Shell model method for Gamow-Teller transitions in deformed nuclei

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β-decay & electron-capture in stars

- Stellar weak-interaction rates are important for studying astrophysical problems
  - for nucleosynthesis calculations
  - for core collapse supernova modeling
- Evaluation of nuclear matrix element
  - essentially a nuclear structure problem
- Theoretical models
  - Independent particle model (FFN)
  - Spherical shell model (SM)
  - Quasiparticle random phase approximation (QRPA)

Stellar enhancement of β-decay rate

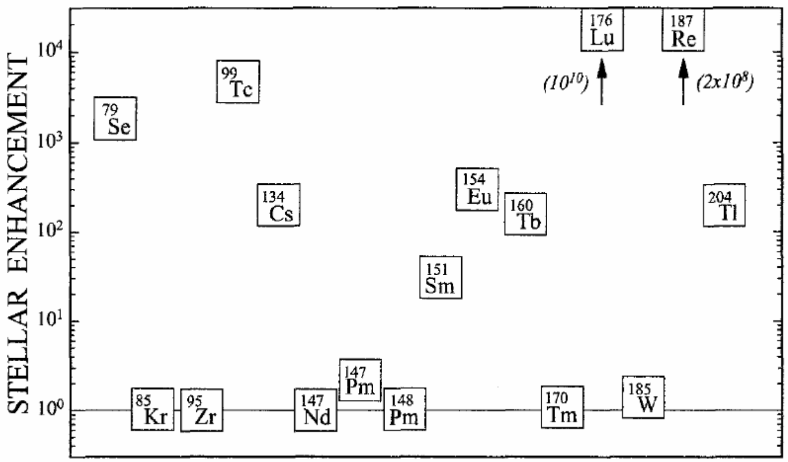
- A stellar enhancement can occur due to thermal population of excited states

$$\lambda_\beta = \sum_i \left(p_i \times \sum_j \lambda_{\beta ij} \right)$$

$$p_i = \frac{(2I_i + 1) \times \exp(-E_i / kT)}{\sum_m (2I_m + 1) \times \exp(-E_m / kT)}$$

- Examples in the s-process

F. Kaeppeler,
Prog. Part. Nucl. Phys. 43 (1999) 419

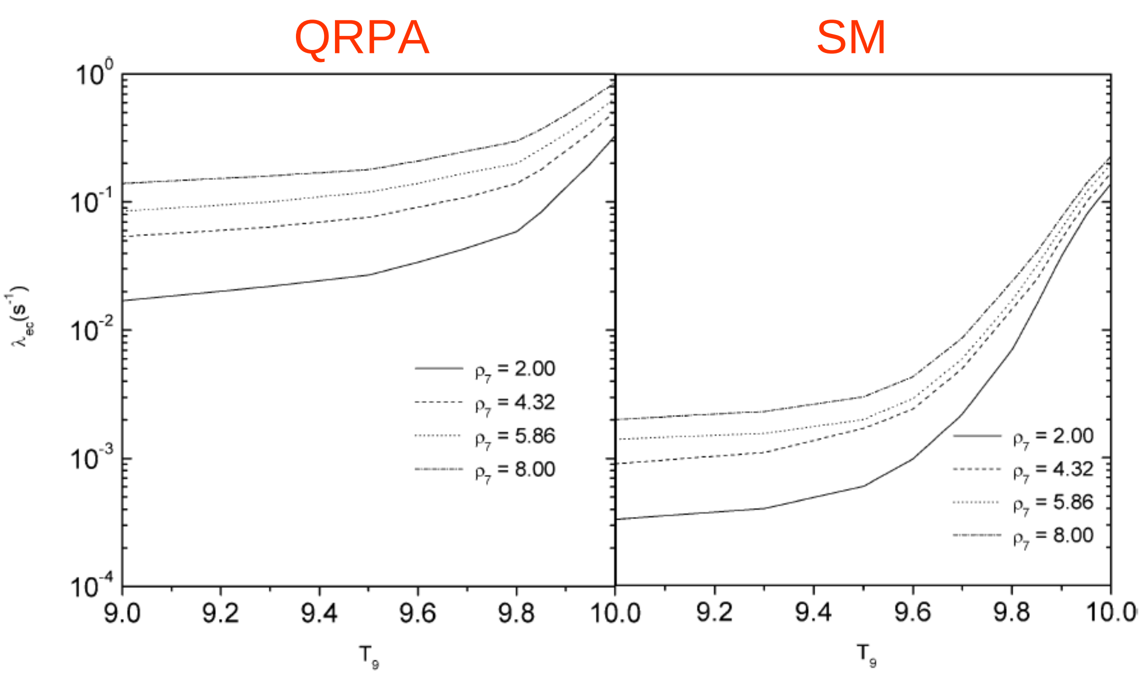


Electron-capture in core collapse supernovae

- Electron-capture rates are needed in simulations of core collapse supernovae
  - In prior simulations, electron capture has been treated in a highly parametrized fashion
  - With realistic treatment of electron capture on heavy nuclei, significant changes in the hydrodynamics of core collapse are found
  - W.R. Hix *et al.*, *Phys. Rev. Lett.* 91 (2003) 201102
- ^{55}Co was ranked as the most important nuclei with respect to the importance for the electron capture process
  - M.B. Aufderheide *et al.*, *Astrophys. J.* Suppl. 91 (1994) 389

Supernova electron capture rates on ^{55}Co by SM and QRPA

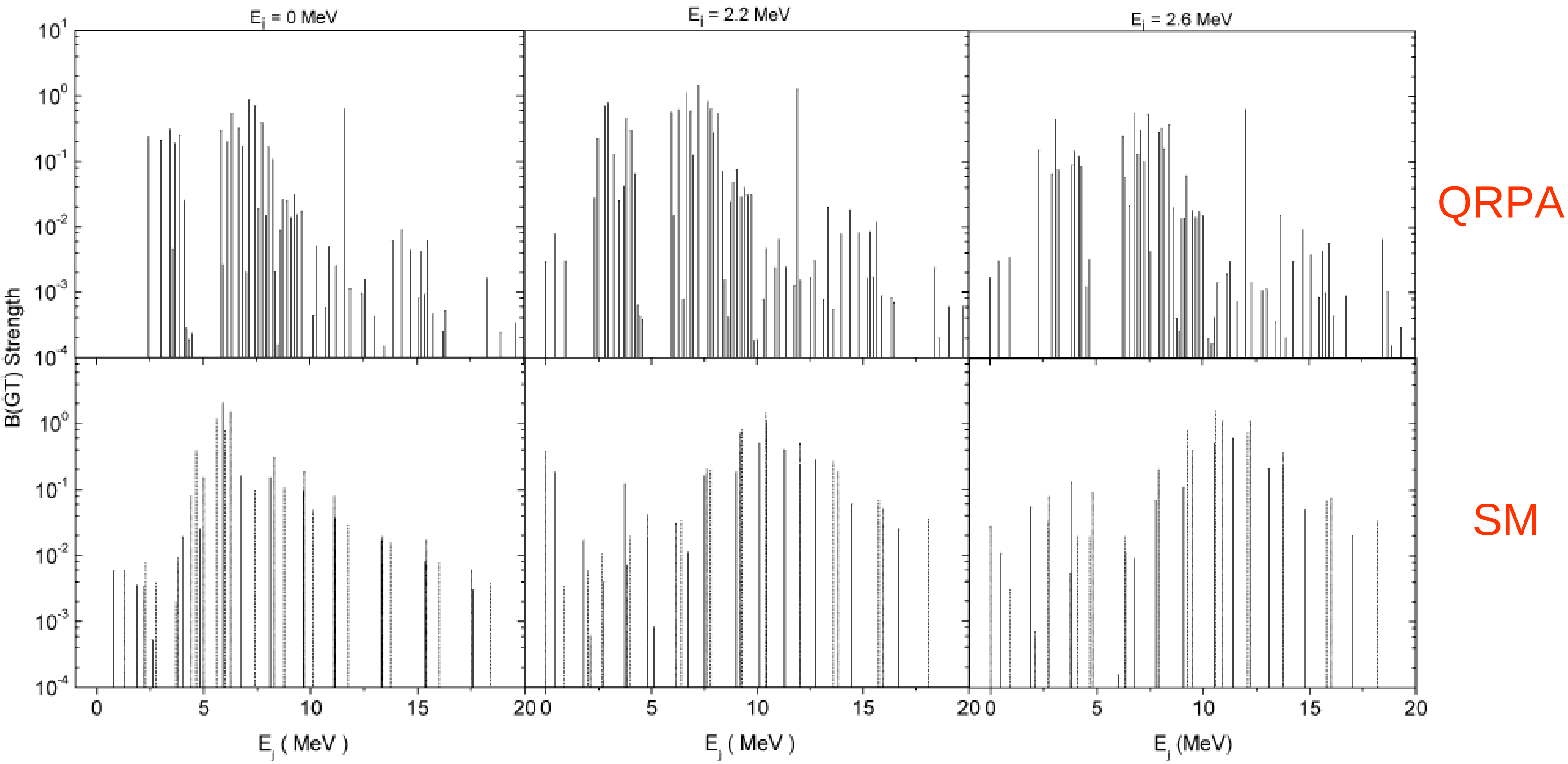
- QRPA rates by two orders of magnitude stronger than SM
  - K. Langanke & G. Martinez-Pinedo, *Phys. Lett.* B436 (1998) 19
  - J.-U. Nabi & M.-U. Rahman, *Phys. Lett.* B612 (2005) 190



This can have a significant astrophysics impact because rate change of lepton-to-baryon ratio changes by 50% due to electron capture on ^{55}Co

B(GT) distribution in SM and QRPA

- G-T strength more fragmented in RQPA than in SM



Comparison of these models

- SM:
  - K. Langanke & G. Martinez-Pinedo, *Phys. Lett.* B436 (1998) 19
  - Small model space in one major *pf* shell, allowing only a few particles excited from $f_{7/2}$, renormalization introduced
  - For electron-capture rates at finite temperature, G-T strength is not calculated, but the Brink-hypothesis is used
  - Deficiency: insufficient model space
- QRPA:
  - J.-U. Nabi & M.-U. Rahman, *Phys. Lett.* B612 (2005) 190
  - Calculated nuclear states are not angular momentum states, but *K*-states (which are mixtures of *I*-states)
  - Deficiency: not a shell model, may contain spurlous in the wavefunction

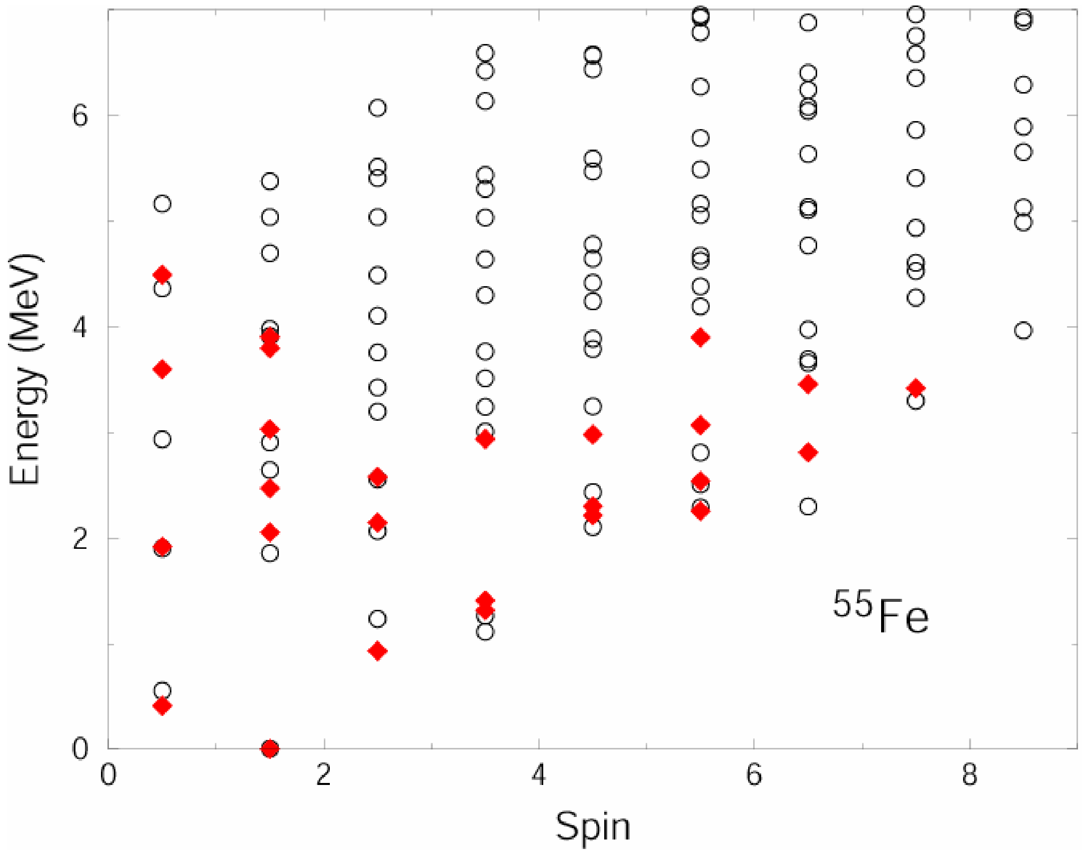
The projected shell model (PSM)

- Take a set of deformed (quasi)particle states (e.g. solutions of HF, HFB or HF + BCS)
- Build up configurations (qp vacuum + multi-qp states)
- Project them onto good angular momentum (if necessary, also parity, particle number) to form a basis in lab frame
- If necessary, superimpose configurations belonging to different qp representations (the GCM-concept)
- Diagonalize a two-body Hamiltonian in projected basis

Hara & Sun, *Int. J. Mod. Phys*. E4 (1995) 637

Energy spectrum for ^{55}Fe: validating the PSM application

- Projected shell model calculation for ^{55}Fe energy levels



Nuclear matrix elements in the projected basis

- Gamow-Teller rate $$B(GT)=\frac{2I_f+1}{2I_i+1}\left\langle \psi_{I_f}\left|\hat{\beta}^{\pm}\right|\psi_{I_i}\right\rangle^2$$
- PSM wavefunction $$\psi^I_M=\sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa}\left|\phi_\kappa\right\rangle$$
- e-e system $$\left|\phi_e(\varepsilon_e)\right\rangle \Rightarrow \left\{ \left|\varepsilon_e\right\rangle, b^+_\nu b^+_\nu\left|\varepsilon_e\right\rangle, b^+_\pi b^+_\pi\left|\varepsilon_e\right\rangle, b^+_\nu b^+_\nu b^+_\pi b^+_\pi\left|\varepsilon_e\right\rangle, \cdots\right\}$$
- o-o system $$\left|\phi_o(\varepsilon_o)\right\rangle \Rightarrow \left\{ a^+_\nu a^+_\pi\left|\varepsilon_o\right\rangle, a^+_\nu a^+_\nu a^+_\nu a^+_\pi\left|\varepsilon_o\right\rangle, a^+_\nu a^+_\pi a^+_\pi a^+_\pi\left|\varepsilon_o\right\rangle, \cdots\right\}$$
- Overlapping matrix element (Gao, Sun, Chen, *PRC* 74 (2006) 054303).

$$\left\langle \phi_o(\varepsilon_o)\right|\hat{O}\hat{P}^I_{K_oK_e}\left|\phi_e(\varepsilon_e)\right\rangle \sim \int d\Omega D^I_{K_oK_e}(\Omega)\left\langle \phi_o(\varepsilon_o)\right|\hat{O}\hat{R}(\Omega)\left|\phi_e(\varepsilon_e)\right\rangle$$

The interactions

- Total Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_{QP} + \hat{H}_{GT}$

- Quadrupole + monopole-pairing + quadrupole-pairing

$$\hat{H}_{QP} = -\frac{1}{2}\chi_{QQ}\sum_{\mu}\hat{Q}^{\dagger}_{2\mu}\hat{Q}_{2\mu} - G_M\hat{P}^{\dagger}\hat{P} - G_Q\sum_{\mu}\hat{P}^{\dagger}_{2\mu}\hat{P}_{2\mu}$$

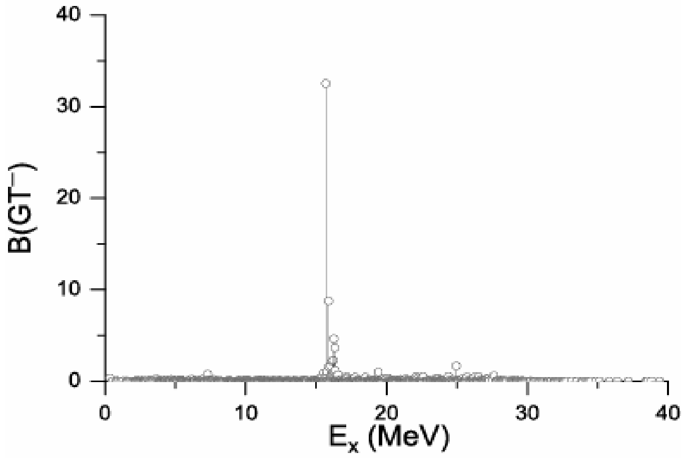
- Charge-exchange (Gamow-Teller)

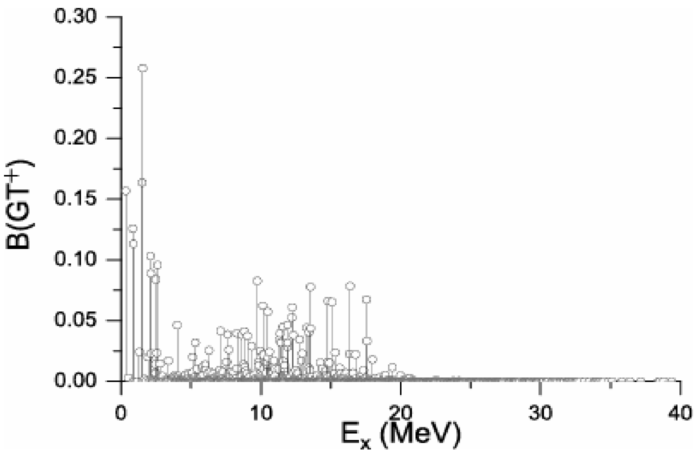
$$\hat{H}_{GT} = +2\chi_{GT}\sum_{\mu}\hat{\beta}^{-}_{1\mu}(-1)^{\mu}\hat{\beta}^{+}_{1-\mu} - 2\kappa_{GT}\sum_{\mu}\hat{\Gamma}^{-}_{1\mu}(-1)^{\mu}\hat{\Gamma}^{+}_{1-\mu}$$

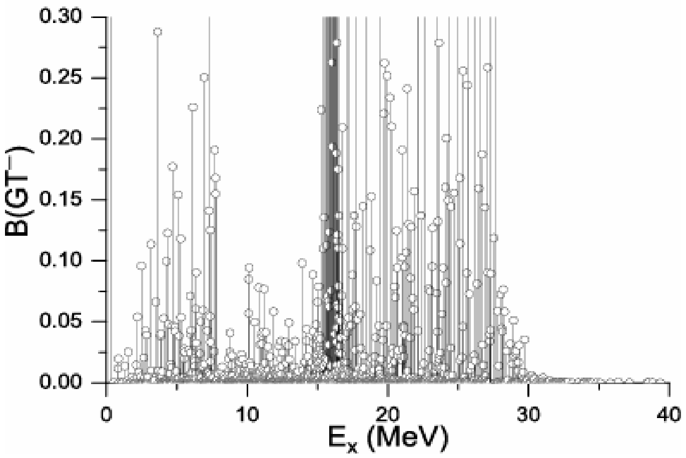
  - Kuz'min & Soloviev, *Nucl. Phys*. A 486 (1988) 118

B(GT) distribution

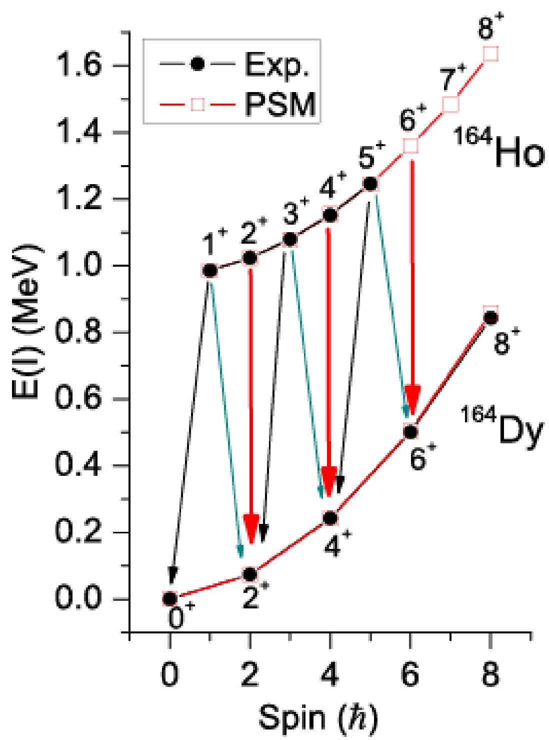
- Initial state: ground state of even-even nucleus
- Final states: all 1^+ states in odd-odd nucleus
- Ikeda sum-rule fulfilled

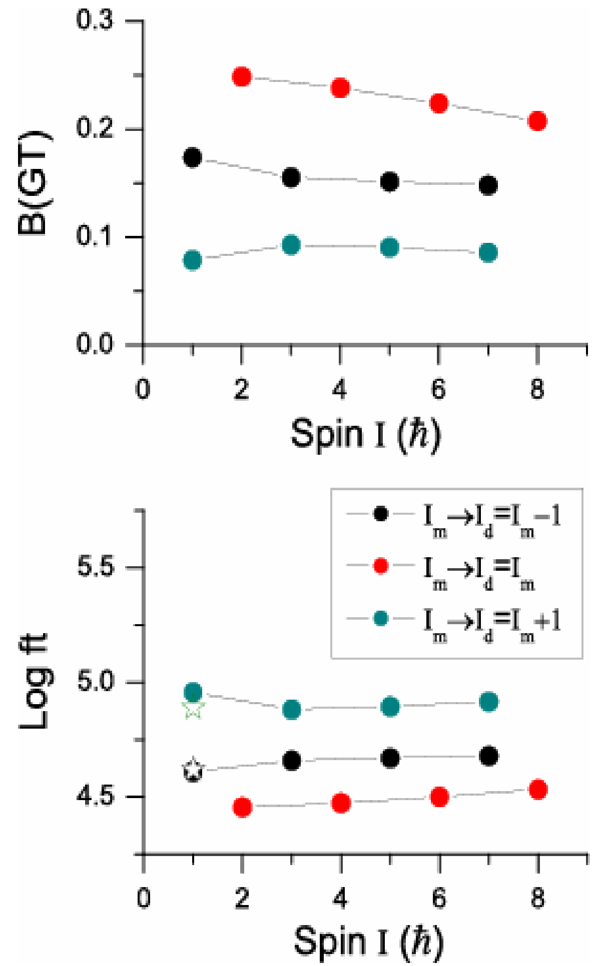






B(GT) and log*ft* in ^{164}Ho → ^{164}Dy





Gao, Sun, Chen, *Phys. Rev.* C 74 (2006) 054303

B(GT) in Projected Shell Model

- PSM wave functions contain correlations beyond mean-field and are written in the laboratory frame, having definite quantum numbers
- A state-by-state evaluation of B(GT) is feasible (no Brink-hypothesis needed)
  - The PSM single-particle state is big, but dimension of the shell model space is small (usually in the range of 10^2–10^4)
- PSM is a multishell shell model and can calculate forbidden transitions
  - Calculation of forbidden transitions is not possible for most of conventional shell models working in one-major shell basis

Conclusion

- Angular momentum projection is an efficient way of truncating shell model space to perform shell model calculations for heavy, deformed nuclei.
- The projection technique is well developed. Projected shell model is a practical example.
- A method of using projected wave functions to calculate weak interaction rates is developed, and can be applied to nuclear astrophysics.
- The method may overcome the deficiencies in SM and QRPA