Monte Carlo Reaction Rates

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Some history on “experimental” laboratory rate evaluations…

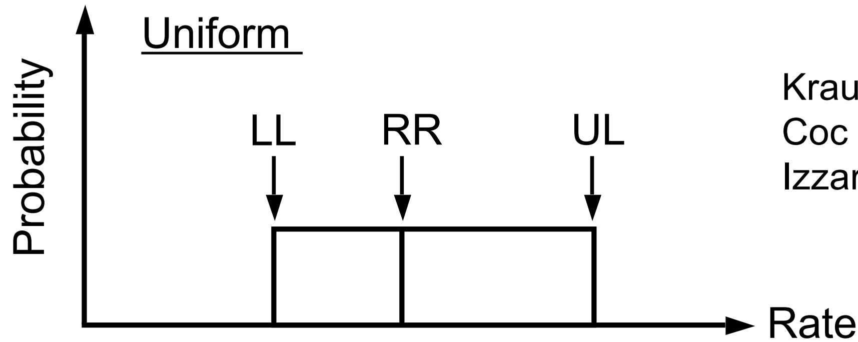
Caughlan & Fowler (1988)	: 128 reactions (no uncertainties)
NACRE, NPA (1999)	: 86 reactions (incl. uncertainties)
Iliadis et al., ApJS (2001)	: 55 reactions (incl. unstable targets)

What is the statistical meaning of published rates?

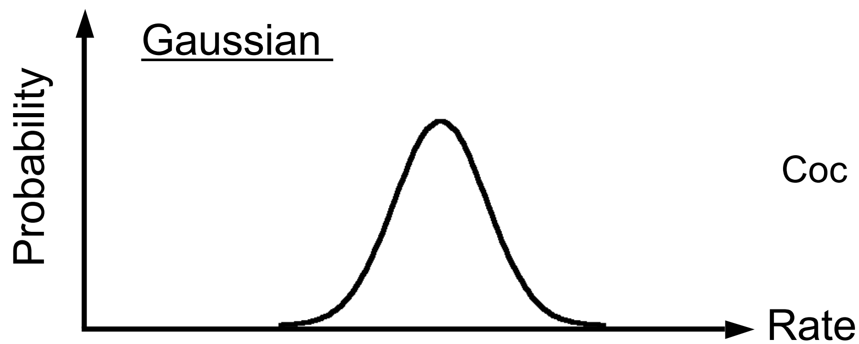
recommended rate?
“lower rate limit”?
“upper rate limit”?

What is the probability density function (PDF) of total rates?

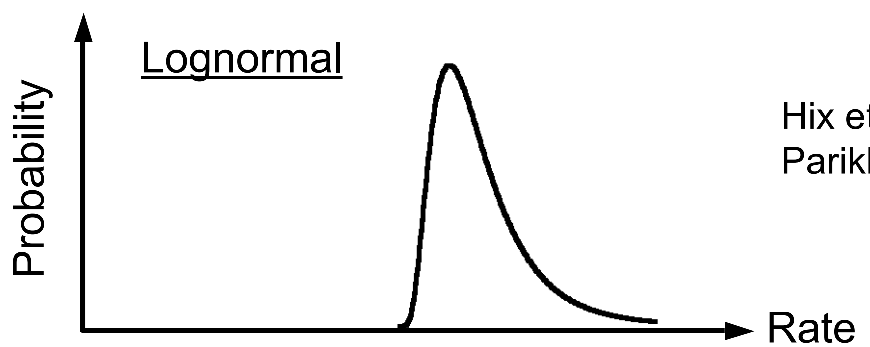
Classical interpretation of rate PDF:



Krauss & Chaboyer, Science 299, 65 (2003)
Coc et al., PRD 65, 043510 (2002)
Izzard et al., AA 466, 641 (2007)

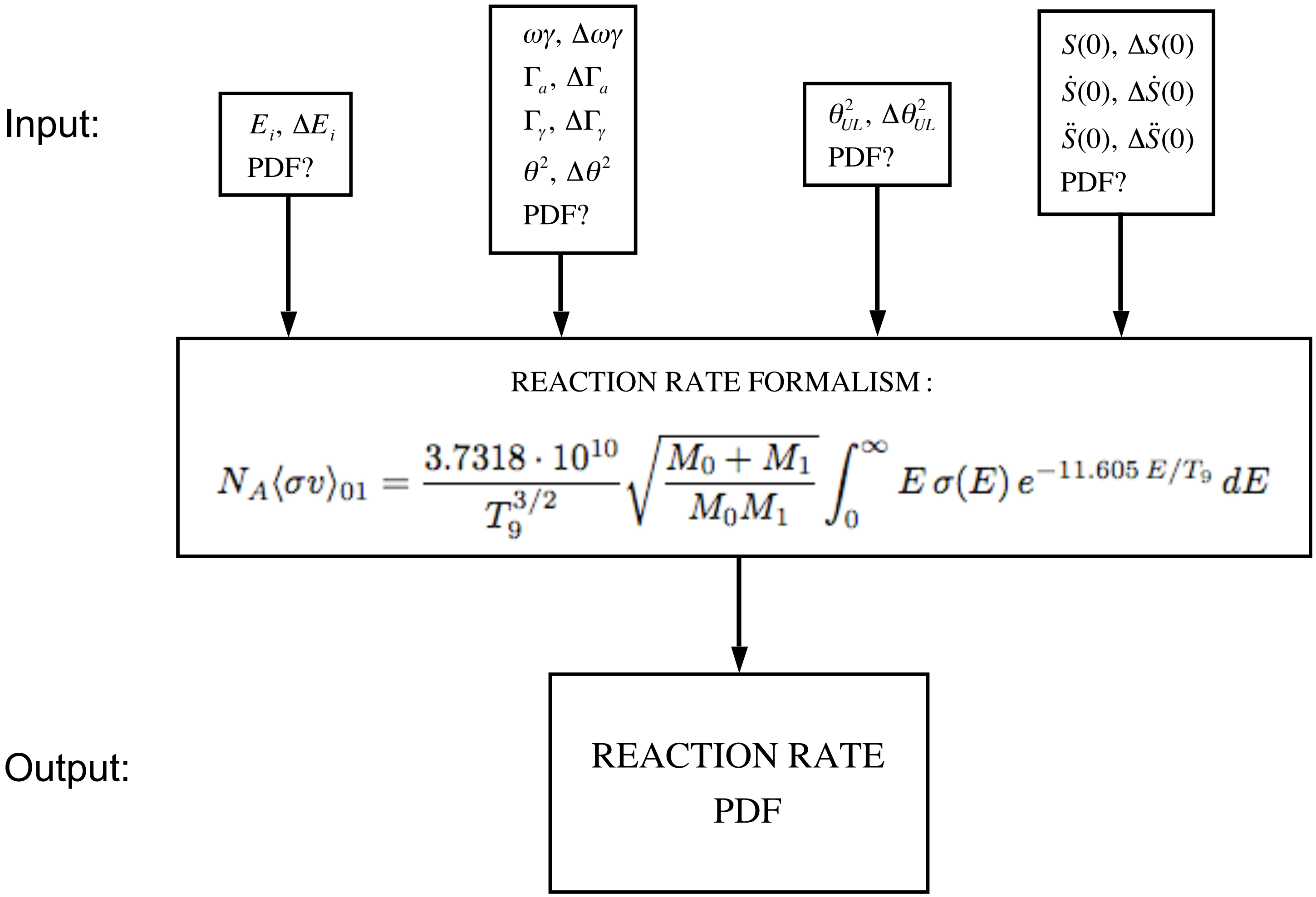


Coc et al., ApJ 600, 544 (2004)



Hix et al., NPA 718, 620 (2003)
Parikh et al., ApJS (submitted)

Monte Carlo method: A=16 and up…



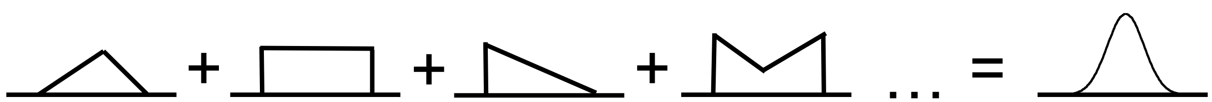
What is the (input) PDF for a resonance energy?

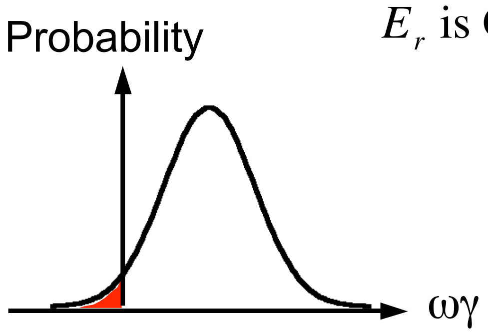
$$E_r,\ \Delta E_r \quad \rightarrow 50\% \text{ of thick - target yield curves}$$

$$E_r = E_x - Q \quad \rightarrow \gamma\text{ - ray spectroscopy}$$

Central limit theorem of statistics:

sum of n independent random variables x_i becomes a Gaussian random variable in the limit of large n, independent of the form of the individual PDFs of the x_i





E_r is Gaussian distributed:

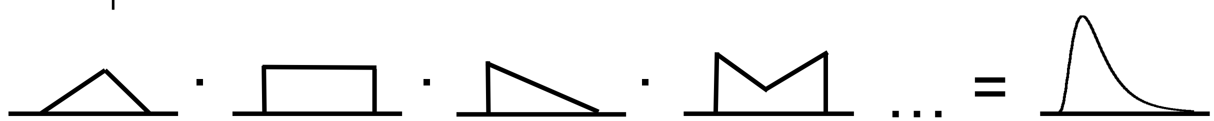
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$E[x] = \mu, \quad V[x] = \sigma^2$$

What is the (input) PDF for a resonance strength or partial width?

Central limit theorem of statistics:

product of n independent random variables x_i becomes a lognormal random variable in the limit of large n, independent of the form of the individual PDFs of the x_i

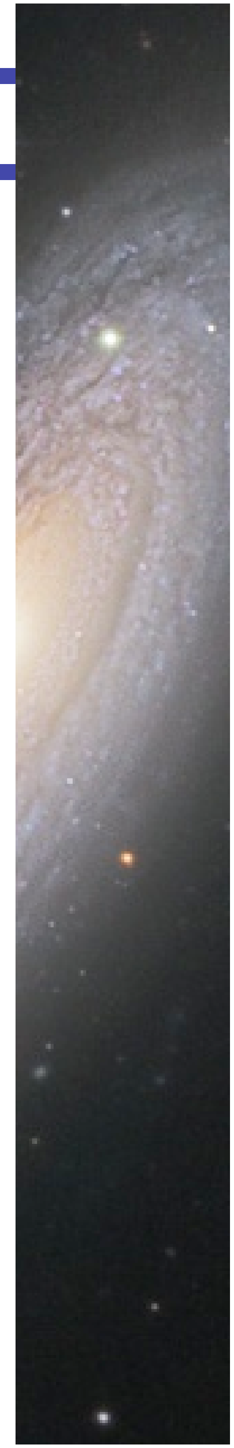


$\omega\gamma$, Γ_i are lognormally distributed:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{x} e^{-(\ln x - \mu)^2/(2\sigma^2)}$$

$$\mu = \ln(E[x]) - \frac{1}{2}\ln\left(1 + V[x]/E[x]^2\right), \quad \sigma = \sqrt{\ln\left(1 + V[x]/E[x]^2\right)}$$

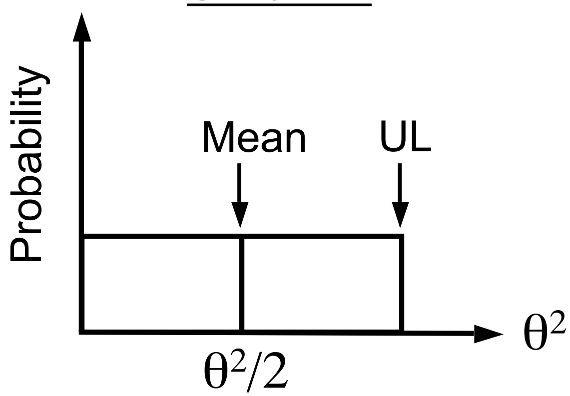
If y is Gaussian distributed, then $x = e^y$ will be lognormally distributed



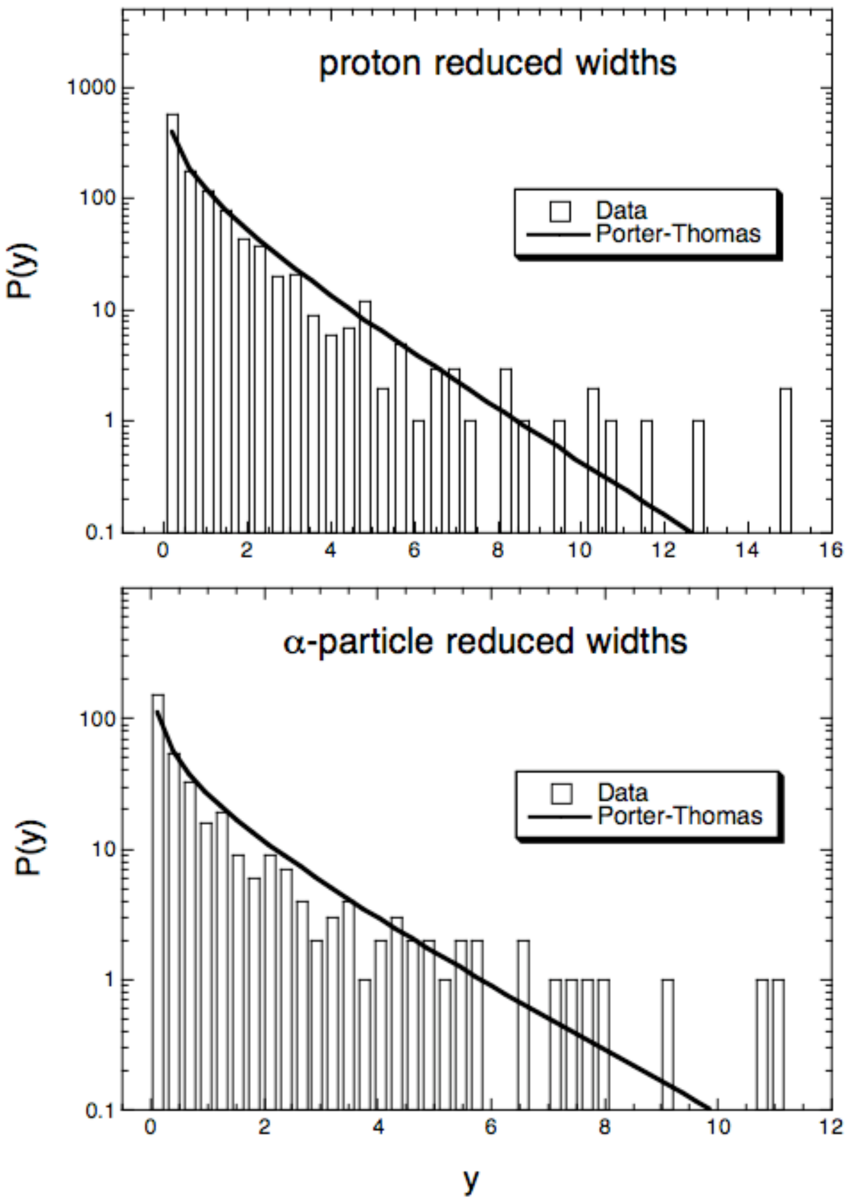
Upper limits of resonance strengths or partial widths I

Meaning of : $\theta_a^2 \leq 0.1$? Need confidence level! What is PDF?

Uniform?



Upper limits of resonance strengths or partial widths II

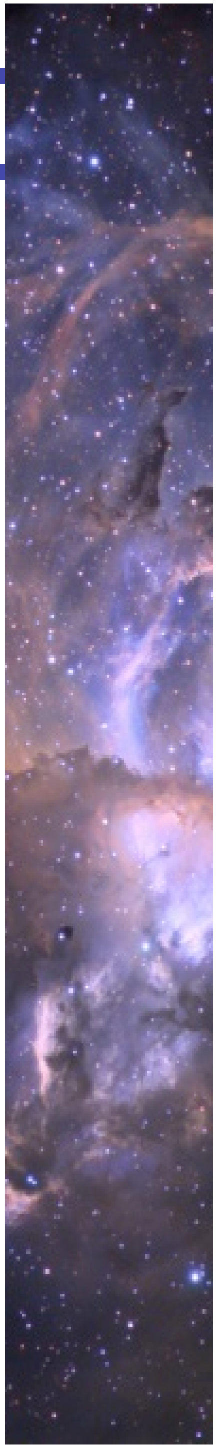


$$y = \frac{\theta^2}{\langle \theta^2 \rangle}$$

$\langle \theta^2 \rangle$ = local mean value for ℓ and A

(p,p), (p,α) on ^{24}Mg, ^{28}Si, ^{36}Ar, ^{40}Ca

Mitchell et al. (1975 - 95), analyzed using R - matrix code MULTI



Upper limits of resonance strengths or partial widths III

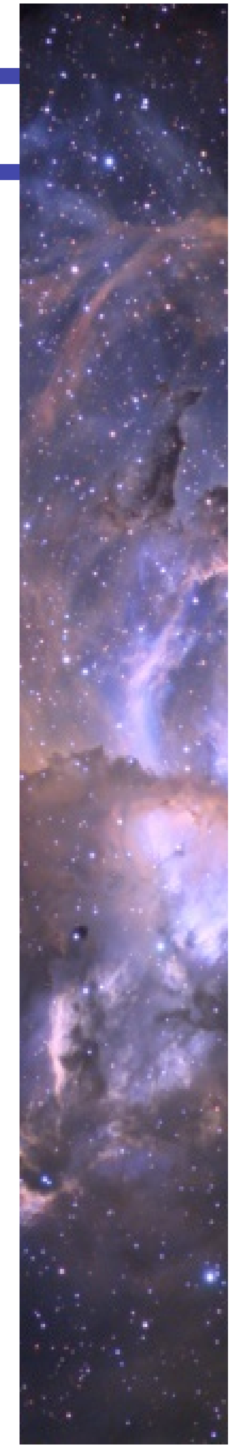
θ_a^2 is Porter - Thomas distributed:

$$f(\theta^2) = \frac{c}{\sqrt{\theta^2}} e^{-\theta^2/(2\langle\theta^2\rangle)}$$

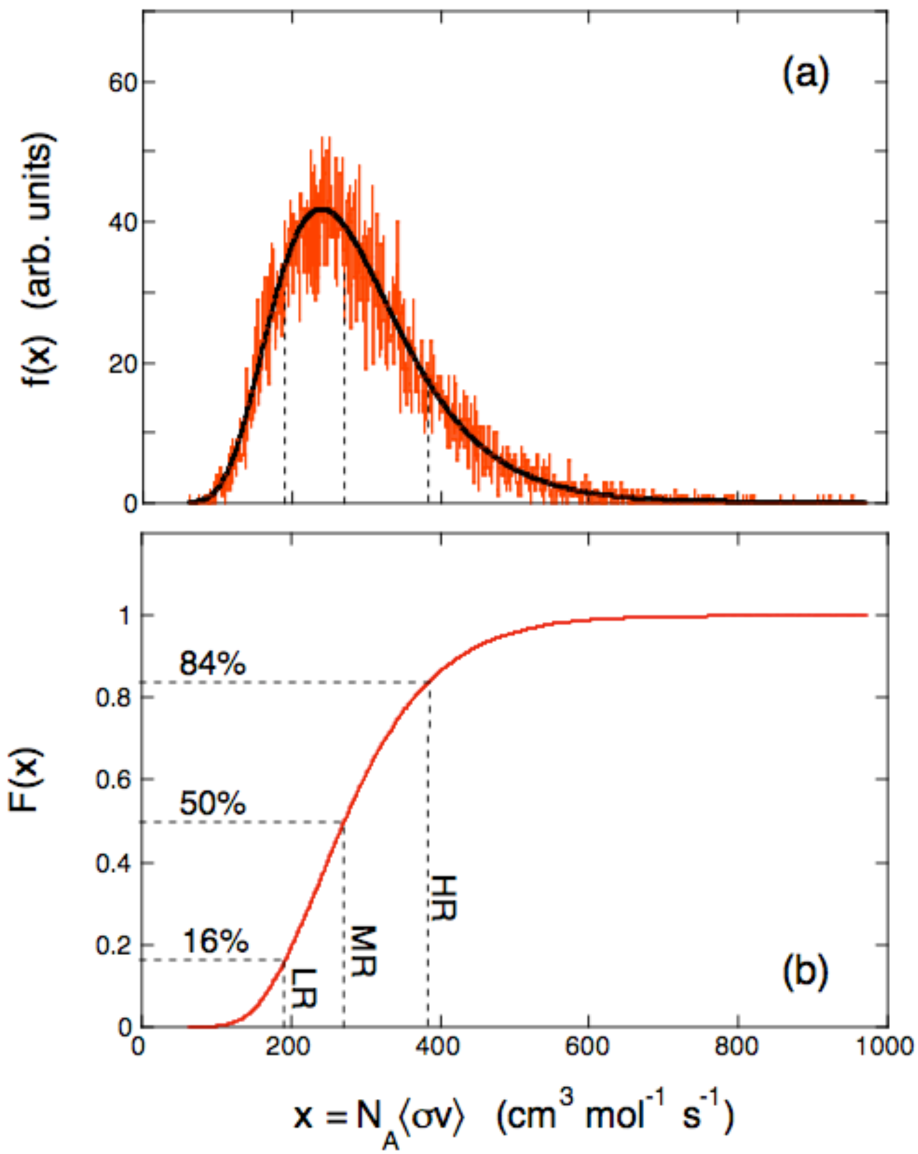
Porter and Thomas, PR 104, 483 (1956)

Porter - Thomas distribution = chi - squared distribution with
with one degree of freedom

If x is Gaussian distributed, then x^2 is chi - squared distributed



Monte Carlo reaction rates



Schematic (not real) example:

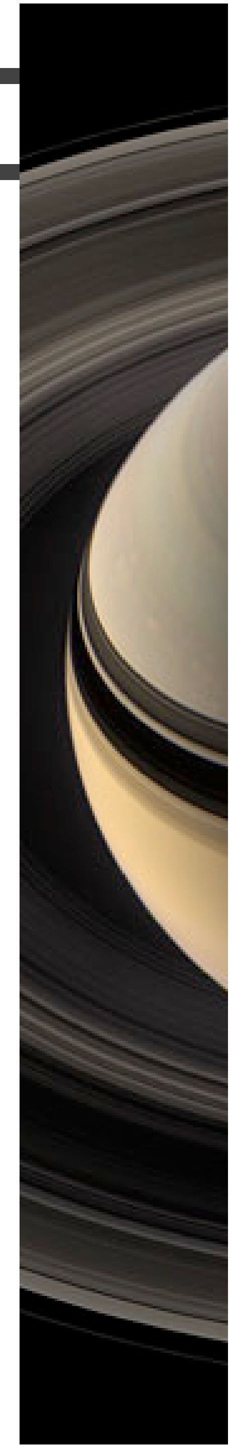
$^{20}\mathrm{Ne}(\alpha,\gamma)^{26}\mathrm{Mg}$

$T = 0.5$ GK

$E_r = 300 \pm 15$ keV

$\omega\gamma = 4.1 \pm 0.2$ eV

10,000 samples



Charged-Particle Thermonuclear Reaction Rates: I. Monte Carlo Method and Statistical Distributions

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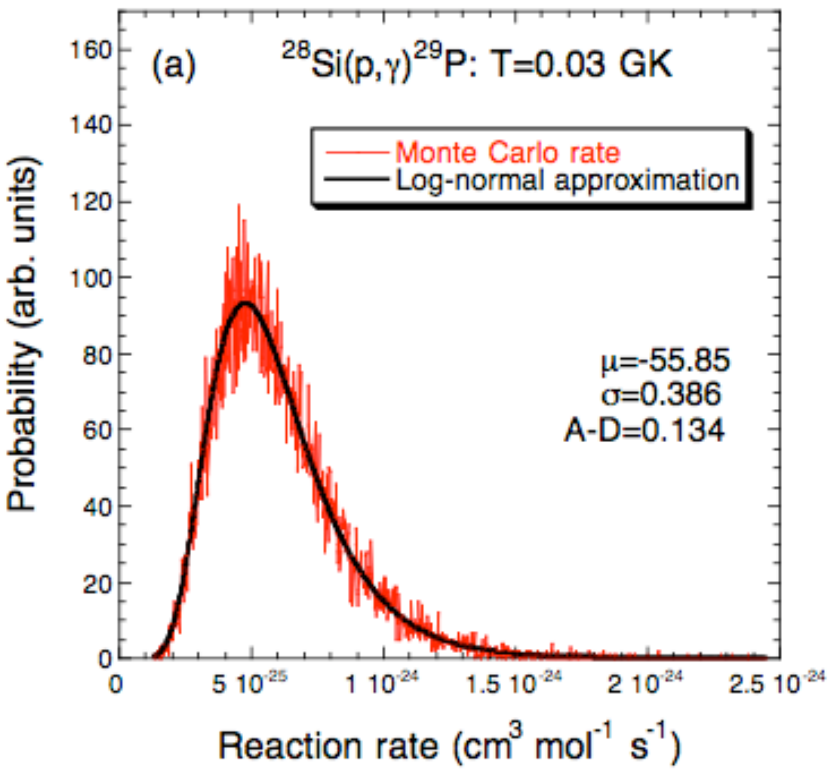
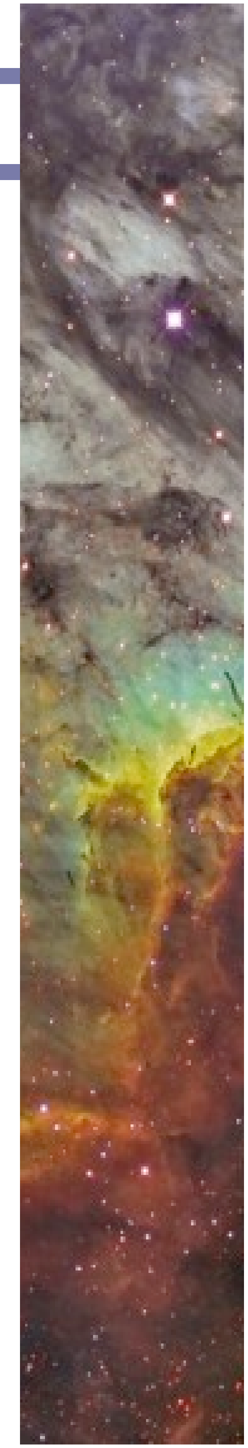
Real example: $^{28}Si(p,\gamma)^{29}P$

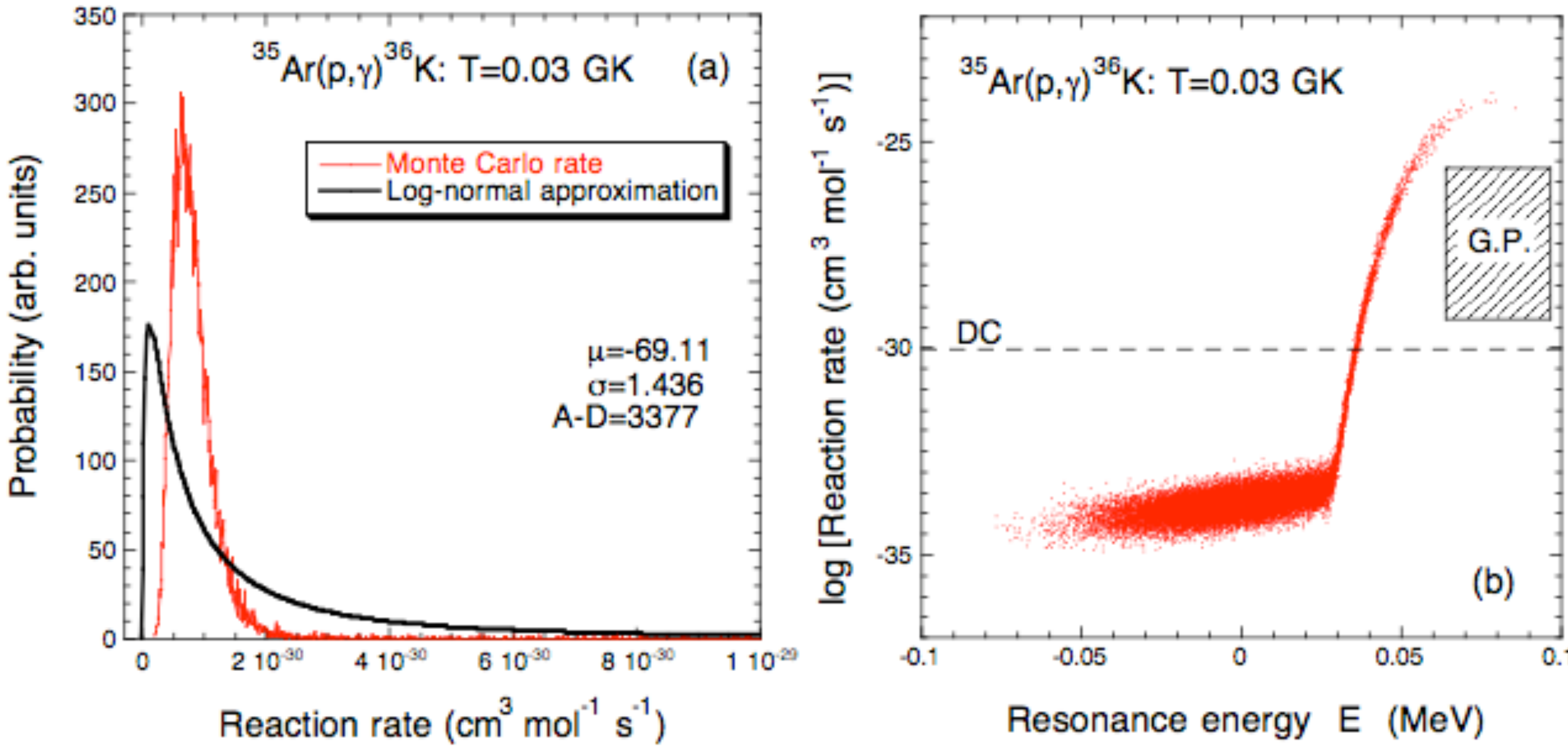


Most (but not all) reaction rates are lognormally distributed

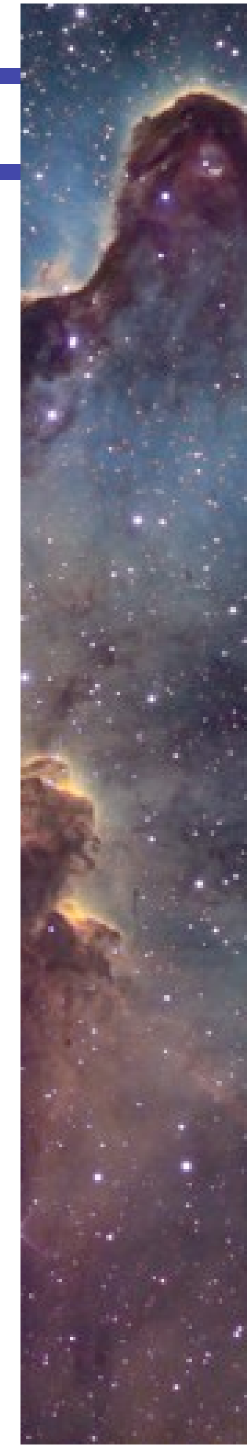


Real example: $^{35}Ar(p,\gamma)^{36}K$ at T=0.03 GK

Rates dominated by single resonance at 2±21 keV and direct capture



Reaction rate is NOT lognormally distributed



Summary

1. New Monte Carlo method is the foundation for new charged-particle reaction rate evaluation, to be published in near future

2. Monte Carlo rates have a statistical meaning

3. New Monte Carlo rates can be used directly in subsequent Monte Carlo nucleosynthesis calculations

4. New rates will refine our understanding of experimental needs