^{12}C(α, γ)^{16}O: the theoretical challenge

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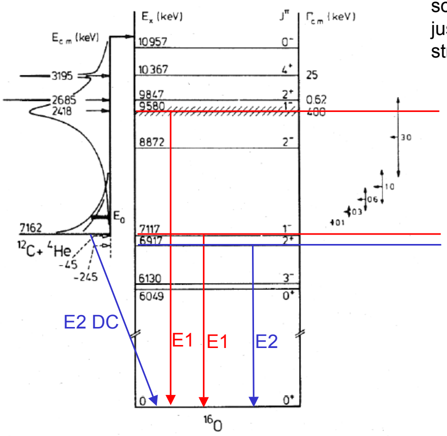
- E1 part: hybrid R-matrix
- E2 part: microscopic approaches
- future approaches: theoretical hopes?
- ...

The challenge



some tails of resonances just make the reaction strong enough …

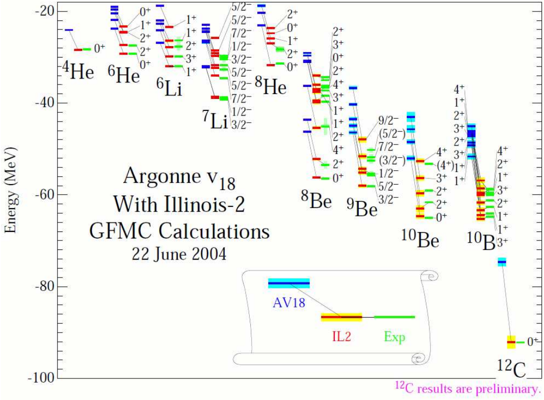
resonance (high lying)

resonance (sub threshold)

resonance (sub threshold)

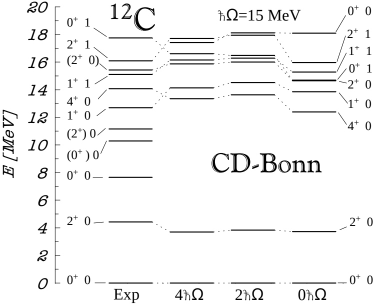
complications:
- very low cross section makes direct measurement impossible
- subthreshold resonances cannot be measured at resonance energy
- Interference between the E1 and the E2 components

Green's Function Monte Carlo



Impressive agreement for light nuclei, but no spectrum for ^{12}C and ^{16}O is yet out of reach.

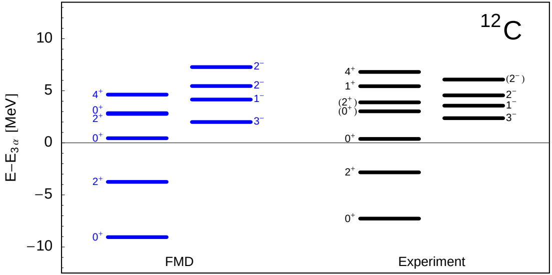
No-core shell model: ^{12}C



Hoyle state is not reproduced in model space; it is too strongly clustered.

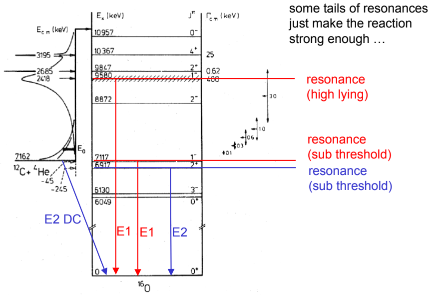
B. Barrett, P. Navratil, E. Ormand

Fermionic Molecular Dynamics: ^{12}C



Flexible model space of shell model and cluster states; calculation based on realistic nucleon-nucleon interaction —
Hoyle state is reproduced in model space.
H. Feldmeier, T. Neff, R. Roth

E1 part: general remarks



complications:
- very low cross section makes direct measurement impossible
- subthreshold resonances cannot be measured at resonance energy
- Interference between the E1 and the E2 components

16

- the 7.12 MeV state is a good shell model state
- the 9.5 MeV resonance has a pronounced α structure
- however, NO $T = 0 \rightarrow T = 0$ E1 transitions
- $T = 1$ admixtures matter; how large???

The hybrid R-matrix model

Incorporates potential model description of scattering states, including the 9.5 MeV resonance, with flexibilty of R-matrix parametrization of 7.12 MeV state

$^{12}C(\alpha,\gamma)^{16}O$ reaction (E1 part)

Hybrid R-matrix (S.E. Koonin, T. Tombrello)

parametrized cross section and phase shift:

$$\sigma_{E1}(\alpha,\gamma) = \frac{6\pi}{k^2} P \left| \frac{R_{\alpha\gamma}}{1-(S+iP)R_{\alpha\alpha}} \right|^2$$

with $\delta = -\phi + \arctan\left(\frac{R_{\alpha\alpha} P}{1 - R_{\alpha\alpha}(S-B)}\right)$

} conventional R-matrix

in terms of R-matrices:

$$R_{\alpha\alpha} = R^0_{\alpha\alpha} + R^1_{\alpha\alpha} \qquad R_{\alpha\gamma} = R^1_{\alpha\gamma} + R^1_{\alpha\gamma}$$

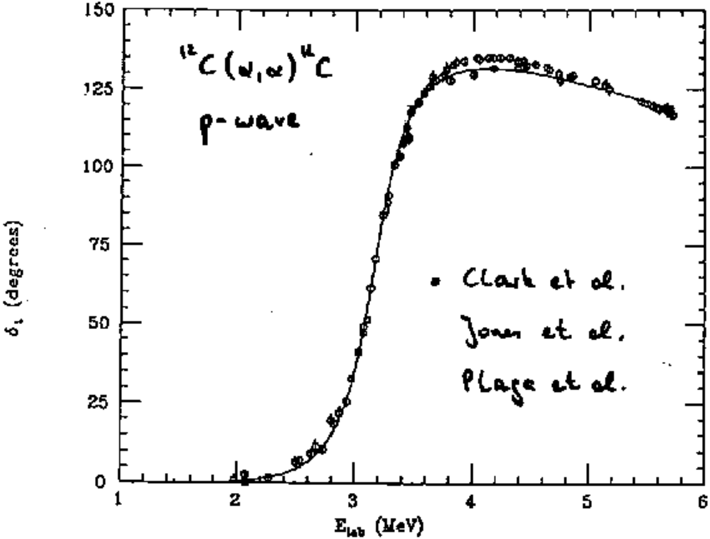
bound state: $R^1_{\alpha\alpha} = \frac{\gamma_\alpha^2}{E_1 - E}$; $R^1_{\alpha\gamma} = \frac{\gamma_\alpha \Gamma_\gamma^{1/2}}{E_1 - E}$

potential part: $R^0_{\alpha\alpha}$, $R^0_{\alpha\gamma}$ } determined from $\alpha + {}^{12}C$ potential (R_0, a_0, V_0 + effective dipole strength)

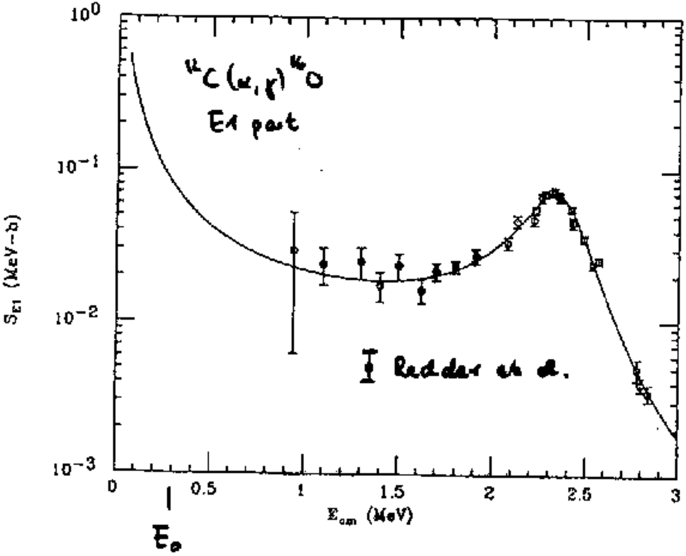
(determines resonance + background + interfer.)

Simultaneous fit to (α,γ) cross sections + p-wave phase shifts to determine parameters

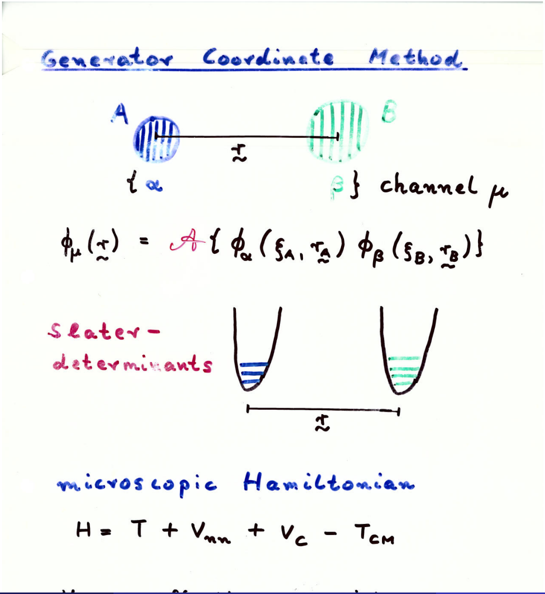
Hybrid R-matrix: phase shift fit



Hybrid R-matrix: E1 cross section



Microscopic models: general idea



Generator Coordinate Method

Equation of motion

$$\psi = \sum_{\mu} \int d^3\mathbf{r}\; \phi_\mu(\mathbf{r})\, f_\mu(\mathbf{r})$$

with f_μ determined by

$$\langle \phi_\mu(\mathbf{r}) \,|(H - E)|\, \psi \rangle = 0$$

$$\Downarrow$$

$$\sum_{\mu'} \int d^3\mathbf{r}' \left\{ H_{\mu\mu'}(\mathbf{r},\mathbf{r}') - E\, N_{\mu\mu'}(\mathbf{r},\mathbf{r}') \right\} f_{\mu'}(\mathbf{r}') = 0$$

for all $\mu, \mathbf{r}$.

$$\begin{matrix} H_{\mu\mu'}(\mathbf{r},\mathbf{r}') \\ N_{\mu\mu'}(\mathbf{r},\mathbf{r}') \end{matrix} = \left\langle \phi_\mu(\mathbf{r}) \left| \begin{matrix} H \\ 1 \end{matrix} \right| \phi_{\mu'}(\mathbf{r}') \right\rangle$$

GCM: asymptotic conditions

2) wavefunction of relative motion

$$\vec{g} = N^{1/2} \cdot \vec{f}$$

asymptotic form

$$g_\mu(\underset{\sim}{x}) = \int T_\mu(\underset{\sim}{x}, \underset{\sim}{r}) f_\mu(\underset{\sim}{r})\, d^3 \underset{\sim}{r}$$

scattering boundary condition

$$g_\mu^\alpha(\underset{\sim}{x}) \rightarrow \delta_{\alpha\mu} F_\mu(\underset{\sim}{x}) + K_{\alpha\mu} G_\mu(\underset{\sim}{x})$$

for the entrance channel α

$$K = i \, \frac{1 - S}{1 + S}$$

$\Downarrow$

S - matrix

GCM approach to the E1 part

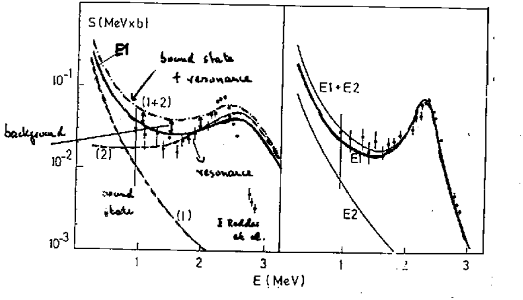
The problem: simultaneous description of dominant configurations

- α-like structures: scattering states, 9.5 MeV resonance
- single-particle states: 7.12 MeV resonance
- $T = 1$ admixtures!

Attempt: α+^{12}C and p+^{15}N wave functions,
where p+^{15}N can be coupled to $T = 1$

Strategy: replace GCM results by data, if available

Generator Coordinate Method: E1 part



P. Descouvemont and D. Baye

Microscopic potential model

<u>microscopic potential model</u>

microscopic: use of antisymmetrized many-body wave function

$$\left(\Psi = \mathcal{A} \{ \phi_1 \; \phi_2 \; g(\underset{\sim}{x}) \} \right)$$

flexible: adjustment of potential to reproduce relevant experimental data

Schrödinger eq. of relative motion:

$$\left\{ -\frac{\hbar^2}{2\mu} \underset{\sim}{\Delta}_x + V_\ell(x) - E \right\} g(\underset{\sim}{x}) = 0$$

$V_\ell(x)$ is local in a good approximation

Microscopic potential model

Caution!

equation of motion in cluster theories

$$\boxed{(h - E n)\, g = 0}$$

$$n^{1/2}\left(n^{-1/2} h\, n^{-1/2} - E\right) n^{1/2}\, g = 0$$

$$\boxed{\Lambda\,(\tilde{h} - E)\,\tilde{g} = 0}$$ orthogonalized equation of motion

with
$$\tilde{h} = T + V_N(x) + V_c(x)$$
$$g = n^{-1/2}\,\tilde{g} \quad \left(= \int n^{-1/2}(x,x')\,\tilde{g}(x')\,dx'\right)$$
$$= \sum_N \mu_N^{-1/2} \langle u_N | \tilde{g} \rangle\, u_N$$

steps towards solution

a) determination of potential parameters

b) calculation of $\tilde{g}$ from orthogonalized equation of motion

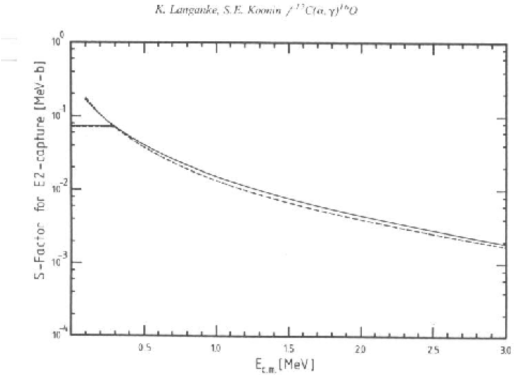
c) construction of many particle wave function

Microscopic potential model: E2 part

based on microscopic α+^{12}C wave functions
potential constrained to:

- binding energies of ground state and 6.92 MeV state
- lifetime of 2^+ state at 6.92 MeV
- energy and width of 4^+ resonance at 10.35 MeV
- B(E2) transitions: $2_1 \rightarrow 0_2$, $4_1 \rightarrow 2_1$
- α+^{12}C phase shifts
- the microscopically correct number of nodes in g(r)

Microscopic potential model: E2 part



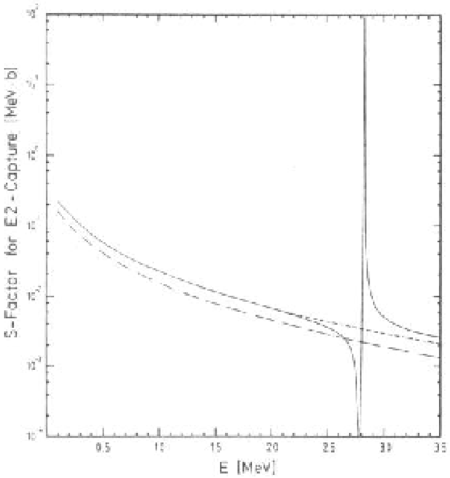
S_{E2} (300 keV) $\approx$ 70 keV b

Microscopic coupled-channel model: E2 part

based on microscopic α+^{12}C, α+^{12}C(2^+) wave functions
potential constrained to:

- binding energies of ground state and 6.92 MeV state
- lifetime of 2^+ state at 6.92 MeV
- energy and width of 2^+ resonance at 9.74 MeV
- energy and width of 4^+ resonances at 10.35 and 11.09 MeV
- B(E2) transitions: $2_1 \rightarrow 0_2$, $4_1 \rightarrow 2_1$
- α+^{12}C phase shifts
- the microscopically correct number of nodes in g(r)

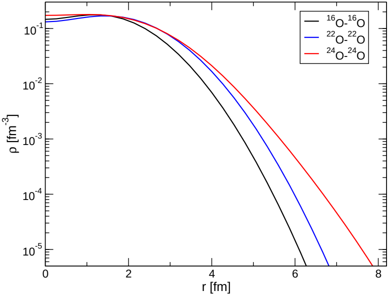
Coupled-channel potential model: E2 part



Funck et al., S_{E2} (300 keV) $\approx$ 100 keV b

Microscopic Nucleus-Nucleus Reactions

- Fermionic Molecular Dynamics (FMD) many-body states
- Realistic effective nucleon-nucleon interaction

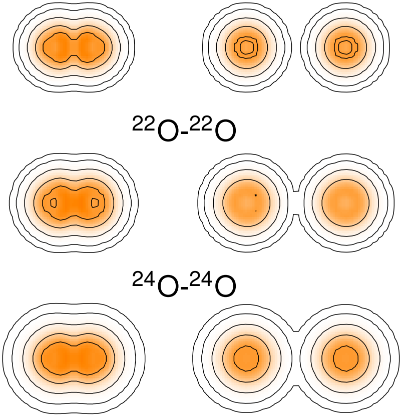


charge distributions

$^{16}O-^{16}O$

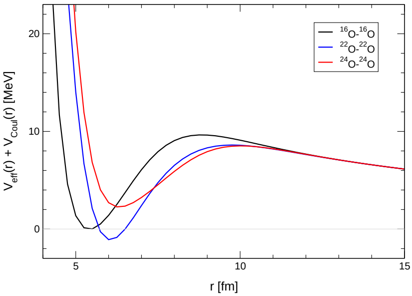
$^{22}O-^{22}O$

$^{24}O-^{24}O$



Thomas Neff, Hans Feldmeier (GSI)
Robert Roth (TUD)

Astrophysical S-Factors

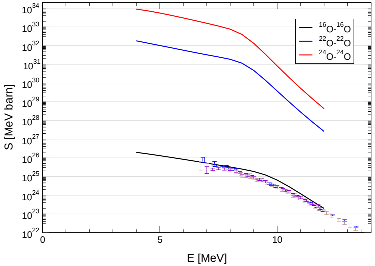


Microscopically derived nucleus-nucleus potentials

astrophysical S-factor

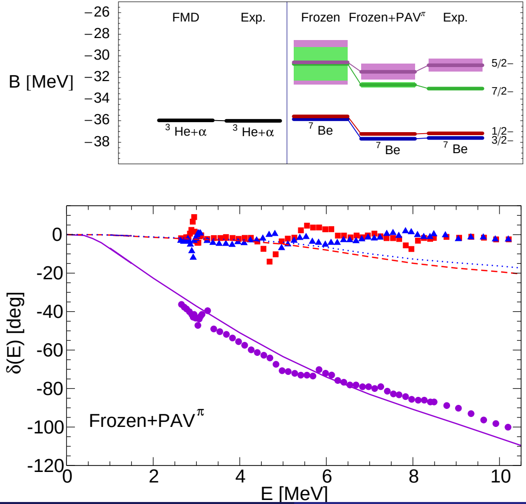
$$S(E) = \sigma(E)\, E\, e^{2\pi\eta}$$

Sommerfeld parameter η



Reaction theory within FMD

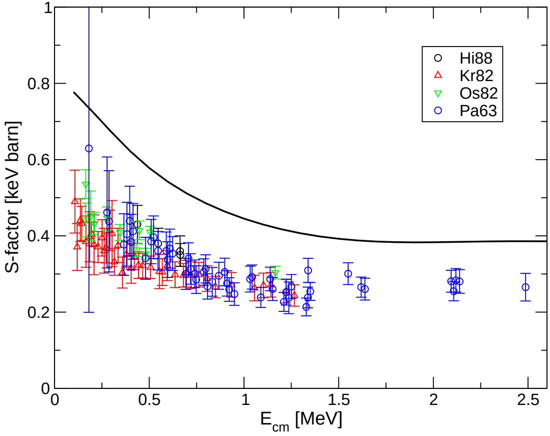
FMD: consistent description of bound and continuum states



^{7}Be energy spectrum

^{3}He+^{4}He phase shifts

^{3}He$(\alpha,\gamma)^{7}$Be S-factor: first attempt



Better description of short-ranged ^{7}Be correlations needed

Outlook

Use ability of FMD to descibe simultaneously bound and scattering states accounting for most relevant nucleonic correlations from realistic nucleon-nucleon interaction

- electromagnetic capture reactions like ^{3}He(^{4}He,γ)^{7}Be
- photodissociation reactions like ^{6}Li $\rightarrow$ ^{4}He + d
- transfer reactions like ^{7}Li(p,^{4}He)^{4}He
- heavy-ion fusion like ^{12}C+^{12}C
- ... and ^{12}C$(\alpha,\gamma)^{16}$O

Promise: results at Charlie's 90th birthday!