rho N_A}$$

$$\Rightarrow \dot{Y}_i = -\lambda_i Y_i \qquad \dot{Y}_m = \lambda_i Y_i$$

general: N's count number of particles produced/distroyed in the reaction (positive/negative)

$$\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} \frac{N_{j,k}^i}{1+\delta_{jk}} \rho N_A < \sigma v >_{j;k} Y_j Y_k.$$

General compound cross section



$$\sigma_i(j,o) = \frac{\pi}{k_j^2}\frac{(1+\delta_{ij})}{(2I_i+1)(2I_j+1)}\sum_{J,\pi}(2J+1)\frac{T_j(E,J,\pi)T_o(E,J,\pi)}{T_{tot}(E,J,\pi)}$$

www.nucastro.org for statist. model cross sections

including spin and parity dependence

Reverse rates

$$\sigma_m(o,j)_J = \frac{\pi}{k_o^2}\frac{(1+\delta_{om})(2J+1)}{(2I_m+1)(2I_o+1)}\frac{T_oT_j}{T_{tot}}$$

$$\sigma_i(j,o)_J = \frac{\pi}{k_j^2}\frac{(1+\delta_{ij})(2J+1)}{(2I_i+1)(2I_j+1)}\frac{T_jT_o}{T_{tot}}$$

$$k_o = \frac{p_o}{\hbar} = \frac{\sqrt{2\mu_{om}E_{om}}}{\hbar} \qquad k_j = \frac{p_j}{\hbar} = \frac{\sqrt{2\mu_{ij}E_{ij}}}{\hbar}$$

$$g_x = (2I_x+1) \qquad E_{ij} = E_{om} + Q_{o,j}.$$

Detailed Balance

$$\frac{\sigma_i(j,o)_J}{\sigma_m(o,j)_J} = \frac{1+\delta_{ij}}{1+\delta_{om}}\frac{g_og_m}{g_ig_j}\frac{k_o^2}{k_j^2}$$

going through a specific state J in the compound nucleus

but true for any state at that energy

$$\sigma_i(j,o;E_{ij}) = \frac{1+\delta_{ij}}{1+\delta_{om}}\frac{g_og_m}{g_ig_j}\frac{k_o^2}{k_j^2}\sigma_m(o,j;E_{om})$$

Reverse rates

$$\langle\sigma v\rangle_{i;j,o} = \frac{1+\delta_{ij}}{1+\delta_{om}} \left(\frac{8}{\mu_{ij}\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E_{ij} \frac{g_o g_m}{g_i g_j} \frac{k_o^2}{k_j^2} \sigma_m(o,j;E_{om})$$

$$\times \ \exp(-E_{ij}/kT)dE_{ij}$$

$$= \frac{1+\delta_{ij}}{1+\delta_{om}} \frac{g_o g_m}{g_i g_j} \left(\frac{\mu_{om}}{\mu_{ij}}\right)^{3/2} \ \exp(-Q_{o,j}/kT)$$

$$\times \left(\frac{8}{\mu_{om}\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E_{om}\sigma_m(o,j;E_{om}) \ \exp(-E_{om}/kT)dE_{om}$$

$$\langle\sigma v\rangle_{i;j,o} = \frac{1+\delta_{ij}}{1+\delta_{om}} \frac{G_m g_o}{G_i g_j} \left(\frac{\mu_{om}}{\mu_{ij}}\right)^{3/2} \ \exp(-Q_{o,j}/kT) \ \langle\sigma v\rangle_{m;o,j}$$

containing the Q-value of the reaction (nuclear mass differences)

Reverse photodisintegrations

$$r_{i\gamma} = n_i \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{\sigma_i(\gamma, o; E_\gamma) E_\gamma^2}{\exp(E_\gamma/kT) - 1} dE_\gamma$$
$$= n_i \lambda_{i;\gamma,o}(T)$$

$$\lambda_{i;\gamma,o}(T) = \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{\sigma_i(\gamma, o; E_\gamma) E_\gamma^2}{\exp(E_\gamma/kT) - 1} dE_\gamma.$$

photodisintegration rates only T-dependent!

$$k_\gamma = \frac{w}{c} = \frac{\hbar w}{\hbar c} = \frac{E_\gamma}{\hbar c} \qquad g_\gamma = 2$$

$$k_o = \frac{p}{\hbar} = \frac{\sqrt{2\mu_{om} E_{om}}}{\hbar} \qquad E_\gamma = E_{om} + Q_{o,\gamma}$$

$$\sigma_i(\gamma, o; E_\gamma) = \frac{g_o g_m}{(1+\delta_{om}) g_i} c^2 \frac{\mu_{om} E_{om}}{E_\gamma^2} \sigma_m(o, \gamma; E_{om})$$

$$\lambda_{i;\gamma,o} = \frac{1}{\pi^2 c^2 \hbar^3} \int_0^\infty \frac{g_o g_m}{(1+\delta_{om}) g_i} c^2 \frac{\mu_{om} E_{om}}{E_\gamma^2} \sigma_m(o, \gamma; E_{om}) E_\gamma^2$$
$$\times \exp(-E_\gamma/kT) dE_\gamma$$
$$= \frac{1}{\pi^2 \hbar^3} \frac{g_o g_m}{g_i} \mu_{om} \exp(-Q_{o,\gamma}/kT)$$
$$\times \int_0^\infty E_{om} \sigma_m(o, \gamma; E_{om}) \exp(-E_{om}/kT) dE_{om}$$

$$\lambda_{i;\gamma,o}(T) = \frac{g_o G_m}{(1+\delta_{om}) G_i} \left(\frac{\mu_{om} kT}{2\pi\hbar^2}\right)^{3/2} \exp(-Q_{o,\gamma}/kT) \langle\sigma v\rangle_{m;o} \square$$

relation between photodisintegration rate and reverse capture rate

Reaction equilibria

reaction network for i(j,o)m

$$\dot{Y}_i = \dot{Y}_j = -\rho N_A \langle\sigma v\rangle_{i;j,o} Y_i Y_j + \rho N_A \langle\sigma v\rangle_{m;o,j} Y_o Y_m$$

$$\dot{Y}_m = \dot{Y}_o = -\dot{Y}_i$$

$$\dot{Y}_i = \dot{Y}_j = -\rho N_A \langle\sigma v\rangle_{i;j,\gamma} Y_i Y_j + \lambda_{m;\gamma,j} Y_m$$

$$\dot{Y}_m = -\dot{Y}_i.$$

in this case o is a photon

if forward and backward reaction are in equilibrium, we have for all indices

$$\dot{Y} = 0$$

this leads to the following abundance relations

$$\frac{Y_m}{Y_i} = \frac{Y_j}{Y_o}\frac{\langle\sigma v\rangle_{i;j,o}}{\langle\sigma v\rangle_{m;o,j}} = \frac{Y_j}{Y_o}\frac{g_o G_m}{g_j G_i}\left(\frac{m_o m_m}{m_i m_j}\right)^{3/2} \exp(Q_{j,o}/kT)$$

$$\frac{Y_m}{Y_i} = \frac{\rho N_A \langle\sigma v\rangle_{i;j,\gamma}}{\lambda_{m;\gamma,j}} Y_j = \rho N_A Y_j \frac{G_m}{g_j G_i}\left(\frac{m_m}{m_i m_j}\right)^{3/2}\left(\frac{2\pi\hbar^2}{kT}\right)^{3/2} \exp(Q_{j,\gamma}/kT)$$

The same results would have been obtained, if the equations for chemical equilibria would have been utilized which include the chemical potentials!!

Nuclear Statistical Equilibriuim (NSE)

$$\bar{\mu}(Z,N) + \bar{\mu}_n = \bar{\mu}(Z,N+1)$$
$$\bar{\mu}(Z,N) + \bar{\mu}_p = \bar{\mu}(Z+1,N)$$

i.e. neutron or proton captures on nucleus (Z,N) are in chemical equilibium with the reverse photodisintegrations.
If this is the case for all neutron and proton captures on all nuclei (hot enough to overcome all Coulomb barriers as well as having high energy photons...) this leads to

$$N\,\text{neutrons} + Z\ \text{protons} \rightleftharpoons (Z,N)$$

$$N\bar{\mu}_n + Z\bar{\mu}_p = \bar{\mu}_{Z,N}.$$

with

$$\bar{\mu}_i = kT\ln\left(\frac{\rho N_A Y_i}{G_i}\left(\frac{2\pi\hbar^2}{m_i kT}\right)^{3/2}\right) + m_i c^2$$

Solving NSE

$$kT \ln\left(\frac{\rho N_A Y(Z,N)}{G_{Z,N}}\left(\frac{2\pi\hbar^2}{m_{Z,N}kT}\right)^{3/2}\right) + m_{Z,N}c^2$$

$$= N\left[kT\ln\left(\frac{\rho N_A Y_n}{g_n}\left(\frac{2\pi\hbar^2}{m_n kT}\right)^{3/2}\right) + m_n c^2\right]$$

$$+Z\left[kT\ln\left(\frac{\rho N_A Y_p}{g_p}\left(\frac{2\pi\hbar^2}{m_p kT}\right)^{3/2}\right) + m_p c^2\right]$$

$$\times \ln\left(\frac{\rho N_A Y(Z,N)}{G_{Z,N}}\left(\frac{2\pi\hbar^2}{m_{Z,N}kT}\right)^{3/2}\right)$$

$$-N\ln\left(\frac{\rho N_A Y_n}{g_n}\left(\frac{2\pi\hbar^2}{m_n kT}\right)^{3/2}\right) - Z\ln\left(\frac{\rho N_A Y_p}{g_p}\left(\frac{2\pi\hbar^2}{m_p kT}\right)^{3/2}\right)$$

$$= \frac{1}{kT}(N m_n c^2 + Z m_p c^2 - m_{Z,N}c^2) = B_{Z,N}/kT.$$

Solving NSE

with $A = N + Z \quad m_n \approx m_u \quad m_p \approx m_u \quad m_{Z,N} \approx A m_u$

this leads to

$$Y(Z,N) = G_{Z,N} (\rho N_A)^{A-1} \frac{A^{3/2}}{2^A} \left(\frac{2\pi\hbar^2}{m_u kT} \right)^{\frac{3}{2}(A-1)} \exp(B_{Z,N}/kT) Y_n^N Y_p^Z$$

and can be solved via two equations (mass conservation and total proton to nucleon ratio Y_e) for neutron and proton abundances

$$\sum_i A_i Y_i = 1$$

$$\sum_i Z_i Y_i = Y_e$$

Brief Summary of Burning Stages (Major Reactions)

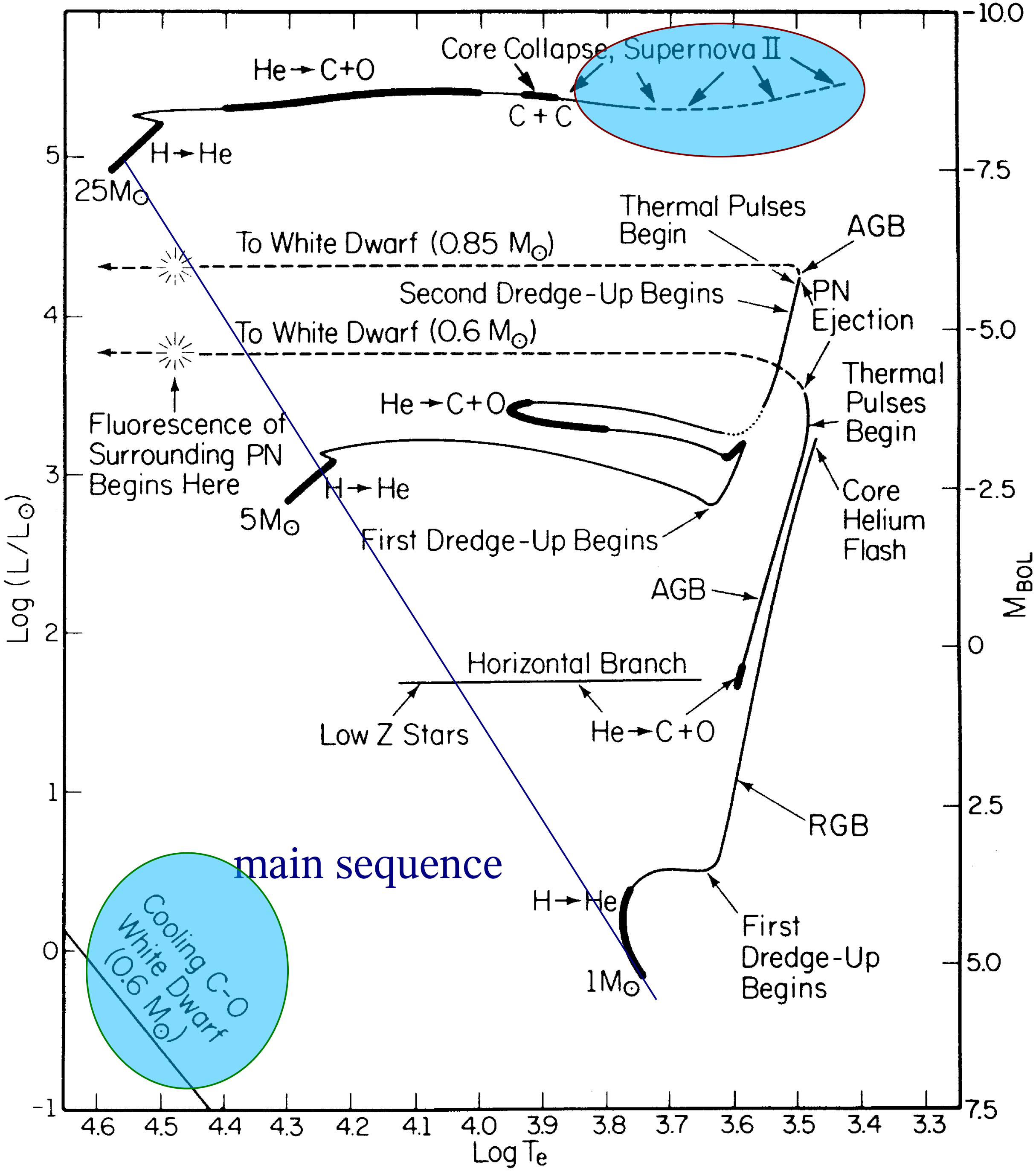
1. Hydrogen Burning — $T = (1\text{-}4)\times10^7$K
   - pp-cycles -> $^1H(p,e^+\nu)^2H$
   - CNO-cycle -> slowest reaction $^{14}N(p,\gamma)^{15}O$
2. Helium Burning — $T=(1\text{-}2)\times10^8$K
   - $^4He+^4He \Leftrightarrow {}^8Be$ — $^8Be(\alpha,\gamma)^{12}C[(\alpha,\gamma)^{16}O]$
   - $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)^{25}Mg$ (n-source, alternatively $^{13}C(\alpha,n)^{16}O$)
3. Carbon Burning — $T=(6\text{-}8)\times10^8$K
   - $^{12}C(^{12}C,\alpha)^{20}Ne$ — $^{23}Na(p,\alpha)^{20}Ne$
   - $^{12}C(^{12}C,p)^{23}Na$ — $^{23}Na(p,\gamma)^{24}Mg$
4. Neon Burning — $T=(1.2\text{-}1.4)\times10^9$K
   - $^{20}Ne(\gamma,\alpha)^{16}O$
   - $^{20}Ne(\alpha,\gamma)^{24}Mg[(\alpha,\gamma)^{28}Si]$ — 30kT = 4MeV
5. Oxygen Burning — $T=(1.5\text{-}2.2)\times10^9$K
   - $^{16}O(^{16}O,\alpha)^{28}Si$ — $^{31}P(p,\alpha)^{28}Si$
   - $......,p)^{31}P$ $...,n)^{31}S(\beta^+)^{31}P$ — $^{31}P(p,\gamma)^{23}S$
6. "Silicon" Burning — $T=(3\text{-}4)\times10^9$K

(all) photodisintegrations and capture reactions possible

⇒ thermal (chemical) equilibrium

ongoing measurements of key fusion reactions at low energies

Astrophysical Sites



Hertzsprung-Russell Diagram of Stellar Evol-ution from Iben, showing as end stages

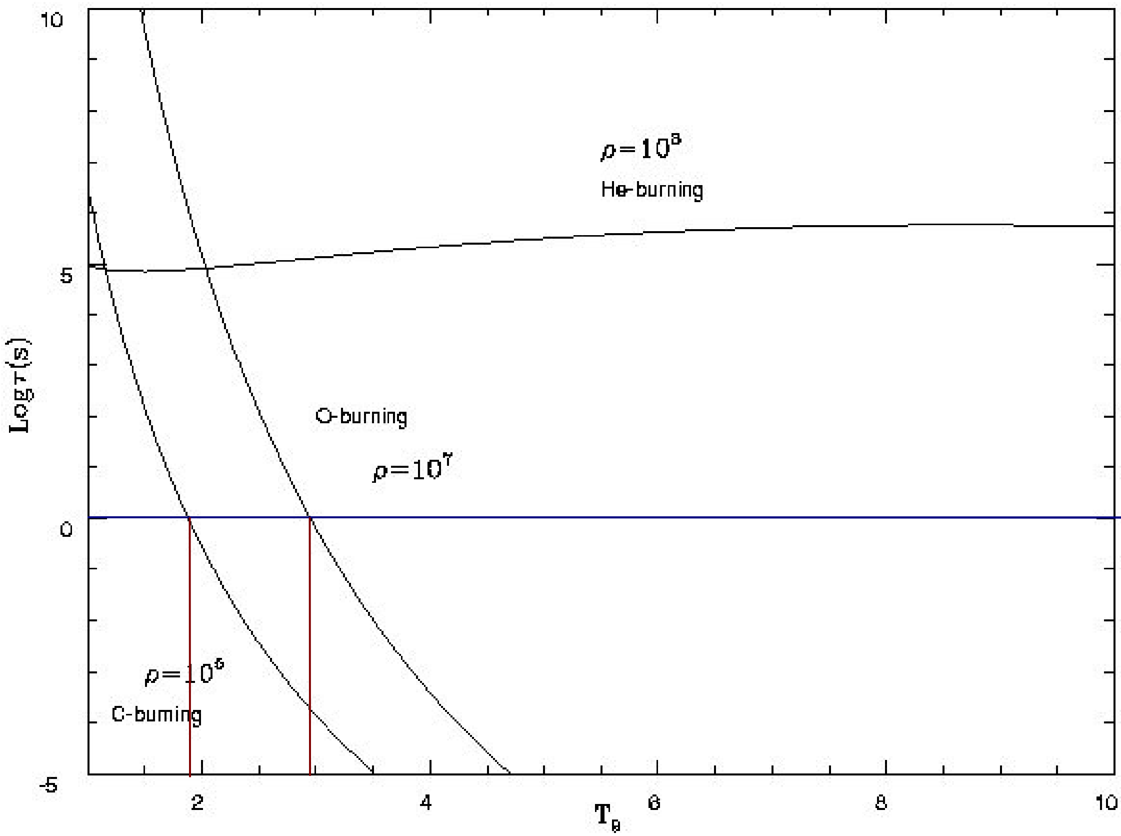
- white dwarfs

and

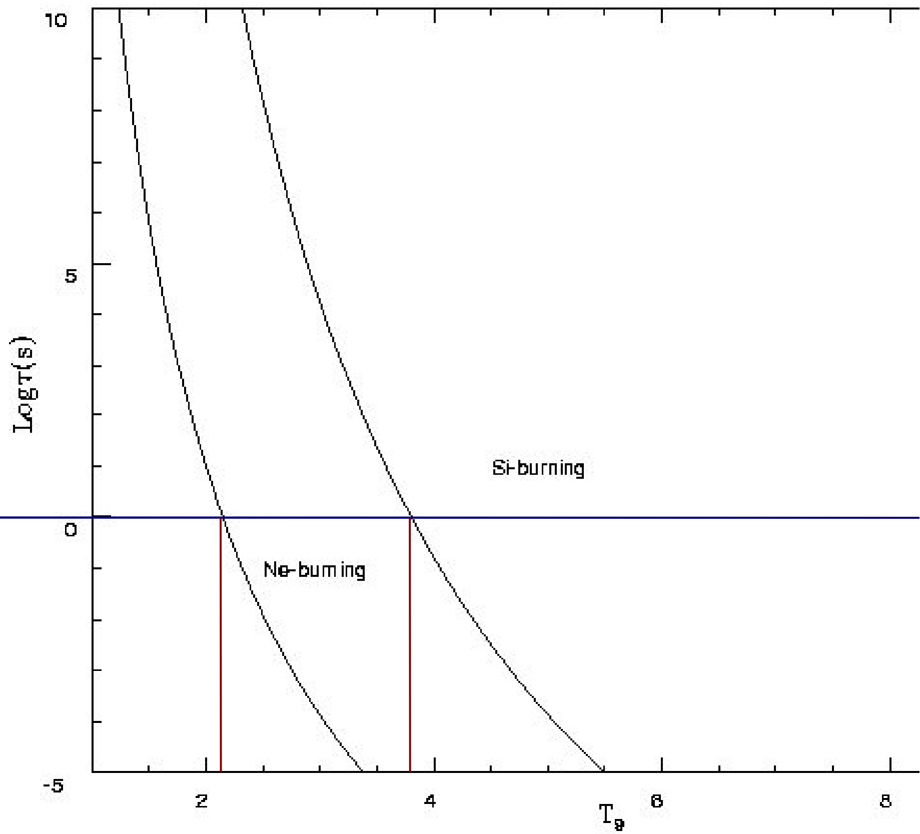
- core collapse (supernovae/neutron stars, black holes, GRBs?)

influence of reaction cross sections, e-capture in late burning stages, metallicity, rotation, magnetic fields, stellar winds on final outcome

Explosive Burning



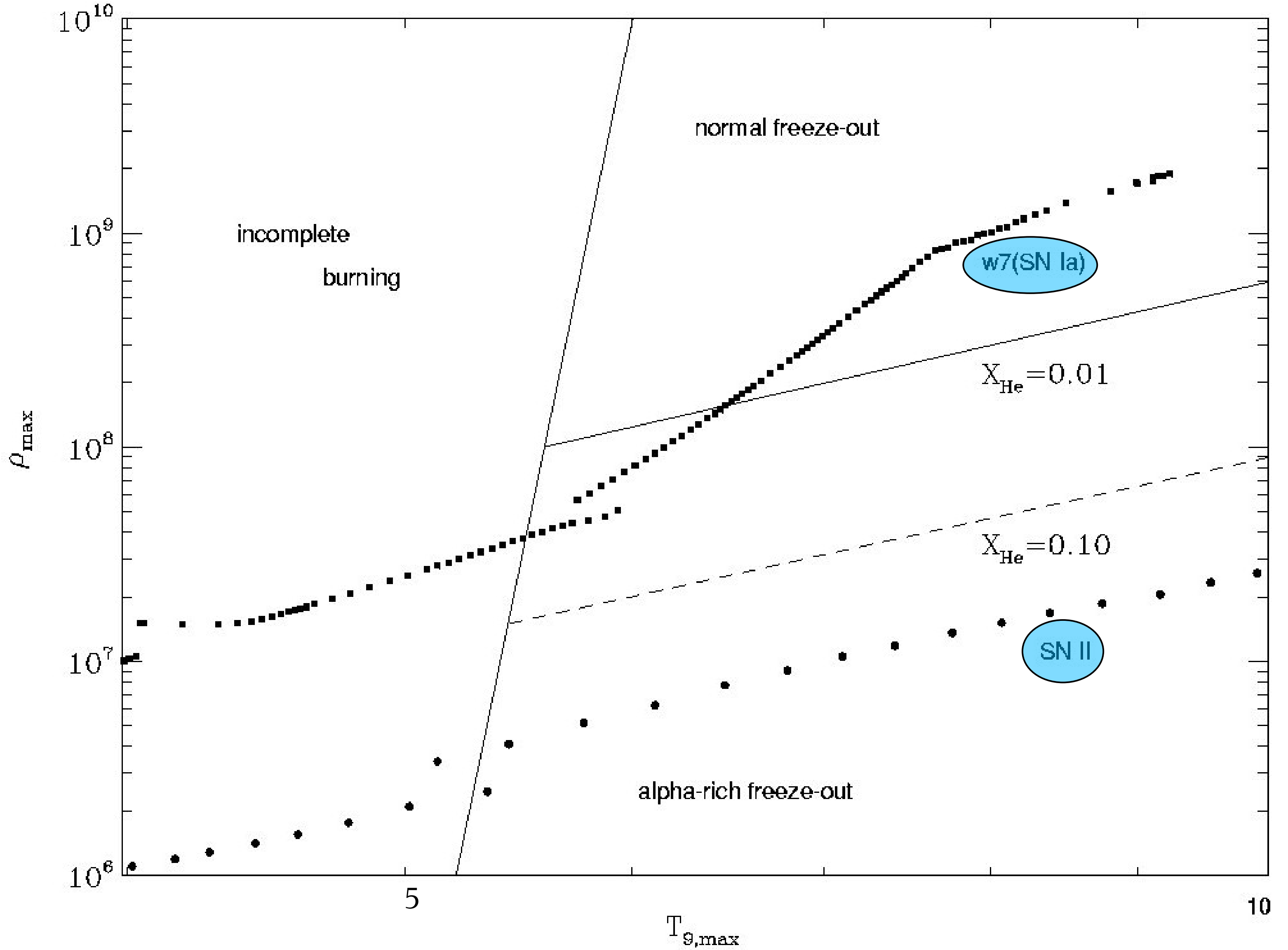
Ne C



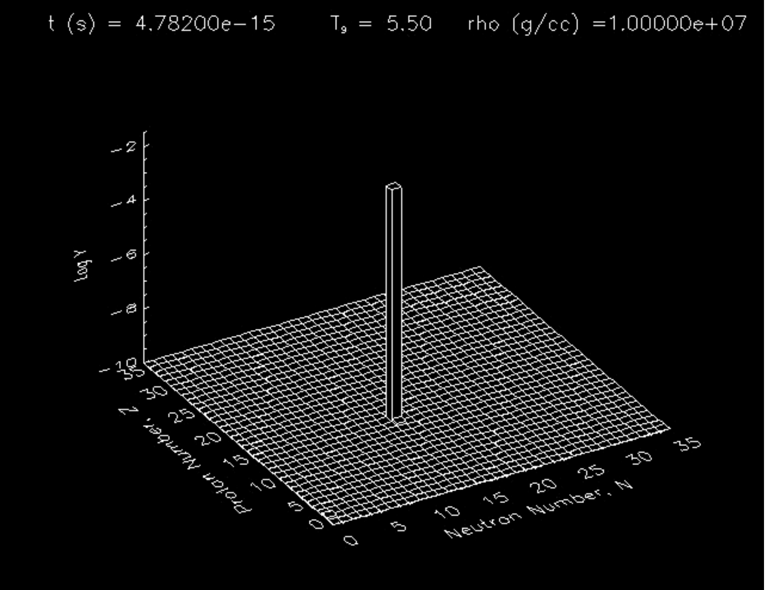
Ne Si

typical explosive burning process timescale order of seconds: fusion reactions (He, C, O) density dependent (He quadratic, C,O linear) photodisintegrations (Ne, Si) not density dependent

Explosive Si-Burning



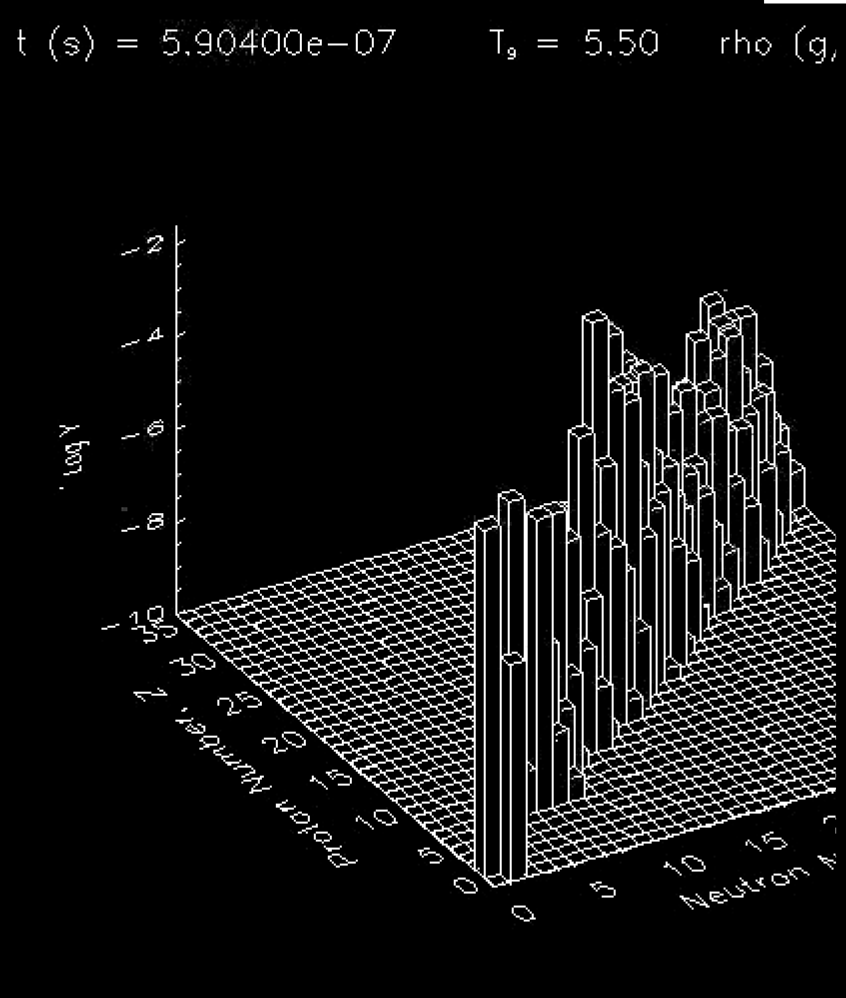
Explosive Burning above a critical temperature destroys (photodisintegrates) all nuclei and (re-)builds them up during the expansion. Dependent on density, the full NSE is maintained and leads to only Fe-group nuclei (normal freeze-out) or the reactions linking ^{4}He to C and beyond freeze out earlier (alpha-rich freeze-out).

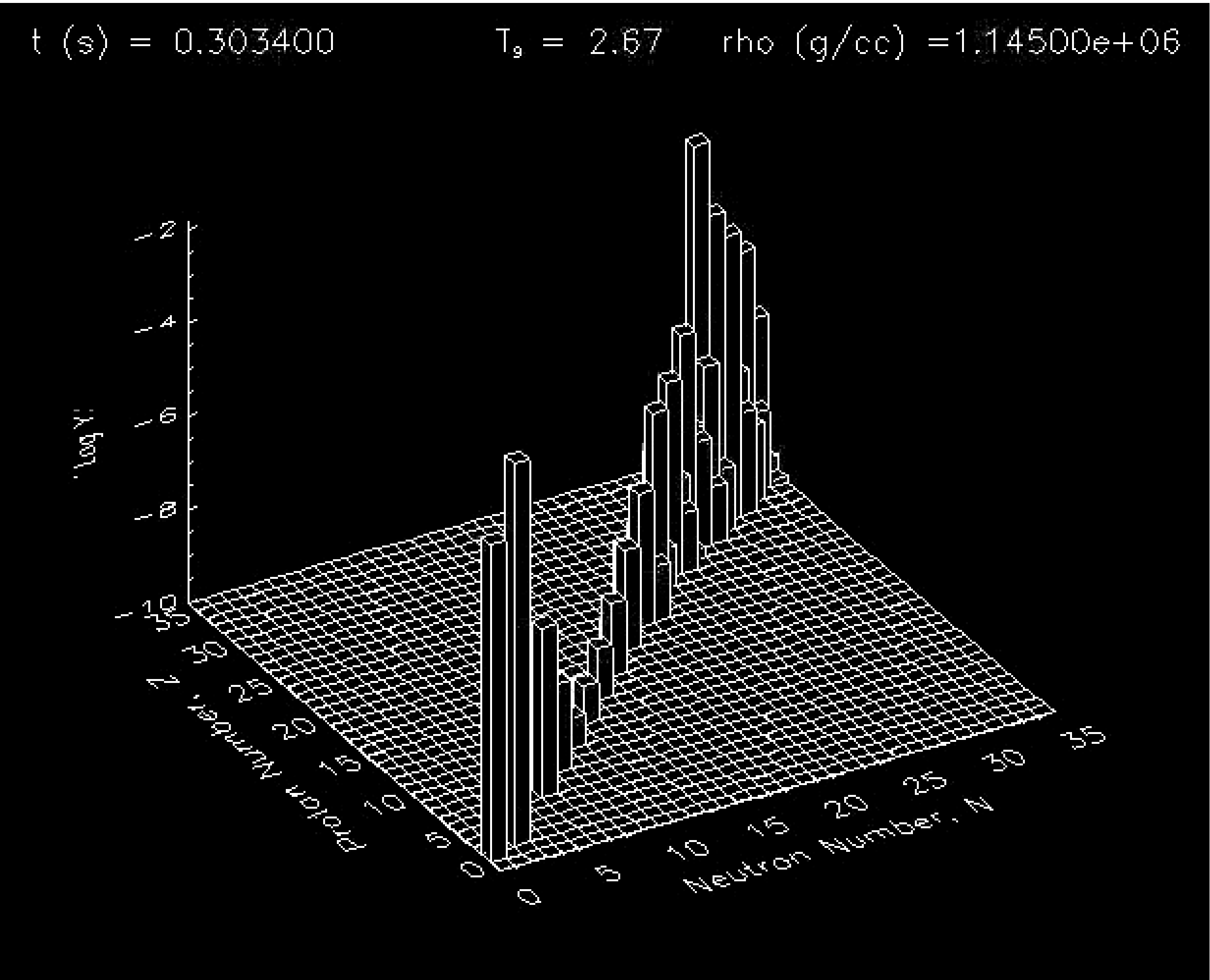


Explosive Si-burning

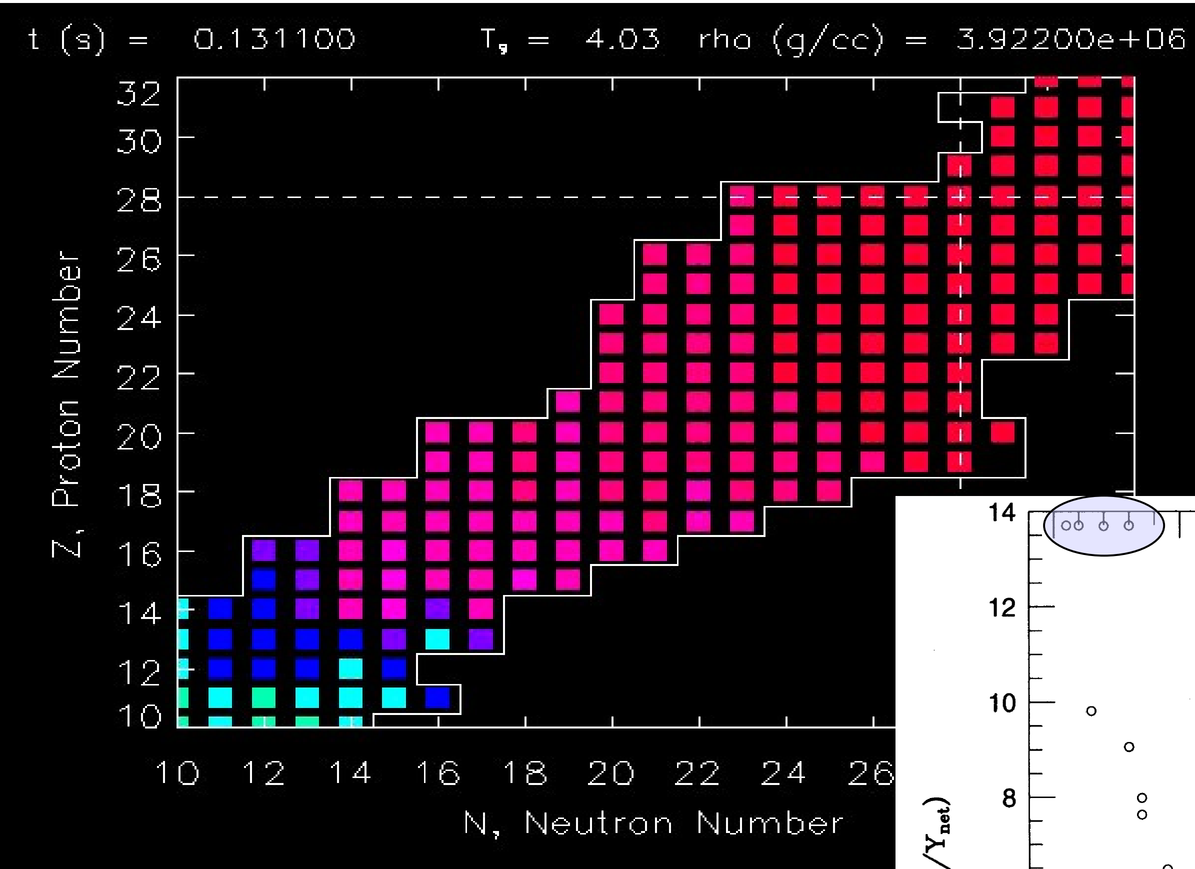
initially only ^{28}Si, fully burned, finally alpha-rich freeze-out

visualization: B.S. Meyer

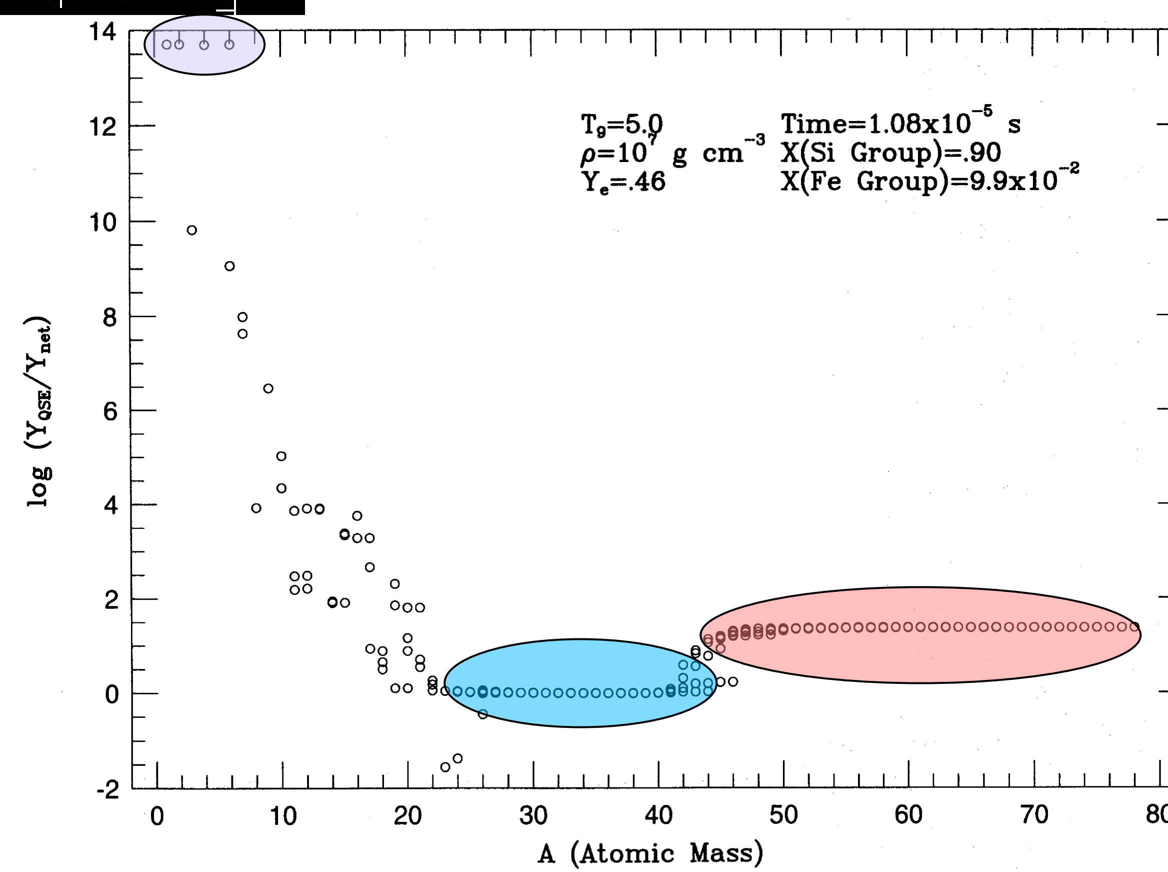




Quasi-Equilibrium (QSE)



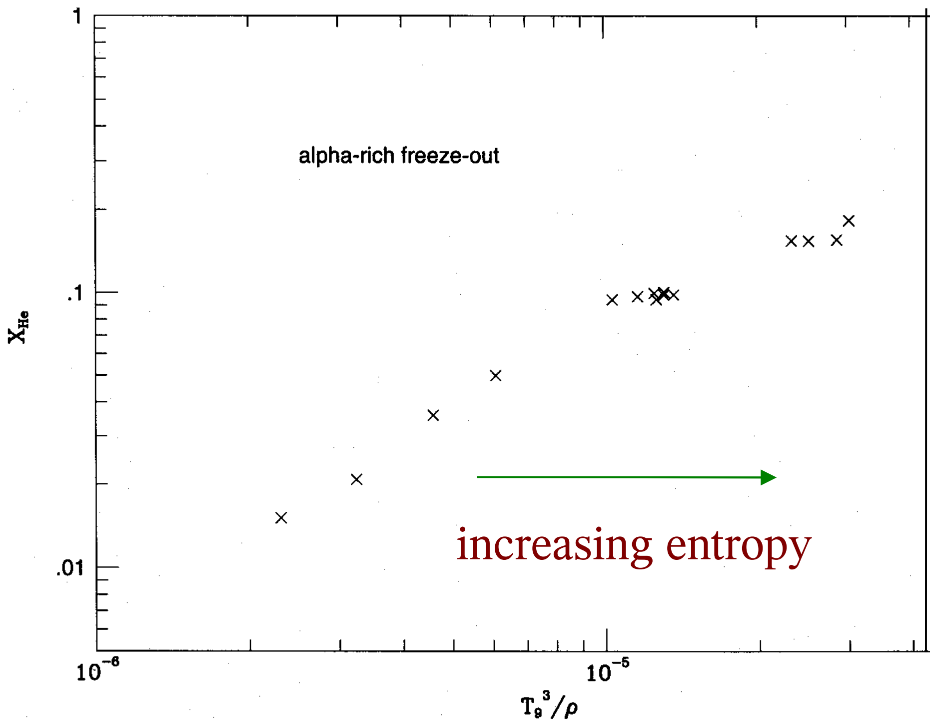
full NSE is not attained, but there exist equilibrium groups around ^{28}Si, ^{56}Ni and n,p,^{4}He, which are separated by slow reactions

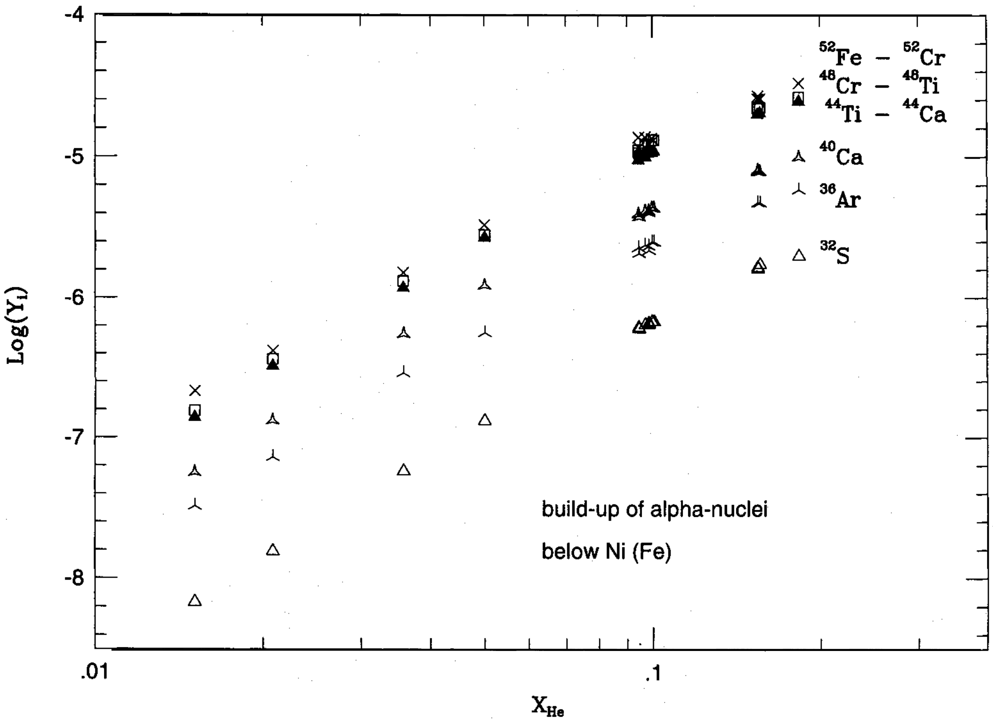


sample Calculations from

- B.S. Meyer and
- Hix and Thielemann
- Hix, Parete-Koon, Freiburghaus, FKT (2007, advantages in 3D nucleosynthesis)

alpha-rich freeze-out





Thielemann et al. (1996) increasing entropy

alpha-rich freeze-out occurs at high temperatures and/or low densities and is a function of entropy S in radiation-dominated matter

- it leads to the enhancement of "alpha-elements"
- and also to the extension of the Fe-group to higher masses (^{56}Ni to ^{64}Ge and for very high entropies up to A=80)

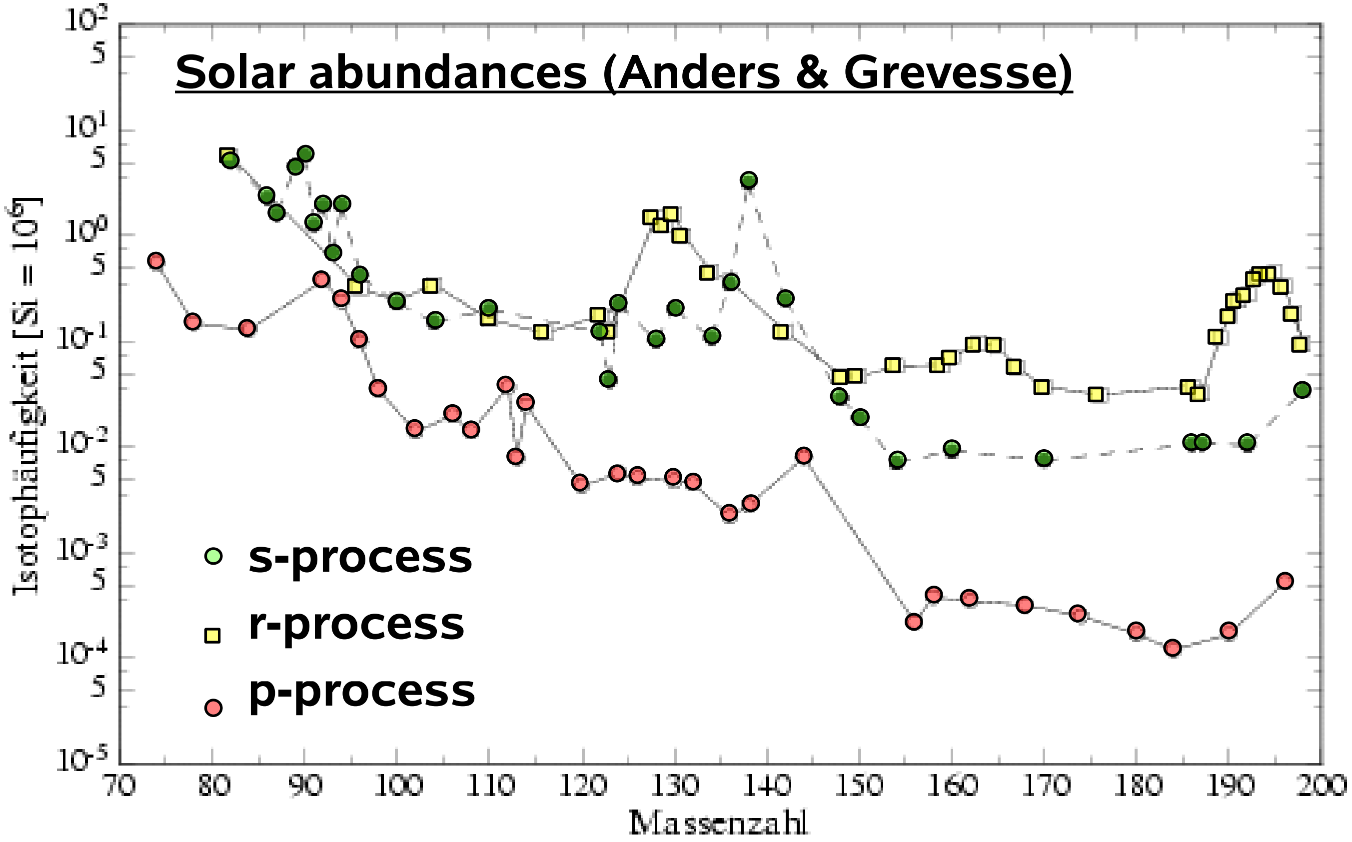
“Historical” Burning Processes (B^2FH)

- H-Burning
- He-Burning
- alpha-Process
- e-Process
- s-Process
- r-Process
- p-Process
- x-Process

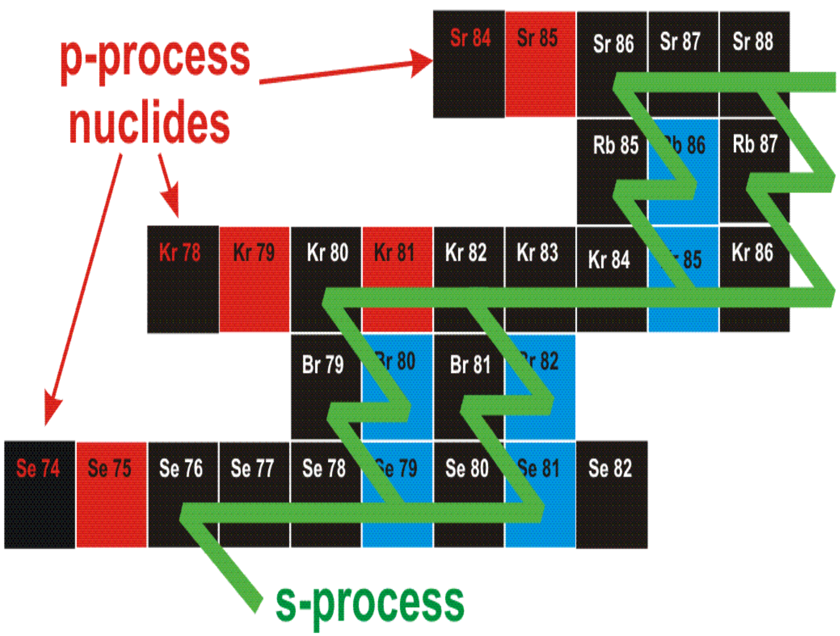
Present Understanding

- H-Burning
- He-Burning
- expl. C, Ne, O-Burning, incomplete Si-Burning
- explosive Si-Burning
- about 70% normal freeze-out, Y_e=0.42-47, about 30% alpha-rich freeze-out, Y_e=0.5
- s-Process (core and shell He-burning, neutrons from alpha-induced reactions on ^{22}Ne and ^{13}C)
- r-Process (see below)
- p-Process (see below)
- x-Process (light elements D, Li, Be, B [big bang, cosmic ray spallation and neutrino nucleosynthesis])
- rp-Process and νp-Process not yet known

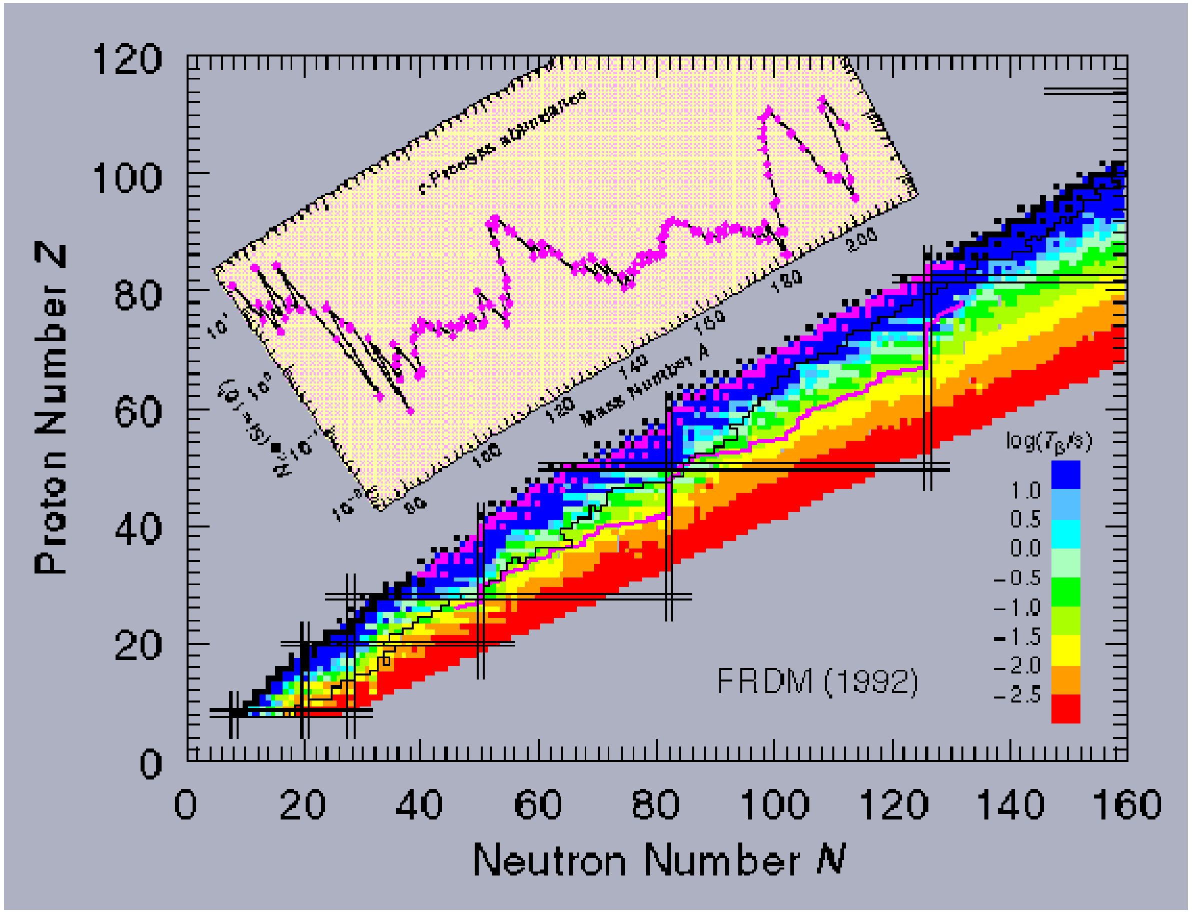
The Heavy Elements



s-, r- and p-Process

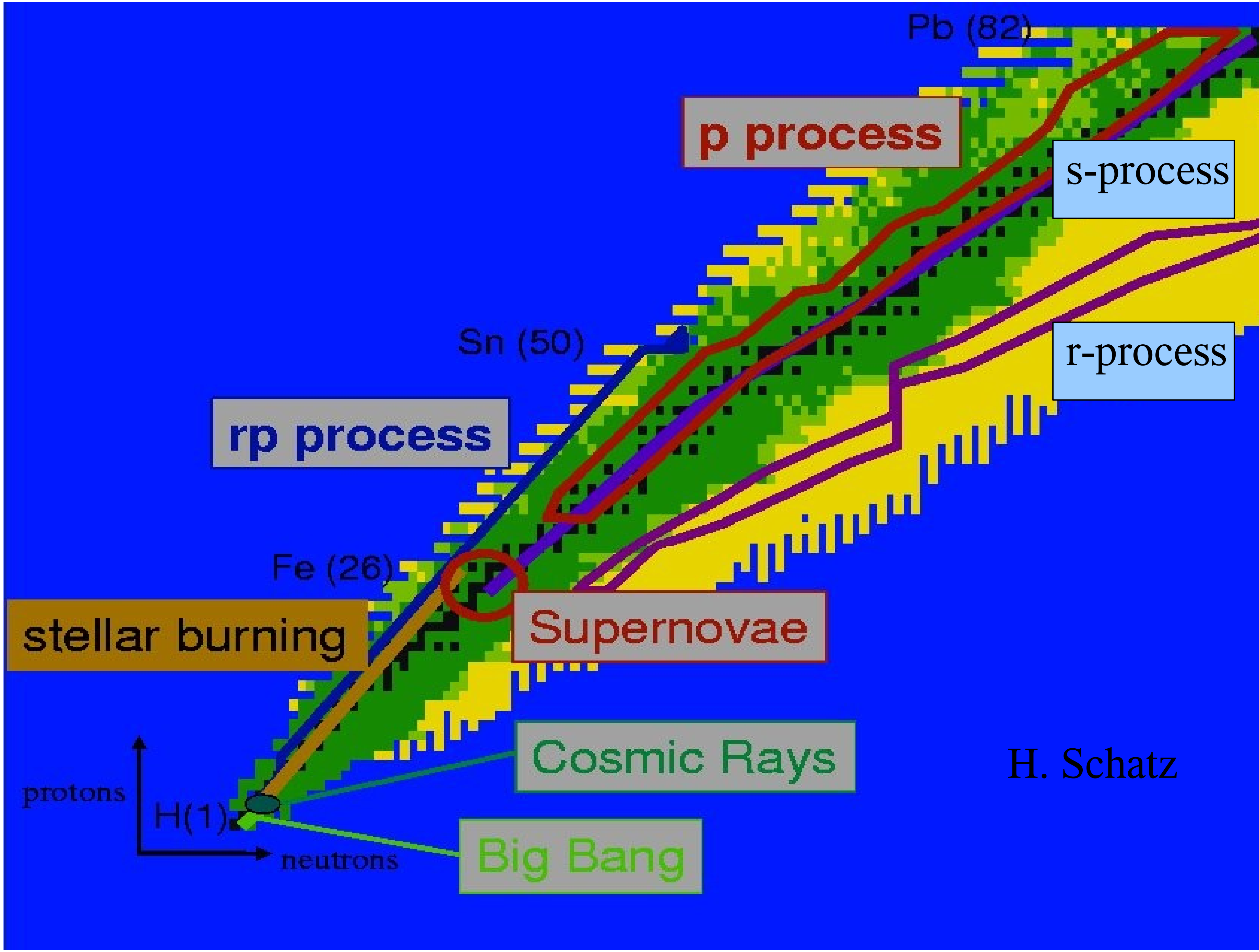


F. Käppeler



P. Möller

Processes in the Nuclear Chart



Types of Equilibria

- Steady Flow of Reactions
- Chemical Equilibrium of Reactions
- Complete Chemical Equilibrium (NSE)
- Clusters of Chemical Equilbrium (QSE)
- QSE Clusters linked by Steady Flow

CNO(I)-Cycle in Steady Flow

The CNO-Cycles in Hydrogen Burning

cycle	reaction sequence
CNOI	$^{12}C(p,\gamma)^{13}N(e^+\nu)^{13}C(p,\gamma)^{14}N\ (p,\gamma)^{15}O(e^+\nu)^{15}N(p,\alpha)^{12}C$
CNOII	$^{15}N(p,\gamma)^{16}O(p,\gamma)^{17}F\ (e^+\nu)^{17}O(p,\alpha)^{14}N$
CNOIII	$^{17}O(p,\gamma)^{18}F(e^+\nu)^{18}O(p,\alpha)^{15}N$
CNOIV	$^{18}O(p,\gamma)^{19}F(p,\alpha)^{16}O$

the network entry for nuclei with mass numbers A=12, 13, 14, 15 is governed in each case by a production reaction (proton reaction on A-1) and a distruction reaction (proton reaction on A). In case of a steady flow they cancel and lead to Y=0 for all A, linking all of these terms and identical to (A=14 is useful as this encounters the slowest reaction and essentially all mass assembles in ^{14}N)

$$
\begin{aligned}
\dot{Y}_1 &= -\rho N_A \langle 12,1\rangle Y_{12}Y_1 - \rho N_A \langle 13,1\rangle Y_{13}Y_1 - \rho N_A \langle 14,1\rangle Y_{14}Y_1 \\
&\quad - \rho N_A \langle 15,1\rangle Y_{15}Y_1 \\
&= -4C_{CNO} = -4\rho N_A \langle 14,1\rangle Y_{14}Y_1 = -\frac{1}{\tau_{1,14}}Y_1
\end{aligned}
$$

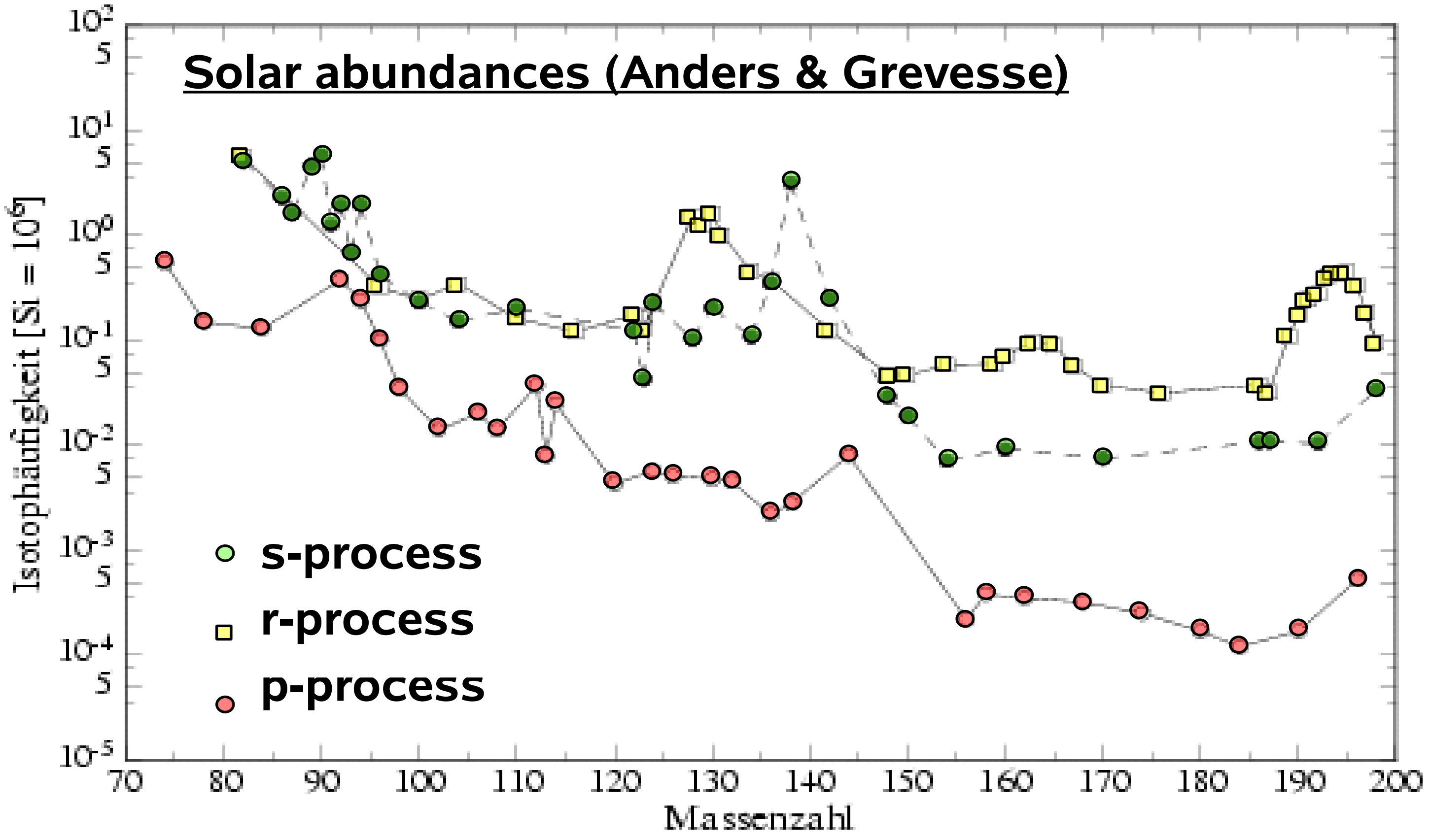
$$\dot{Y}_4 = \rho N_A \langle 15,1\rangle Y_{15}Y_1 = C_{CNO}$$

$$C_{CNO} = \rho N_A \langle 14,1\rangle Y_{14}Y_1$$

$$Y_{14} \approx \frac{1.4\times 10^{-2}}{14}$$

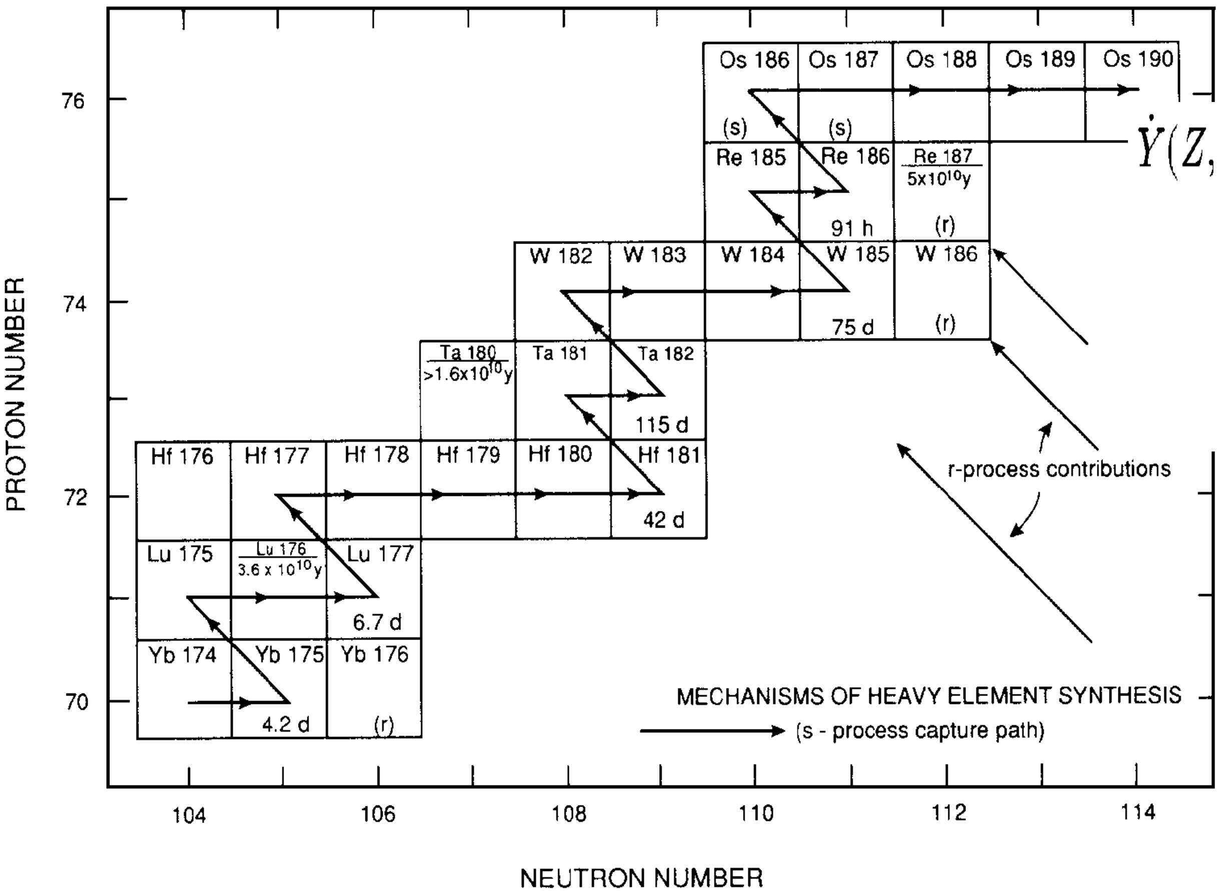
summing all mass fractions of CNO nuclei for solar metallicity

s-process and steady flow



shown are s-, r-, and p-only nuclei!

s-process and steady flow



possible destruction of nucleus (Z,A)

$$\dot{Y}(Z,A) = -\lambda_{\beta^-}(Z,A)Y(Z,A) - \rho N_A < \sigma v >_{n,\gamma} Y_n Y(Z,A)$$
$$= -\lambda_{\beta^-}(Z,A)Y(Z,A) - < \sigma v >_{n,\gamma} n_n Y(Z,A)$$
$$= -\frac{1}{\tau_\beta}Y(Z,A) - \frac{1}{\tau_{n,\gamma}}Y(Z,A).$$

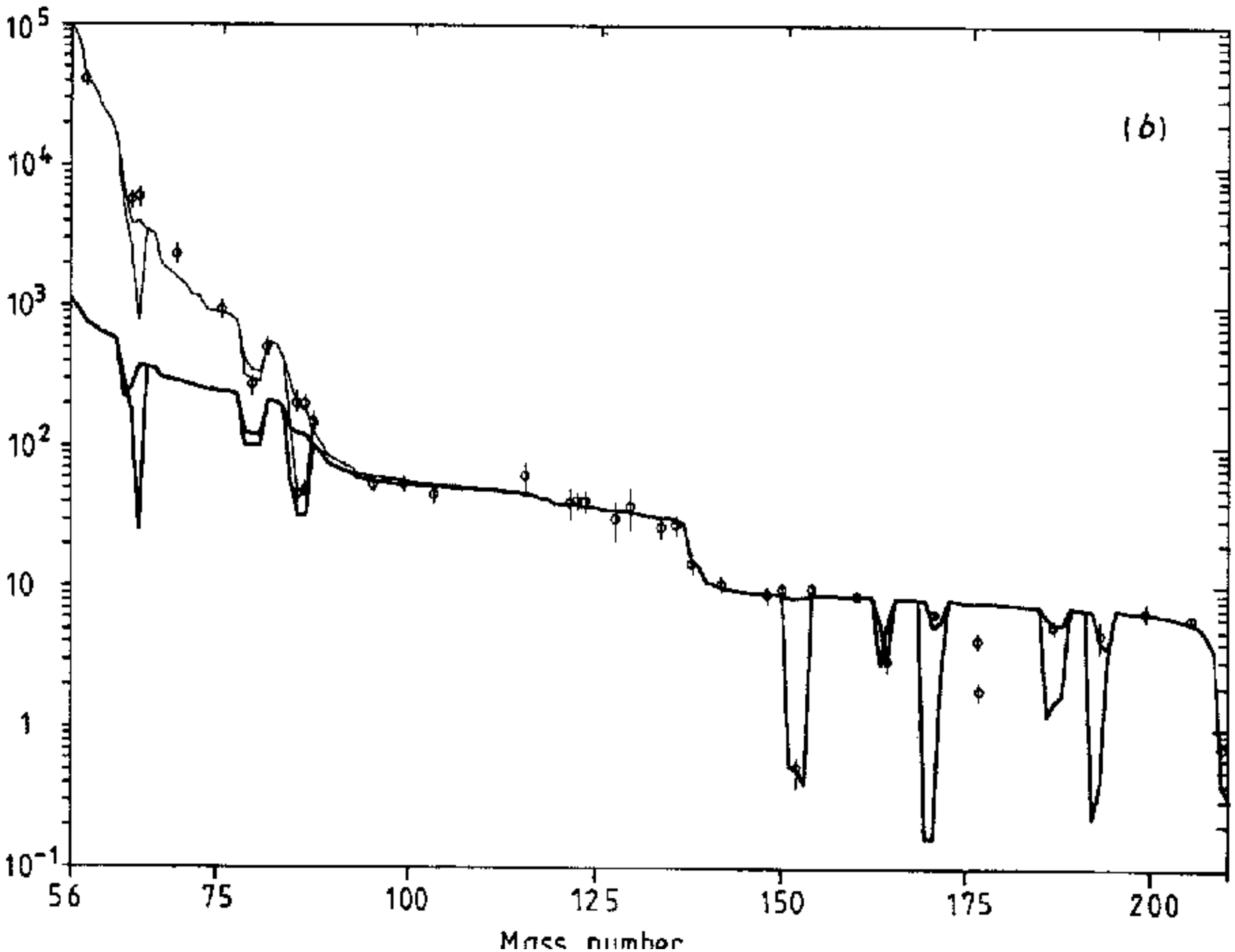
$\tau_n > \tau_\beta$ beta-decay to (Z+1,A)

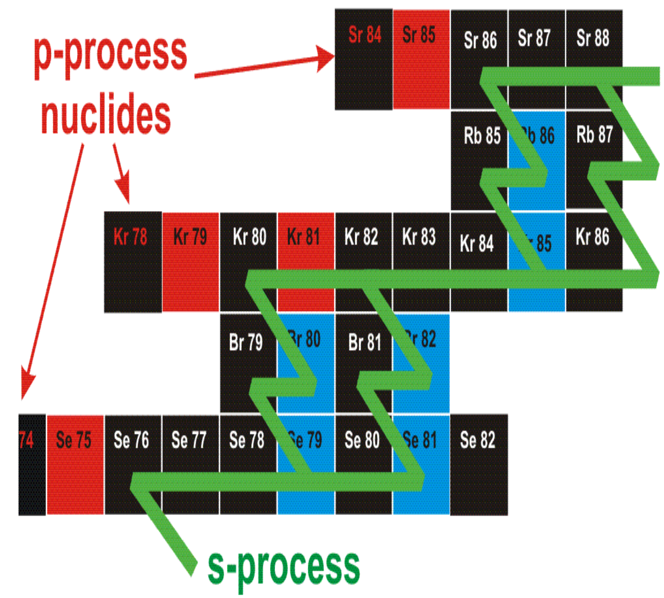
only one nucleus per A needs to be considered!

$$\dot{Y}(A) = n_n < \sigma v >_{n,\gamma} Y(A-1) - n_n < \sigma v >_{n,\gamma} Y(A)$$ in case of steady flow =0

$\sigma \approx 1/v, < \sigma v > = \sigma(v)v$ therefore $\sigma(A-1, 30 \text{ keV})Y(A-1) = \sigma(A, 30 \text{ keV})Y(A)$

The sigma*N-curve

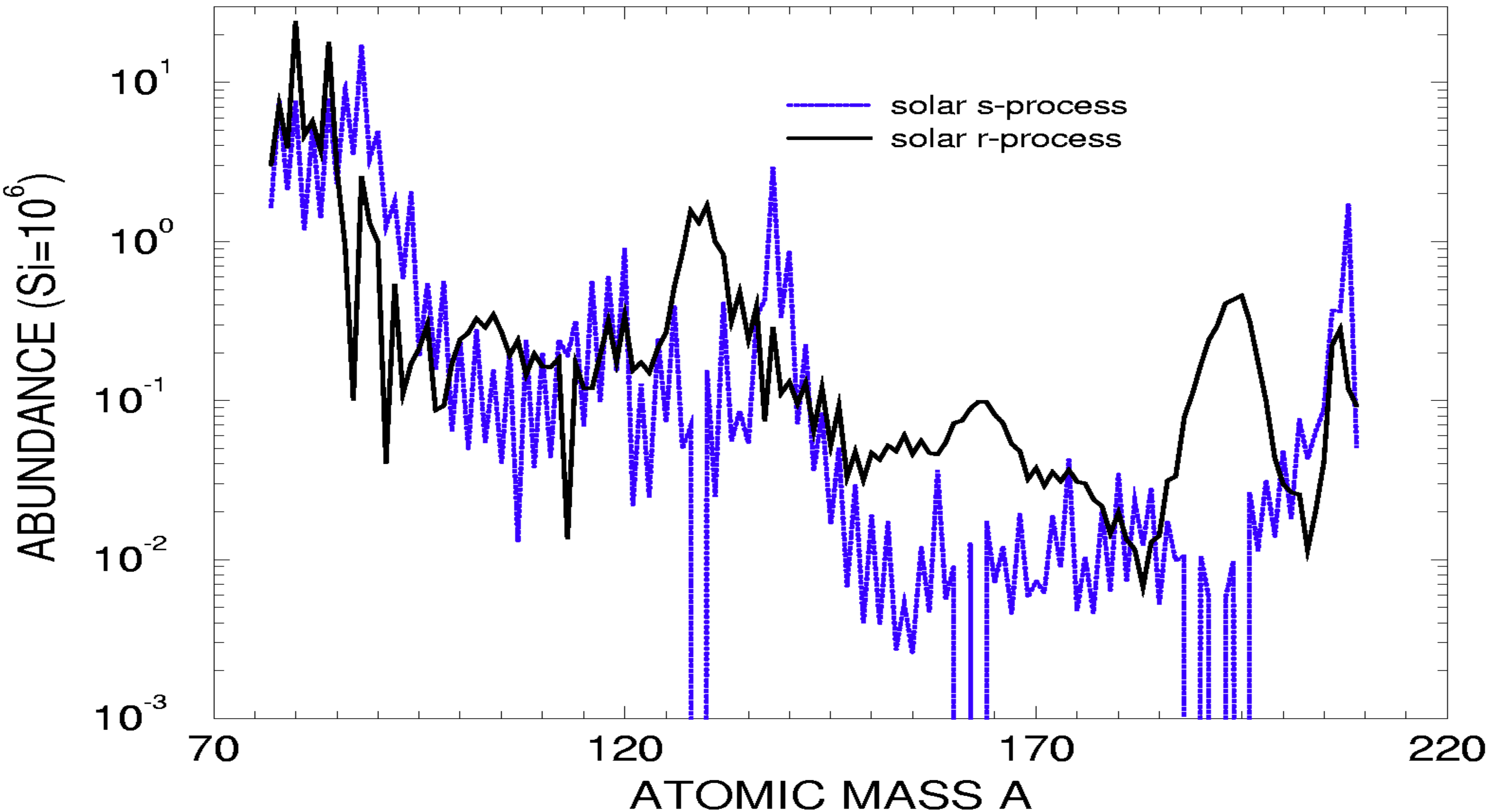




double values due to branchings

a complete steady flow is not given, but in between magic numbers (where the neutron capture cross sections are small) almost attained!

s- and r-decomposition



the almost constant sigma*N-curve leads to a large odd-even staggering in the abundances (due to the odd-even staggering in n-capture cross sections!

Steady flows and chem. equilibrium in stellar burning

pp-cycles and CNO-cycle lead to steady flows in H-burning

1. Hydrogen Burning $T = (1\text{-}4) \times 10^7 K$

   pp-cycles -> $^1H(p,e^+\nu)^2H$

   CNO-cycle -> slowest reaction $^{14}N(p,\gamma)^{15}O$

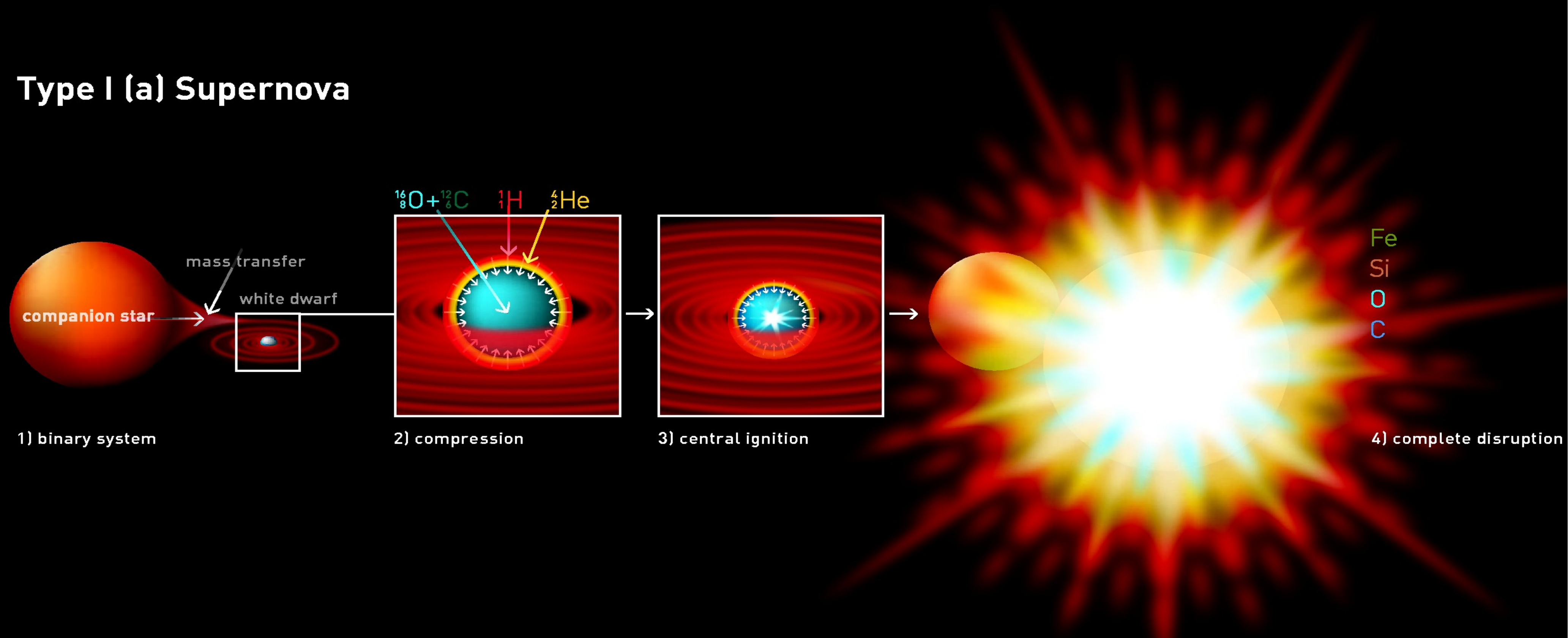
2. Helium Burning $T = (1\text{-}2) \times 10^8 K$

   $^4He + ^4He \Leftrightarrow ^8Be$ $^8Be(\alpha,\gamma)^{12}C[(\alpha,\gamma)^{16}O]$

   $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)^{25}Mg$

$^4He + ^4He \longleftrightarrow ^8Be$ is in chemical equilibrium
released neutrons lead to steady flow in neutron capture

Type Ia Supernovae from Accretion in Binary Stellar Systems

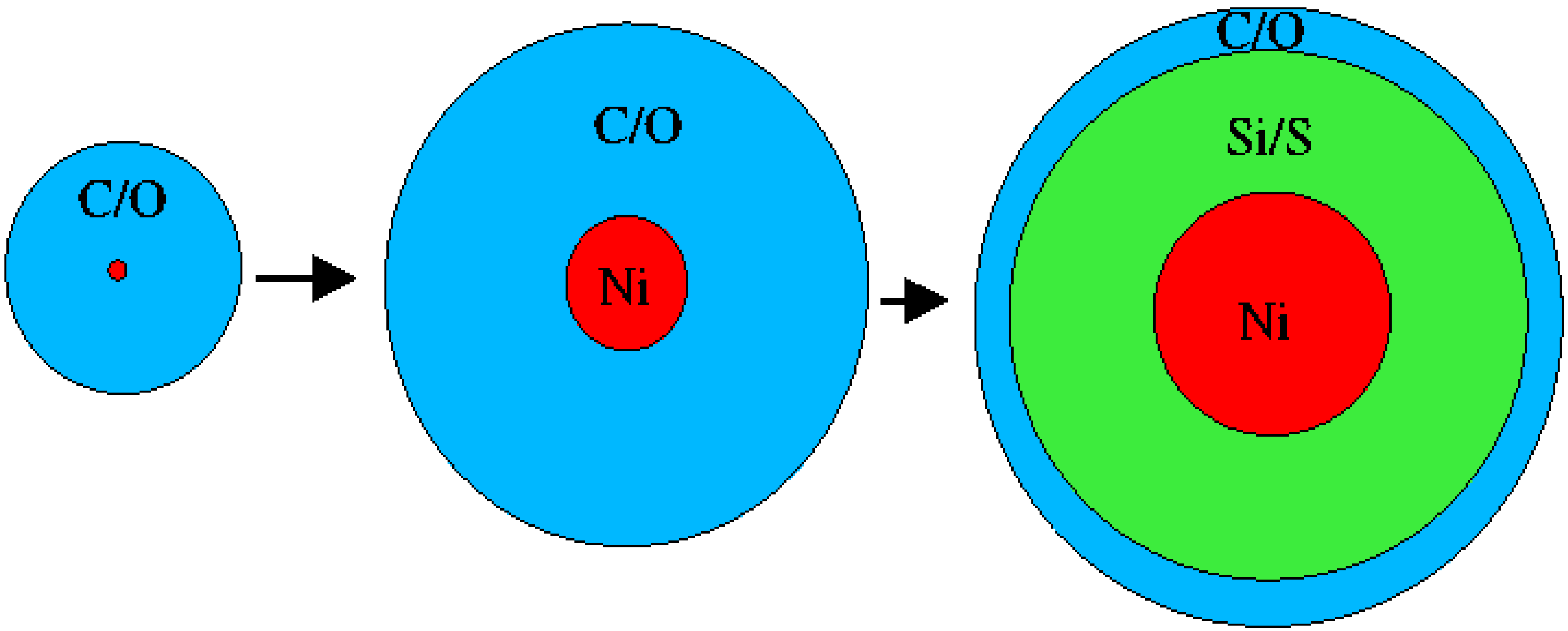


binary systems with accretion onto one compact object can lead (depending on accretion rate) to explosive events with thermonuclear runaway (under electron-degenerate conditions)

- white dwarfs (novae, type Ia supernovae)
- neutron stars (type I X-ray bursts, superbursts?)

Back of the Envelope SN Ia

e.g. W7 (Nomoto, Thielemann, Yokoi 1984); delayed detonations (Khokhlov, Höflich, Müller; Woosley et al.)



$M_{ch} \approx 1.4 M_\odot$ of $^{12}C/^{16}O=1$ WD $\rightarrow 1.398776\ M_\odot\ ^{56}Ni$

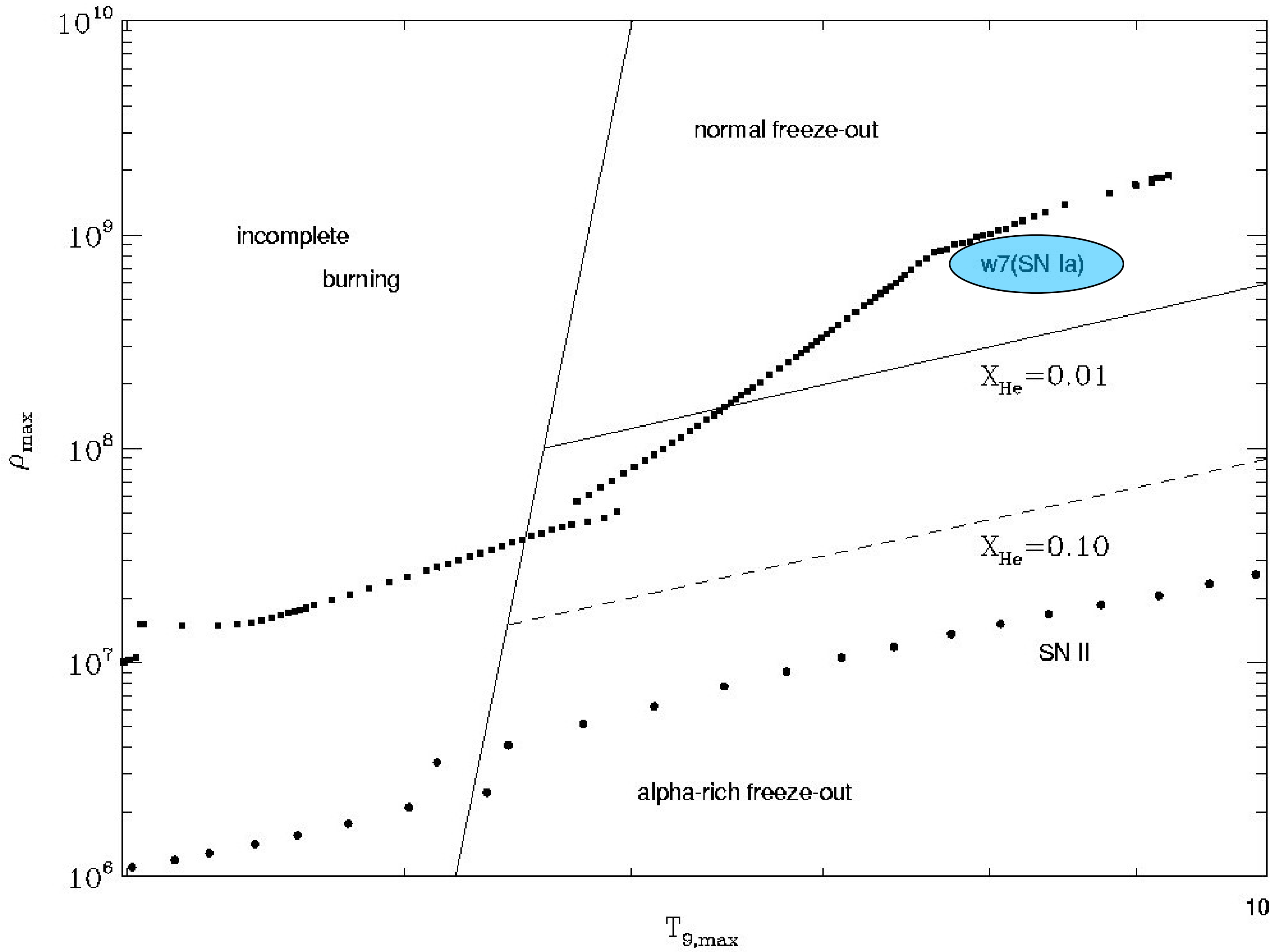
$\rightarrow 2.19 \times 10^{51}$ erg - $E_{grav} \approx (5-6) \times 10^{50}$ erg

reduction due to intermediate elements like Mg, Si, S, Ca

$\rightarrow 1.3 \times 10^{51}$ erg

in spherically symmetric models description of the burning front propagation (with hydrodynamic instabilities) determines outcome!

Complete chem. equilibrium (NSE)



Si-burning in stellar evolution and expl. Si-burning at high densities lead to NSE!

Neutronization via electron capture

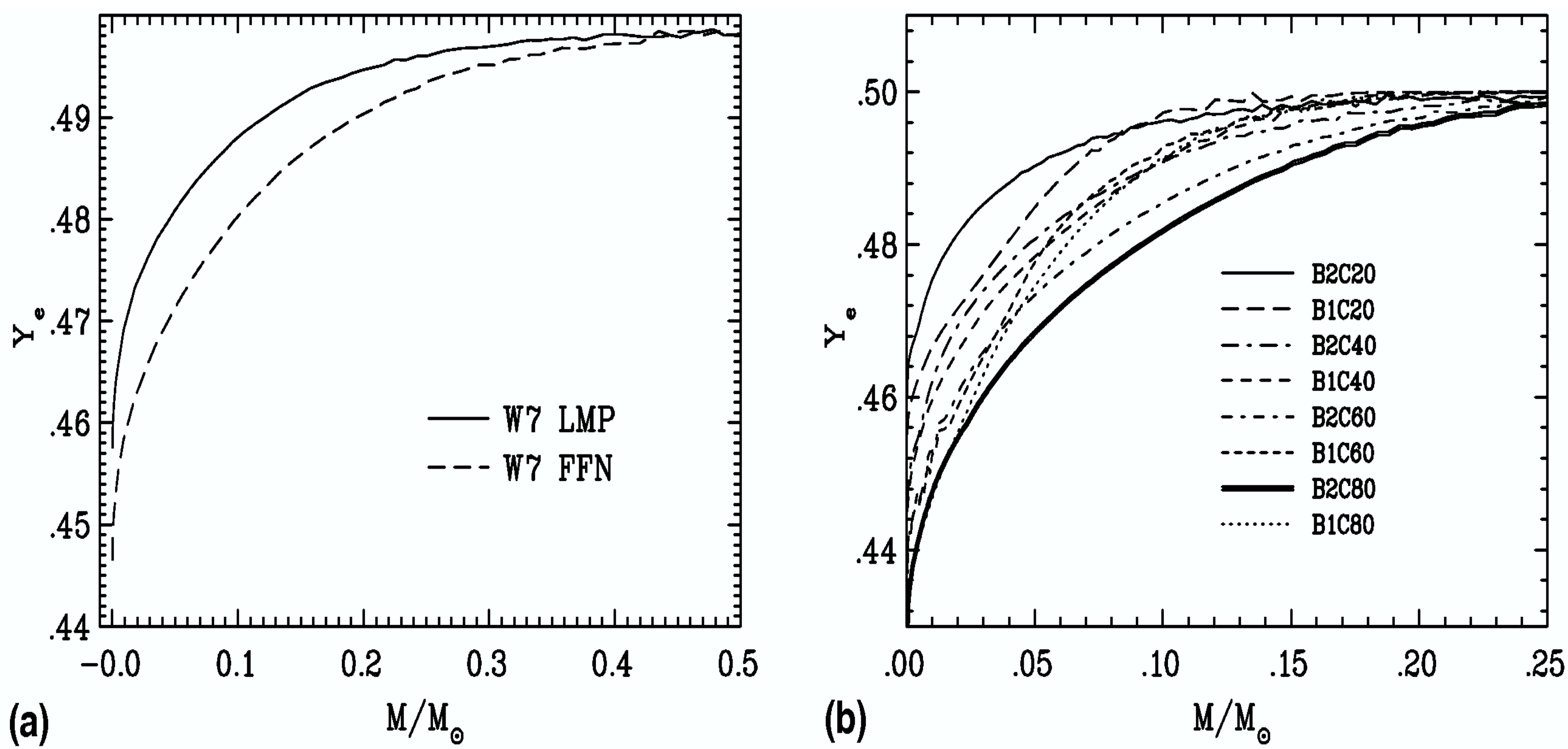
(high Fermi energies at central densities)

^{56}Ni

^{54}Fe, ^{58}Ni

^{56}Fe

^{50}Ti, ^{54}Cr



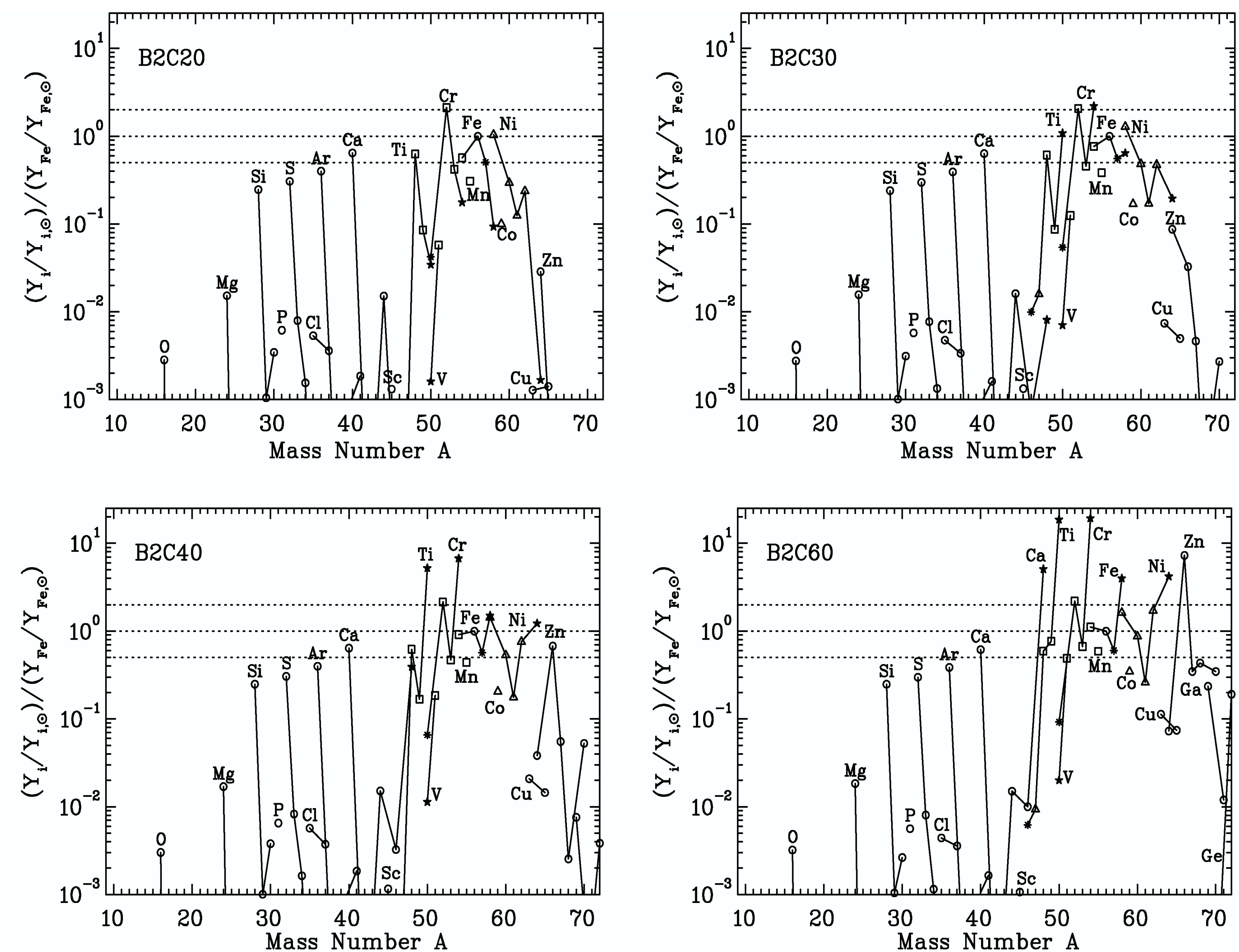
(a) Test for influence of new shell model electron capture rates (including pf-shell Langanke, Martinez-Pinedo 2003)

(b) Test for burning front propagation speed (Brachwitz et al. 2001)

direct influence on dominant Fe-group composition resulting from SNe Ia

Ignition density determines Ye and neutron-richness of (60-70% of) Fe-group

FKT et al. (2004)



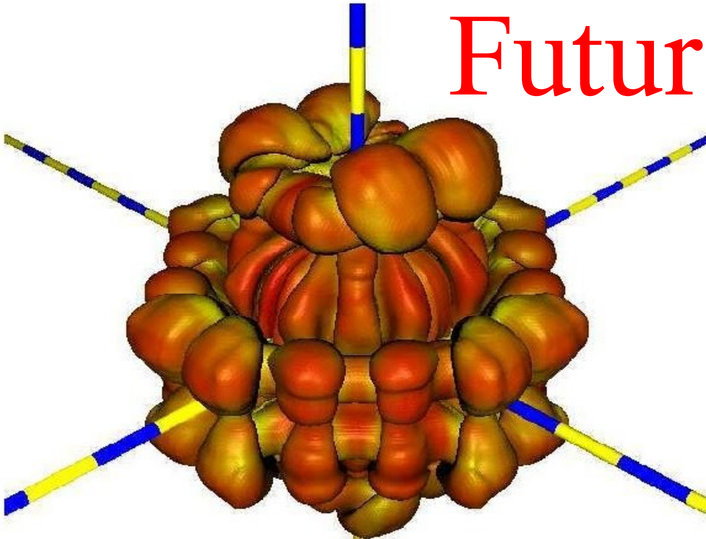
results of explosive C, Ne, O and Si-burning:
Fe-group to alpha-elements 2/1-3/1

SNe Ia dominate Fe-group, over-abundances by more than factor 2 not permitted

→

maximum central density 3 10^9 gcm^{-3}

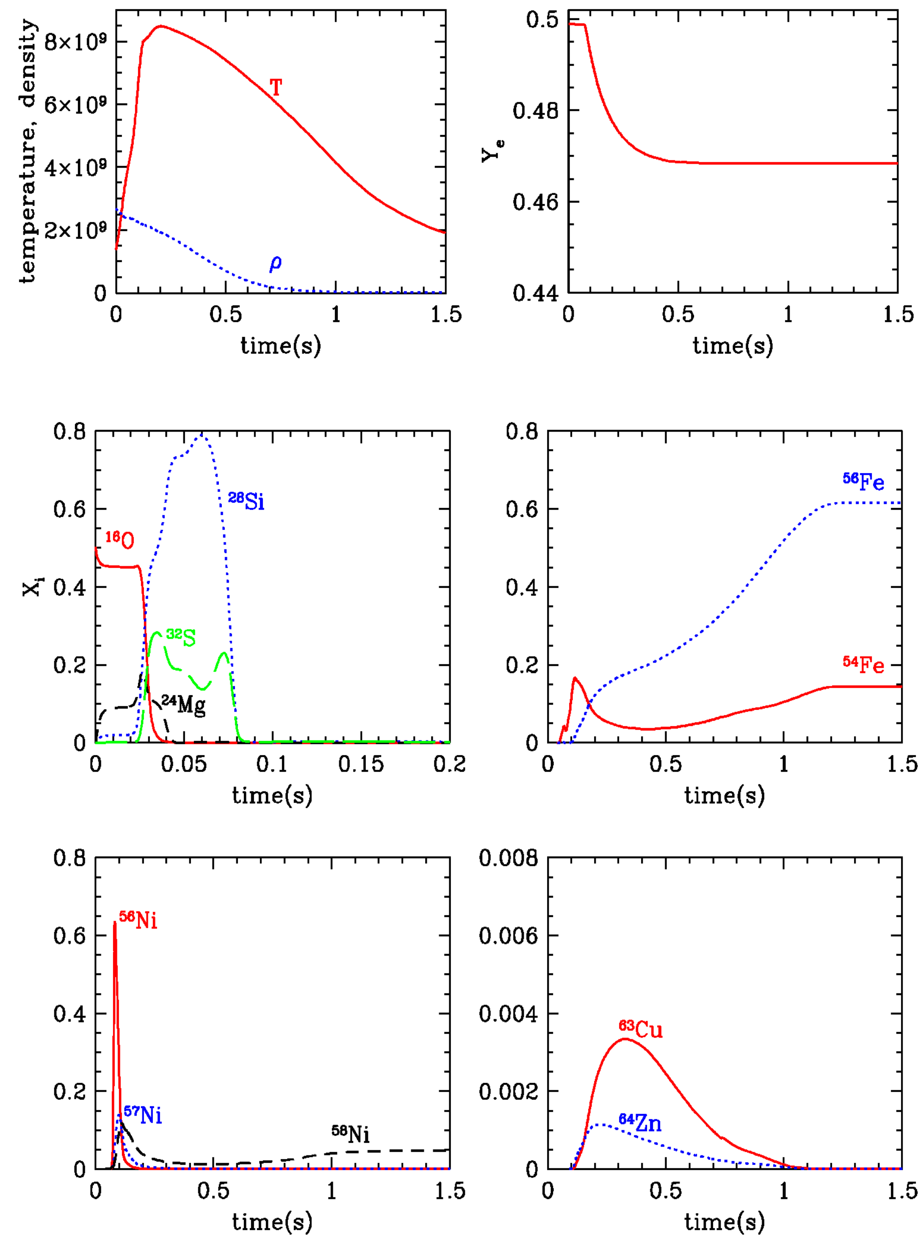
Future 3D Models



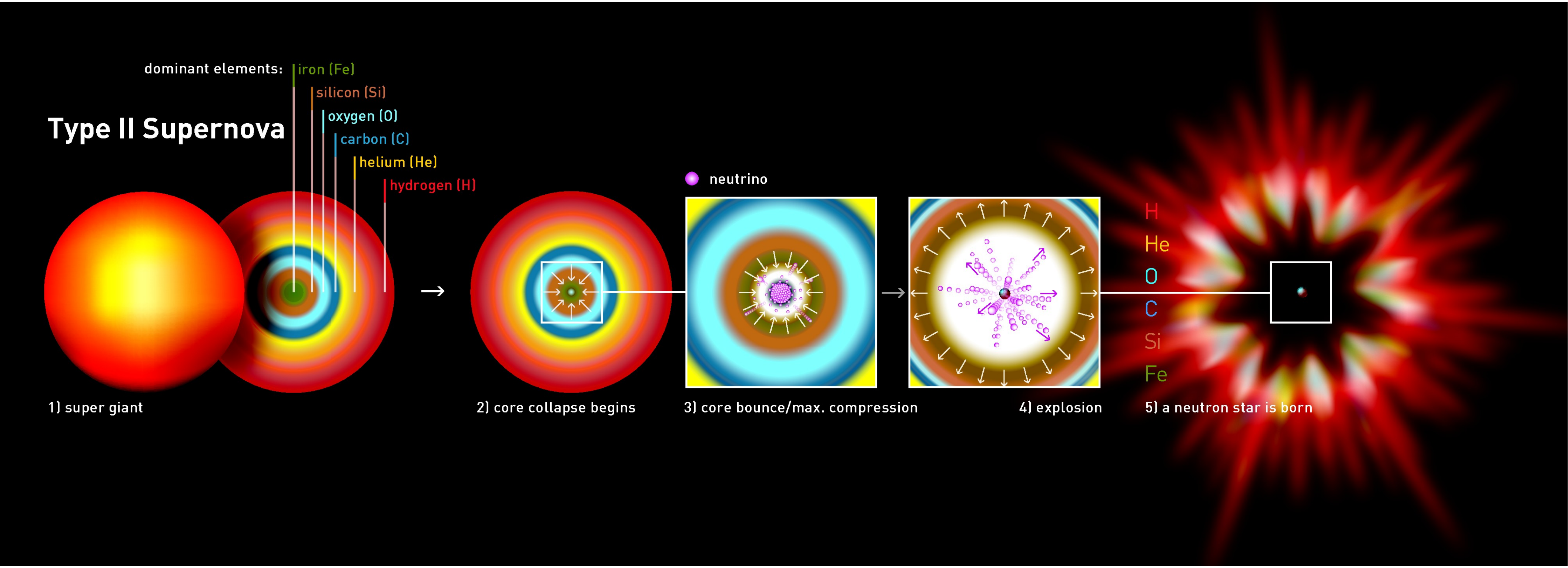
Travaglio, Reinecke, Hillebrandt, FKT (2004, 3D nucleosynthesis with tracer particles)

consistent treatment needed instead of parametrized spherical propagation, MPA Garching (Röpke et al. 2007), U. Chicago/SUNY Stony Brook (Calder et al. 2007)

- *distribution of ignition points uncertain*
- *hydrodynamic instabilities determine propagation of burning*
- *deflagration/detonation transition*

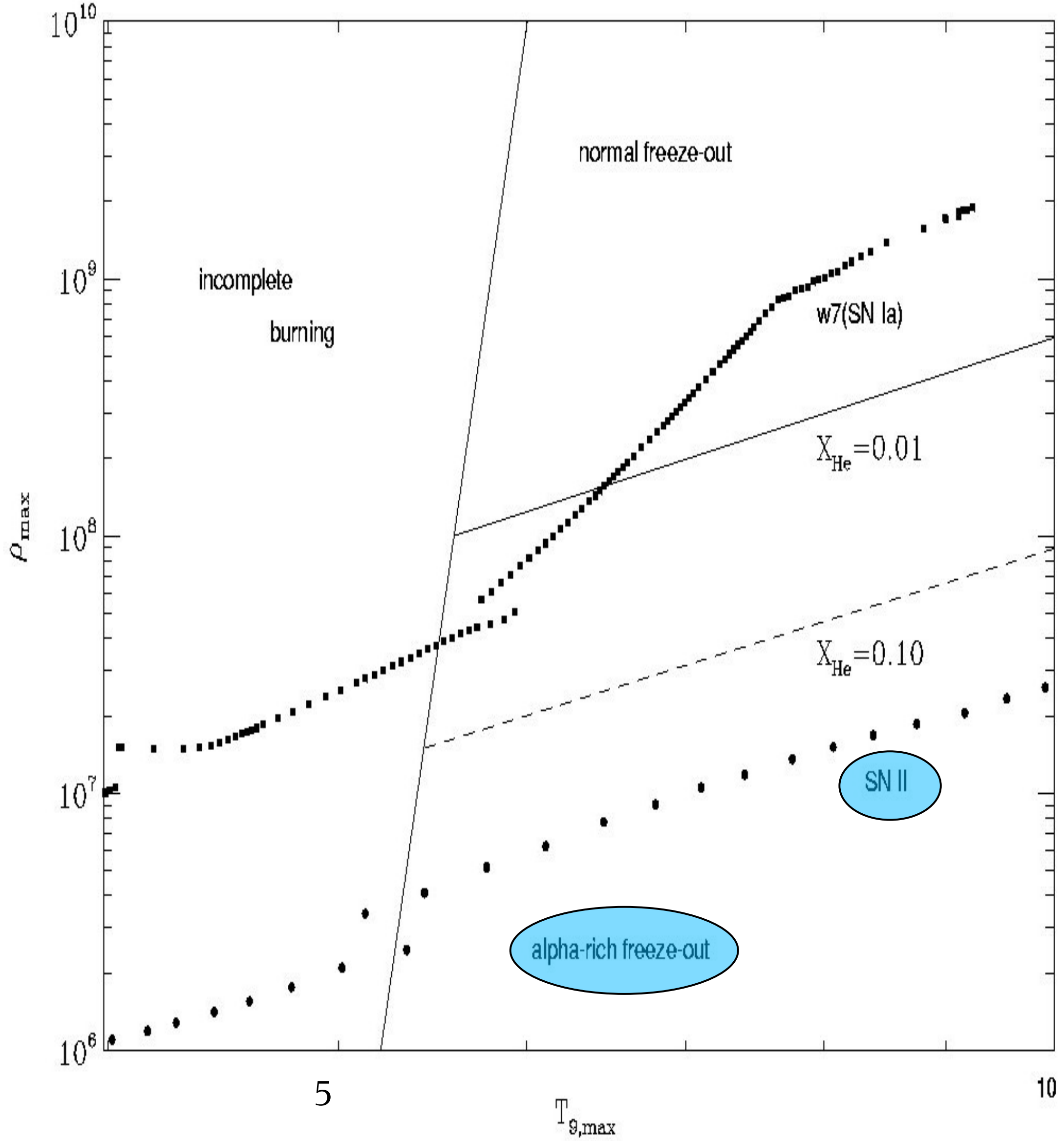


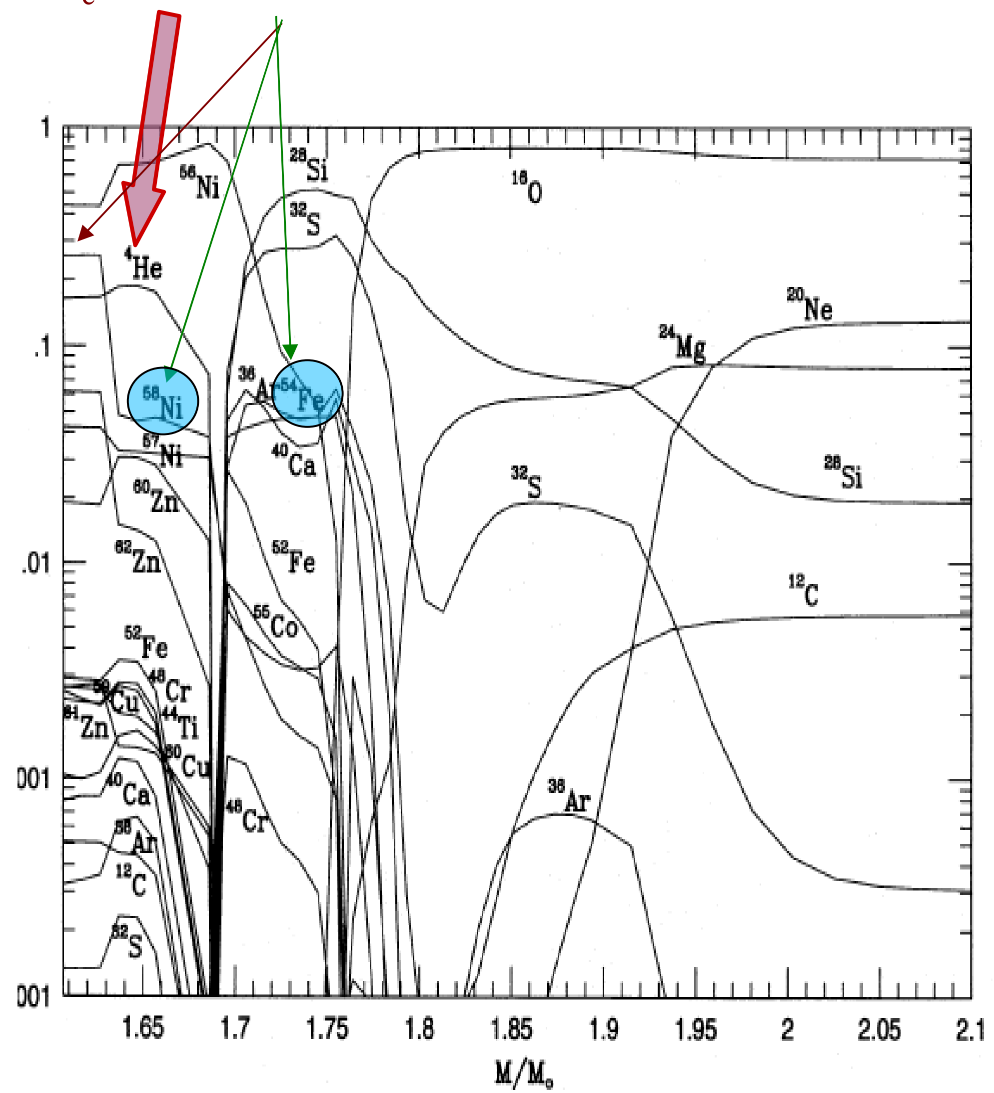
Core Collapse Supernovae from Massive Stars



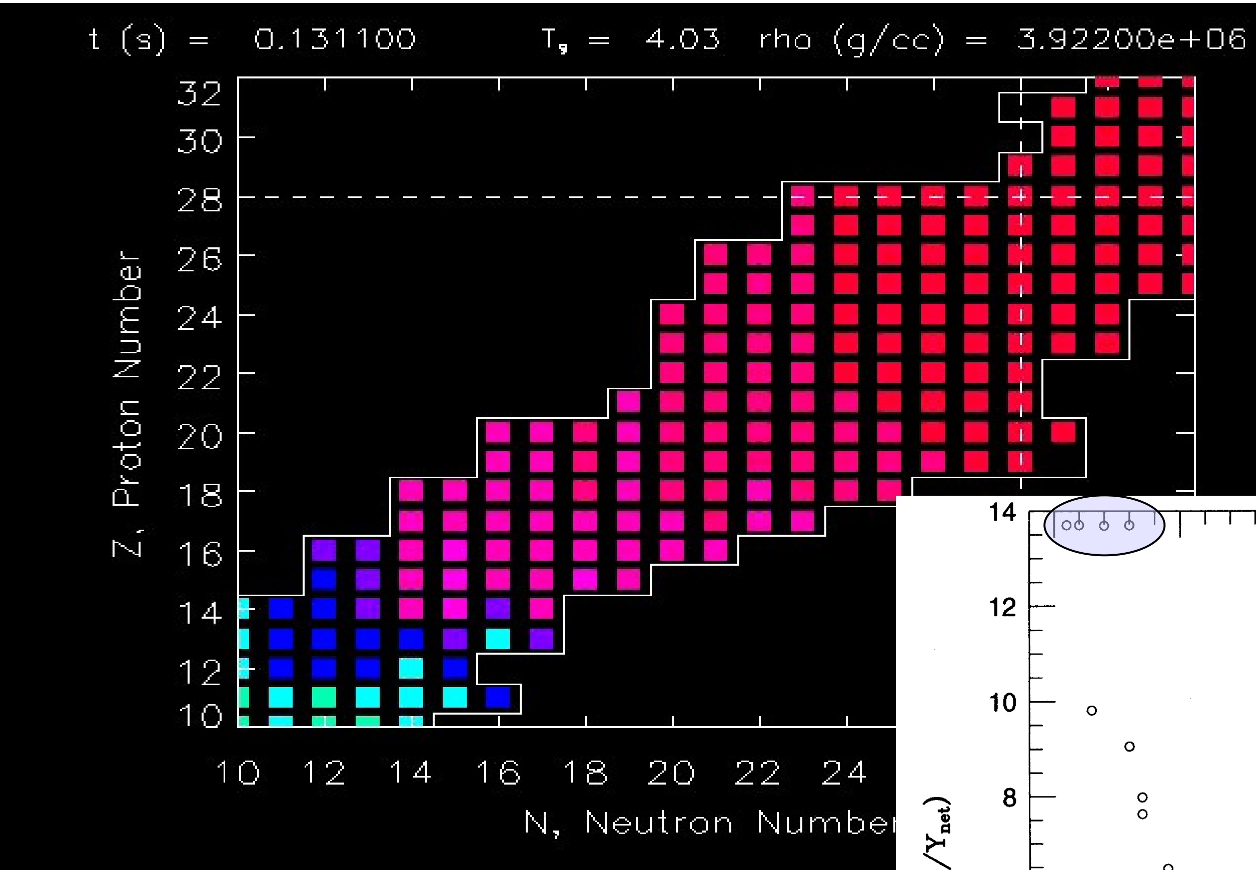
QSE in low density expl. Si-burning

disconnected light element (n,p,He) and Si-Fe QSE-cluster, high alpha-abundance prefers alpha-rich nuclei (^{58}Ni over ^{54}Fe), Y_e determines dominant QSE-isotope.

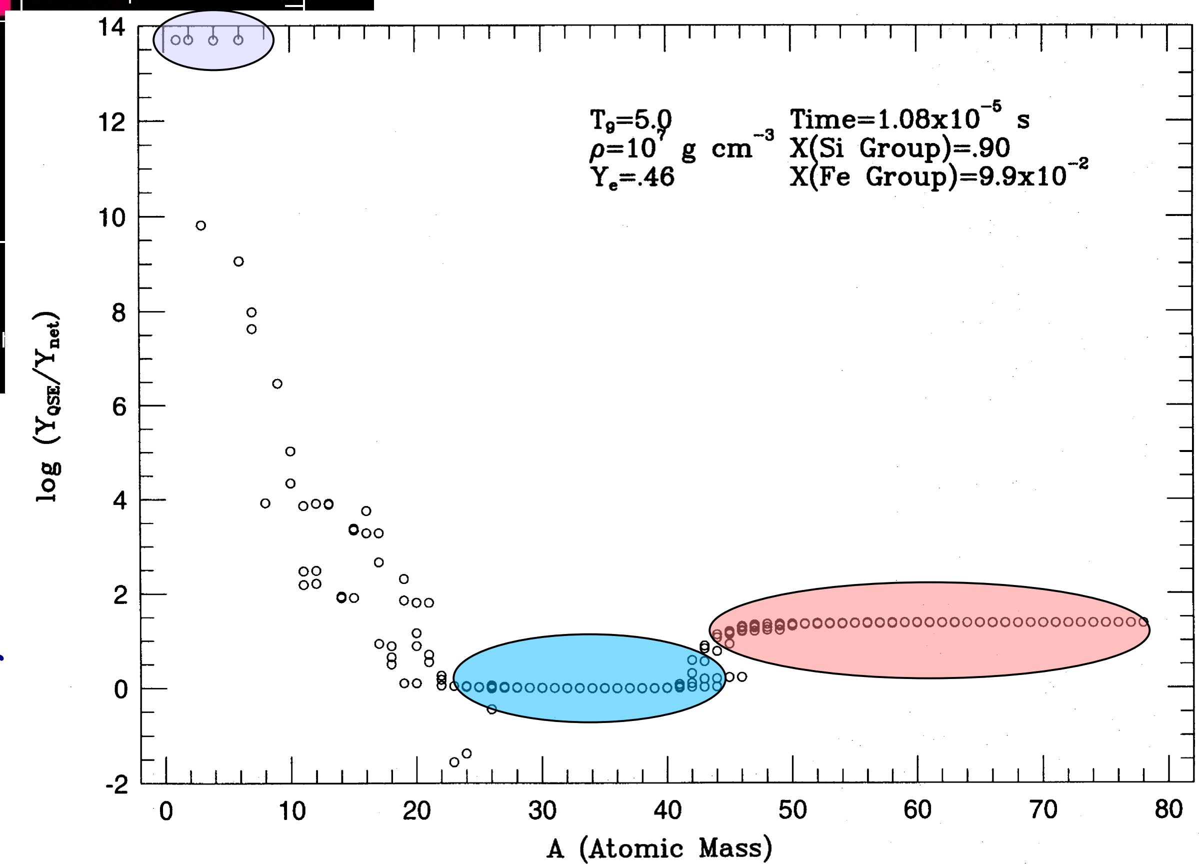




QSE in explosive Si-burning



full NSE is not attained, but there exist equilibrium groups around ^{28}Si, ^{56}Ni and n,p,^{4}He, which are separated by slow reactions



Sample Calculations from

- B.S. Meyer and
- Hix and Thielemann

small Q-values of reactions out of Z=20, N=20 cause small cross sections and hold up equilibrium

QSE Formalism

light group

$$Y_{NSE}(^AZ) = C(^AZ) Y_n{}^N Y_p{}^Z$$

Si-group

$$Y_{QSE,\mathrm{Si}}(^AZ) = \frac{C(^AZ)}{C(^{28}\mathrm{Si})} Y(^{28}\mathrm{Si}) Y_p^{Z-14} Y_n^{N-14}$$

Ni/Fe-group

$$Y_{QSE,\mathrm{Ni}}(^AZ) = \frac{C(^AZ)}{C(^{56}\mathrm{Ni})} Y(^{56}\mathrm{Ni}) Y_p^{Z-28} Y_n^{N-28}$$

$$C(^AZ) = \frac{G(^AZ)}{2^A} \left(\frac{\rho N_A}{\theta}\right)^{A-1} A^{\frac{3}{2}} \exp\left(\frac{B(^AZ)}{k_B T}\right)$$

$$Y_{NG} = \sum_{i \in Lt\ group} N_i Y_i + \sum_{i \in Si\ group} (N_i - 14) Y_i + \sum_{i \in Fe\ group} (N_i - 28) Y_i ,$$

$$Y_{ZG} = \sum_{i \in Lt\ group} Z_i Y_i + \sum_{i \in Si\ group} (Z_i - 14) Y_i + \sum_{i \in Fe\ group} (Z_i - 28) Y_i ,$$

$$Y_{SiG} = \sum_{i \in Si\ group} Y_i ,$$

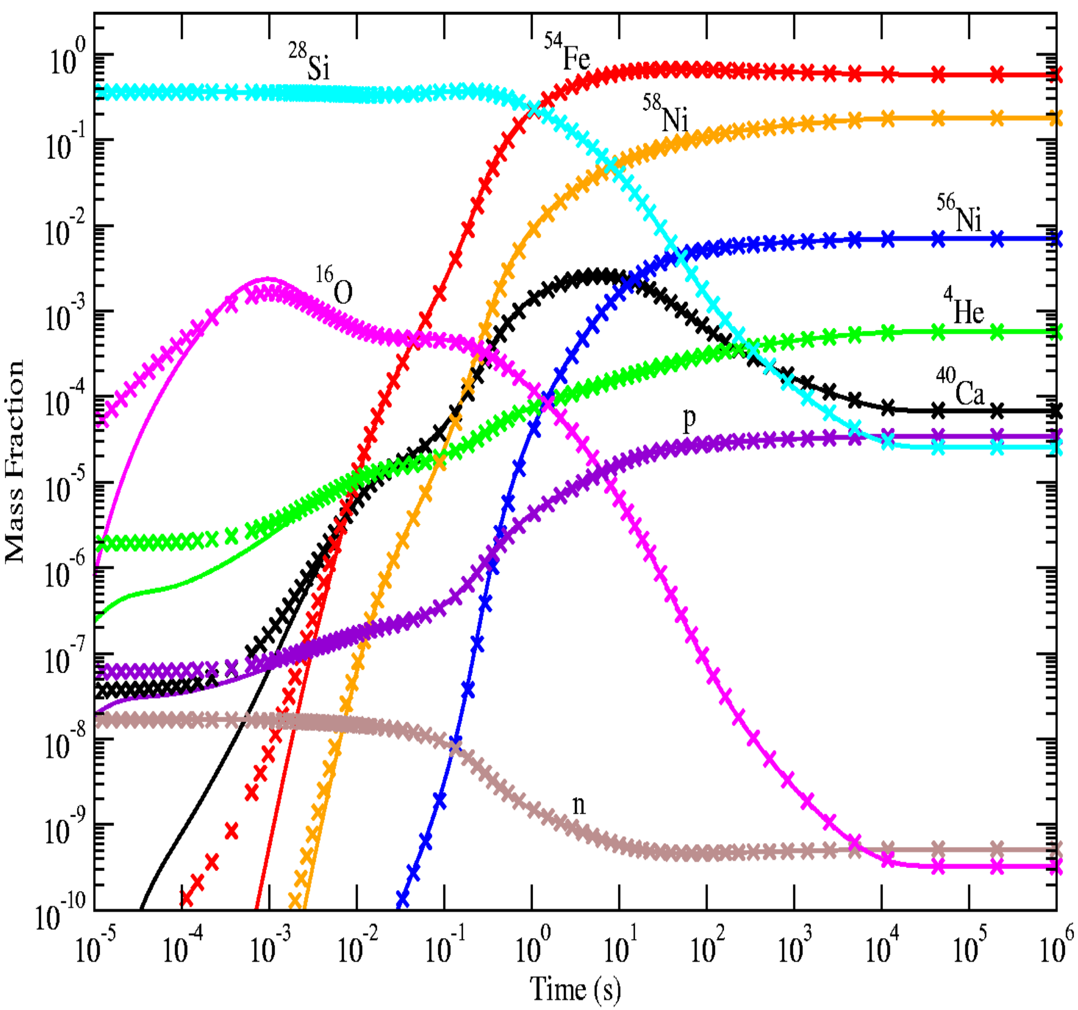
$$Y_{FeG} = \sum_{i \in Fe\ group} Y_i .$$

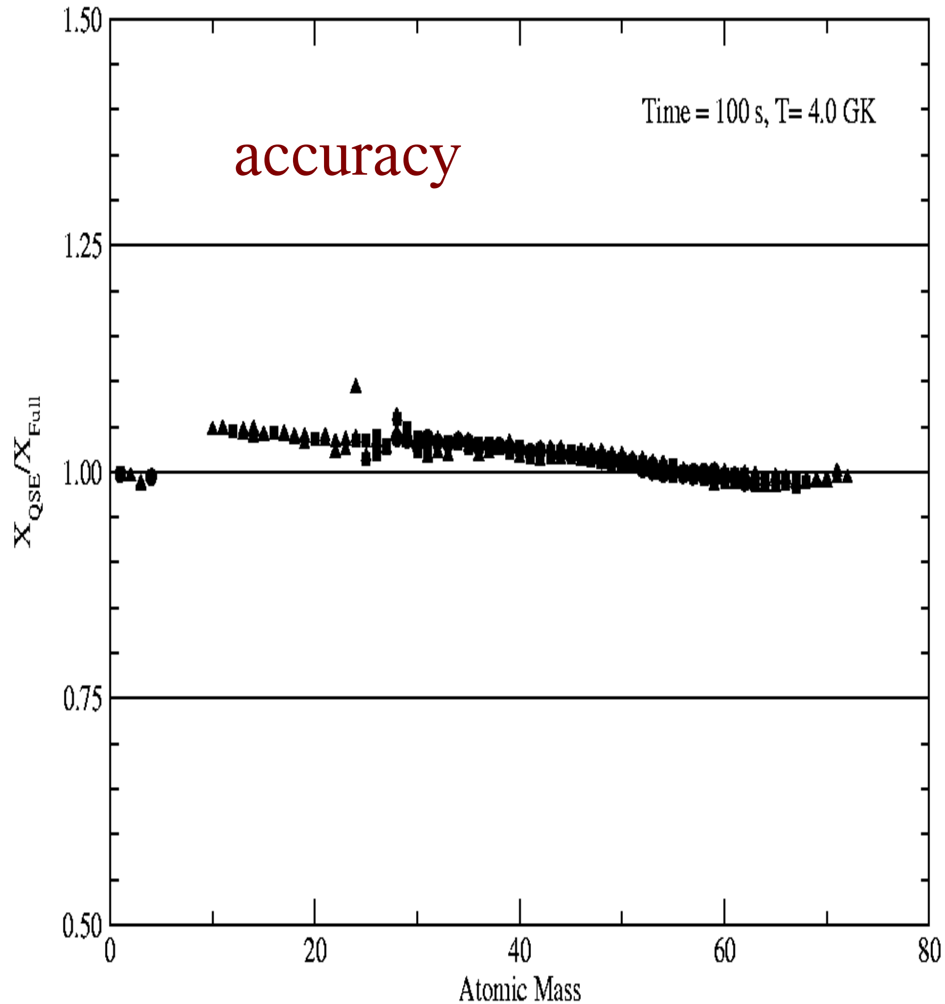
$$\theta = \left(\frac{m_u k_B T}{2\pi\hbar^2}\right)^{3/2}$$

binding energy differences, i.e. masses enter directly

time evolution for those quantities which are in equilibrium and the individual abundances of nuclei with slow reactions which link equilibrium groups (Hix, Parete-Koon, Freiburghaus, Thielemann 2007)

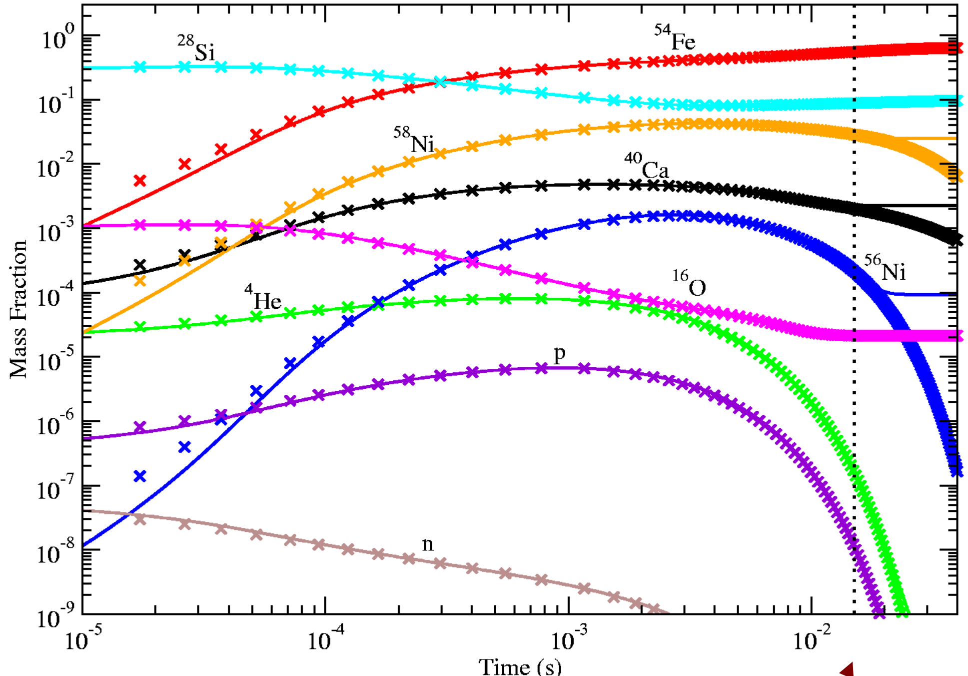
Obtaining quilibrium at high T

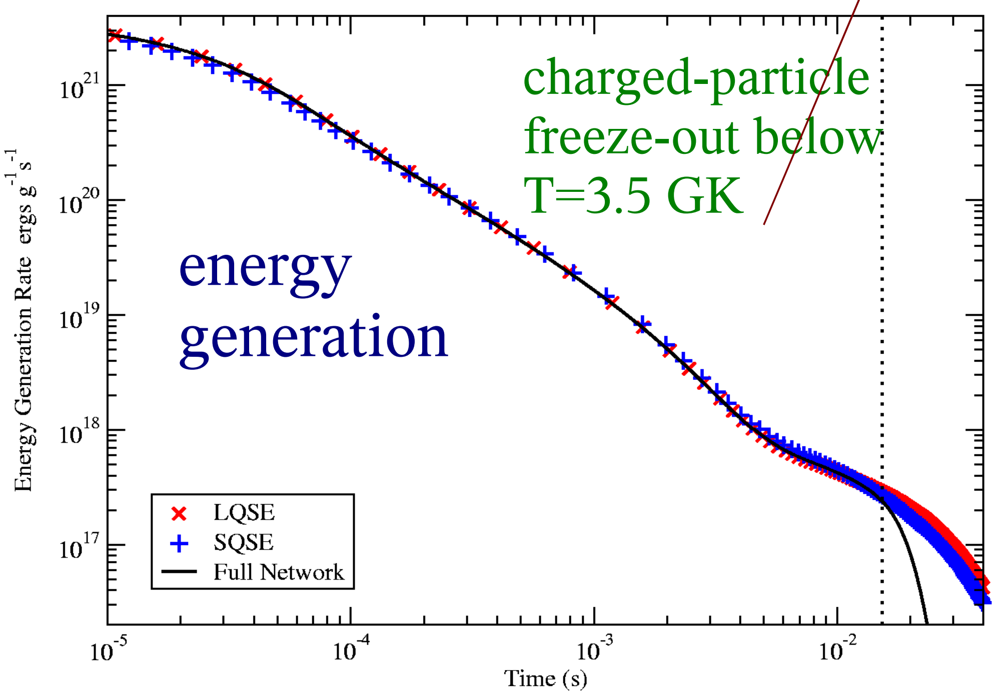


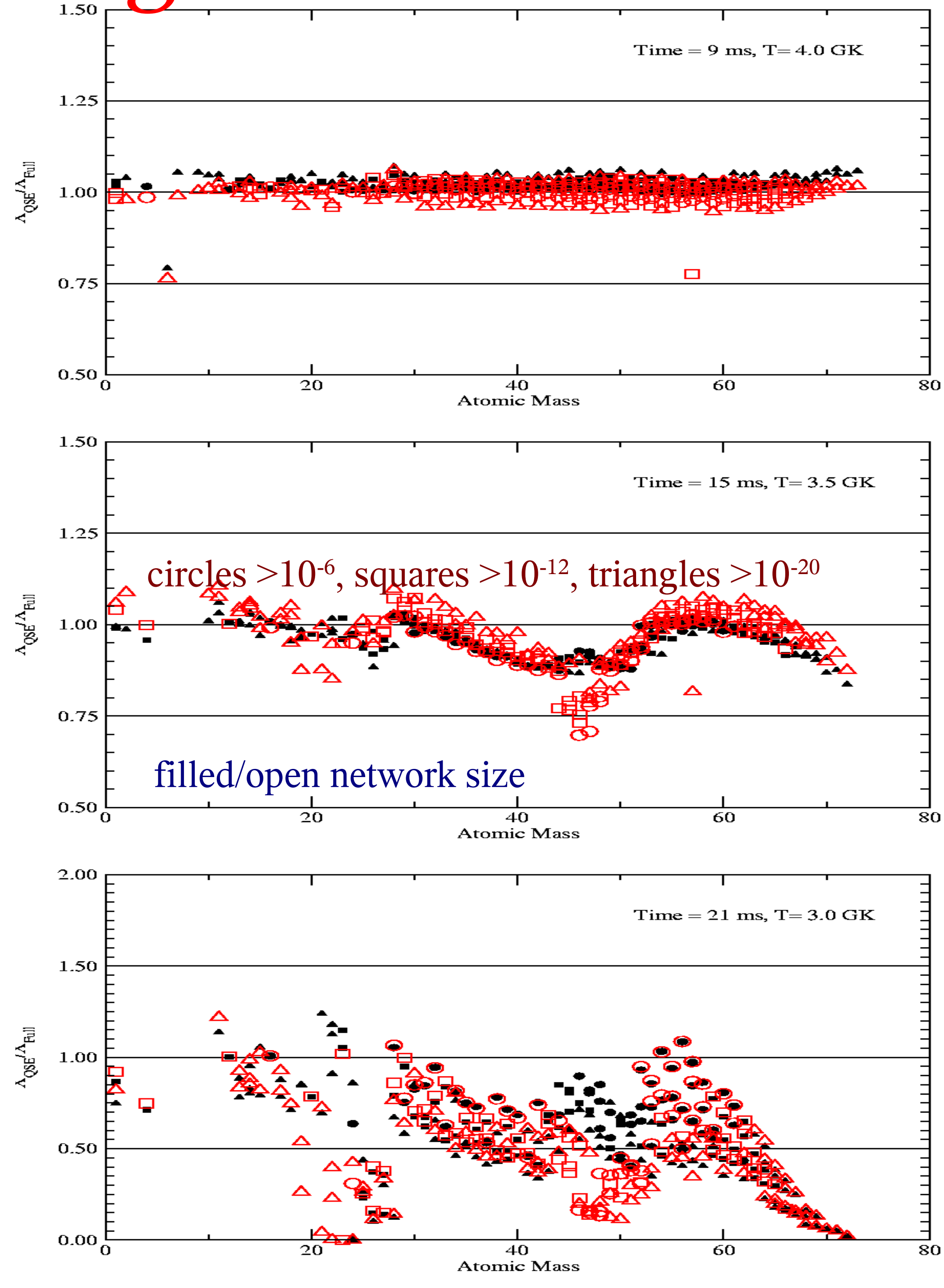


at T=4 GK the equilibrium description is correct after about 10^{-3} s!

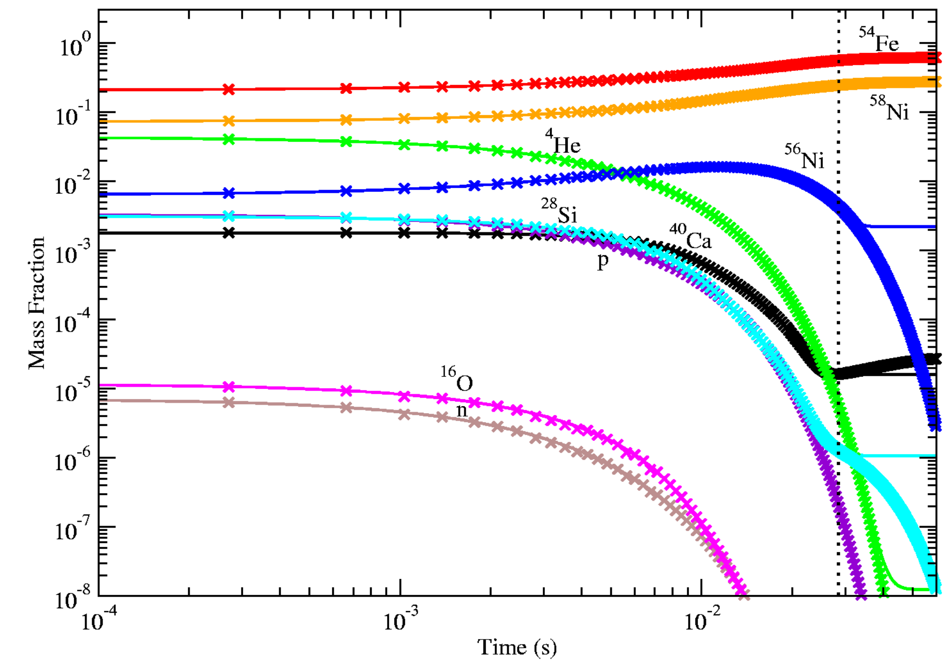
Incomplete Si-burning with freeze-out

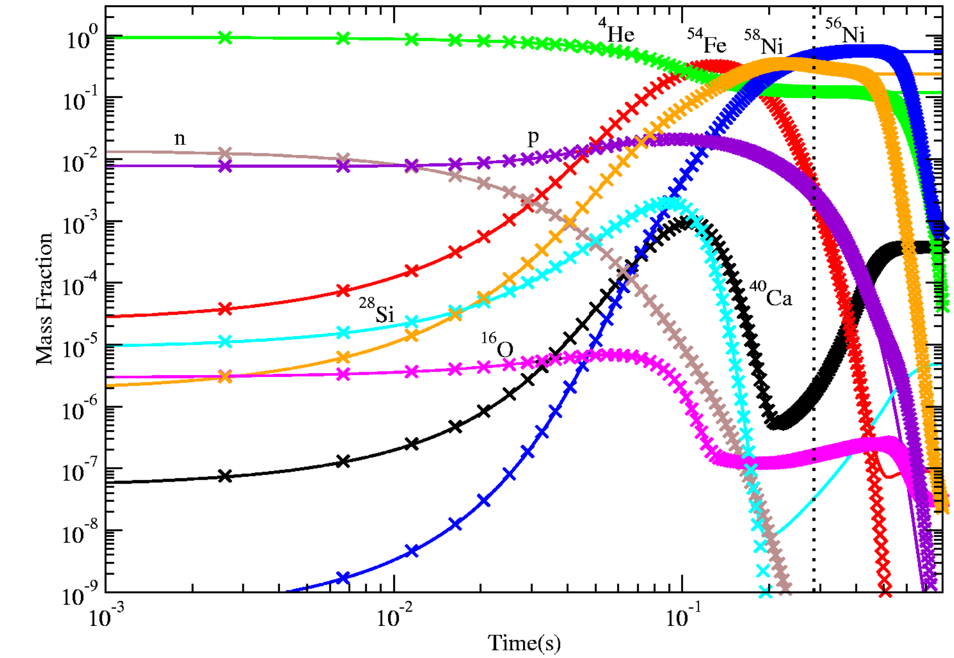


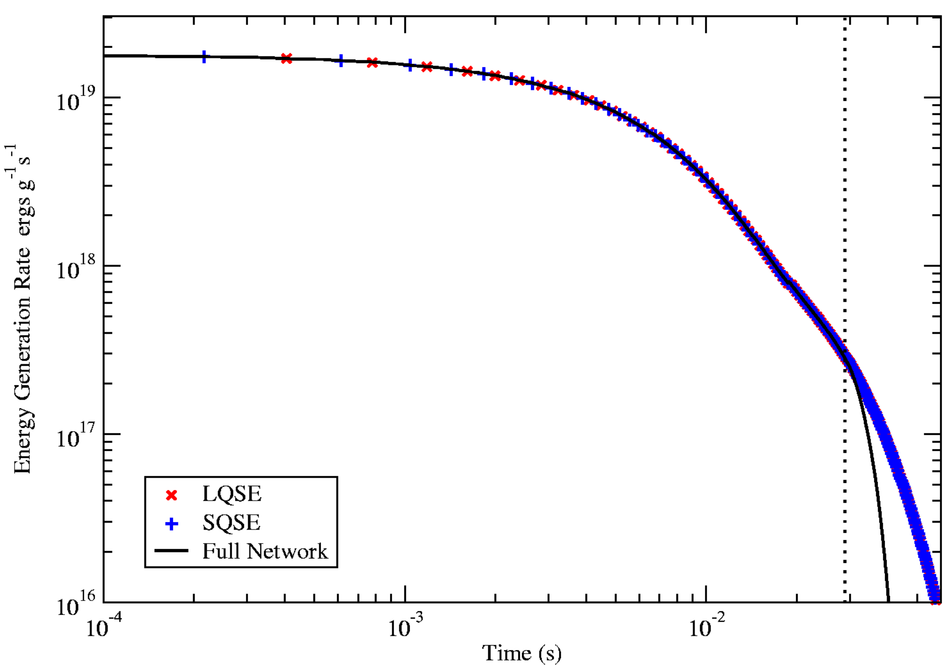


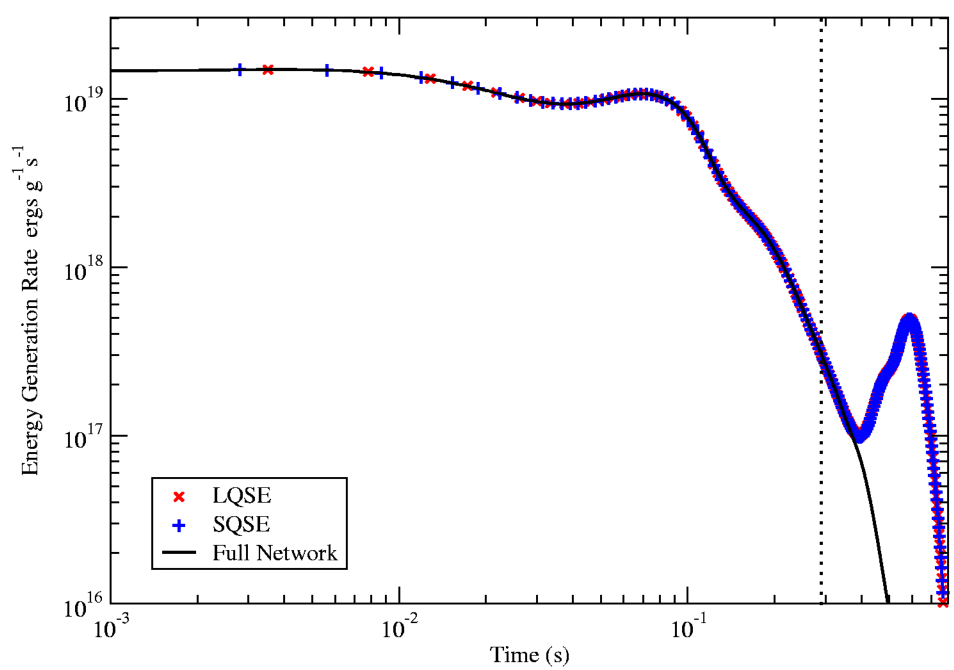


Normal and alpha-rich freeze-out









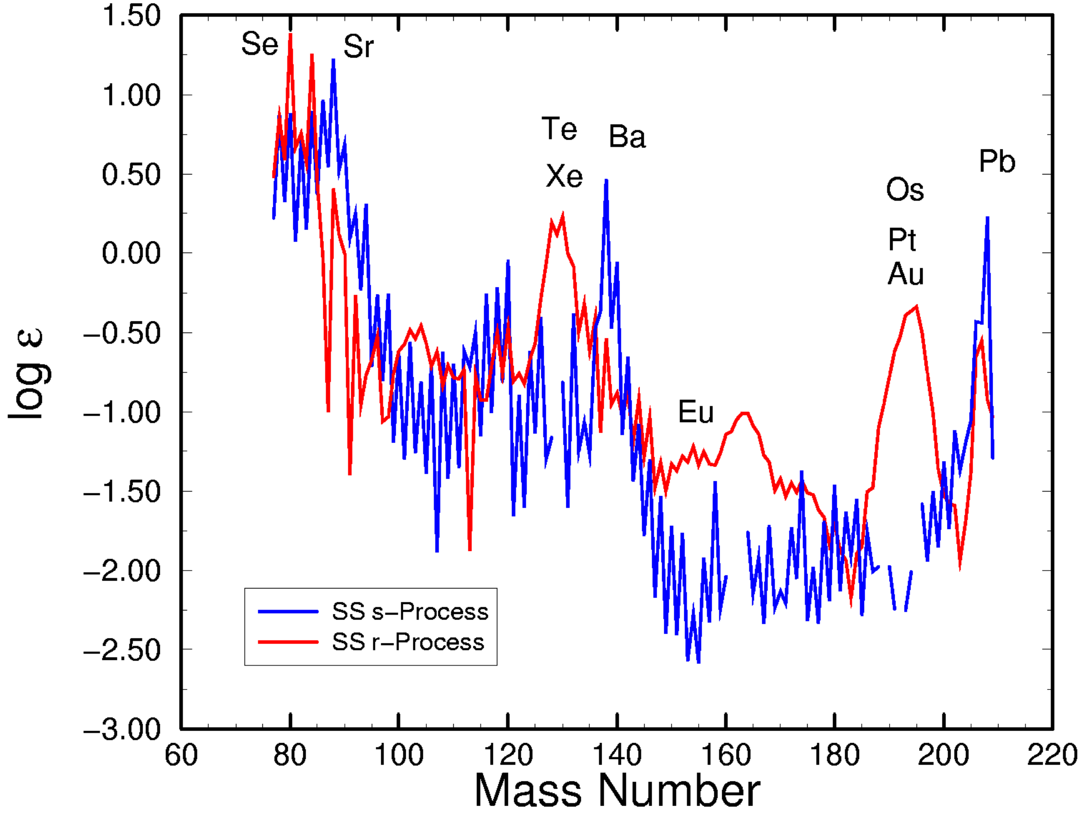
Interim conclusions

- steady flows are approached in many hydrostatic burning stages during stellar evolution, including the s-process. They are determined by rates (often the smallest ones), which are/can be related to small Q-values.
- NSE/QSE equilibria are obtained in hydrostatic Si-burning and in explosive burning. Abundance distribution depends directly on mass differences, but for these applications mostly close to stability.
- How about QSE-equilibria linked by steady flows (and far from stability)?

The classical r-process

- Assume conditions where after a charged-particle freeze-out the heavy QSE-group splits into QSE-subgroups containing each one isotopic chain Z, and a high neutron density is left over
- these QSE-groups are connected by beta-decays from Z to Z+1
- neutrons are consumed to form heavier nuclei
- is a steady flow of beta-decays conceivable?

s- and r-decoposition

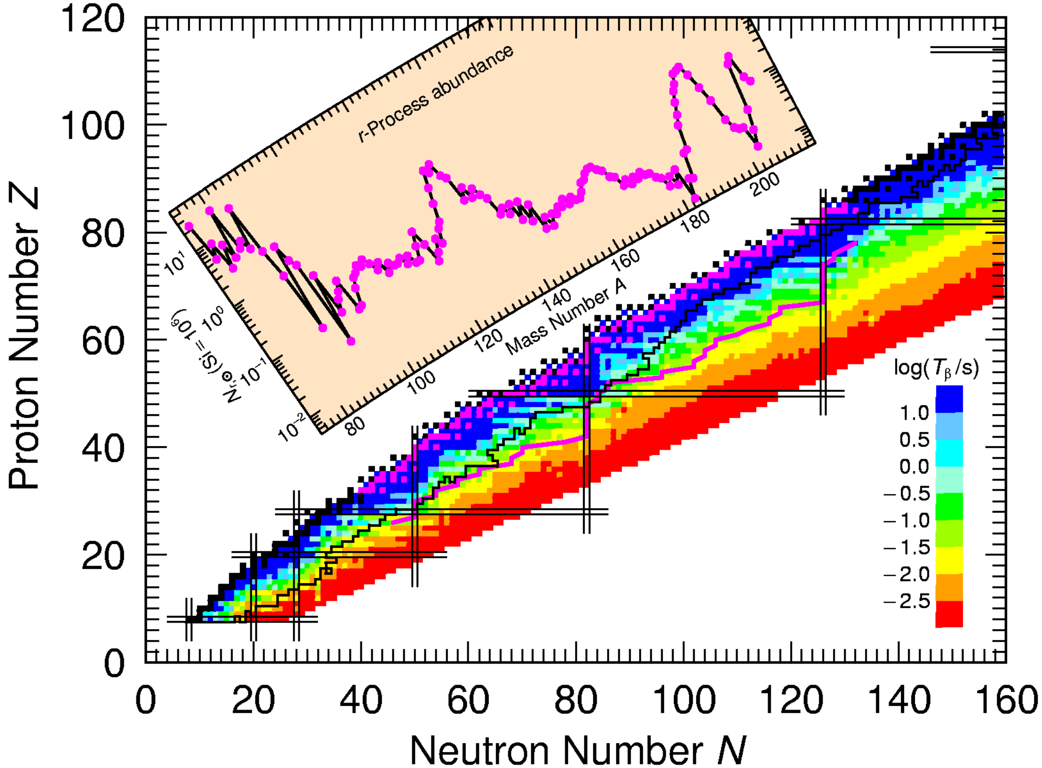


$$\dot{Y}(Z,A) = -\lambda_{\beta^-}(Z,A)Y(Z,A) - \rho N_A <\sigma v>_{n,\gamma} Y_n Y(Z,A)$$
$$= -\lambda_{\beta^-}(Z,A)Y(Z,A) - <\sigma v>_{n,\gamma} n_n Y(Z,A)$$
$$= -\frac{1}{\tau_\beta}Y(Z,A) - \frac{1}{\tau_{n,\gamma}}Y(Z,A).$$

which timescale is shorter? neutron capture inverse proportional to n_n !

Heavy Elements are made by slow and rapid neutron capture events

High neutron densities lead to nuclei far from stability



Nuclear Reactions to be considered: (n,γ), (γ,n)

(β,xn), (β,f), (n,f), inelastic ν-scattering, (ν_e,e^-) ......

The classical r-process

How to predict abundance changes?

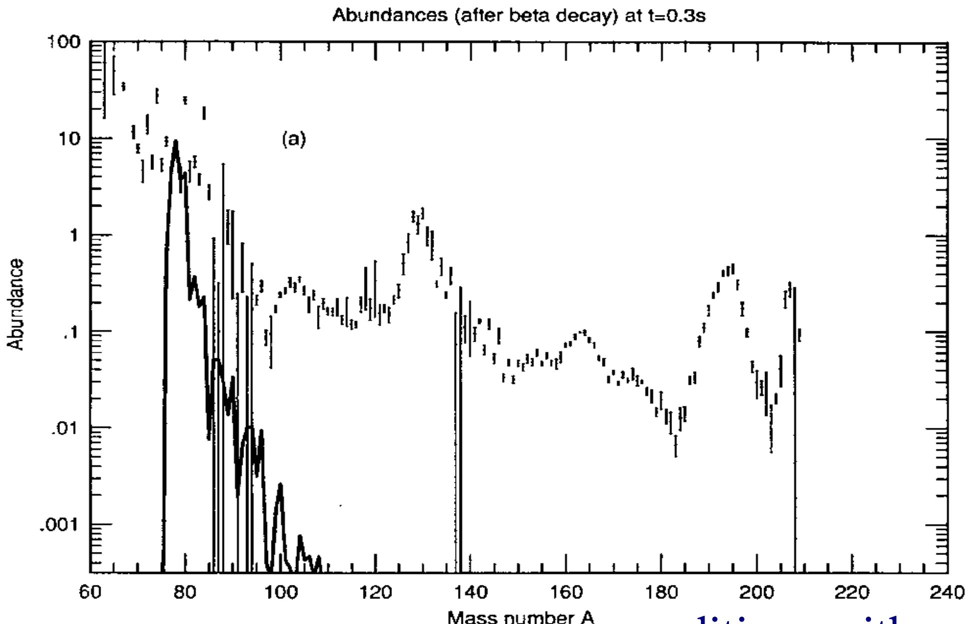
- $\dot{Y}(Z,A) = \sum \lambda_{Z',A'} Y_{Z',A'} + \sum \rho N_A < \sigma v >_{Z',A'} Y_{Z',A'} Y_n$ with $n_n = \rho N_A Y_n$
- $\dot{Y}(Z,A) \approx \lambda_\gamma(Z,A+1) Y(Z,A+1) - < \sigma v >_{Z,A} Y_{Z,A} n_n$ in case (n,γ), (γ,n) rates dominate
- $\dot{Y}(Z,A) = 0$ in chemical equilibrium, $Y(Z,A+1)/Y(Z,A) = f(n_n, T, S_n)$ due to detailed balance relation between $\lambda_\gamma(Z,A+1)$ and $< \sigma v >_{Z,A}$
- abundance maxima for all Z's at same S_n
- $\dot{Y}(Z) = \lambda_\beta(Z-1) Y(Z-1) - \lambda_\beta(Z) Y(Z)$ for summed abundances in isotopic chain and averaged decay rates

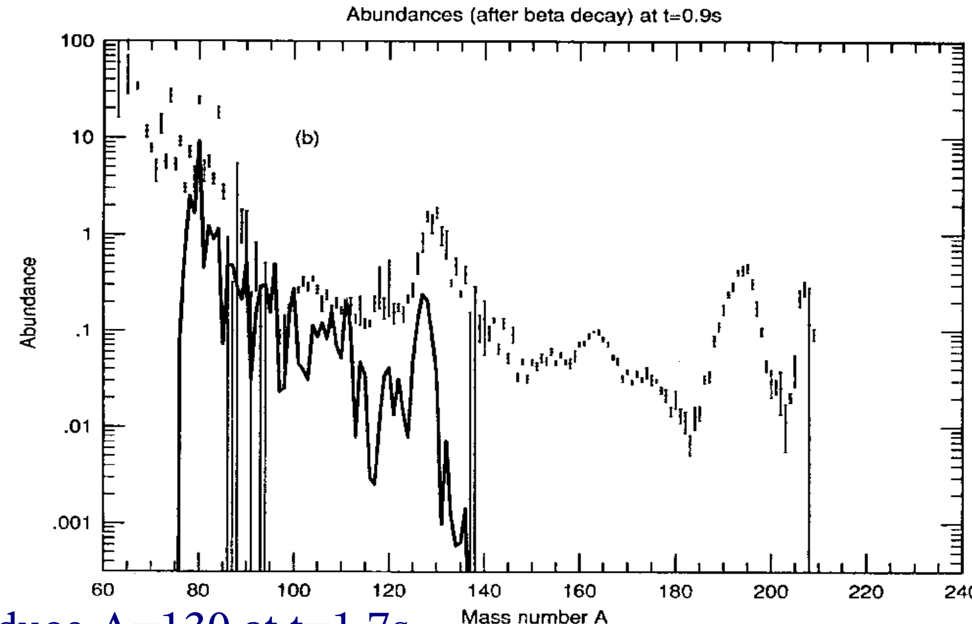
$$\frac{Y(Z,A+1)}{Y(Z,A)} = \frac{< \sigma v >_{n,\gamma}(A)}{\lambda_{\gamma,n}(A+1)} n_n$$

$$\lambda_{\gamma,n}(A+1) = \frac{2G(Z,A)}{G(Z,A+1)} \left[\frac{A}{A+1}\right]^{3/2} \left[\frac{m_u kT}{2\pi\hbar^2}\right]^{3/2} \langle\sigma v\rangle_{n,\gamma}(A) \exp(-S_n(A+1)/kT)$$

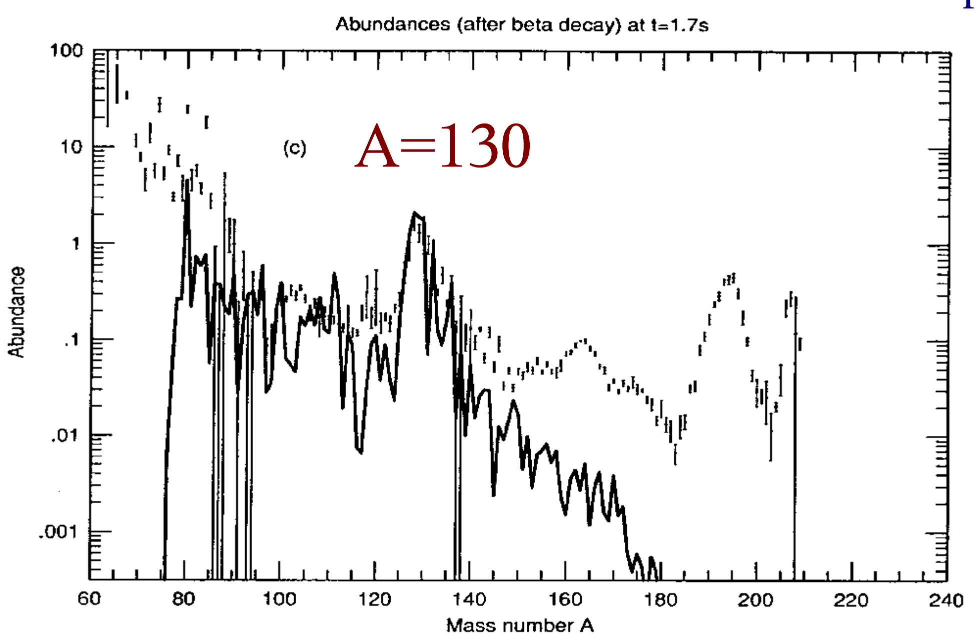
$$\frac{Y(Z,A+1)}{Y(Z,A)} = n_n \frac{G(Z,A+1)}{2G(Z,A)} \left[\frac{A+1}{A}\right]^{3/2} \left[\frac{2\pi\hbar^2}{m_u kT}\right]^{3/2} \exp(S_n(A+1)/kT)$$

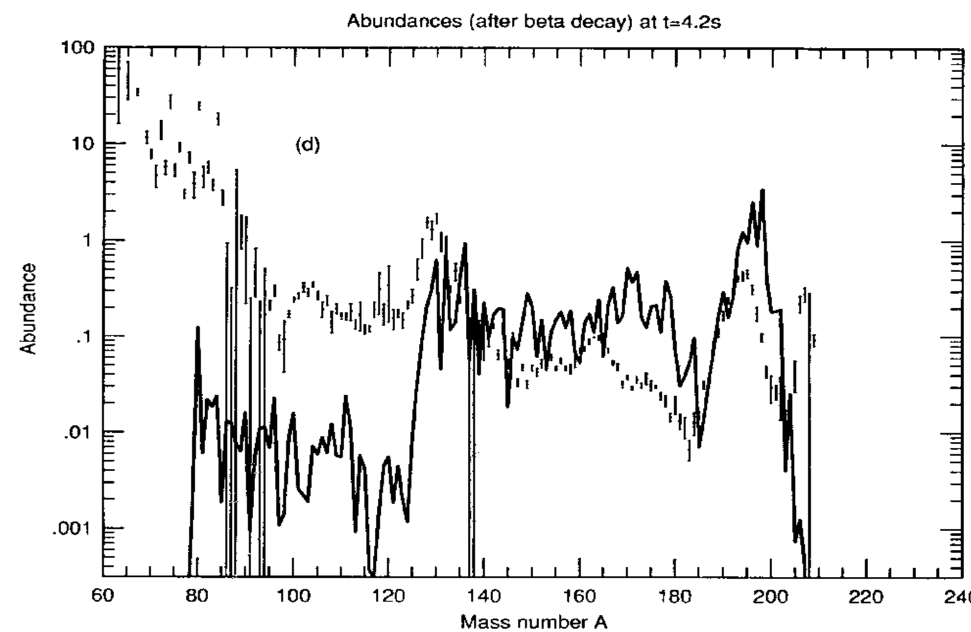
classical calculation with n_n=const and T=const



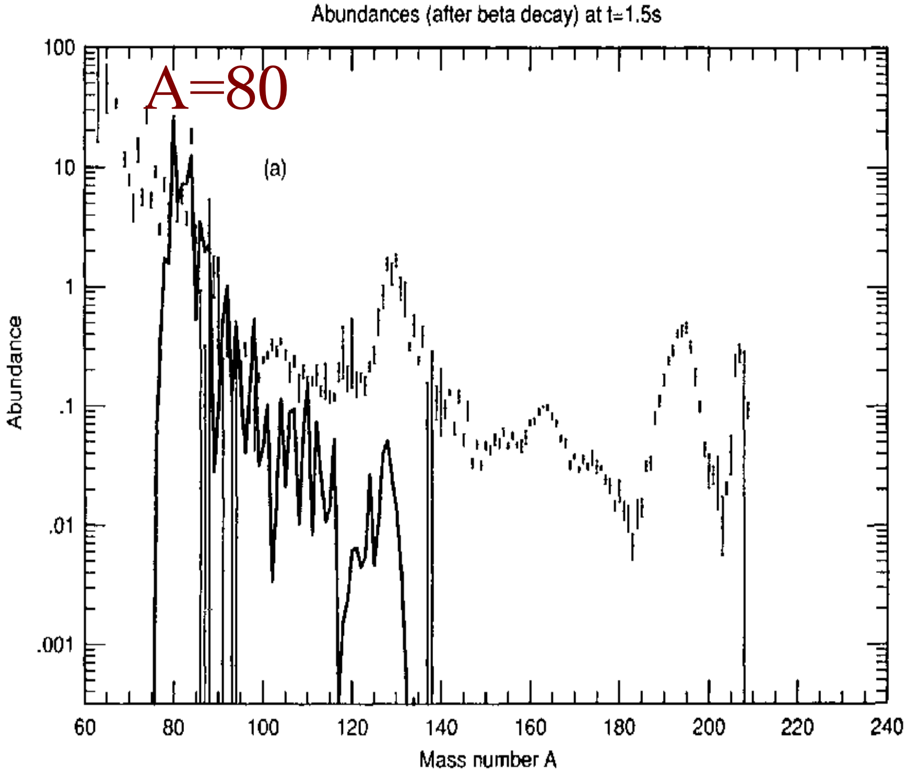


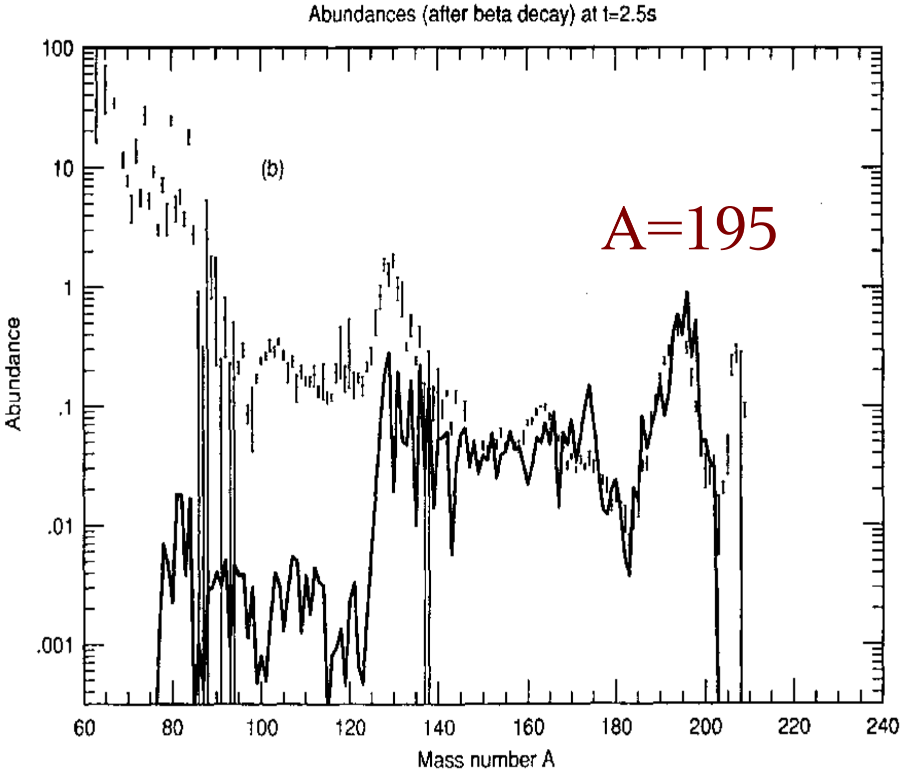
conditions with reproduce A=130 at t=1.7s





A=80 and 195 peaks





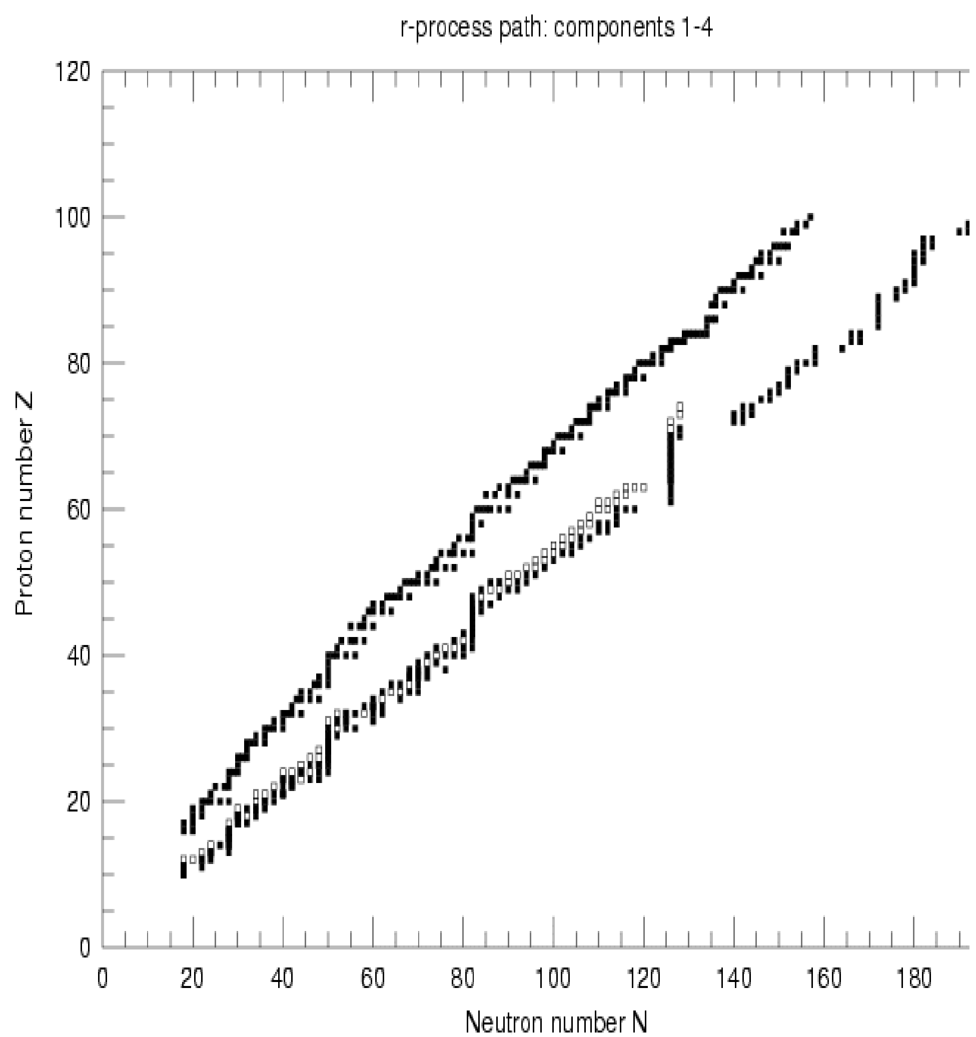
t=1.5s

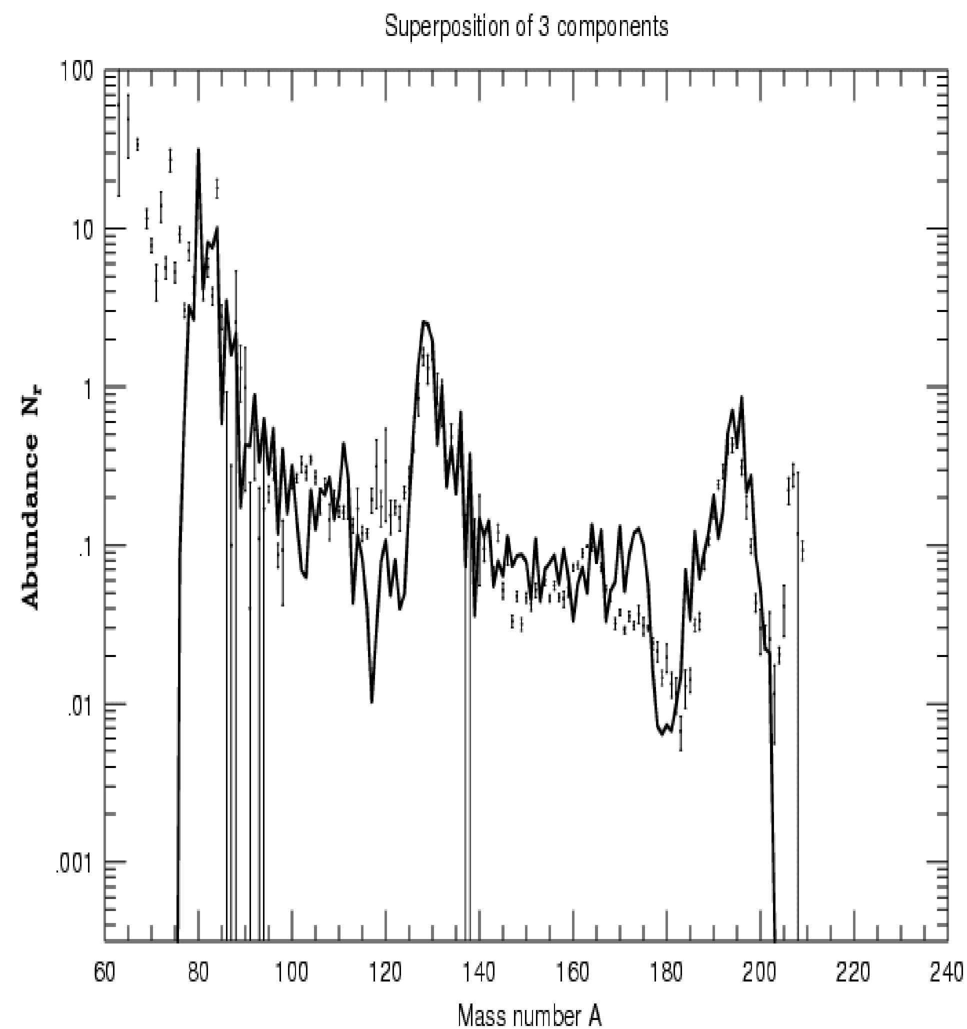
t=2.5s

three components produce the A=80, 130, and 195 peaks during "comparable" timescales (for the first time experimental half-lives and masses are known in the r-process path at A=80 and 130)!

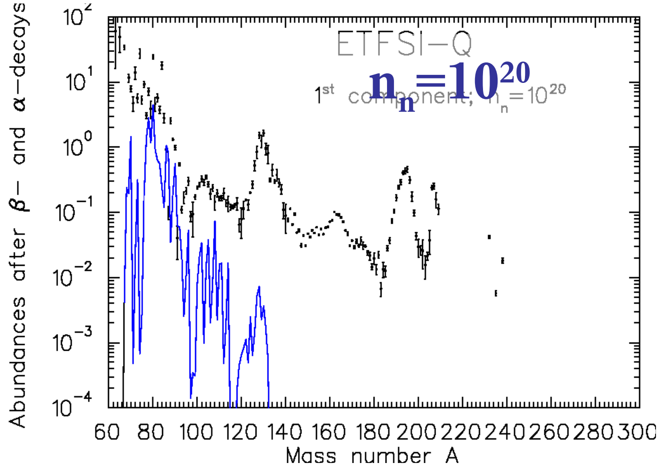
Following three S_n's for timescales t_1,t_2,t_3

Kratz, Bitouzet, Thielemann, Möller, Pfeiffer and permutations 1993-1999



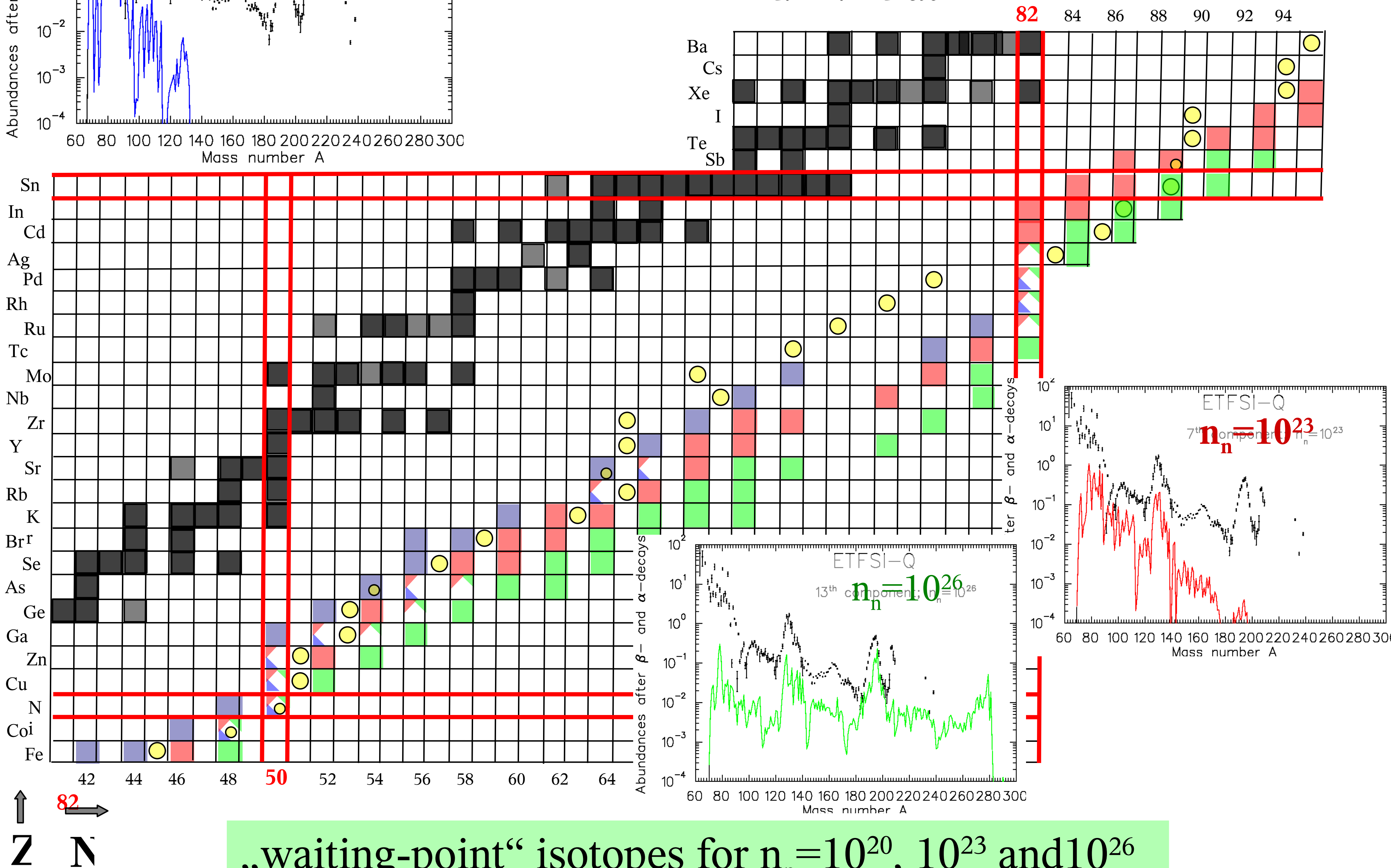


constant n_n and T for timescale t and afterwards instantaneous beta-decay



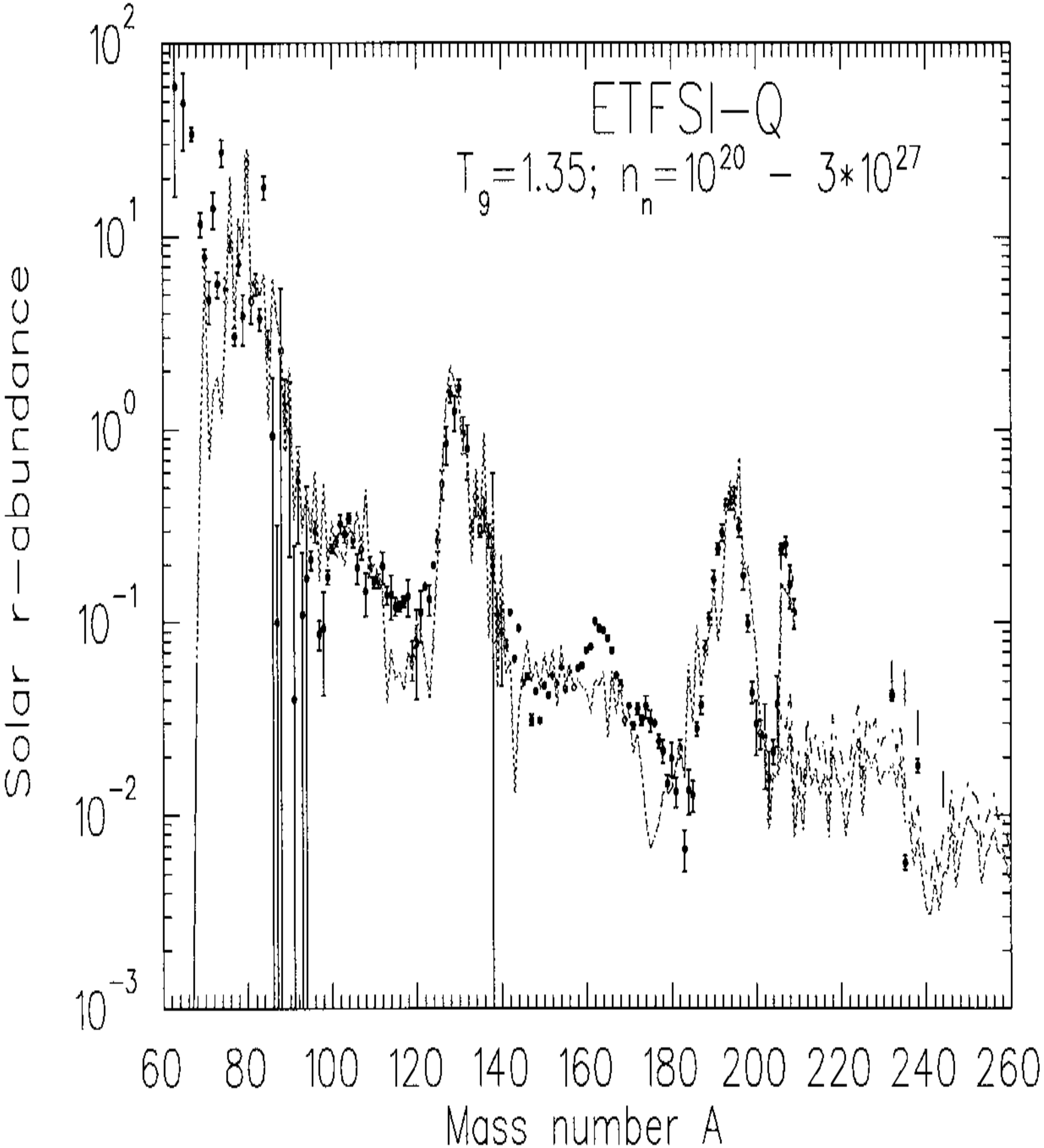
r-Process paths for $n_n=10^{20}$, 10^{23} and 10^{26}

K.-L. Kratz

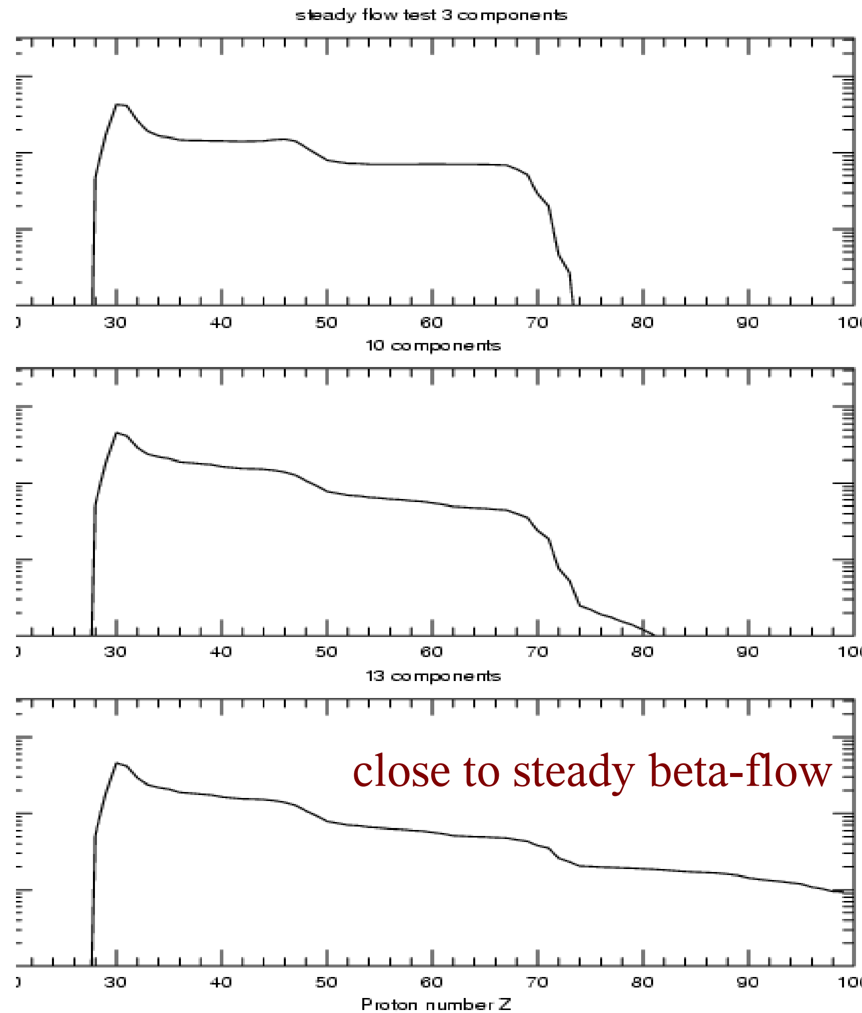


„waiting-point“ isotopes for $n_n=10^{20}$, 10^{23} and10^{26}

Multi-components and steady beta-flow



superposition with weights
$w(n_n)=8.36\ 10^6 n_n^{-0.247}$ and $t(n_n)=6.97\ 10^{-2}\ n_n^{0.062}s$



decay rate of complete Z-chain multiplied with total abundance of Z-chain close to constant in between magic numbers (where long half-lives are encountered).

Varying the Superposition Range

This is a fit, there is freedom in choosing the superpositions. We do not know which components exist in a realistic astrophysical environments, whether the whole r-process abundance range comes from one event or whether there are „weak“ and „strong“ r-process components!