E1 Measurements of $^{12}C(\alpha,\gamma)$ and future visions underground

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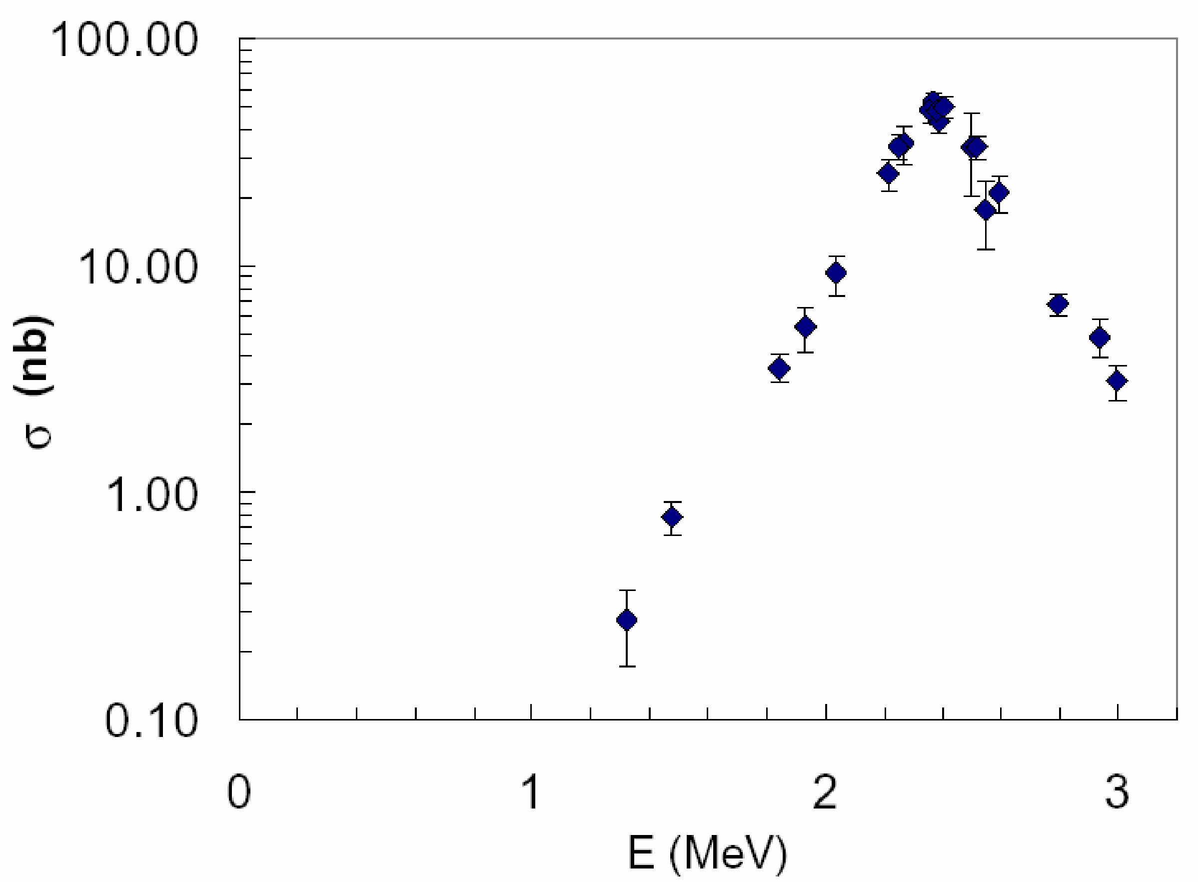
Measurement of the E1 component: experiments and extrapolation



$\sigma(1^-)$=37-55 nb

R-matrix fit to the σ_{E1}

- Cross sections include contributions from few levels
- Level parameters from experimental data
- Global fit
- Least square method:
- $\chi^2=\chi^2_\beta+\chi^2_{\delta_1}+\chi^2_{\delta_3}+\chi^2_\gamma$
- Extrapolation
- Uncertainty on extrapolation and fitted parameters

- ^{16}N -> ^{16}O -> $^{12}C+\alpha$ data
- $W_\alpha(E)=F(E, a_\ell, A_{\lambda\ell}, \gamma^2_{\lambda\ell}, E_{\lambda\ell})$
- $\ell=1,3$; $\lambda=1,2,3$

- $^{12}C(\alpha,\gamma)^{16}O$
- $\sigma_{E1}(E)=H(E, a_\ell, \gamma^2_{\lambda\ell}, \Gamma^2_{\lambda\ell}, E_{\lambda\ell})$
- $\ell=1$; $\lambda=1,2,3$

- $^{12}C(\alpha,\alpha)^{12}C$
- $\delta_\ell(E)=G(E, a_\ell, \gamma^2_{\lambda\ell}, E_{\lambda\ell})$
- $\ell=1,3$; $\lambda=1,2,3$

Least square method – correlated data

- Measurement of Y in conjunction with X -> (x_i,y_i) i=1,…,n
- Model $Y=f(X, A_1,\ldots,A_m)$
- $Q=\Sigma_{ij}[y_i-f(x_i;a_1,\ldots,a_m)]V^{-1}_{i,j}[y_j-f(x_j;a_1,\ldots,a_m)]$
- if $\mathrm{cov}(y_i,y_j) = E[(y_i-\langle y_i\rangle)(y_j-\langle y_j\rangle)]=\delta_{ij}$, uncorrelated data, usual definition
- $\mathrm{cov}(y_i,y_j) = E[(y_i-\langle y_i\rangle)(y_j-\langle y_j\rangle)]=V_{ij}\neq 0$ for $i\neq j$
- Minimization

Building the covariance matrix for normalized data

★ Measurement of Y in conjunction with X -> (x_i,y_i) i=1,…,n

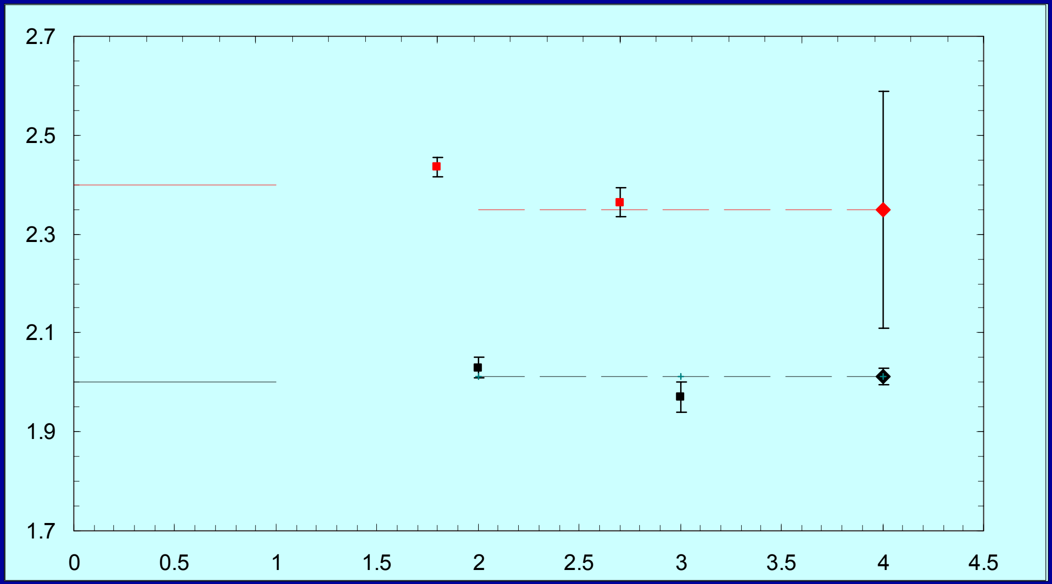
★ $y_i=c \cdot z_i$, c is a normalization constant with error σ_c

$z_1=2.03\pm0.02$
$z_2=1.97\pm0.03$
$\langle z\rangle=2.018\pm0.018$

$c=1.20\pm0.10$
$y_1=c\cdot z_1=2.44$
$y_2=c\cdot z_2=2.36$

$y=2.3\pm0.2$

so, biased estimator



Alternative:
first fit unnormalized data,
then propagate the normalization error:
$y=2.4\pm0.2$.
But what about the case of many data sets?

G. D'Agostini, NIM A 346(1994)306

A method for dealing with normalization errors and different data sets

- ★ let $y_{i_k,k}=c_k\, z_{i_k,k}$ $k=1,..,n$ be n measurements with $i_k=1,..,n_k$ points each
- ★ Model $Y_k=f_k\,(X_k;\, A_1,\ldots,A_m)$
- ★ $Q=\Sigma_k \{\Sigma_{i_k} [z_{i_k,k} - f(x_i;a_1,\ldots,a_m)/c_k]^2/\sigma_{y_{ik}}^{\;2}+(c_k-a_{m+1})^2/\sigma_{c_k}^{\;2}]\}$
- ★ hopefully: Q->χ^2 distribution with $\nu= \Sigma_k\, n_k - m - n$ degrees of freedom

An example

★ $Y_1=f_1(X;A_1,A_2,A_3)=A_1+\mathrm{sqrt}(A_2)\cdot X$

★ $Y_2=f_2(X;A_1,A_2,A_3)=A_1+A_2\cdot X/3+A_3^3 \cdot X^{-2}$

★ Experiment:

★ $\sigma_{y_{1i}}/y_{1i}=0.01$

★ $\sigma_{c_1}/c_1=0.1$

★ $\sigma_{y1i}/y_{1i}=0.1$

★ $\sigma_{c_2}/c_2=0.01$

★ True parameter values

★ $A_1=1;\ A_2=2;\ A_3=3;$

★ Normalization:

★ $y_1=c_1\cdot y_1$

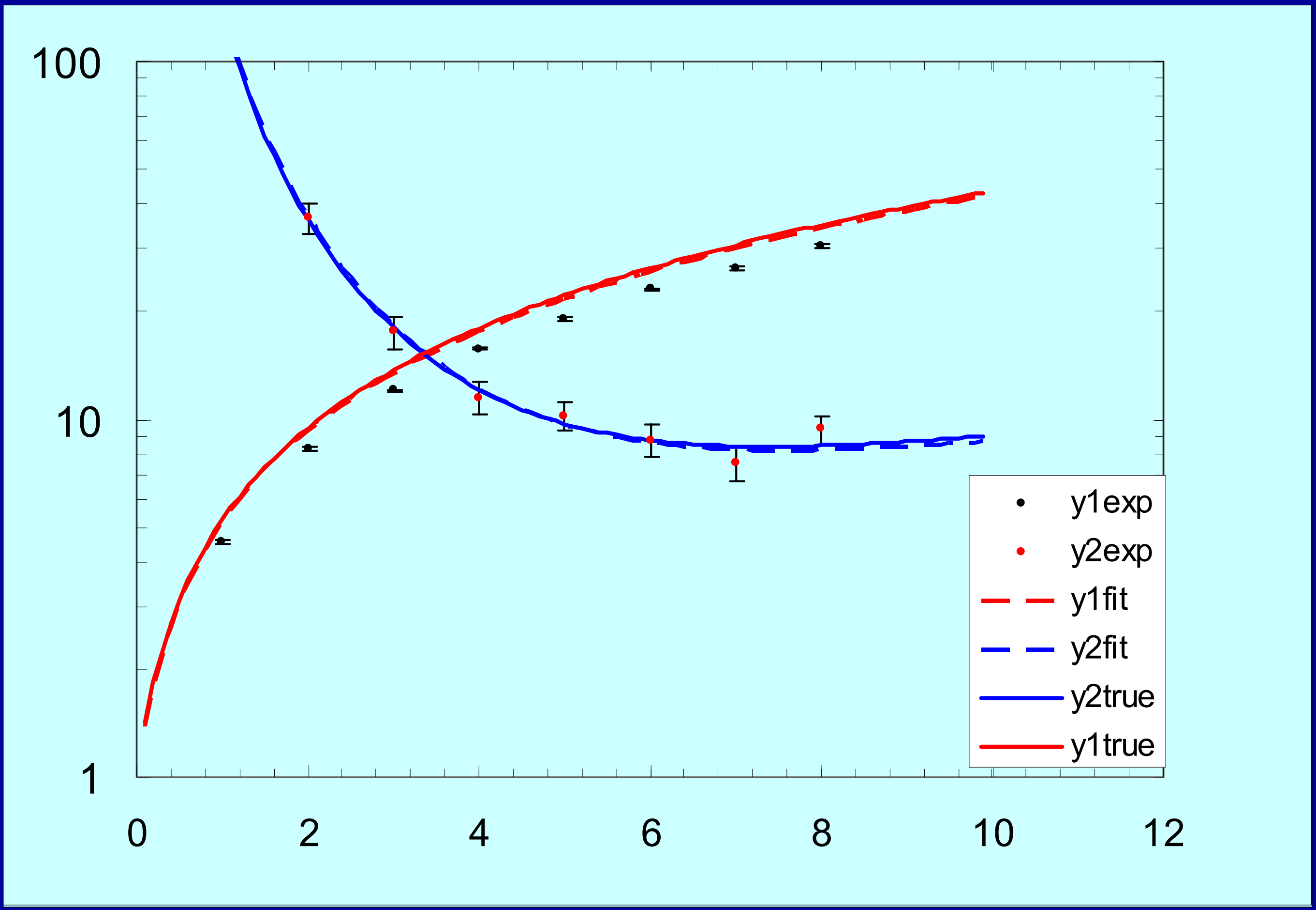
★ $y_2=c_2\cdot y_2$

★ True normalization values

★ $c_1=1.2;\ c_2=1$

Fit without normalization constants

Fit with normalization constants



- fit:
- $A_1=0.87(1)$
- $A_2=1.91(2)$
- $A_3=3.08(3)$

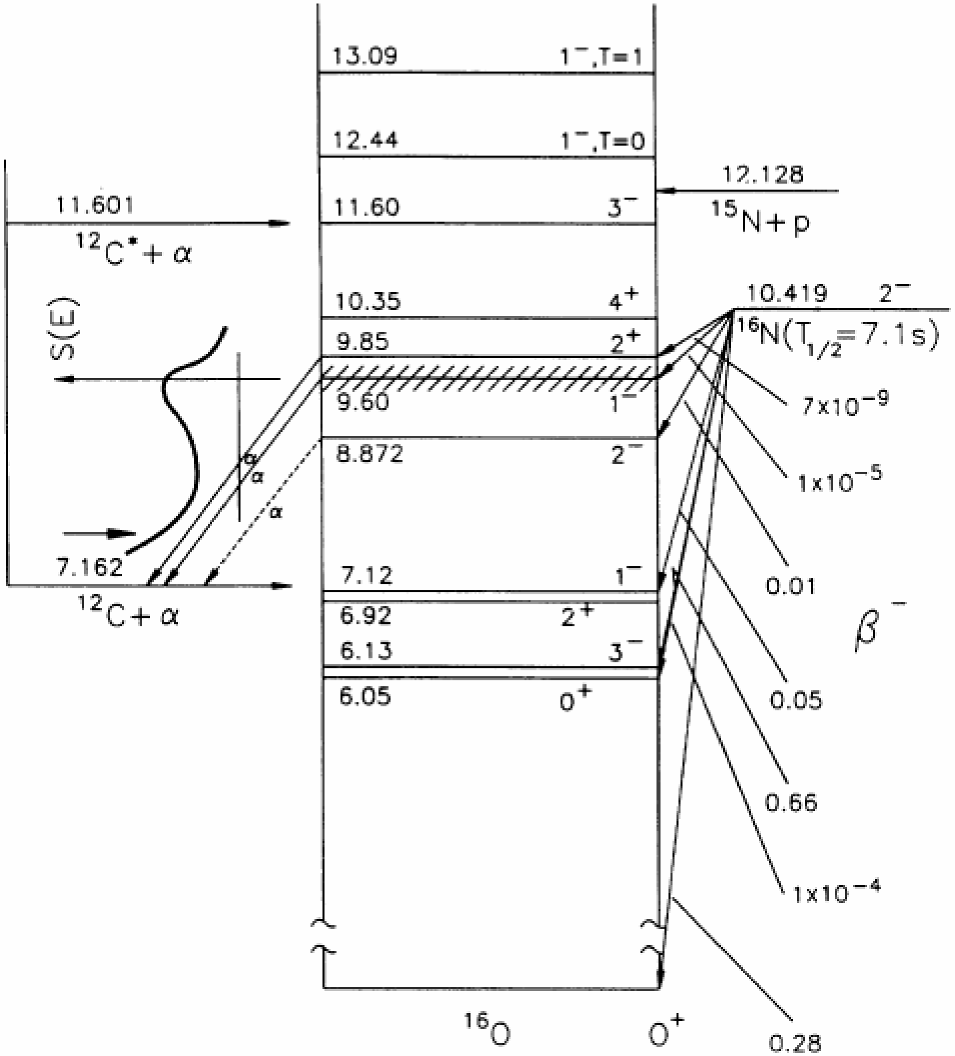
- $\chi^2=9.4$
- $\nu=11$
- $\chi^2/\nu=0.84$

fitted:

- $c_1=1.07(1)$
- $c_2=0.99(1)$

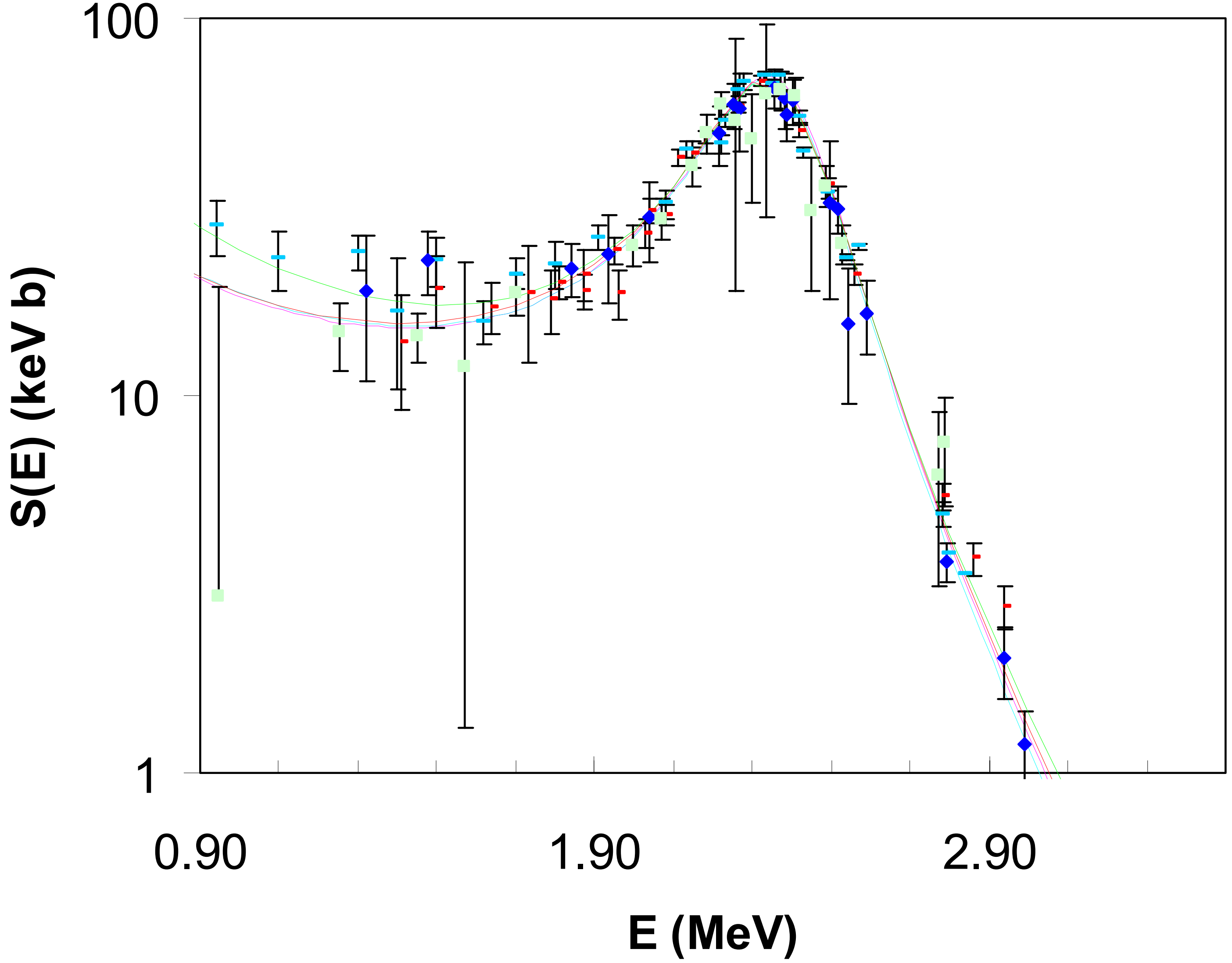
experimental:

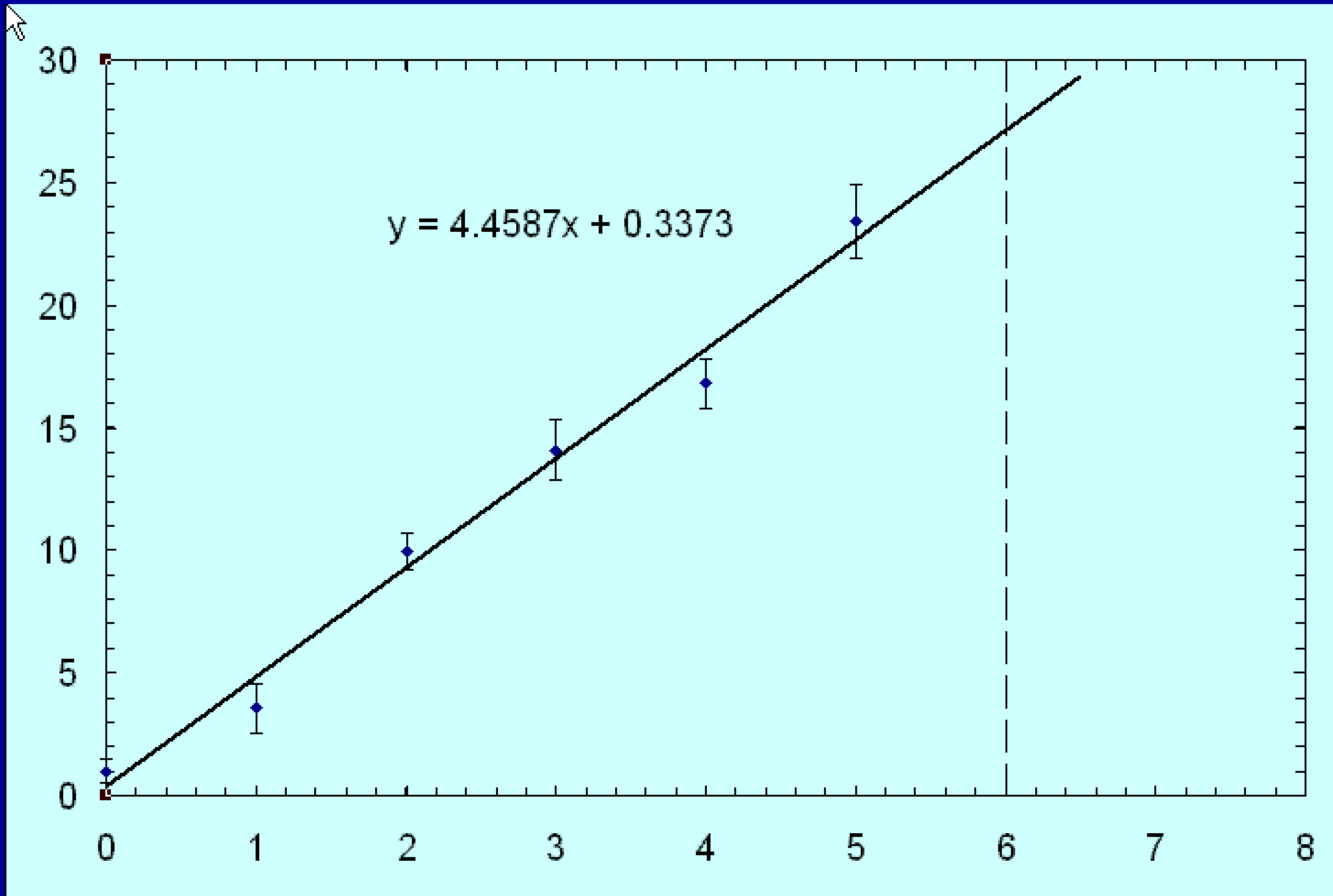
- $c_1=1.4\pm0.2$ (1.2)
- $c_2=0.99\pm0.01$ (1)



Final state		Branch (%)
^{16}O* (MeV)	J^π	
0	0^+	26 ± 2 [c]
6.05	0^+	$(1.2 \pm 0.4) \times 10^{-2}$ [d]
6.13	3^-	68 ± 2 [c]
7.12	1^-	4.9 ± 0.4 [c]
8.87	2^-	1.0 ± 0.2 [c]
9.63	1^-	$(1.20 \pm 0.05) \times 10^{-3}$ [e]
9.85	2^+	$(6.5 \pm 2.0) \times 10^{-7}$ [f]

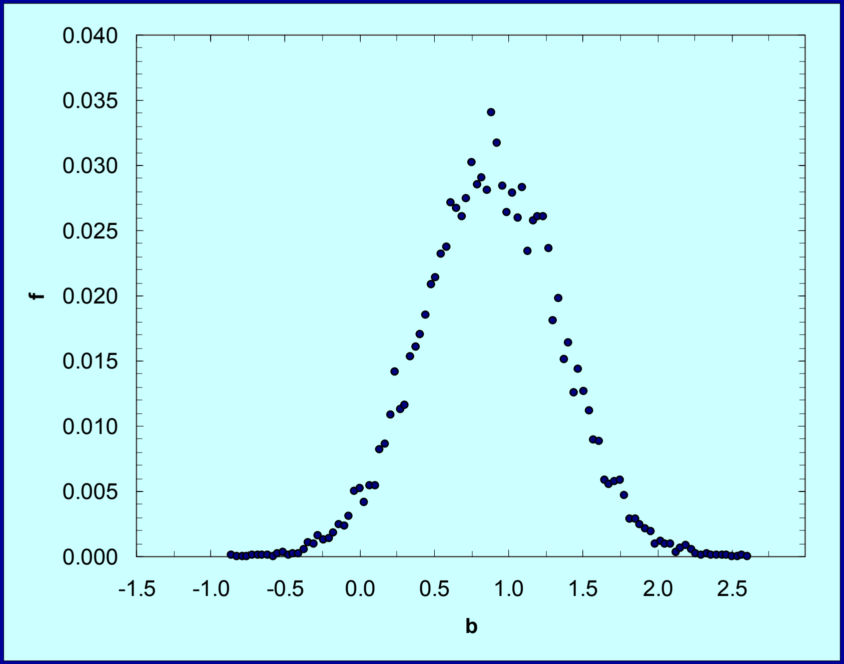
Relative BR	S300 (keV·b)
4000 fixed	80
3650 fitted	85
3800 fitted + norm	86
3800 fitted + norm + select	90
1550 free (just for fun)	154

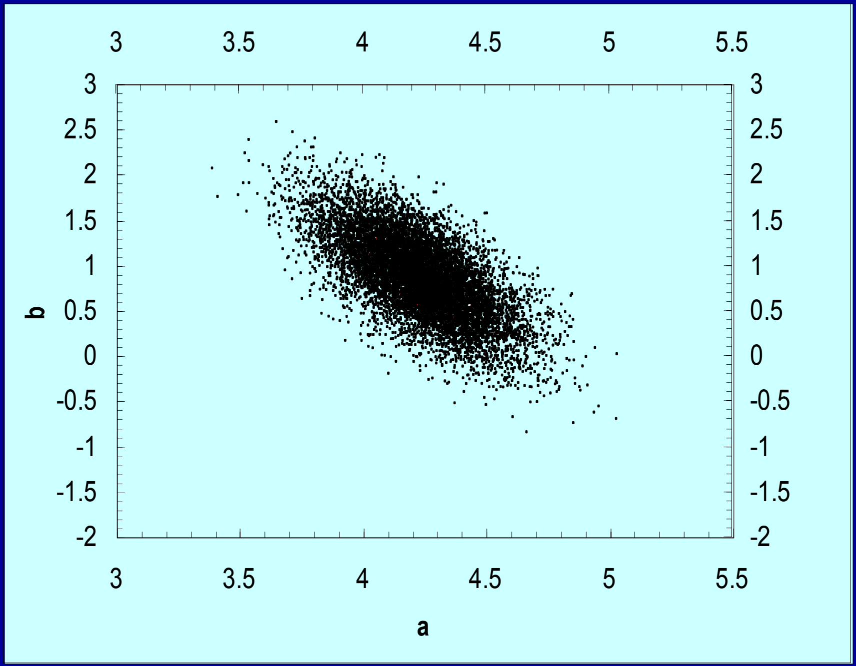

100
10
1
S(E) (keV b)
0.90
1.90
2.90
E (MeV)



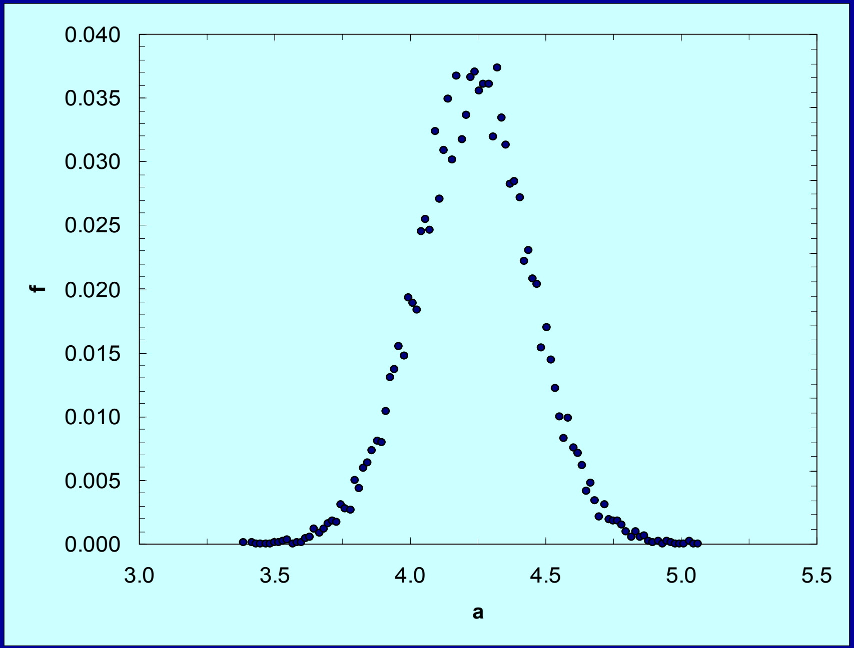
y = 4.4587x + 0.3373
30
25
20
15
10
5
0
0
1
2
3
4
5
6
7
8

Example: linear case – Monte Carlo

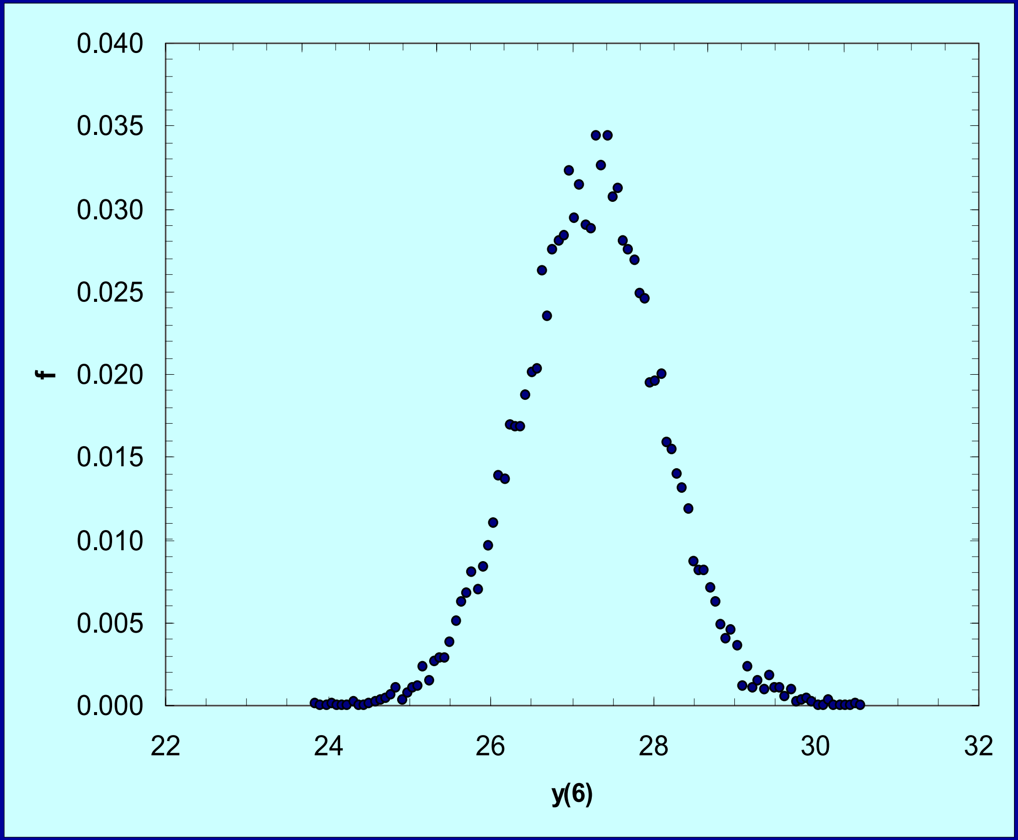




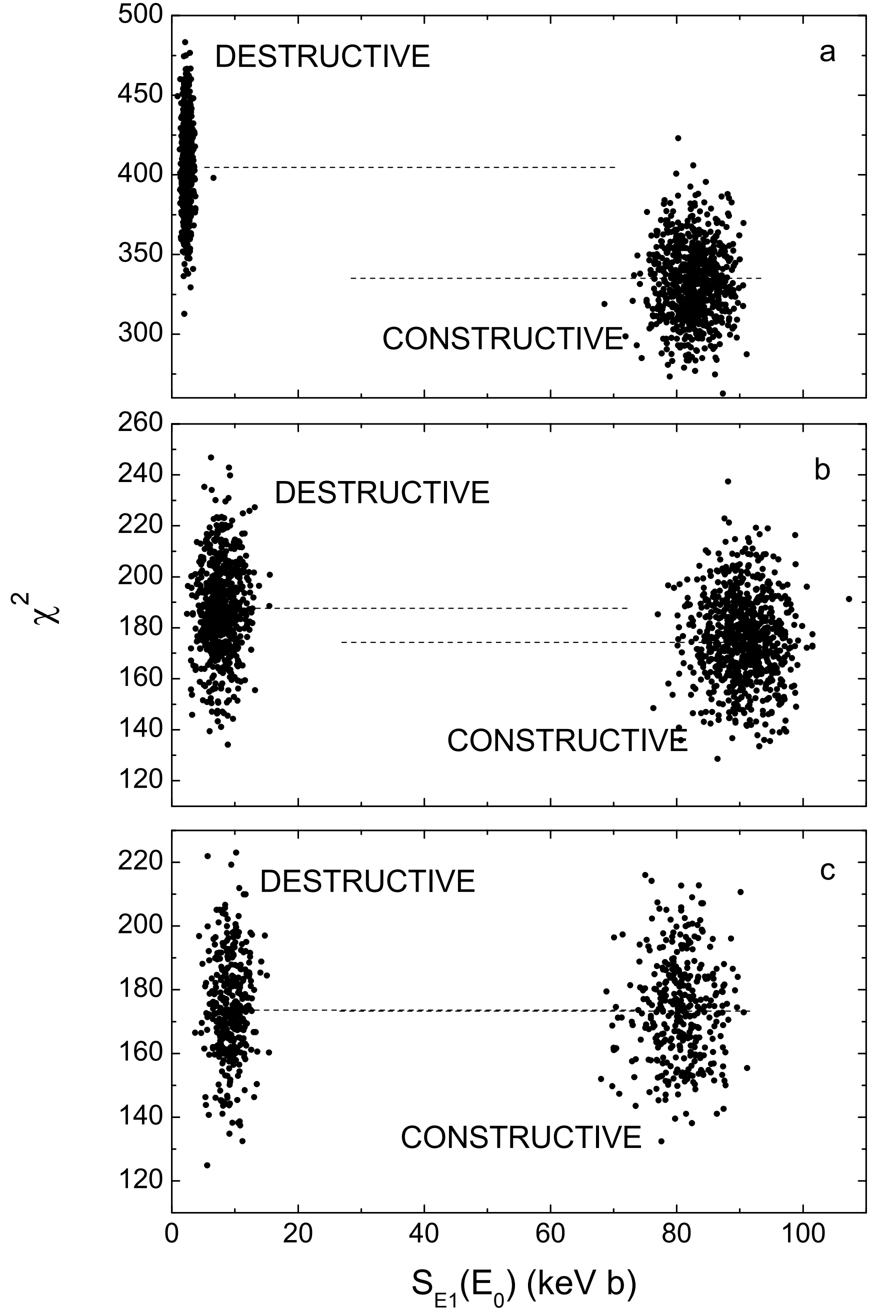
- N=10000
- a=4.231 ; b=0.873
- σ^2_a= 0.044 ; σ^2_b = 0.200; cov(a,b)=-0.0631
- analytical: a=4.227 ; b=0.879
- σ^2_a= 0.044 ; σ^2_b = 0.203; cov(a,b)=-0.0629



Example: linear case – Monte Carlo



- $x^* = 6;\ y^* = y(x^*) = 26.26$
- $\sigma_{y^*} = 1.02$
- analytical: $\underline{y^*} = \underline{26.24}$;
- $\sigma_{y^*} = 1.00$



all

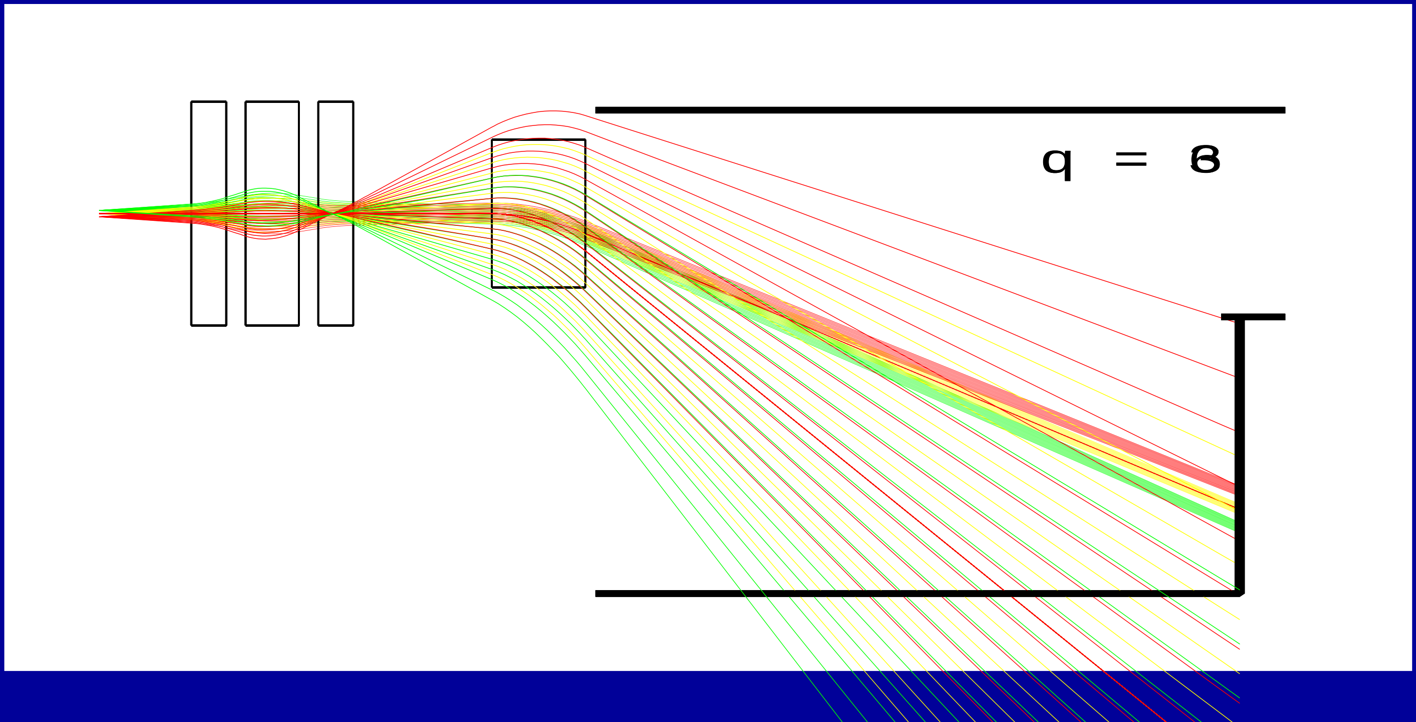
gialanella

kremer

S_{E1}= 86±15 keV·b

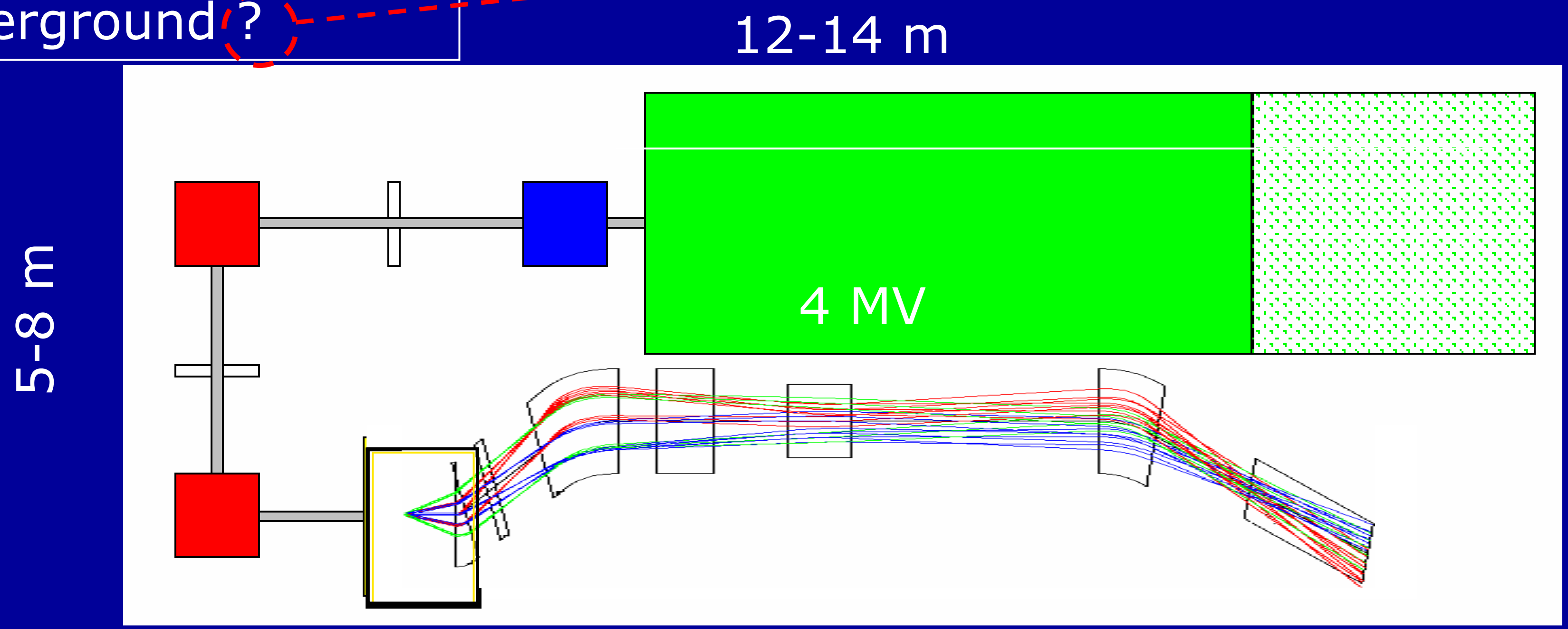
Visions

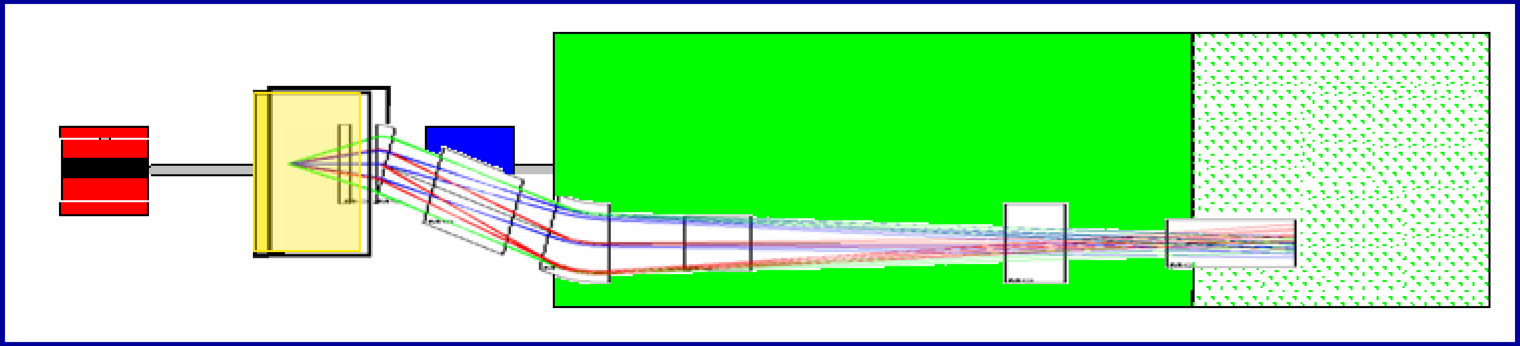
RMS approach: background and leaky beams



Possible layout:
- -Single stage (3.5 MV)
- -ECR source
- -Quadrupole Free RMS
- -Underground ?

Angular acceptance = 50 mrad
Energy acceptance= ±15%
Beam suppression = $10^{-?}$
Gamma bkg suppr =10^3-10^5



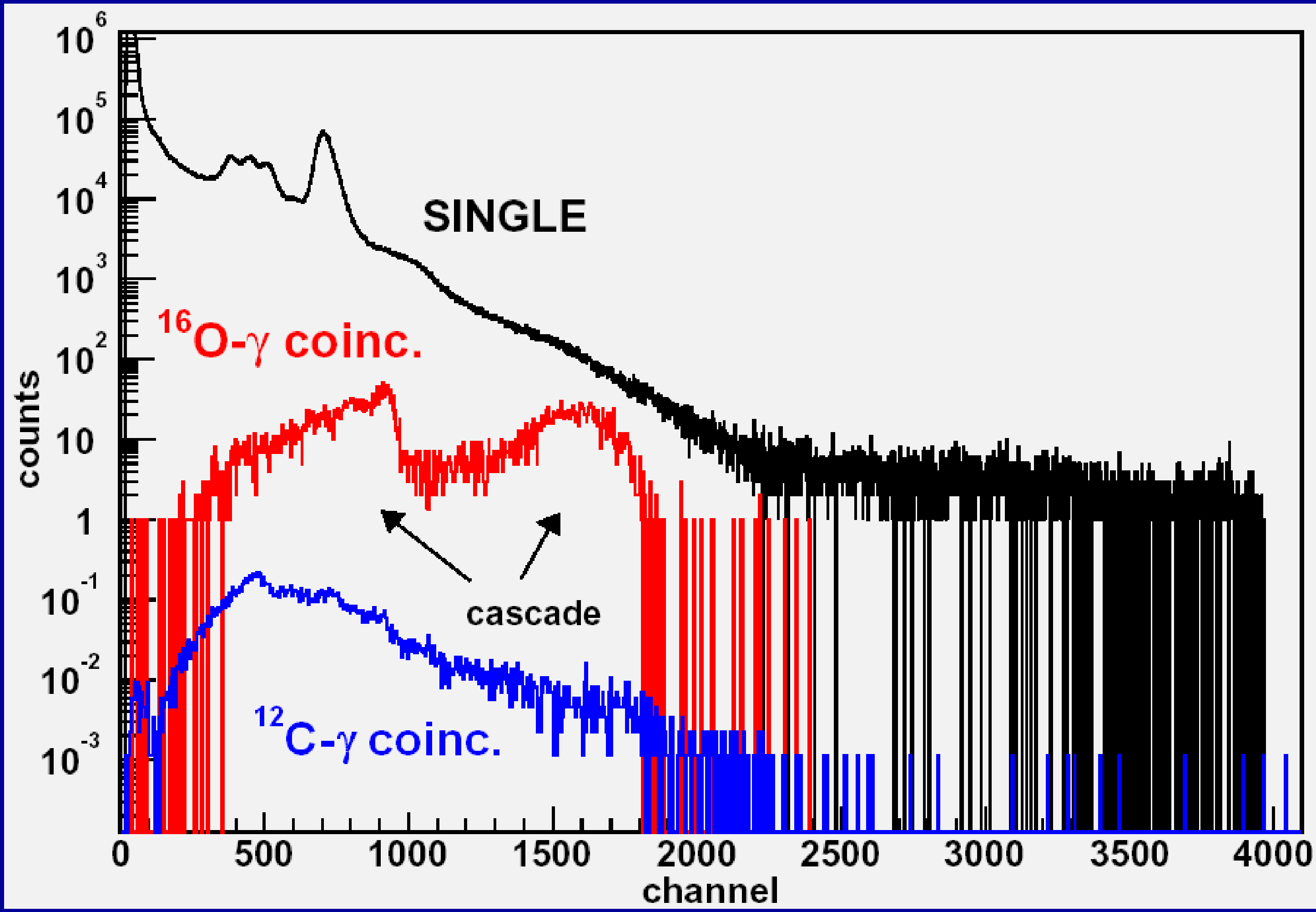


Should one go underground?

$^{12}C(\alpha,\gamma)^{16}O$
E_{cm}=3.2 MeV
$\sigma \approx 4\ 10^{-6}$ b

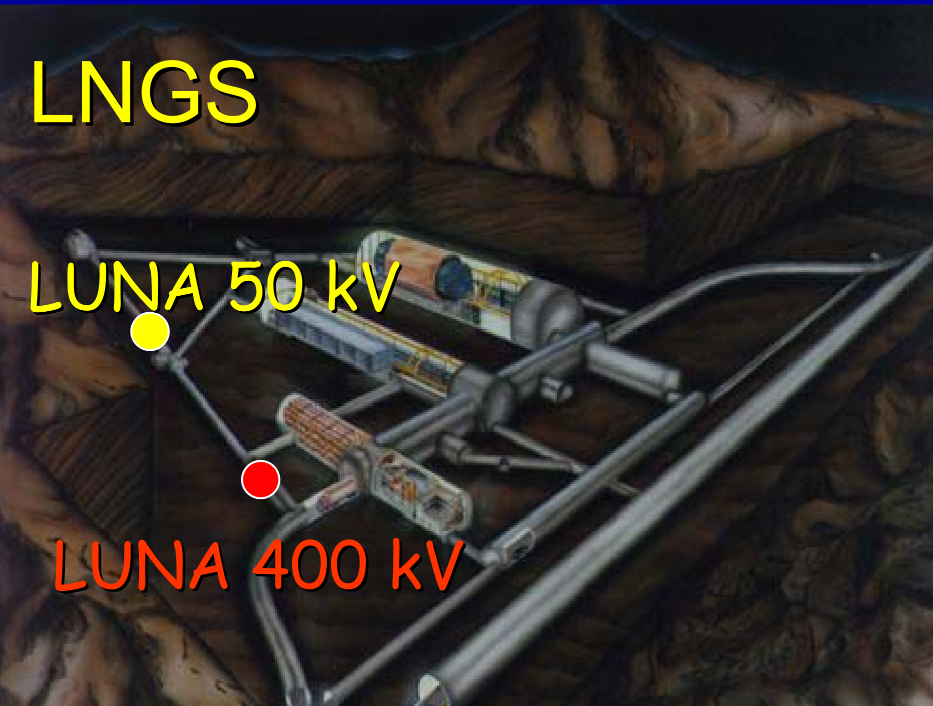
Back. Suppression ca 10^{-6}

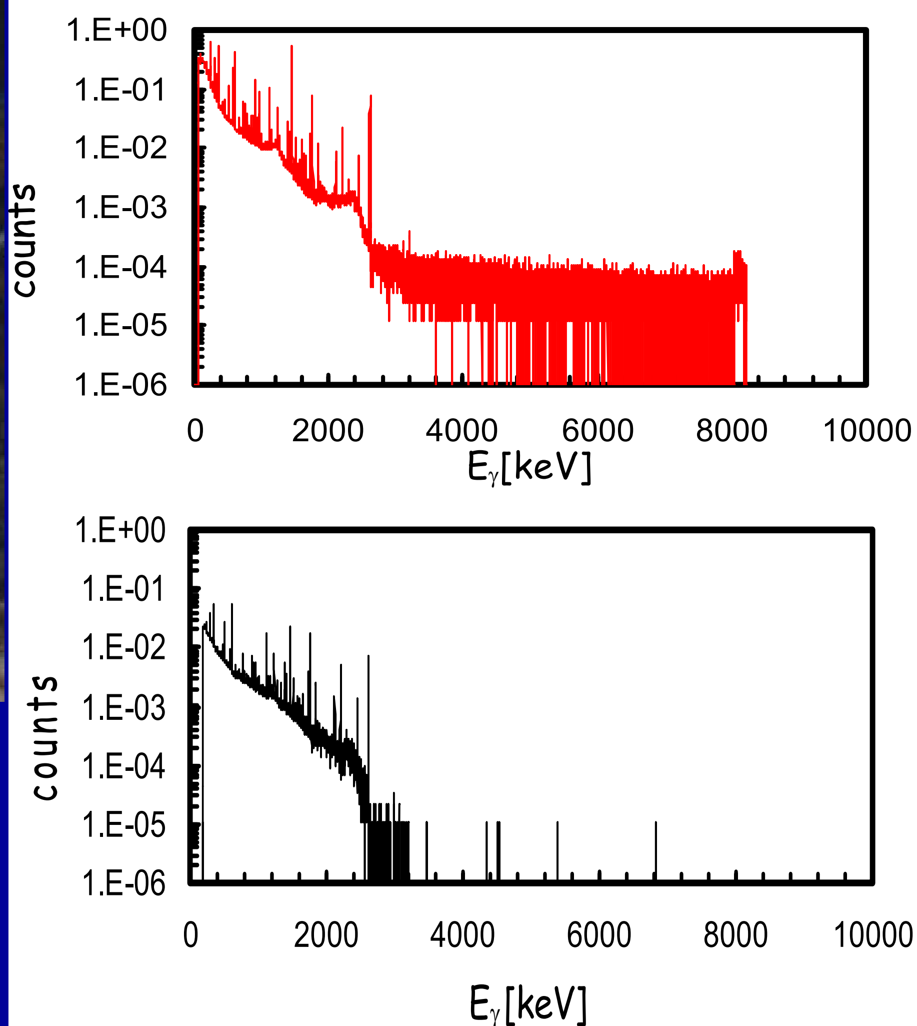
Signal/ background: ca 10^{3}



$E_{cm,eff}$ = 475 keV → $\sigma_{tot} = 2 \times 10^{-5}$ nb (ca 10 cpd) -> if gamma coincidence is needed, one should go underground

Underground laboratory



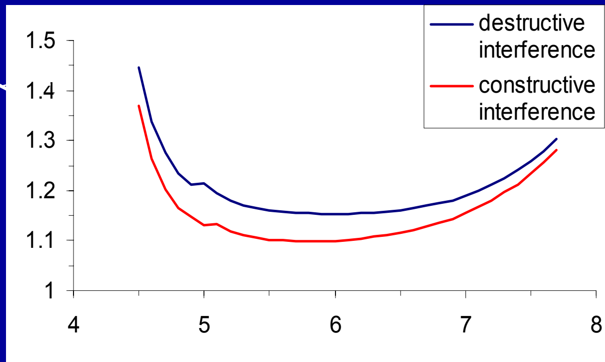


3MeV < E_γ < 8MeV
0.5 Counts/s

3MeV < E_γ < 8MeV
0.0002 Counts/s

Interaction radius

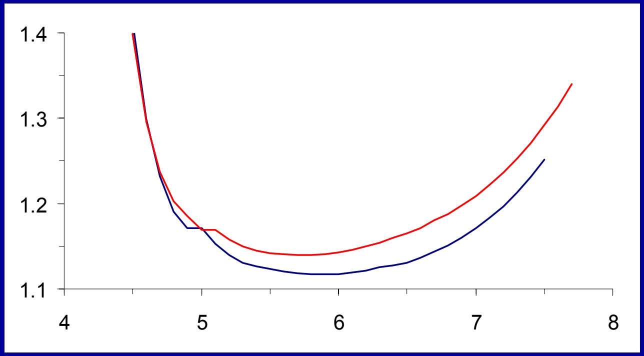
χ^2



r (fm)

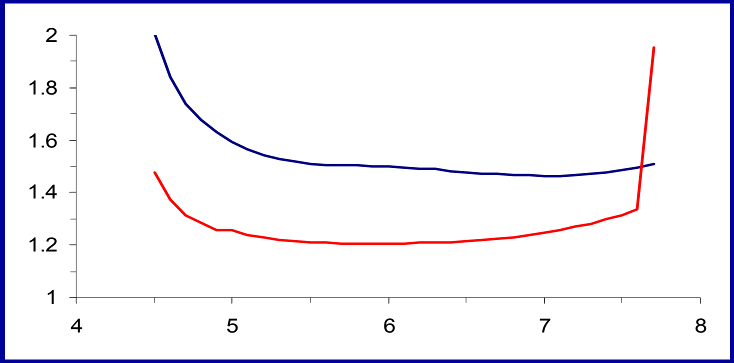
χ^2_{rid}

Kremer



r (fm)

χ^2_{rid}



Redder

r (fm)