



# ***PYCNONUCLEAR REACTIONS in dense stellar matter***

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## ***Main collaborators:***

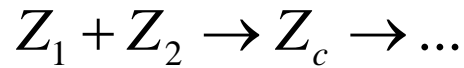
**L. Gasques – JINA, Notre Dame  
P. Haensel – CAMK, Warsaw  
K.P. Levenfish – Ioffe Institute  
M. Wiescher – JINA, Notre Dame**

## ***Talk outline:***

- **Thermonuclear reactions**
- **Pycnonuclear reactions**
- **From thermo to pycno**
- **Applications**

**Notre Dame, JINA, August 21, 2007**

# THERMONUCLEAR REACTIONS



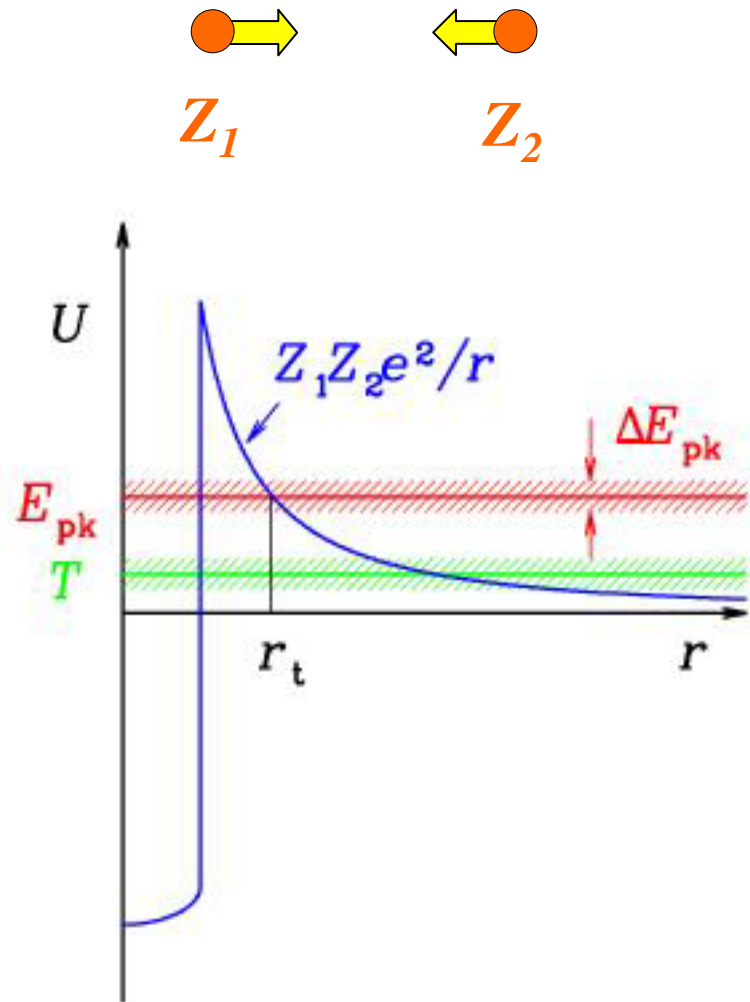
**Reaction rate:**

$$R = \frac{n_1 n_2}{1 + \delta_{12}} \langle v \sigma \rangle \quad \frac{\text{reactions}}{\text{cm}^3 \text{ s}}$$

$$\int_0^\infty \mathbf{d}v \quad v^3 \quad \mathbf{e}^{-E/T} \sigma(E)$$

$$\sigma(E) = \frac{S(E)P(E)}{E}, \quad P(E) = \exp(-\eta)$$

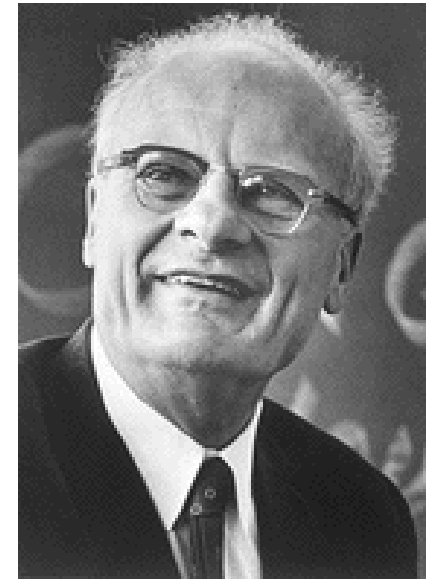
$$\eta(E) = \frac{2}{\hbar} \int_b^a \mathbf{d}r \quad |p(r)| = \frac{2\pi Z_1 Z_2 e^2}{\hbar v}$$



$$\langle v\sigma \rangle = 4 \sqrt{\frac{2E_{pk}}{3M}} \frac{S(E_{pk})}{T} \exp(-\tau)$$

$$\tau = \frac{3E_{pk}}{T} = \left( \frac{27\pi^2 M Z_1^2 Z_2^2 e^4}{2T\hbar^2} \right)^{1/3} \gg 1$$

***Reaction rate  $R$  depends mainly on  $T$***



**Nobel Prize, 1967**

- **History** → **G. Gamow, H. Bethe, C. Critchfield, E. Salpeter, 1938**  
**C. Von Weizsacker, A. Cameron, W. Fowler, G. Rivers**
- **Test** → ***Stellar evolution***
- **Example** → ***Carbon burning:***  $^{12}\text{C} + ^{12}\text{C} \Rightarrow ^{24}\text{Mg}^* \Rightarrow \dots$

$$\rho = 10^9 \text{ g cm}^{-3}, \quad t = n_i / R = \text{burning time}$$

$$T = 10^9 \text{ K} \quad \rightarrow \quad t \sim 1 \text{ min}$$

$$T = 10^8 \text{ K} \quad \rightarrow \quad t \sim 10^{36} \text{ yr}$$

***No burning at low  $T$ !***

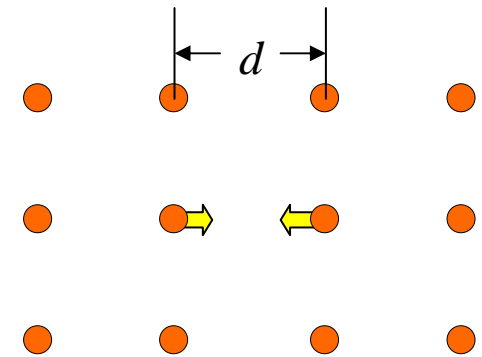
# PYCNONUCLEAR REACTIONS

Coulomb lattice of nuclei,  $T=0$

$$E_n = \hbar \omega \left( n + \frac{1}{2} \right)$$

Zero-point vibrations,  $E \sim \hbar \omega$ ,  $r_0 \sim \sqrt{\frac{\hbar}{m\omega}}$

$$\omega \sim \omega_p = \sqrt{\frac{4\pi Z^2 e^2 n_i}{m}} \quad \text{=ion plasma frequency}$$

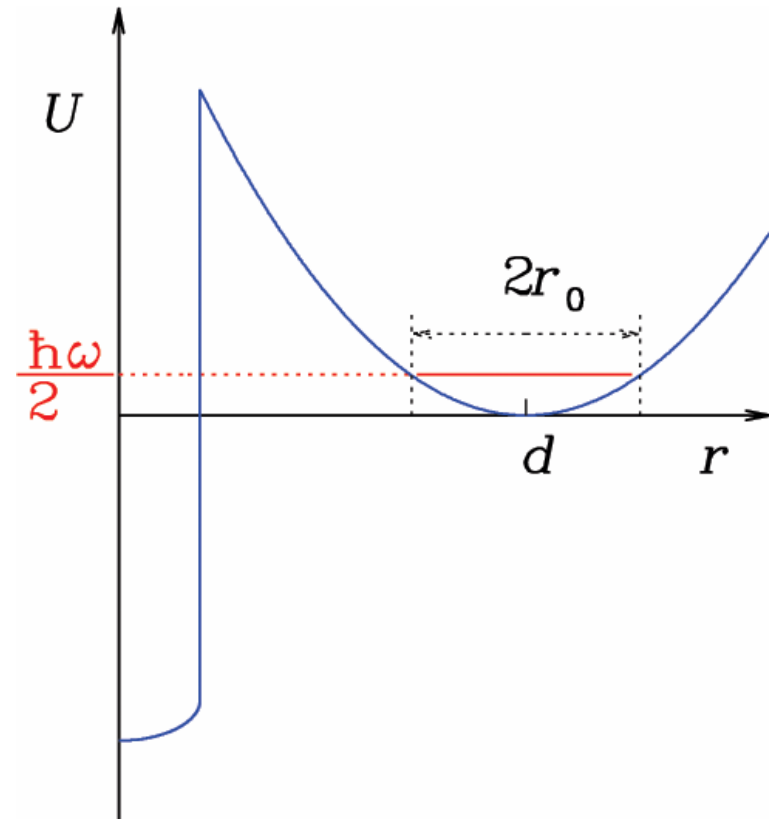


WKB tunneling:

$$\sigma(E) = \frac{S(E)P(E)}{E}, \quad P(E) = \exp(-\eta)$$

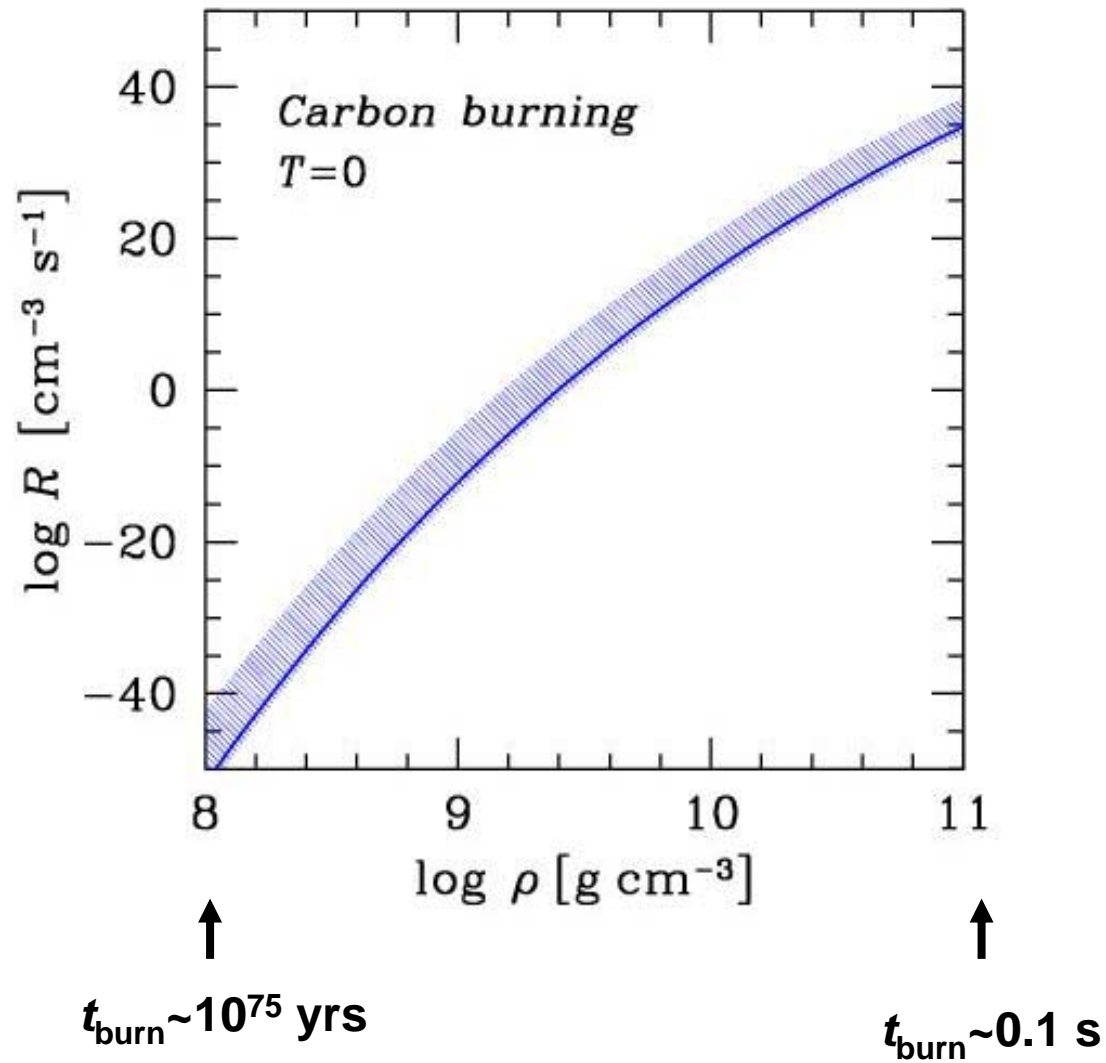
$$\eta(E) = \frac{2}{\hbar} \int_b^a dr \quad |p(r)| = \alpha \left( \frac{d}{r_0} \right)^2 \propto \frac{1}{\rho^{1/6}}$$

$$\alpha \sim 1 = ?$$



The reaction rate exponentially increases with growing density!

# DENSITY DEPENDENCE OF CARBON BURNING RATES



# Gamow and Wildhack

APRIL 15, 1939

PHYSICAL REVIEW

VOLUME 55

## Physical Possibilities of Stellar Evolution

G. GAMOW

George Washington University, Washington, D. C.

(Received November 18, 1938)

The evolution of gaseous bodies, caused by different physical processes happening in their interior and serving as energy sources, is considered qualitatively and partially quantitatively in view of possible applications for the explanation of various observed states of known stars. It is shown that the part of evolution during which the main source of energy is given by *thermonuclear reactions* leads to a steadily increasing luminosity and goes over continuously into the *contractive stage* where the energy liberation is purely gravitational. The later stages of contraction and the transition into the state of *degenerate gas*

*model* are discussed, in application to the present state of white dwarfs. Some remarks are made about the possibility of *neutron-core formation* in heavier stars, in application to the explosion phenomena observed in supernovae. An attempt is made to explain the energy production in *red giants* as due to thermonuclear reactions of *light elements* (lithium, beryllium, and boron), and the pulsation phenomena observed for *Cepheid variables* is interpreted as due to instability during the transitions from the giant branch into the main sequence.



JANUARY 15, 1940

PHYSICAL REVIEW

VOLUME 57

## The Proton-Deuteron Transformation As a Source of Energy in Dense Stars\*

W. A. WILDHACK

George Washington University, Washington, D. C.

(Received November 4, 1938)

The rates of energy evolution due to the transformation to helium, starting with the reaction  $H+H=D+e^+$ , in hydrogen at densities of  $10^4$  to  $10^8$  g/cm<sup>3</sup>, were calculated on the basis of complete degeneracy, and the assumption of a crystal-like spacing of the protons. The results indicate that any considerable amount of hydrogen in white dwarf stars would lead to much higher luminosities than those observed. Thus the low effective molecular weight (1.5) as calculated for some of these stars from the accepted white dwarf model, cannot be due to a high content of hydrogen. It might be explained as due to very large content ( $\sim 100$  percent) of the helium isotope  $He^3$  but it is very difficult to see how such large amounts of this isotope could be present in these stars. It appears that the paradox can be removed only by revision of the observational data concerning the white dwarf radii.

# Later History

Т. 33. Журнал экспериментальной и теоретической физики. Вып. 4 (10)

1957

## О ЯДЕРНЫХ РЕАКЦИЯХ В СВЕРХПЛОТНОМ ХОЛОДНОМ ВОДОРОДЕ

Я. Б. Зельдович

Показано, что ядерные реакции, происходящие подбарьерно в холодном водороде при плотностях  $10^4$ — $10^6$  г/см<sup>3</sup>, идут с вполне заметной для астрофизических масштабов вероятностью. Это обстоятельство кладет предел возможному сжатию холодного водорода, так как уже при плотности  $0,7 \cdot 10^5$  г/см<sup>3</sup> небесное тело не может прожить более  $10^8$  лет. Такая плотность в холодном водороде достигается под действием гравитации при массе, близкой к массе Солнца.

Известно [1-3], что в звездах осуществляются термоядерные реакции  $p + p = D + e^+ + \tilde{\nu}$ ,  $p + D = \text{He}^3 + \gamma$  и далее при высокой температуре  $\text{He}^3 + \text{He}^3 = \text{He}^4 + p + p$ , а при высокой плотности  $\text{He}^3 + e^- = \text{T} + \nu$ ,  $\text{T} + p = \text{He}^4 + \gamma$ .

Впервые Шатцман [1] отметил, что при высокой плотности и низкой тем-



**Zeldovich (1957)**

**Cameron (1959) – “pycnos”**

**Kirzhnits (1960)**

**Kopyshev (1964)**

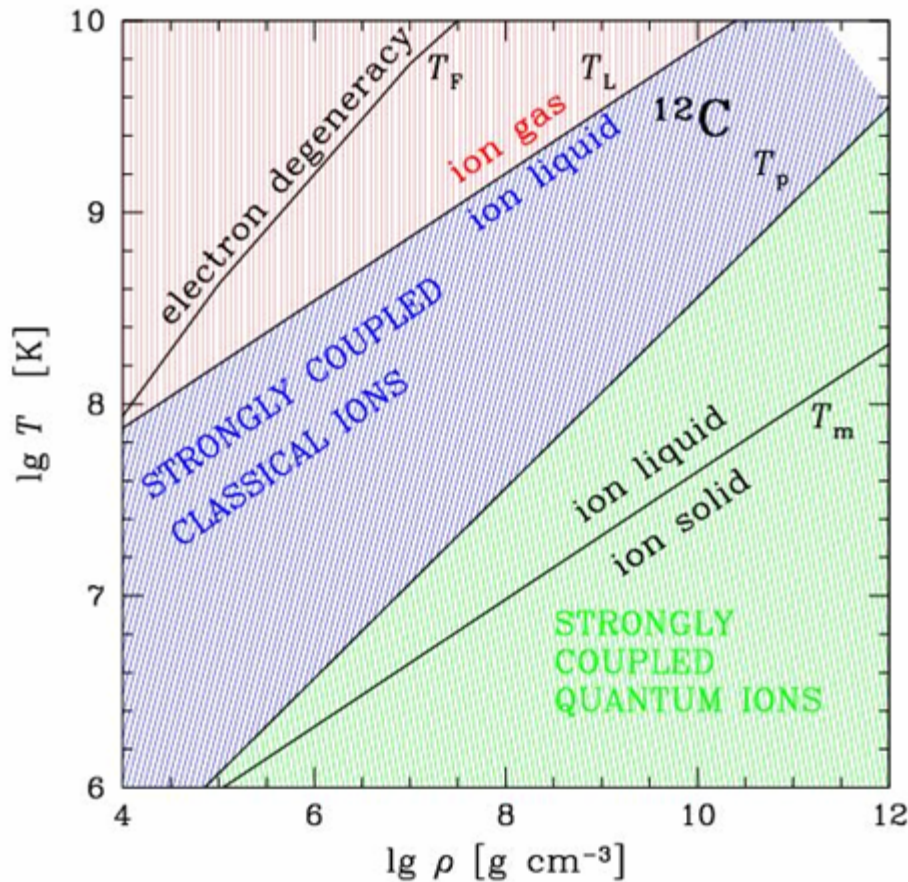
**Wolf (1965)**

**Van Horn (1966)**

**Salpeter & Van Horn (1969)**

**Schramm & Koonin (1990)**

# PHYSICAL CONDITIONS



$$Z_1 + Z_2 \rightarrow Z_c \rightarrow \dots$$

$$\Gamma = \frac{Z^2 e^2}{aT}, \quad a = \left( \frac{3}{4\pi n_i} \right)^{1/3}.$$

$$T_L = \frac{Z^2 e^2}{a} \Rightarrow \Gamma = 1; \quad T_m \Rightarrow \Gamma = 175$$

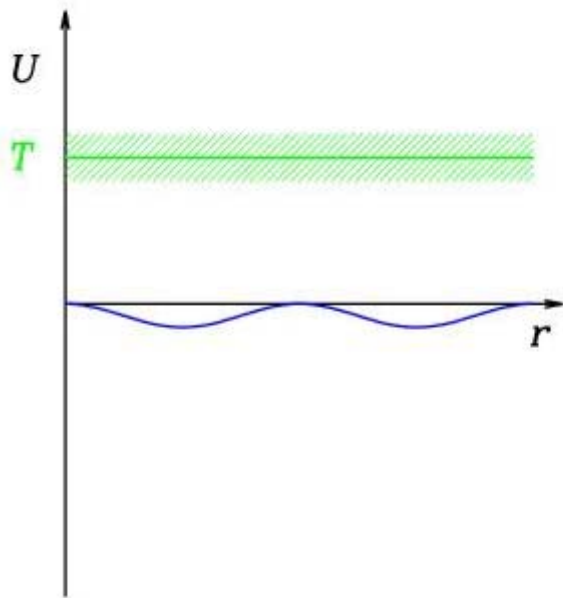
$$T_p = \hbar \omega_p, \quad \omega_p = \sqrt{\frac{4\pi Z^2 e^2 n_i}{m_i}}$$

# PHYSICAL STATES OF IONS

## Gas

$$T \gg \frac{Z^2 e^2}{a}$$

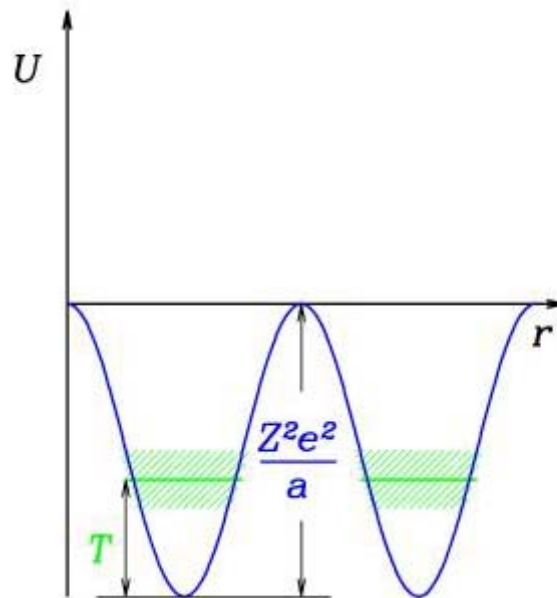
$$\Gamma \ll 1$$



## Strongly coupled classical system

$$\hbar \omega_p \ll T \ll \frac{Z^2 e^2}{a}$$

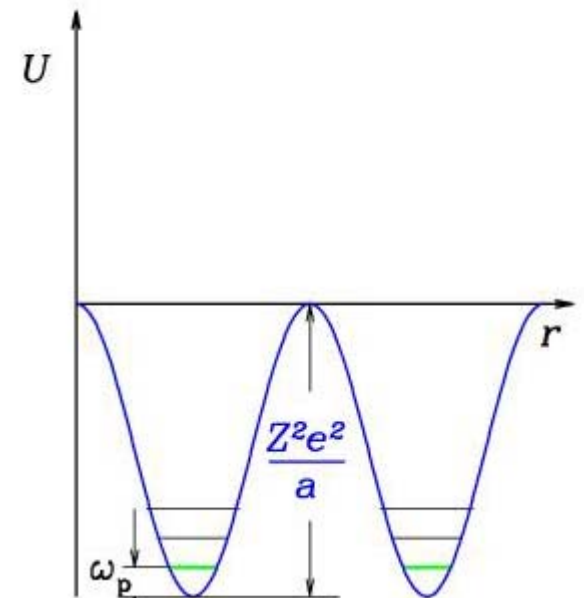
$$\Gamma \gg 1$$



## Strongly coupled quantum system

$$T \ll \hbar \omega_p \ll \frac{Z^2 e^2}{a}$$

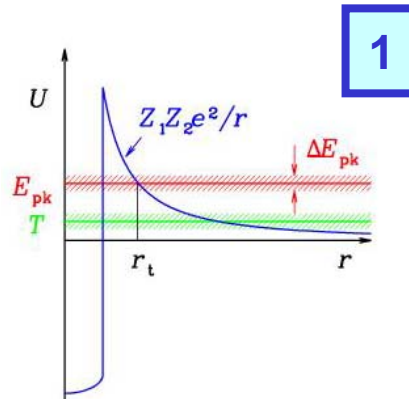
$$\Gamma \gg 1$$



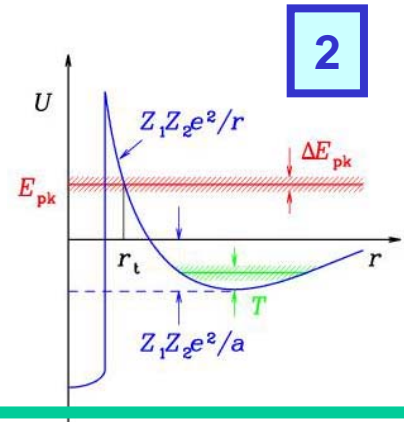
# FIVE REGIMES OF BURNING IN DENSE MATTER

Thermonuclear →

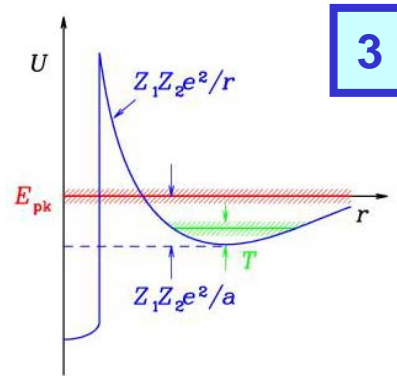
Classical



Strong screening

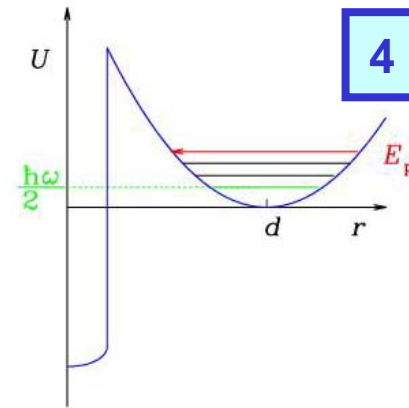


Intermediate →

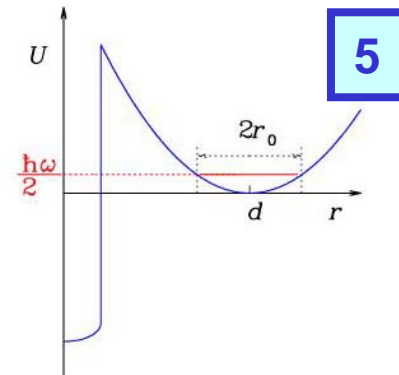


Pycnonuclear →

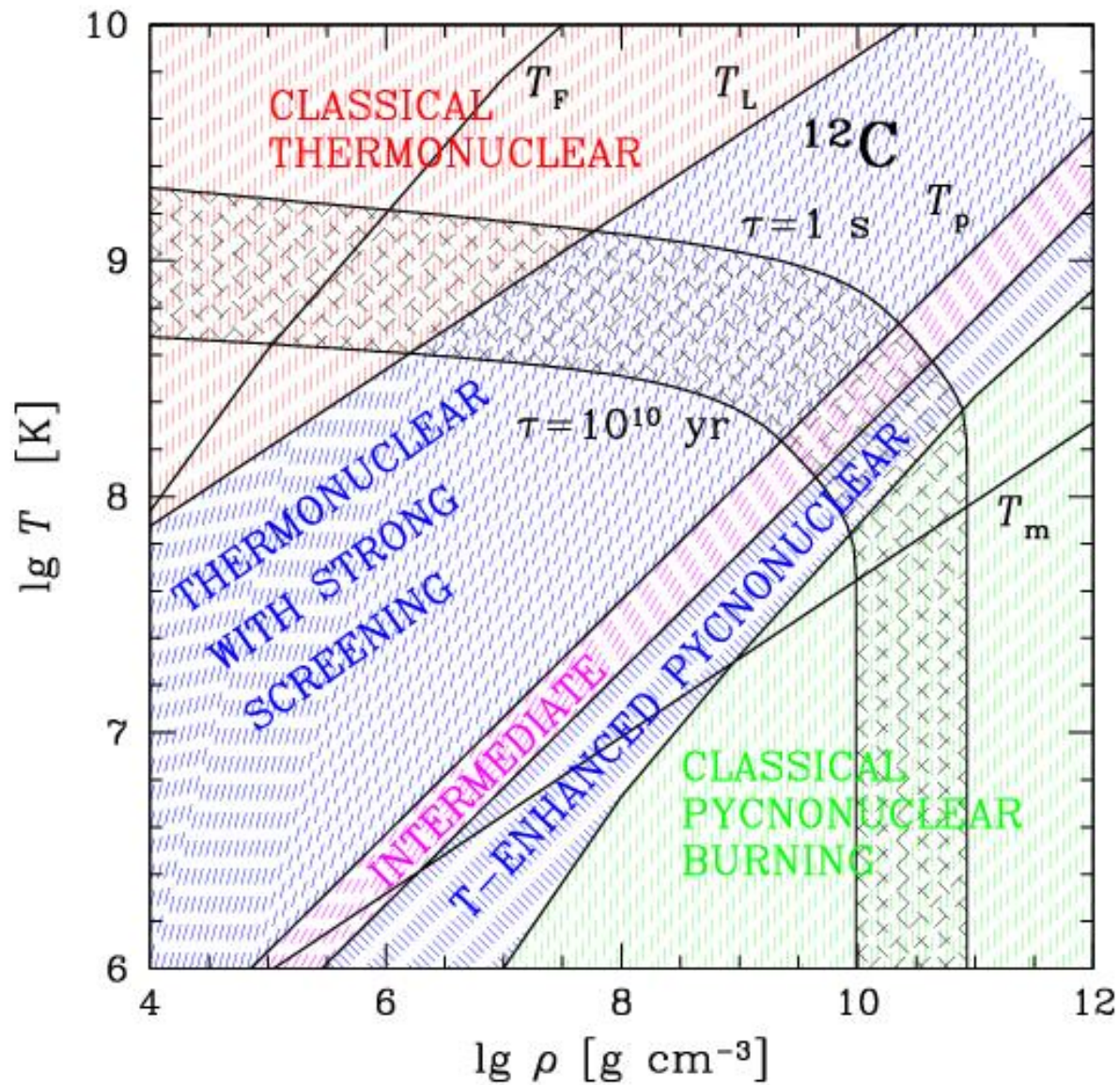
T-enhanced



$T=0$

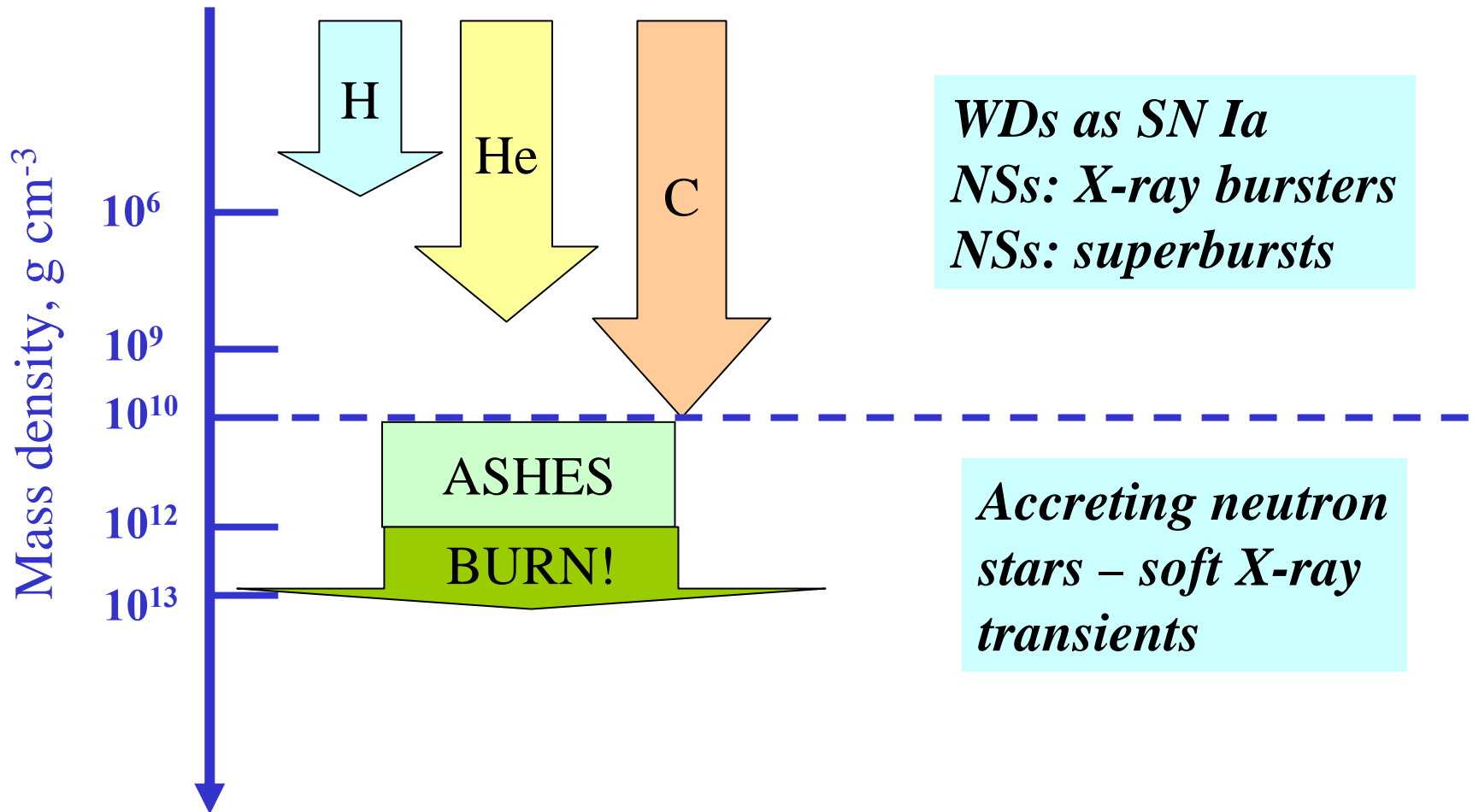


# GENERAL OUTLOOK for Carbon Burning

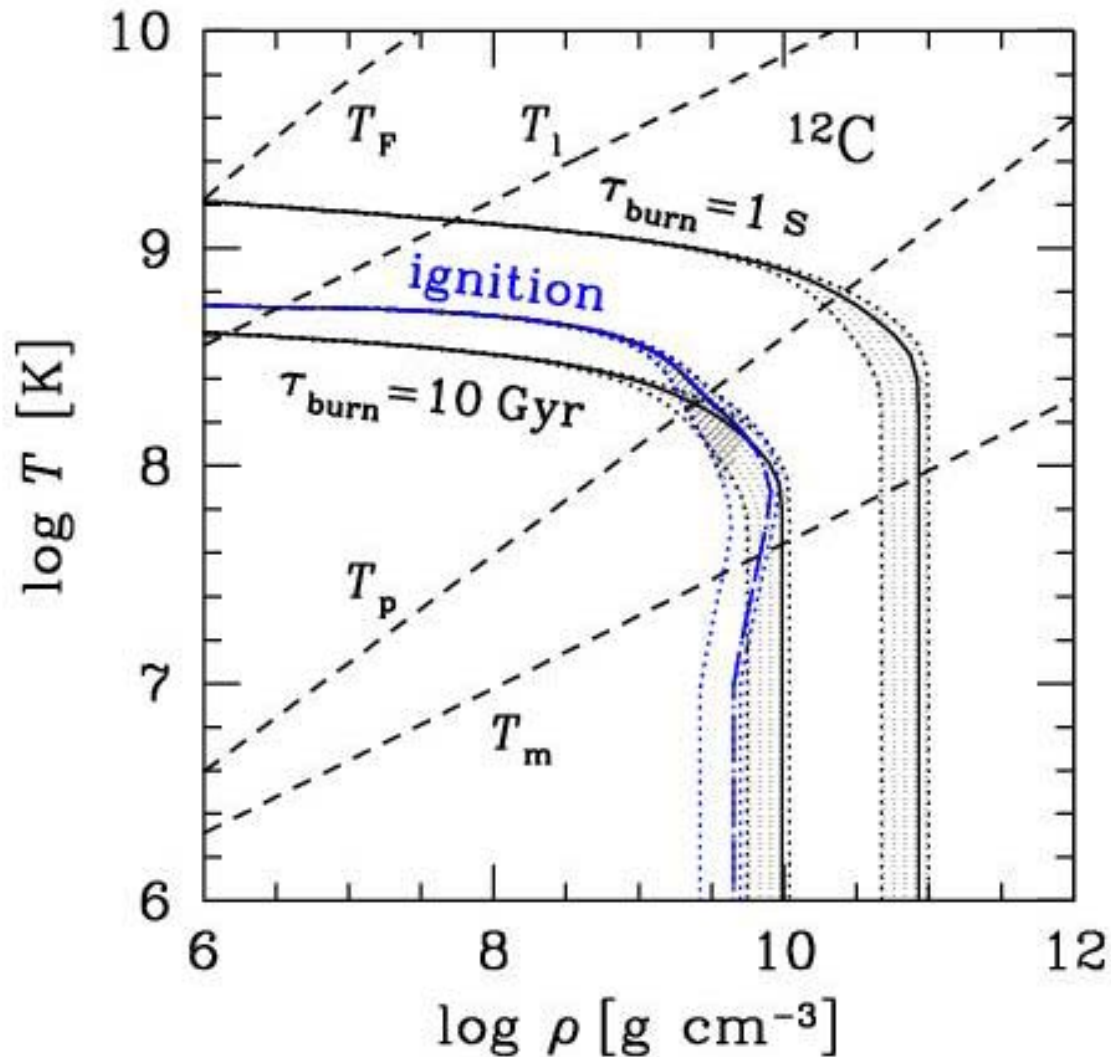


# Applications of pycnonuclear burning

*White dwarfs and neutron stars (particularly, accreting)*

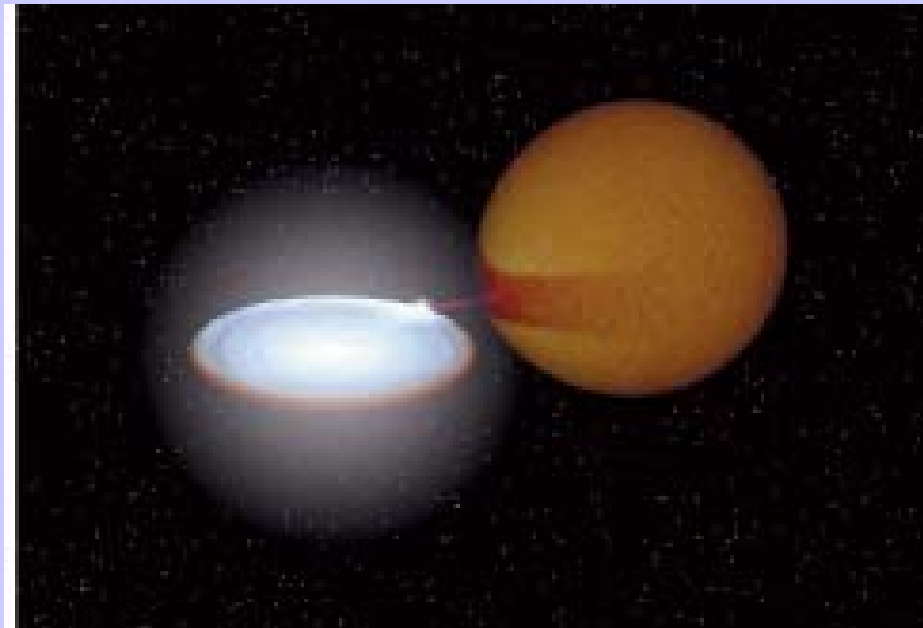


# CARBON IGNITION CURVE IN WHITE DWARFS



Talk by  
David Chamulak

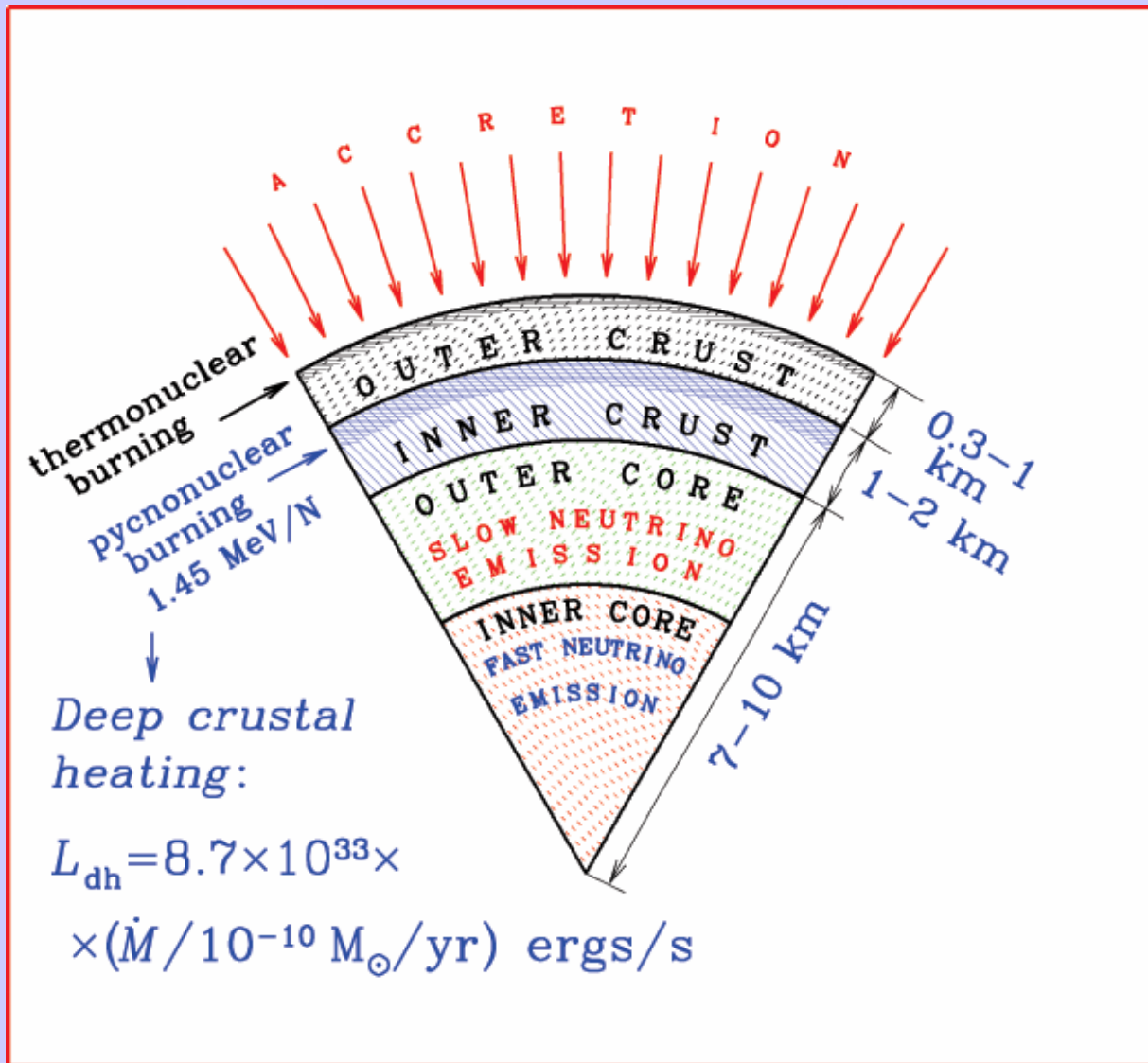
# Soft X-ray transients



- **Active states:**  $L_X = 10^{36} - 10^{39}$  erg/s  
*weeks – months – years; X-ray bursts*
- **Quiescent states:**  $L_X = 10^{30} - 10^{34}$  ergs/s  
*years - decades*
- **SXRTs belong to LMXRBs**  
Donor star: main-sequence or  
subgiant,  $M \leq M_{\text{SUN}}$
- $P_{\text{ORB}}$ : a few hours – a few days
- $\langle \dot{M} \rangle \approx 10^{-11} - 10^{-9} M_{\text{SUN}}/\text{yr}$

*Brown, Bildsten & Rutledge (1998):*  
**Aql X-1  $T_s \sim 10^6$  K**




# Accreted Crust



Reactions:

1. Beta captures
2. Neutron emission and absorption
3. Pycnonuclear reactions

# Accreted crust: Starting from $^{56}\text{Fe}$ (Haensel & Zdunik 1990)

$P$ (dyn cm $^{-2}$ )	$\rho$ (g cm $^{-3}$ )	Process	$\Delta\rho/\rho$	$q_{\text{tot}}$ (MeV)	$q$ (MeV)
$7.23 \times 10^{26}$	$1.49 \times 10^9$	$^{56}\text{Fe} \rightarrow ^{56}\text{Cr} - 2e^- + 2\nu_e$	0.08	0.04	0.01
$9.57 \times 10^{27}$	$1.11 \times 10^{10}$	$^{56}\text{Cr} \rightarrow ^{56}\text{Ti} - 2e^- + 2\nu_e$	0.09	0.04	0.01
$1.15 \times 10^{29}$	$7.85 \times 10^{10}$	$^{56}\text{Ti} \rightarrow ^{56}\text{Ca} - 2e^- + 2\nu_e$	0.10	0.05	0.01
$4.78 \times 10^{29}$	$2.50 \times 10^{11}$	$^{56}\text{Ca} \rightarrow ^{56}\text{Ar} - 2e^- + 2\nu_e$	0.11	0.05	0.01
$1.36 \times 10^{30}$	$6.11 \times 10^{11}$	$^{56}\text{Ar} \rightarrow ^{52}\text{S} + 4n - 2e^- + 2\nu_e$	0.12	0.06	0.05
$1.980 \times 10^{30}$	$9.075 \times 10^{11}$	$^{52}\text{S} \rightarrow ^{46}\text{Si} + 6n - 2e^- + 2\nu_e$	0.07	0.13	0.09
$2.253 \times 10^{30}$	$1.131 \times 10^{12}$	$^{46}\text{Si} \rightarrow ^{40}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.18	0.14	0.10
$2.637 \times 10^{30}$	$1.455 \times 10^{12}$	$^{40}\text{Mg} \rightarrow ^{34}\text{Ne} + 6n - 2e^- + 2\nu_e$	0.39	0.16	0.12
 $3.204 \times 10^{30}$	$1.951 \times 10^{12}$	$^{34}\text{Ne} + ^{34}\text{Ne} \rightarrow ^{68}\text{Ca}$ $^{68}\text{Ca} \rightarrow ^{62}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.39	0.09	0.40
$3.216 \times 10^{30}$	$2.134 \times 10^{12}$	$^{62}\text{Ar} \rightarrow ^{56}\text{S} + 6n - 2e^- + 2\nu_e$	0.45	0.09	0.05
$3.825 \times 10^{30}$	$2.634 \times 10^{12}$	$^{56}\text{S} \rightarrow ^{50}\text{Si} + 6n - 2e^- + 2\nu_e$	0.50	0.09	0.06
$4.699 \times 10^{30}$	$3.338 \times 10^{12}$	$^{50}\text{Si} \rightarrow ^{44}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.55	0.09	0.07
 $6.043 \times 10^{30}$	$4.379 \times 10^{12}$	$^{44}\text{Mg} \rightarrow ^{36}\text{Ne} + 8n - 2e^- + 2\nu_e$ $^{36}\text{Ne} + ^{36}\text{Ne} \rightarrow ^{72}\text{Ca}$ $^{72}\text{Ca} \rightarrow ^{66}\text{Ar} + 6n - 2e^- + 2\nu_e$	0.61	0.14	0.28
$7.233 \times 10^{30}$	$5.839 \times 10^{12}$	$^{66}\text{Ar} \rightarrow ^{60}\text{S} + 6n - 2e^- + 2\nu_e$	0.70	0.04	0.02
$9.238 \times 10^{30}$	$7.041 \times 10^{12}$	$^{60}\text{S} \rightarrow ^{54}\text{Si} + 6n - 2e^- + 2\nu_e$	0.73	0.04	0.02
$1.228 \times 10^{31}$	$8.980 \times 10^{12}$	$^{54}\text{Si} \rightarrow ^{48}\text{Mg} + 6n - 2e^- + 2\nu_e$	0.76	0.04	0.03
 $1.602 \times 10^{31}$	$1.127 \times 10^{13}$	$^{48}\text{Mg} + ^{48}\text{Mg} \rightarrow ^{96}\text{Cr}$	0.79	0.004	0.11
$1.613 \times 10^{31}$	$1.137 \times 10^{13}$	$^{96}\text{Cr} \rightarrow ^{88}\text{Ti} + 8n - 2e^- + 2\nu_e$	0.80	0.02	0.01

# ***SUMMARY***

- **Cold dense matter undergoes pycnonuclear burning**
- **Pycnonuclear burning is not efficient in ordinary stars but can be important in cores of white dwarfs and envelopes of neutron stars**
- **Applications of pycnonuclear burning include:**
  - (a) explosions of white dwarfs as Ia type supernovae**
  - (b) deep crustal heating of transiently accreting neutron stars**
  - (c) superbursts of accreting neutron stars**
- **The theory of pycnonuclear burning is not perfect; new theories, implementations, observations and theoretical interpretation are ahead**