Nuclear Level Densities in Hauser-Feshbach Calculations

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Some Nomenclature...

- general reaction shall be $i(j,o)m$ or

$$i + j \longrightarrow m + o$$

- "target" is the target nucleus i
- "daughter" is the residual nucleus m
- "compound" is the formed compound nucleus
- a "known" state is a experimentally (nearly unambiguously) known state in a nucleus

Definition

SMOKER/MOD-SMOKER the transmission probabilities as **decay probabilities** from the compound into the daughter and the target! (principle of detailed balance)

Hauser-Feshbach Formula

From Hauser-Feshbach theory, the cross section for the reaction $i^{\mu}(j,o)m^{\nu}$ is proportional to

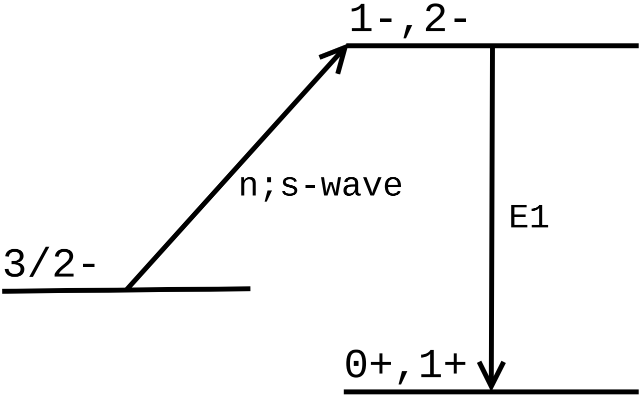
Hauser-Feshbach Crosse Section

$$\sigma_{jo}^{\mu\nu}(E_{ij}) \propto \sum_{J,\pi}(2J+1)\frac{T_j^{\mu}(a;b)\cdot T_o^{\nu}(a;c)}{\sum_d T_d(a)}$$

here $a=(E,J,\pi)$ depicts the compound state properties; b the same for the target; c the same for the daughter

Note...

The Hauser-Feshbach model itself includes π & J-dependence: the transmission coefficients should come from a many body method that is sensitive to these quantum numbers!
$\Longrightarrow$ but there are no such models!



The parity is important because SMOKER gives the same transmission coefficient for $3/2^+ \rightarrow 1^-$ and $3/2^- \rightarrow 1^+$! But what happens if there is no 1^- at this energy?

Level Densities

The *back-shifted Fermi gas* level density $\rho(U)$ is a pure statistical level density. The excitation energy for its calculation is back-shifted to include pairing effects: $U = E - \delta$

BSFG level density

In this approach the J- and π-dependence is included via multiplicative factors:

1. $F(U,J) = \frac{2J+1}{2\sigma^2} e^{\frac{-J(J+1)}{2\sigma^2}}$ gives the J-dependence
2. $\Pi(U,\pi)$ this is the parity factor! In case of equal distributed parities this factor becomes $1/2$.

$$\rho(U,J,\pi) = \Pi(U,\pi) \cdot F(U,J) \cdot \rho(U)$$

$\Pi = 1/2$: both parities are qually distributed $\Rightarrow$ rather good approximation for high energies

Parity Dependence

However, it has been known for some time that this approximation is not very good for low energies on certain nuclei!

Where does the parity dependence enter?

1. **Calculation of the TC:**

$$T_k(a) = \sum^{\omega} T_k^{\nu}(a;c) + \int_{E^{\omega}}^{E-S_o} \sum_{J_o,\pi_o} T_k(a;c)\rho(c)dE_o$$

2. **Compound Sum:** within the sum over J, π in the cross section formula since the optical model does not give a J and π dependence in case of SMOKER!
$\Longrightarrow$ therefore we have to find a description that takes the **non-existence** of a certain parity (and maybe certain spin J) at a given energy into account!

Parity Dependence in TC

We now need a functional form of $\Pi(E, \pi)$:

1. positive parity: $\Pi(E, +) = \frac{\rho(E,+)}{\rho(E,+)+\rho(E,-)}$
2. negative parity: $\Pi(E, -) = \frac{\rho(E,-)}{\rho(E,+)+\rho(E,-)}$

NOTE: $\Pi(E, \pi)$ does NOT depend on J! (cancels out)

Warning!

This is an **implicit** description of the parity projected level density! Therefore we need to obtain these $\rho(E, +)$ and $\rho(E, -)$ in another way.

Linear Dependency

The important connection between the transmission coefficients and the resonance widths is:

$$T_c(E,J,\pi) = 2\pi \frac{\langle \Gamma_c \rangle}{D_{J,\pi}} = 2\pi\rho(E,J,\pi) \langle \Gamma_c \rangle$$

this combined with

$$\sigma_{jo}^{\mu\nu}(E_{ij}) \propto \sum_{J,\pi} (2J+1) \frac{T_j^{\mu}(a;b) \cdot T_o^{\nu}(a;c)}{\sum_d T_d(a)}$$

results in a **linear** influence of ρ!

IDEA: Weighting Factors!

$$\sigma_{jo}^{\mu\nu}(E_{ij}) \propto \sum_{J,\pi} \beta(E,J,\pi)(2J+1) \frac{T_j^{\mu}(a;b) \cdot T_o^{\nu}(a;c)}{\sum_d T_d(a)}$$

The Compound Weights

Advantage...

These compound weights β produce the relevant physcis and we can retain a simple potential model to calculate the transmission coefficients!

MOD-SMOKER Compound Weights

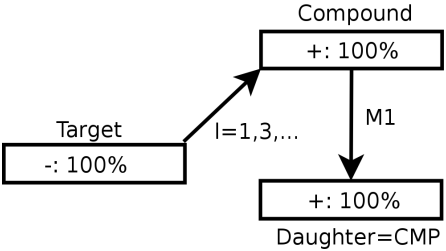
These βs have to be calculated similar to the Π in the level density.

$$\beta(E, \pi; J) = \frac{2 \cdot \rho(E, J, +)}{\rho(E, J, +) + \rho(E, J, -)}$$

NOTE: we still assume equally distributed J-values; only an implicit J dependence is left since $\rho(+, J_1)/\rho(-, J_1) \neq \rho(+, J_2)/\rho(-, J_2)$ generally

Influence From The CMP Weights

What happens in the compound sum? Let's assume a target nucleus with negative parity states only at a low energy:



- the neutron capture leads to a nucleus where the positive parity dominates thus s-wave capture is heavily suppressed
- also the decay transitions are suppressed since only M1 transitions (because positive parity dominates) can occur

Influence From The CMP Weights

The combination of the two effects gives the relevant physics!

Beware

The example gave reduction (enhancement of weak transitions $M1$, p-wave)! But enhancing is also possible, since:

If one transition type (E1,even-L) is REDUCED the counterpart transition is always ENHANCED (M1, odd-L) and vice versa!

Why?: because the total number of states is NOT changed - only the distribution over parity is changed!

$\Longrightarrow$ correlating the both effects means, they can ...

1. compensate each other
2. enhance each other

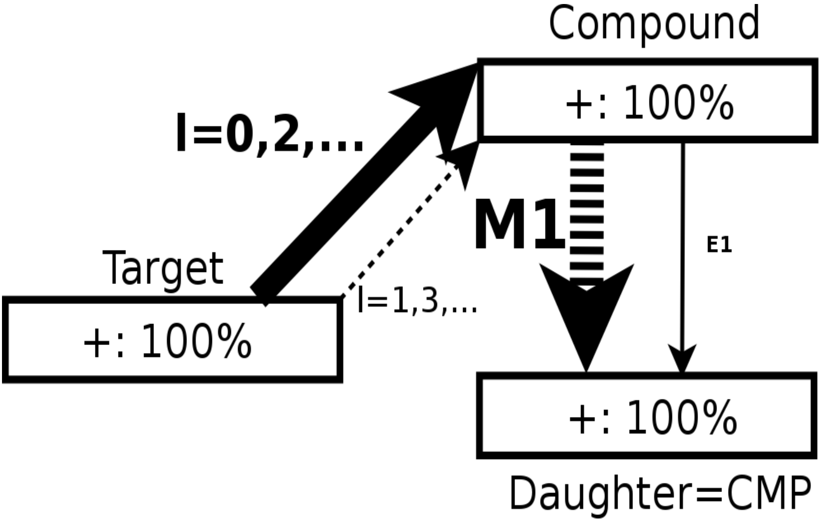


Figure: example for the “compensation” of both effects

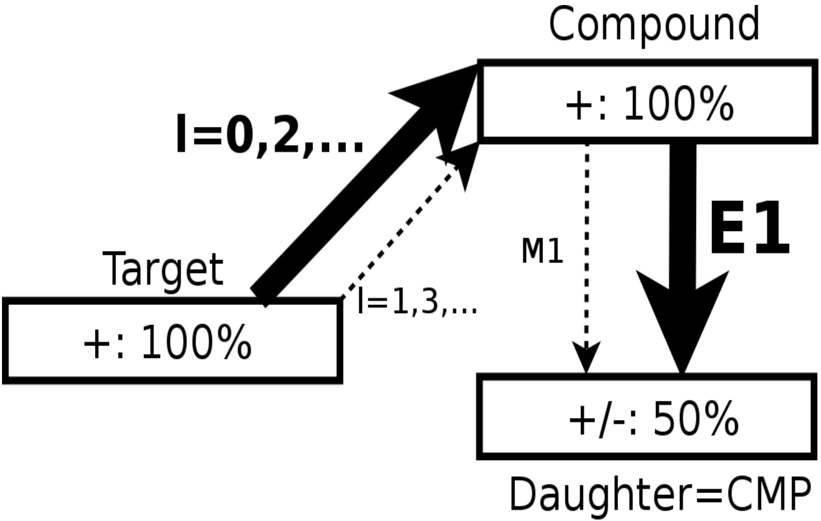


Figure: example for an enhancement of the total cross section due to the combination of the compound weighting and the TC parity dependence

Hilaire's Level Densities

- Hilaire et al.: recently published a paper (Nucl. Phys. A 779 [2006]) where they calculated π- and J-projected level densities for Z=8 to Z=114
- $\Rightarrow$ MOD-SMOKER interpolates these level densities J- and π-dependent on its internal energy grid for a specific nucleus!
- $\Rightarrow$ That way it obtains the $\rho(E, J, +)$ and $\rho(E, J, -)$!

Renormalisation

The level densities that were used are NOT renormalised to the experimental level scheme and the neutron resonance spacings near S_n!

Hilaire's Level Density - Compound Weights

Beware...

There are now two ways to use this input data...

1. use the interpolated level densities to calculate the $\Pi(E,\pi)$. The rest is the BSFG level density. The compound weighting factors are given by

$$\beta(E,\pi) = \frac{2 \cdot \rho(E,\pi)}{\rho(E)}$$

   $\rho(E,\pi) = \sum_J \rho(E,J,\pi)$ - NOTE: no back-shift used since LDs already contain pairing effects!

2. throw away the BSFG level density and use the Hilaire LDs instead! The compound weighting factors are given by

$$\beta(E,\pi;J) = \frac{2 \cdot \rho(E,J,\pi)}{\rho(E,J)}$$

   $\rho(E,J) = \rho(E,J,+) + \rho(E,J,-)$

Some More Nomenclature...

BSFG Notation:

- **normal:** calculated with Rauscher et al. (1997) BSFG LDs; mass model: FRDM; no parity!
- **Tparity:** calculated with Rauscher et al. (1997) BSFG LDs; mass model: FRDM; $\Pi(E,\pi)$ from Hilaire LDs; parity dependence only for TCs
  NOTE: similar approach with different method done by D. Mocelj et al.
- **Fullparity:** Tparity + parity dependent compound weighting $\beta(E,\pi)$

In ratio plots generally the ratios $\rho(\text{Tparity})/\rho(\text{normal})$ or $\rho(\text{Fullparity})/\rho(\text{normal})$ are given!

Some More Nomenclature 2

DIRECT Notation:

DIRECT-normal: Hilaire et al. (2006) LDs replace BSFG LDs;
mass model: FRDM; $\rho(E,J,\pi) = \frac{\rho(E,J,+)+\rho(E,J,-)}{2}$

DIRECT-Tparity: Hilaire et al. (2006) LDs replace BSFG LDs;
only parity dependence in TCs

DIRECT-Fullparity: DIRECT-Tparity + parity dependent
compound weighting $\beta(E,J,\pi)$

In ratio plots generally the ratios
ρ(DIRECT-Tparity)/ρ(DIRECT-normal) or
ρ(DIRECT-Fullparity)/ρ(DIRECT-normal) are given!

NOTE: all reaction rates are STELLAR reaction rates!

Ni-chain with BSFG + FRDM masses

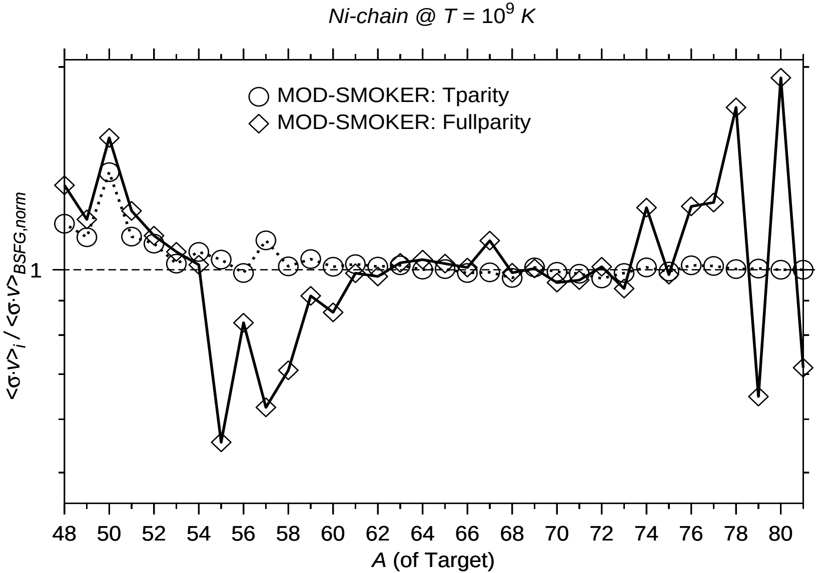


Figure: Nickel chain from ^{48}Ni to ^{81}Ni

Ni-chain DIRECT + FRDM masses

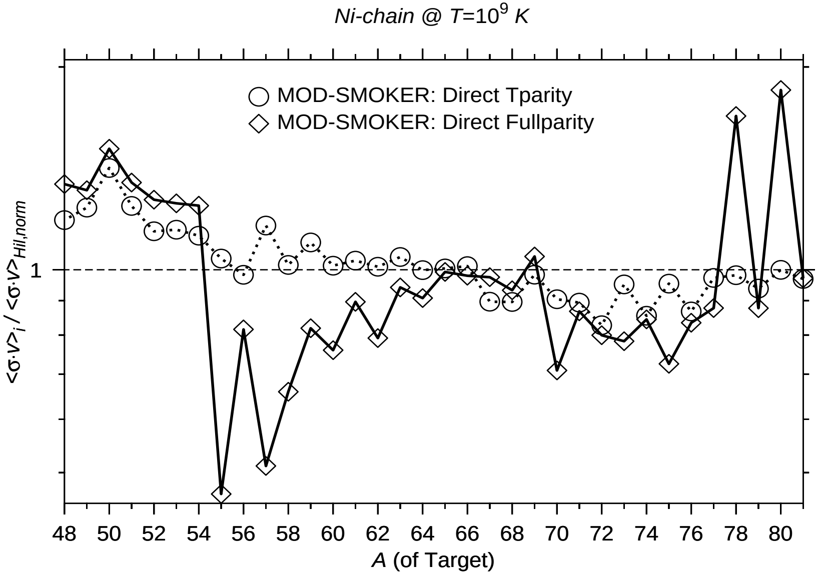
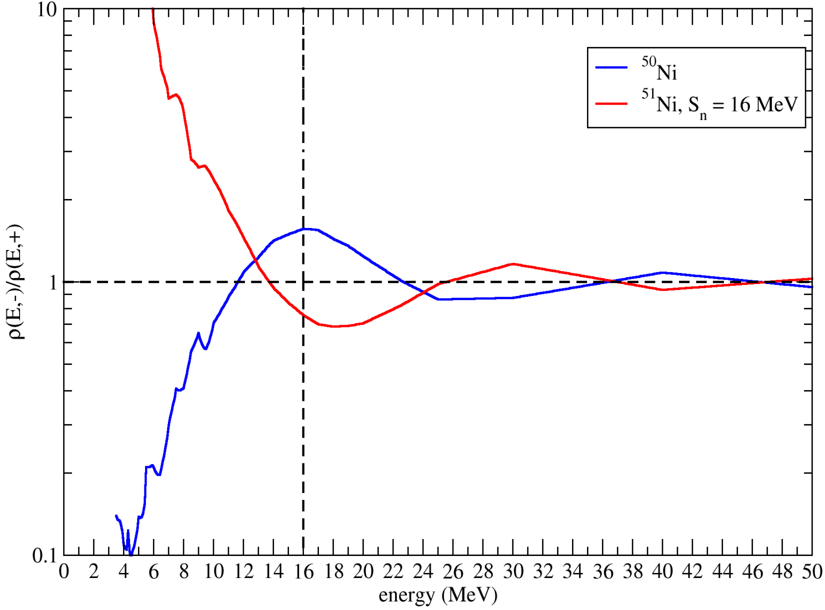
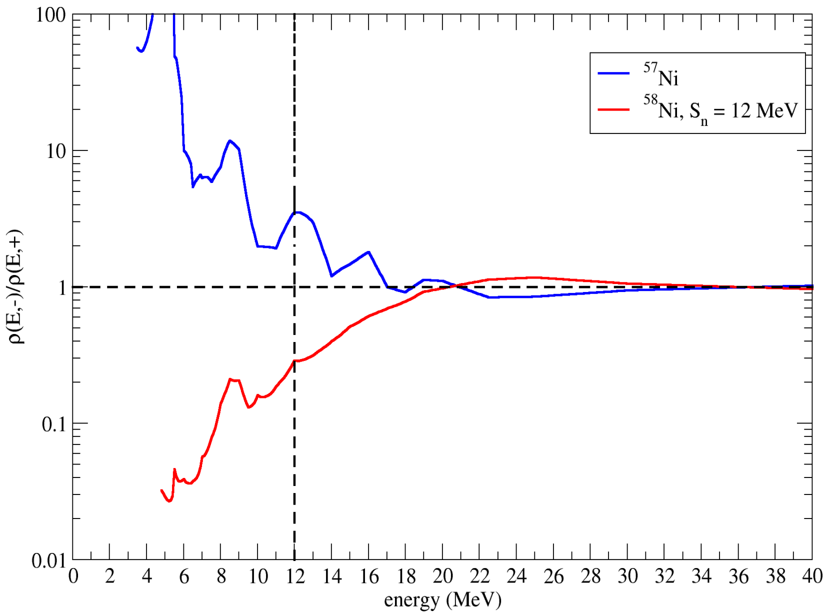


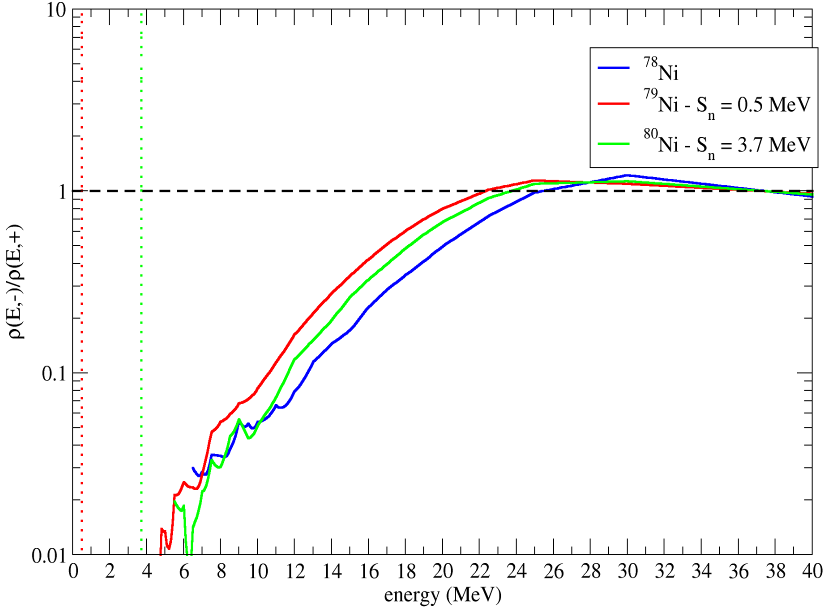
Figure: Nickel chain from ^{48}Ni to ^{81}Ni



^{50}Ni
^{51}Ni, S_n = 16 MeV
ρ(E,-)/ρ(E,+)
energy (MeV)
10
1
0.1
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
44
46
48
50



100
10
1
0.1
0.01
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
energy (MeV)
ρ(E,-)/ρ(E,+)
57Ni
58Ni, Sn = 12 MeV



10
1
0.1
0.01
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
energy (MeV)
ρ(E,-)/ρ(E,+)
78Ni
79Ni - Sn = 0.5 MeV
80Ni - Sn = 3.7 MeV

Sn-chain BSFG + FRDM masses

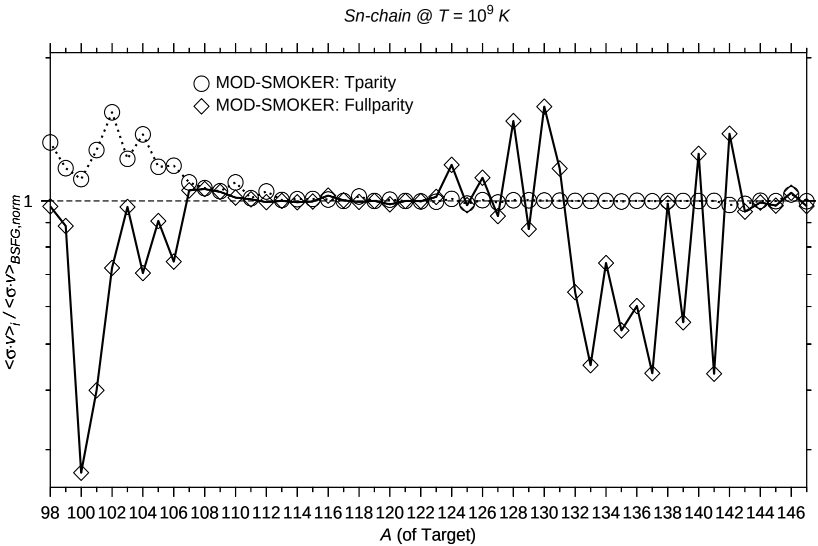


Figure: Tin chain from ^{98}Sn to ^{147}Sn

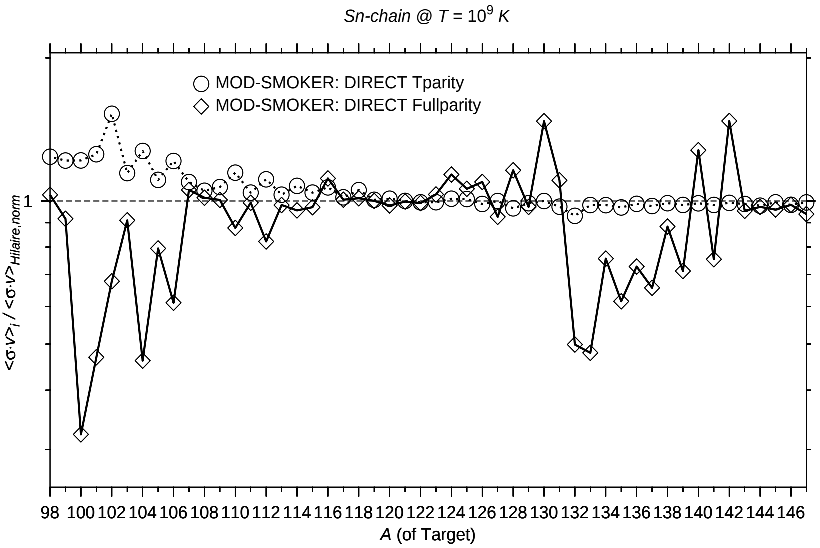


Figure: Tin chain from ^{98}Sn to ^{147}Sn

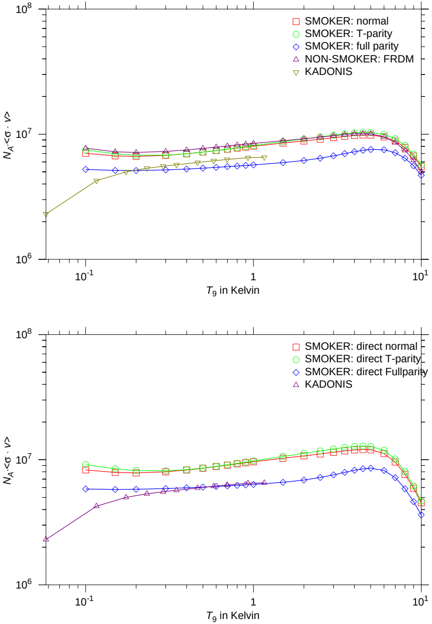
MOD-SMOKER & KADONIS

The parity effect might be a nice effect in theory, but what about experimental data? To examine that let's compare to the KADONIS dataset!

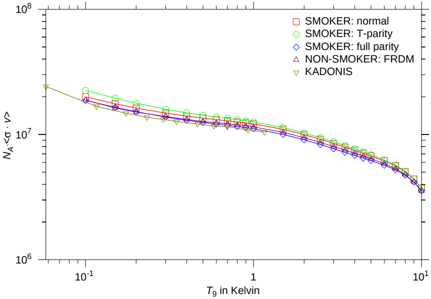
Crucial points...

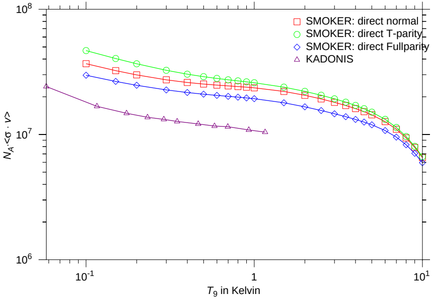
- In general: semi-empirical data
- In case that there is no experimental data: KADONIS gives theoretical values only! These are (in our case) mostly based upon NON-SMOKER from Thomas Rauscher
- Comparability only for stable nuclei since KADONIS' concern is s- and p-process data!

$^{58}Ni(n,\gamma)^{59}Ni$

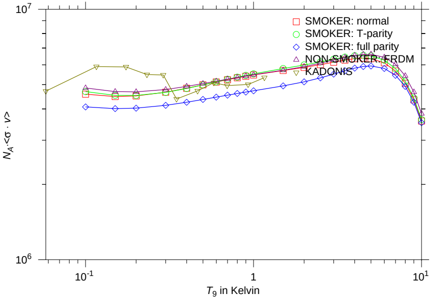


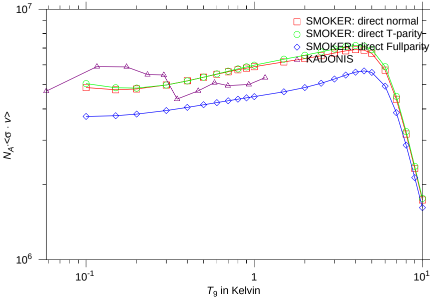
$^{59}Ni(n,\gamma)^{60}Ni$



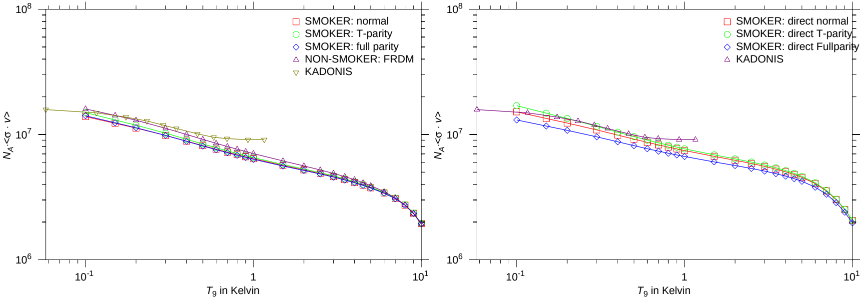


$^{60}Ni(n,\gamma)^{61}Ni$

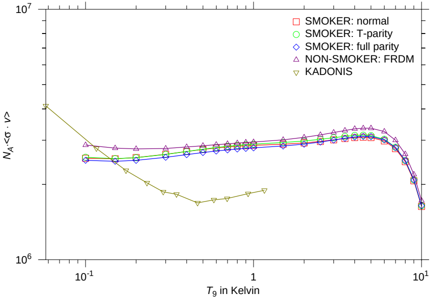


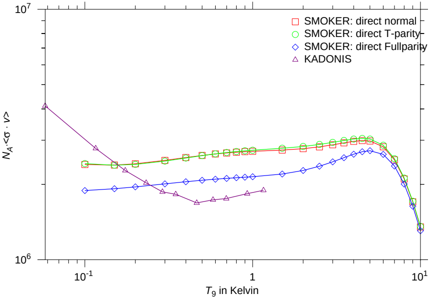


$^{61}Ni(n,\gamma)^{62}Ni$

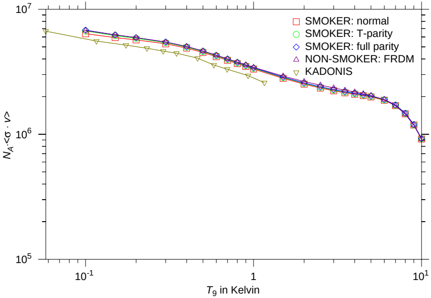


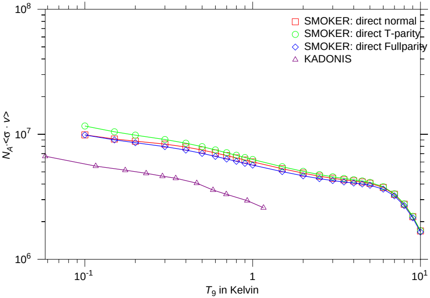
$^{62}Ni(n,\gamma)^{63}Ni$



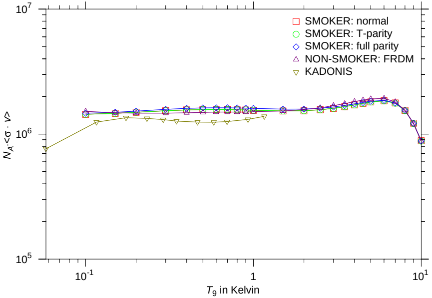


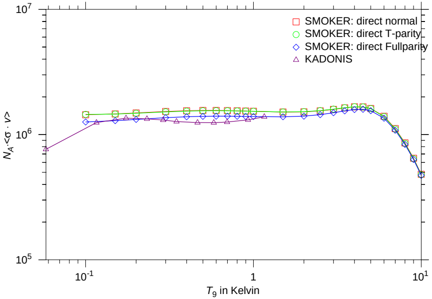
$^{63}Ni(n,\gamma)^{64}Ni$





$^{64}Ni(n,\gamma)^{65}Ni$





Conclusions...

- for some nuclei the parity non-equidistribution has a big effect!
- there is work ahead to examine the influence of J-dependence
- the most desirable treatment would be π-dependent (and J-dependent) many-body-method, so that the parity dependence does not have to be included artificially
- concerning the J-dependence such a many-body-method is not available at the moment
- concerning the π-dependence: there has been research on π-dependent optical models
- nevertheless other uncertainties can have a bigger influence such as the mass model, the level density treatment itself or the optical model

  concerning mass model: separation energies of neutron rich nuclei

The End!

- Thanks for your attention -