SUMMARY LECTURE 1

1. EXTERNAL REGION WAVE FNS.

   $\Psi, \psi, \Phi, \psi_\alpha \Rightarrow$ Total, rel, CM and Spin

2. Radial Wave Equation

   For $V = \frac{Z_1 Z_2 e^2}{r}$ using $\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$

   Standard Eqn.

   Dimensionless Eqn.

3. SOLUTIONS TO RADIAL EQN

   $I_c^+, O_c^+ \Rightarrow F_c$ and G_c

4. Relationship Between $I, O \Rightarrow F G$

   $I_c^+ = (G_c - iF_c)e^{i\omega_c}$, $O_c^+ = (G_c + iF_c)e^{-i\omega_c}$

5. Complete Channel Wave fns. , 5.a. INTERNAL WAVE FNS

   Ingoing and Outgoing — Groups of SAME JM !!!

   $\psi_{JM} = \sum_\lambda A_{\lambda J} X_{\lambda JM}$.

6. Surface Wave Function

   $\varphi_s = \frac{1}{r\alpha}(i^l Y_m^l(\Omega))$

7. Surface Functions

   V_c and D_c

8. Surface OverLap Integrals.

9. EXTERNAL FNS.

   $P_c^+, S_c^+, S_c^-, \Omega_c^+, L_c^+, \mathcal{L}_c^+, B_c^+, \omega_c$.

   9.A NON-ZERO B

   $S_l^* = S_l^0 - B_l$

10. Defn. of Reduced Width γ and R-Function.

11. COMMENTS ON SPECTRA --- Interference, etc.

12. Collision Function $\Longleftrightarrow$ R-Function.

    $U_l = \frac{I_l}{O_l} \frac{1 - L_l^* R_l}{1 - L_l R_l}$, $R_l = \frac{I_l - U_l O_l}{\rho(I_l' - U_l O_l')}$

13. Phase Shift

    $\delta_l = \tan^{-1}\left[\frac{R_l P_l}{1 - R_l S_l}\right] - \varphi_l + \omega_l$

Lane & Thomas

A20P LT

III. 1. Definitions and Notations — Channels. $\left.\begin{array}{l} a=\alpha \\ a=\alpha' \end{array}\right\}$ Reaction

The "α" index p. 266.

Channel Characterization. Reduced Mass, k, v, η, σ_c $\rho = k r_\alpha$

$$\lambda = \left(\frac{\hbar}{2m}\right)^{1/2}$$

III 2. Wave Functions External Region. p. 268.

Ψ = total channel wave fn.

ψ = Rel. Motion wave fn.

Φ = Centroid Motion Wave fn. — Not used in CM.

ψ_α = Spin wave fn.

Channel Spin Wave fn.

$$\psi_{\alpha s \nu} = \sum_{i_1+i_2=\nu} (I_1 I_2 i_1 i_2 | s \nu)\, \psi_{\alpha_1 I_1 i_1}\, \psi_{\alpha_2 I_2 i_2} \qquad \int \psi^*_{\alpha s\nu}\, \psi_{\alpha' s' \nu'} = 4\pi a^2\, \delta_{\alpha s\nu, \alpha' s' \nu'}$$

Wave Function - Rel. Motion p. 269

$$\psi \sim \underbrace{\frac{1}{r_\alpha} u_{\alpha s l}(r_\alpha)}_{\text{radial wv. f.}} \underbrace{(i^l Y_m^l)}_{\text{angular wave. fn.}}$$ As usual, Separation of Variables.

where $\left[\frac{d^2}{dr_\alpha^2} - \frac{l(l+1)}{r_\alpha^2} - \frac{2M}{\hbar^2}(V_{\alpha s l} - E_\alpha)\right] u_{\alpha s l}(r_\alpha) = 0$ —

Dimensionless Radial Equation $[\rho = k r_\alpha]$ EXERCISE FOR STUDENT I.A

$$\longrightarrow \quad u''_{\alpha l}(\rho_\alpha) - \left[l(l+1)/\rho_\alpha^2 + 2\eta_\alpha \rho_\alpha^{-1} \mp 1\right] u_{\alpha l}(\rho_\alpha) = 0$$ Note absence of index s.

Solns. to the Coulomb Radial Eqn. (Radial Equations)

$$*** — A) \left\{\begin{array}{l} I_c^+ = I_{\alpha l}^+ \sim \exp\left[i\left(\rho_\alpha - \eta_\alpha \log 2\rho_\alpha - \frac{1}{2} l\pi + \sigma_{\alpha 0}\right)\right] \\ \Theta_c^- = I_{\alpha l}^- \sim \exp\left[i\left(\rho_\alpha - \eta_\alpha \log 2\rho_\alpha - \frac{1}{2} l\pi + \sigma_{\alpha 0}\right)\right] \end{array}\right\} \begin{array}{l} \text{Incoming} \\ \text{Outgoing} \end{array}$$

alternatively

$$** — B) \left\{\begin{array}{l} F_c \equiv F_{\alpha l} \sim \sin\left(\rho_\alpha - \eta_\alpha \log 2\rho_\alpha - \frac{1}{2} l\pi + \sigma_{\alpha l}\right) \\ G_c \equiv G_{\alpha l} \sim \cos\left(\rho_\alpha - \eta_\alpha \log 2\rho_\alpha - \frac{1}{2} l\pi + \sigma_{\alpha l}\right) \end{array}\right\} \begin{array}{l} \text{Regular} \\ \text{Irregular} \end{array}$$

ESSENTIAL EQUATIONS:-

F_c } Needed in CODE AZURE – } ALSO FOR NEUTRONS.
G_c } Returned by COUL.FOR

O_c^- — For bound states (Neg. Energy channels.

W_c — In CODE AZURE [COMMENTS re. CB]

O_c^+ — DEFINES L_0, P, S.
in terms of F and G.

$L_0 \Rightarrow$ Magical Diagonal Matrix value

Real $[L_0] \Rightarrow$ total shift
Imag $[L_0] \Rightarrow$ total width.

I_c^+ – For, $\mathcal{L}$, $\mathcal{B}$, w

$\boxed{\text{ANSWER:- In ONE dimension}}$ $\text{grad}(r\psi) = \frac{\partial}{\partial r}(r\psi) = r\frac{\partial \psi}{\partial r} + \psi$

Thus $D = \left(\frac{\hbar^2}{2Ma_c}\right)^{1/2} \int \varphi_c^* (r \,\text{grad}\,\psi + \psi)\, dS$

$$D = \left(\frac{\hbar^2}{2Ma_c}\right)^{1/2} \int \varphi_c^* \psi \, dS + \left(\frac{\hbar^2}{2Ma_c}\right)^{1/2} \int \underbrace{r_c}_{=a_c} \varphi^* \,\text{grad}\,\psi \, dS$$

$$D = V_c + \left(\frac{\hbar^2 a_c}{2M}\right)^{1/2} \int \varphi^* \,\text{grad}\,\psi \, dS$$

Thus Soln's A and B are related

*** $I_c^+ = (G_c - iF_c)\exp(i\omega_c)$

*** $O_c^+ = (G_c + iF_c)\exp(-i\omega_c)$

$\omega_c = \omega_{\alpha\ell} = \sigma_{\alpha\ell} - \sigma_{\alpha 0} = \sum_{n=1}^{\ell} \tan^{-1}\left(\frac{\eta_\alpha}{n}\right)$

TO PROVE ALL OF THIS EXERCISE FOR STUDENT I.B

III 2. Complete Channel Wave Function

p. 270

$$\mathcal{I}^+_{\alpha\ell s\nu m} = \frac{I^+_{\alpha\ell}}{v^{1/2}} \frac{1}{r_\alpha}\left(i^\ell Y^\ell_m\right)\psi_{\alpha s\nu}$$

$$\mathcal{O}^+_{\alpha\ell s\nu m} = \frac{O^+_{\alpha\ell}}{v^{1/2}} \frac{1}{r_\alpha}\left(i^\ell Y^\ell_m\right)\psi_{\alpha s\nu}$$

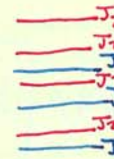
EXERCISE FOR STUDENT

See U.G. Quant. Mech. Courses.

Define Rij as a surface function. [i.e. the surface variation for r_α = fixed]

Surface function (OF EXTERNAL WAVE FN)

$$\varphi_{\alpha\ell s\nu m} = \frac{1}{r_\alpha}\left(i^\ell Y^\ell_m\right)\psi_{\alpha s\nu}$$



***III 3. Wave Functions for the INTERNAL REGION

Compound Nucleus

$H\Psi_{JM} = E\Psi_{JM}$

$$\boxed{\Psi_{JM} = \sum_\lambda{}' A_{\lambda J} X_{\lambda JM}}$$

THE $X_{\lambda JM}$ are a complete set of internal wave fns. [Independent of M]

$X_{\lambda JM} \Rightarrow$ R-M function / levels / Energies
$\Rightarrow$ Proper Function / levels / Energies
$\Rightarrow$ "Pole" Function / levels / Energies

*** III. 4 Wave Function on S

$$V_c = \left(\frac{\hbar^2}{2M a_c}\right)^{1/2} U_c(a_c)$$

$$D_c = \left(\frac{a_c\hbar^2}{2M}\right)^{1/2}\left(\frac{dU_c}{dr_c}\right)_{r_c = a_c}$$

p. 271

Surface Overlap Integrals.

$$V_c = \left(\frac{\hbar^2}{2M a_c}\right)^{1/2}\int_S \varphi_c^* \Psi \, dS$$

where Ψ is the Complete Wave fn. of System (INTERNAL ?)

$$D_c = \left(\frac{\hbar^2}{2M a_c}\right)^{1/2}\int_S \varphi_c^* \,\mathrm{grad}_n\left(r_c\Psi\right) dS$$

II

$$= V_c + \left(\frac{a\hbar^2}{2M}\right)^{1/2}\int_S \varphi_c^* \,\mathrm{grad}\,\Psi \, dS$$

EXERCISE FOR STUDENT

p.271 Cont^{d}.

III 4.a. EXTERNAL FUNCTIONS

$$\boxed{\text{Defn } L_c \equiv \left(\rho_c \frac{O_c'}{O_c}\right)_{r_c=a_c} = S_c + iP_c}$$

where S_c = shift
P_c = penetrability

from eqn. Top. of p.2, calc $\rho_c \frac{O_c'}{O_c}$

$$P_c^+ = \frac{\rho_c}{[F^2+G^2]}\bigg|_{r_c=a_c}$$

$$S_c^+ = P_c^+ [F_c F_c' + G_c G_c']$$

$$S_c^- = \left(\rho_c \frac{W_c'}{W_c}\right)_{r_c=a_c}$$

where W = Whittaker fns.

$$P_c^- = 0.$$

$$\Omega_c^+ = (I_c/O_c)^{1/2}_{a_c}$$

$$\Omega_c^+ = \Omega_{dc}^+ = \exp i(\omega_c - \varphi_c^+)$$

ALL THESE EQNS. USED IN CODE-AZURE III
EXERCISE FOR STUDENT: VERIFY THESE EQNS

TOTAL PHASE SHIFT.

[Exercise — Work out from I_c and O_c]
(HARDPHASE)
where $\varphi_c^+ = \varphi_{c0}^+ = \tan^{-1}(F_c/G_c)$

DEA
see notes on bottom of p.271, LT.

also, introduce, and prove

$$\mathcal{L}_c = \left(\rho_c \frac{I_c'}{I_c}\right)_{a_c} = L_c^{+*}$$

$$\mathcal{B}_c = (\rho_c / I_c O_c)_{a_c} = P_c^{+*}$$

Calc. in same way as $L = \left(\rho_c \frac{d}{O_c}\right)$

$$w_c = i(O_c' I_c - I_c' O_c)_{a_c} = 2i$$

INTERNAL FUNCTIONS

Remember, our complete set of internal fns are $X_{\lambda JM}$

$$\gamma_{\lambda c} = V_{\lambda c} = \left(\frac{\hbar^2}{2M a_c}\right)^{1/2} \int_S \varphi_c^* X_{\lambda JM}\, dS$$

OVERLAP INTEGRAL

DEFN. OF REDUCED WIDTH AMPLITUDE

$$\delta_{\lambda c} = D_{\lambda c} = \left(\frac{a_c^2 \hbar^2}{2M}\right)^{1/2} \int \varphi_c^* \operatorname{grad}_n X_{\lambda JM}\, dS$$

$$\frac{\delta_{\lambda c}}{\gamma_{\lambda c}} = \frac{D_{\lambda c}}{V_{\lambda c}} = B_c$$

Logarithmic derivative quantity.

N.B. ADD 'INTERFERENCE' — Spectra — Pencil of same spin & Parity — hard phase.
'BC FOR LEVEL I'

p. 272

IV ELASTIC SCATTERING SPINLESS PARTICLES

FOR OUR COMPLETE SET [illegible]

1) Derivation of Cross-Section

We need ALL of this page in detail !!

For two solns u_1 and u_2

$$\left(u_2 \frac{du_1}{dr} - u_1 \frac{du_2}{dr}\right) + \frac{2M}{\hbar^2}(E_1 - E_2)\int_0^a u_1 u_2\, dr = 0$$

$$\boxed{\text{Zero deriv. BC.} \quad \left(\frac{du_\lambda}{dr}\right)_a = 0}$$ ——— *

Orthonormal $\int_0^a u_\lambda u_{\lambda'} = \delta_{\lambda\lambda'}$ where $\lambda\lambda'$ are 1 2

At any Energy E

$$u_E = \sum_\lambda A_\lambda u_\lambda(r) \quad \text{Internally.}$$

For two solns u_E and u_λ (E, E_λ)

$$\therefore \quad A_\lambda = \int_0^a u_\lambda u_E \, dr.$$

$$-u_\lambda(a)\left(\frac{du_E}{dr}\right)_a + \left(\frac{2M}{\hbar^2}\right)(E_\lambda - E)\int_0^a u_E u_\lambda \, dr = 0$$

Combining these two.

$$A_\lambda = \frac{\hbar^2}{2M}\cdot\frac{u_\lambda(a)}{E_\lambda - E}\left(\frac{du_E}{dr}\right)_a$$

Hence.

$$u_E(r) = G(r,a)\left(a\frac{du}{dr}\right)_a$$

where

$$G(r,a) = \frac{\hbar^2}{2Ma}\sum_\lambda \frac{u_\lambda(r)\, u_\lambda(a)}{E_\lambda - E}$$

THE R-FUNCTION, DEFINITION

$$\boxed{R = G(a,a) = \sum_\lambda \gamma_\lambda^2/(E_\lambda - E)}$$

p. 273

$$R = \frac{u_E(a)}{a\left(\frac{du_E}{dr}\right)_a}$$

2) The Collision Function Quick & Easy DEFINITION

$$\psi_\ell = I_\ell - U_\ell O_\ell$$

(N.B. Only index is ℓ)

Express U_ℓ in terms of R.

EXERCISE FOR STUDENT IV

$$\left(\frac{u_\ell}{\rho u_\ell'}\right) = R_\ell = \frac{(I_\ell - U_\ell O_\ell)}{\rho(I_\ell' - U_\ell O_\ell')} \Longrightarrow U_\ell = \frac{I_\ell}{O_\ell}\,\frac{1 - L_\ell^* R_\ell}{1 - L_\ell R_\ell}$$ [see eqn 1.5 p. 289]

NOTE SIGNIFICANCE OF ℓ !!! GROUP OF SAME J^π !!!

LECTURES 1 #2

p. 273 Cont$^{d.}$

*** $U_\ell = \exp(2i\delta_\ell)$ [see p. 281 §9]

*** $\delta_\ell = \tan^{-1}\left[\dfrac{R_\ell P_\ell}{(1-R_\ell S_\ell)}\right] - \varphi_\ell + \omega_\ell$

EXERCISE FOR STUDENT $\underline{V}$

NOT USED IN AZURE BUT IS USED IN $^{12}C(\alpha,\alpha)^{12}C$ TISCHHAUSER ET AL

<u>One Level Approximation</u>

* assume $R_\ell \approx \dfrac{\gamma_{\lambda\ell}}{E_\lambda - E}$

* thus $\delta_\ell = \tan^{-1}\left[\dfrac{\frac{1}{2}\Gamma_{\lambda\ell}}{E_{\lambda\ell}+\Delta_{\lambda\ell}-E}\right] - \varphi_\ell$

***** <u>Level Width</u> $\boxed{\Gamma_{\lambda\ell} = 2\gamma_{\lambda\ell}^2 P_\ell}$ — CODE-AZURE

p. 274

<u>Relation Between U and dσ/dΩ</u>

*** $\sigma(\theta) = |A(\theta)|^2 = \frac{1}{4}k^{-2}\left|\sum_\ell (2\ell+1)(1-U_\ell)P_\ell(\cos\theta)\right|^2$ eqn. 1.24

<u>Comparison with N.D. $^{12}C(\alpha,\alpha)^{12}C$.</u>

$$\delta_\ell = \tan^{-1}\left[\frac{P_\ell}{\frac{1}{R_\ell+B} - S}\right] - \varphi_\ell$$

where B = Boundary Condition. Put $B=0$ for comparison

$$\delta_\ell = \tan^{-1}\left[\frac{P_\ell}{\frac{1}{R_\ell} - S}\right] - \varphi_\ell$$

$$\delta_\ell = \tan^{-1}\left[\frac{P_\ell R_\ell}{1-R_\ell S_\ell}\right] - \varphi_\ell$$ Compare with Above.

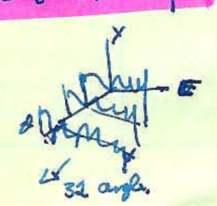
MAYBE RELOCATE THIS TILL AFTER NEXT THREE PAGES

<u>Clark & Schiff</u>

REF? Look up PLAGA.

* $\sigma(\theta) = \frac{1}{k^2}\left|\frac{\eta}{2}\sin^{-2}\left(\frac{\theta}{2}\right)\exp\left(-2i\eta\ln\sin\left(\frac{\theta}{2}\right)\right) + \sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\theta)\exp[i(2\omega_\ell+\delta_\ell)]\cdot\sin\delta\right|^2$ TISCHAUSSER ET AL.

SHOW ND YIELD CURVES



***
***

To change from "ZERO" BC to finite BC.

N.B. B_ℓ does NOT have an index λ ...!!!

For the "zero" BC. we got formula

$$\boxed{\delta_\ell = \tan^{-1}\left[\frac{R_\ell P_\ell}{1 - R_\ell S_\ell}\right] - \varphi_\ell.}$$

Instead of using $\underbrace{\left(\frac{du_\lambda}{dr}\right)_a = 0}_{S^\circ}$, we use $\boxed{\left(\frac{r}{u_\lambda}\frac{du_\lambda}{dr}\right)_a = B_\ell}$ ARBITRARY CHOOSE ONE PARTICULAR λ

This is the "standard" BC we use in LT.

EXERCISE FOR STUDENT. (p. 274 LT)

So that all previous eqns. (1.13 → 1.19) can be re-written to account for our new BC, by replacing the original shift fnla, S°; by $\boxed{S_\ell^\circ = S_\ell - B_\ell}$ EXERCISE FOR STUDENT MODIFY EQN. on p(3) WITH THIS

For example:- the phase shift δ now becomes.

$$\delta = \tan^{-1}\left[\frac{R_\ell P_\ell}{1 - R_\ell(S_\ell - B_\ell)}\right] - \varphi_\ell \quad = \tan^{-1}\left[\frac{P_\ell}{\underbrace{\frac{1}{R_\ell} - (S - B)}_{\text{DIRECTORY (ALL)}}}\right] - \varphi_\ell$$

CHOOSING THE B.C.

(1) First consider the behaviour of δ as a fn of Energy.

$$\delta = \tan^{-1}\left[\frac{P_\ell \cdot \sum_\lambda \frac{\gamma_{\lambda\ell}^2}{E_\lambda - E}}{1 - \sum_\lambda' \frac{\gamma_{\lambda\ell}^2}{E_\lambda - E}\cdot(S_\ell - B_\ell)}\right] - \varphi_\ell$$

TO SEE HOW THIS BEHAVES AS A FN OF ENERGY
Now at EACH RESONANCE, assume contributions for neighboring resonances are negligible

$$\delta_R \sim \tan^{-1}\left[\frac{P_\ell \frac{\gamma_\lambda^2}{E_\lambda - E}}{1 - \frac{\gamma_\lambda^2}{E_\lambda - E}\cdot(S_\ell - B_\ell)}\right]$$

Multiply num & denom by $E_\lambda - E$

$$S_\lambda = \tan^{-1}\left[\frac{P_e}{E_\lambda - E - \gamma_\lambda^2 (S_e - B_e)}\right]$$

Note that $S_e = P_e(FF' + GG')$ calculated AT BOMBARDING ENERGY

$\boxed{\text{CHOOSE} \quad B_e = S_e \text{ for a given LEVEL } \lambda.}$

$\therefore$ Denominator goes through RESONANCE = BOMBARDING ENERGY EXACTLY FOR THE CHOSEN LEVEL λ.

WHEREAS :- ALL OTHER LEVELS WILL EXHIBIT RESONANCES AT SHIFTED ENERGIES !!!

Ψ_λ

**** Relationship Between the PROPER STATES E_λ and the **** experimental states $\boxed{E_R \text{ or } \Psi_R}$

DISCUSSIONS BY E.V. on

PROPER STATES $\Longleftrightarrow$ experimental states.

**** FITTING THEORY TO DATA

1) Only level λ will have resonance of $\overleftrightarrow{E_\lambda} = E_R$
2) ~~All~~ FOR all other levels, the fit ~~will occur for E_λ~~ the proper energies will be ~~adjust~~ found such that the peaks of the ~~yields will be~~ theory will be matched to the actual (exptl) resonances, but these energies E_λ' will NOT be equal to E_R' because of the shift.

ONLY FOR LEVEL λ will $E_\lambda = E_R$

" " " " will γ_λ be the physical values

HOW TO FIND PHYSICAL VALUES FOR THE OTHER RESONANCES

For the next λ of interest choose $S_\lambda = B_\lambda$ for this level.
(It may not be exactly at the exptl E_λ because you haven't yet done a fit)

REPEAT THE FIT. NOW MAYBE YOU HAVE TO ITERATE A BIT UNTIL $E_\lambda = E_R$ (EXPTL)
Now you have ~~a~~ "physical" values of E_λ and γ_λ for this new level.

REPEAT FOR ALL LEVELS UGH!

IS THERE A BETTER WAY? YES

SEE LECTURE 6 BY CARL BRUNE!