Recent Developements in R-Matrix Analysis

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Outline

- Level Shifts and Boundary Conditions
- R-matrix description of β decays
- The channel radius
- R-matrix reduced widths from transfer reactions

Level shifts and boundary conditions

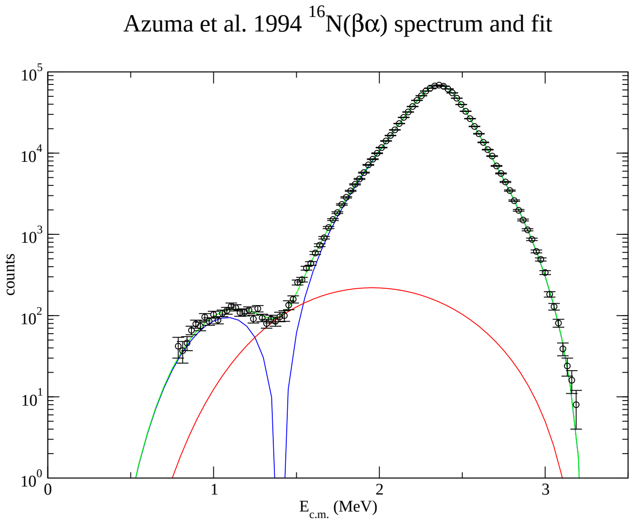
- The level shift in R-matrix theory is well-known to depend upon the choice of boundary conditions.
- F.C. Barker showed in 1972 that the R-matrix are invariant for changes in the B_c, even for a finite number of levels.
- Analogous to rotational invariance or Lorentz transformations.
- R-matrix equations have been formulated which are mathematically equivalent to Lane and Thomas but which have no B_c (Angulo, Descouvemont, Brune).
- I consider this to be a solved problem.

β-Delayed Particles: $A \rightarrow a + b + e + \nu$

- Analysis can supply considerable useful information. The $a + b$ relative energy spectrum is particularly useful.
- But how does one do the analysis?
- For the relative energy spectrum, F.C. Barker has proposed (as opposed to derived!):

$$N_c(E) = f_\beta P_c \left| \sum_{\lambda\mu} B_\lambda \, \gamma_{\mu c} \, A_{\lambda\mu} \right|^2$$

  The B_λ are feeding factors, unfortunate notation...
- What are the underlying assumptions?
- Are the feeding factors B_λ real?
- How does one go beyond the allowed approximation?



What fills in the minimum?

Return to First Principles (with G.M. Hale):

- Use β-decay formalism of H. Behring and W. Bühring, *Electron Radial Wavefunctions and Nuclear Beta Decay* (1982).
- Start from:

$$\begin{aligned} d\Gamma &= (2\pi)^4 \delta^3(\vec{p}_A - \vec{p}_B - \vec{p}_e - \vec{p}_\nu) \\ &\times \delta(E_A + M_A - M_a - M_b - E_B - E_\alpha - W_e - W_\nu) \\ &\times |T|^2 \frac{d^3\vec{p}_a}{(2\pi\hbar)^3} \frac{d^3\vec{p}_b}{(2\pi\hbar)^3} \frac{d^3\vec{p}_e}{(2\pi\hbar)^3} \frac{d^3\vec{p}_\nu}{(2\pi\hbar)^3} \end{aligned}$$

- $T \propto \langle a + b | H_{\text{weak}} | A \rangle$.
- Use an R-matrix expansion for T.
- Integrate over unobserved variables.

Summary of Findings

- Barker's formula for the relative energy spectrum is verified, assuming the allowed approximation and ignoring $e - \nu$ recoil effects.
- The feeding factors B_λ are matrix elements involving the R-matrix eigenfunctions.
- The B_λ are real provided that nuclear currents are time-reversal invariant.
- A formalism for calculating higher-order corrections is provided (e.g., recoil, forbidden transitions).
- It would also be straightforward to include the external contribution (à la Barker-Kajino) to beta decay. This contribution will be important when the β decay of weakly-bound nuclei is analyzed, e.g. $^6\mathrm{He} \to \alpha + d + e + \nu$.

How should one choose the channel radius (a_c)? (I do not have the answer...)

- Lane and Thomas assume the nuclear force vanishes beyond the channel radius $\rightarrow$ maybe we should we should use "large" a_c.
- However, there are practical problems with large a_c:
  - The energies of background pole(s) are lowered.
  - The magnitude of the hard-sphere phase shift is increased. Some would say that this is unphysical.
  - Isospin violation is "artificially" magnified.
  - Dimensionless reduced width lose their meaning. Not a problem for me.
- The tail of the nuclear force is not negligible for channel radii commonly used (Descouvemont, Nollett, Wiringa...). What are the consequences ?

So how do we proceed ?

- It is essential that sensitivity of one's conclusions to the channel radius be studied and that the effects of the uncertainty in the channel radius be included in the error analysis.
- Many studies have found that their conclusions are insensitive to the choice of channel radius, provided that sufficient flexibility is allowed for in the background levels.
  - For elastic scattering there is a trade-off between the hard-sphere phase shift and the background state.
  - For reaction channels there is a similar trade-off for the cross section between resonances.
  - These findings are perfectly natural when considered from the point of view of completeness!
  - I believe this path is the correct one nearly all cases: a strong sensitivity the conclusions (e.g. extrapolated S-factors) likely indicates that additional background states need to be included.
- There is a strong temptation to minimize the number of free parameters (e.g. levels) when carrying out R-matrix analysis. Beware.

A Particular Case: $^{12}\mathrm{C}(\alpha,\alpha)$, Tischhauser *et al.*, Phys. Rev. Lett **88**, 072501 (2002).

- This work fitted the channel radius and found $a = 5.42^{+0.16}_{-0.27}$ for $^{12}\mathrm{C}+\alpha$.
- The same channel radius was used for all $^{12}\mathrm{C}+\alpha$ partial waves.
- The restriction on channel radius very interesting: it indicates physics "beyond Lane and Thomas." Probably either truncation of the number of levels or the impact of the nuclear potential beyond the channel radius.
- It would be very interesting to see which partial wave(s) are responsible for this restriction as well as other details of this fit.

One Approach to these Questions: The Unified R-Matrix-Plus-Potential Model C.H. Johnson, Phys. Rev C **7**, 561 (1973)

- The particular case analyzed is $^{16}\mathrm{O}(n, n)$ and $^{13}\mathrm{C}(\alpha, n)$, an important one for astrophysics...
- Builds upon some suggestions of Vogt; used subsequently by Tombrello, Koonin, Langanke, and others for $^{12}\mathrm{C}(\alpha, \gamma)$.
- Provides a consistent framework for combining the R-matrix and a real optical potential (preservers unitarity).
- Includes effect of the optical potential for $r > a_c$ on penetration and shift functions.
- Should be particularly useful for the analysis of elastic scattering, where the optical potential plays a critical role.
- Allows for a check on the assumptions of phenomenological R-matrix fits which assume the nuclear potential vanishes for $r > a_c$.

Relating Transfer Reactions and R-matrix Reduced Widths

- Transfer reactions measure single-particle (or cluster) wavefunctions at the nuclear surface. R-matrix reduced-width amplitudes are the amplitude of the single-particle (or cluster) wavefunctions at the nuclear surface. The connection is obvious.
- The key for relating the two methods is through the observed width (for unbound states) or the asymptotic normalization (bound states). Formulas have been given by many people over the years (e.g., J.P. Schiffer, H.T. Fortune, C. Iliadis, A. Mukhamedzhanov, C.R. Brune, G. Rogachev).
- Yet, there remains some confusion. For example

$$\Gamma_p = C^2 S \Gamma_{s.p.} \quad \text{with} \quad \Gamma_{s.p.} = \frac{\hbar^2 a_c P_l}{\mu} |R_{s.p.}(a_c)|^2$$

which is used in a study of $^{18}\mathrm{F}(d,p)$ by N. de Sèrèville *et al.*, Nucl. Phys. **A791**, 251 (2007).

Concluding Remarks

- The understanding and application of R-matrix methods continues to improve:
  - better computers
  - more experience
- There is need for further theoretical work:
  - Tests of R-matrix methods with a Toy Model (e.g., RGM: Descouvemont).
  - DWBA analysis of transfer reactions to unbound states.
- A lot of good work has been done in the past, let's try not to overlook it.