The Level Matrix

Notation !!

Equations required

$$W = 1 + B^{1/2}(1-RL_0)^{-1} R B^{1/2} w \quad \text{——— (1)}$$

$$R_{c'c} = \sum_\lambda{}' \frac{\gamma_{\lambda c'}\gamma_{\lambda c}}{E_\lambda - E} \quad \text{——— (2)}$$

or $\left[R = \sum_\lambda (\gamma_\lambda \times \gamma_\lambda) / (E_\lambda - E) \right]$

$\left[\text{this might be written as } R = \sum_\lambda (\gamma_\lambda \gamma_\lambda)_{c'c} / (E_\lambda - E) \right]$

$$R = R^0 + R'$$

$$L' = L_0 (1 - R^0 L^0)^{-1}$$

Basic Assumption

$$\boxed{(1-R'L')^{-1} = 1 + \sum_{\mu\nu}{}' (\gamma_\mu \times \beta_\mu) A_{\mu\nu}} \quad \text{——— IX 1.4 p. 294}$$

WHY WOULD ANYONE WANT TO CHOOSE THIS FORM ?? Compare with $(1-RL_0)^{-1}$ from the R-Matrix section.

see opp. p.9 of lecture 2.

If the above assumption can really be done, then we can solve for the quantities $A_{\mu\nu}$.
To do this we write

$$(1-R'L') = \left(1 - \sum_\lambda \frac{(\gamma_\lambda \times \gamma_\lambda)}{E_\lambda - E} \cdot L'\right) \quad \text{where we can use } \boxed{L' = \frac{\beta_\nu}{\gamma_\nu}}$$

Thus

$$\boxed{(1-R'L') = \left(1 - \sum_\lambda{}' \frac{(\gamma_\lambda \times \beta_\lambda)}{E_\lambda - E}\right)} \quad \text{——— IX 1.6 p. 294}$$

Now Multiply Eq. 1.4 × Eqn. 1.6

$$1 = \left[1 - \sum_\lambda{}' \frac{(\gamma_\lambda \times \beta_\lambda)}{E_\lambda - E}\right]\left[1 + \sum_{\mu\nu}{}' (\gamma_\mu \times \beta_\nu) A_{\mu\nu}\right]$$

$$\text{OR} \quad -\sum_\lambda{}' \frac{(\gamma_\lambda \times \beta_\lambda)}{E_\lambda - E} + \sum_{\mu\nu}{}' (\gamma_\mu \times \beta_\nu) A_{\mu\nu} - \sum_{\lambda\mu\nu}{}' \frac{(\gamma_\lambda \times \beta_\nu)}{E_\lambda - E} \xi_{\lambda\mu} A_{\mu\nu} = 0$$

where $\xi_{\lambda\mu} = (\beta_\lambda, \gamma_\mu)$ [Inner Product]

Now after the application of a Vector Identity

$$\sum_{\mu\nu} (\gamma_\lambda \times \beta_\nu) \left[-\frac{\delta_{\lambda\nu}}{E_\lambda - E} + A_{\lambda\mu} - \sum_\mu \frac{\xi_{\lambda\mu} A_{\mu\nu}}{E_\lambda - E} \right] = 0 .$$

which is satisfied for all λ, ν if quantity $[\;] \equiv 0$, giving

$$\boxed{(E_\lambda - E) A_{\lambda\mu} - \sum_\mu \xi_{\lambda\mu} A_{\mu\nu} = \delta_{\lambda\mu}}$$

$$\Downarrow$$

Let $A = \begin{pmatrix} A_{11} & A_{12} \cdots \\ \vdots & \\ A_{n1} & A_{n2} & A_{nn} \end{pmatrix}$ $\quad e = \begin{pmatrix} E_1 & & 0 \\ & E_2 & \\ 0 & & \ddots \end{pmatrix}$ $\quad \xi_{\lambda\mu} = \begin{pmatrix} \xi_{11} & \cdots & \\ & & \xi_{nn} \end{pmatrix}$

Eigen Energies for Proper Wave functions.

$$\Downarrow$$

$$\boxed{A = (e - E - \xi)^{-1}} \quad (1) \qquad E = \begin{pmatrix} E & & 0 \\ & E & \\ 0 & & \ddots E \end{pmatrix}$$

Bounding Energy.

$$W = 1 + B^{1/2} \Big[(1 - \underbrace{R^0 L^0)^{-1}}_{=0} R^0 + \sum_{\lambda\mu} (\gamma_\lambda \times \gamma_\mu) A_{\lambda\mu} \Big] B^{1/2}{}_{\mu\nu}$$

$$\boxed{W = 1 + B^{1/2} \Big[1 + \sum_{\lambda\mu} (\gamma_\lambda \times \gamma_\mu) A_{\lambda\mu} \Big] B^{1/2}{}_{\mu\nu}} \quad (2)$$

~~with~~ Note here that the β-vectors have been swallowed up in the A^{-1} matrix elements.

Thus we need

$$\boxed{\gamma_{\mu} = (\gamma_{\mu 1} \cdots \cdots \gamma_{\mu n})} \quad (3)$$

$$\boxed{\beta_{\nu} = (\beta_{\nu 1} \cdots \cdots \beta_{\nu n})} \quad (4)$$

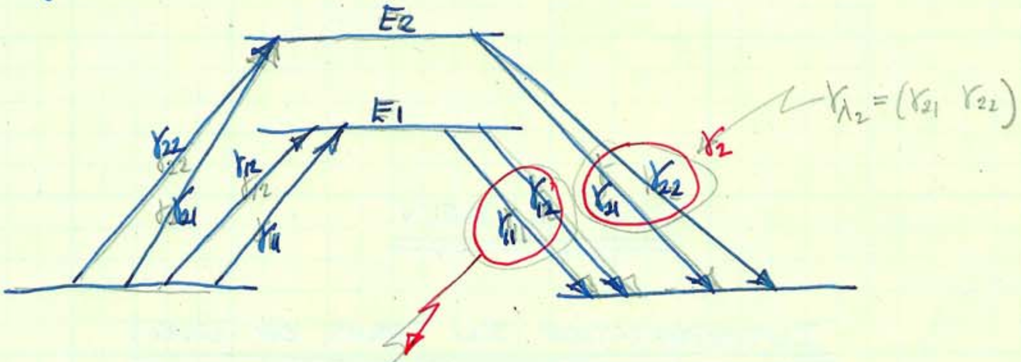
where $\quad \beta_\nu = L_0 \gamma_\nu$

Interpretation of L^0 (diagonal Matrix for exit channels)

$$\beta_\nu = L^0 \gamma_\nu$$

$$\xi_{\lambda\nu} = (\beta_\lambda \cdot \gamma_\nu)$$

We need to understand these three equations in order to do an actual calculation.
Let us consider the example of 2-channels, 2-levels of same J^π group. For simplicity let us assume that we have only elastic Scattering.



$$\vec{\gamma}_1 = (\gamma_{11}\ \gamma_{12})$$
$$\vec{\gamma}_2 = (\gamma_{21}\ \gamma_{22})$$

N.B. The $\vec{\gamma}_\lambda$ vectors are those which have as elements ALL the γ's in the exit channel for ~~one~~ level.
additional

e.g. For future reference, suppose in addition to elastic, we had inelastic with three channels γ_{13}, γ_{14} and γ_{15} for level $\lambda = 1$ and γ_{23}, γ_{24} and γ_{25} for level $\lambda = 2$

$$\vec{\gamma}_1 = (\gamma_{11}\ \gamma_{12}\ \gamma_{13}\ \gamma_{14}\ \gamma_{15})$$
$$\vec{\gamma}_2 = (\gamma_{21}\ \gamma_{22}\ \gamma_{23}\ \gamma_{24}\ \gamma_{25})$$

u.s.w.

Exercise for Student :- Add to above diagram the additional exit channels for this example of inelastic Scattering.

THIS LOOKS PROMISING — MAYBE WE CAN SUM OVER CHANNELS INSTEAD OF LEVELS.

Now what about the vector $\vec{\beta}_\lambda$???

Just below eqn. IX 1.4 (p. 294) we have

$$\beta_\nu = L^0 \gamma_\nu = \begin{pmatrix} L^0_1 & 0 \\ 0 & L^0_2 \end{pmatrix} \begin{pmatrix} \gamma_{\nu 1} \\ \gamma_{\nu 2} \end{pmatrix} = \begin{pmatrix} \gamma_{\nu 1} L^0_1 \\ \gamma_{\nu 2} L^0_2 \end{pmatrix}$$

where L^0 and the γ's are defined for EXIT CHANNELS

Remember, the elements of L^0 are

$L^0_1 \equiv S_1 - B + i P_1$
$L^0_2 \equiv S_2 - B_2 + i P_2$

THEY ARE THE SAME FOR ALL LEVELS OF SAME $J\pi$

In the simplification of the previous page.

$$\vec{\gamma}_1 = (\gamma_{11} \;\; \gamma_{12}) \Longrightarrow \vec{\beta}_1 = (\gamma_{11} L^0_1 \;\; \gamma_{12} L^0_2)$$

$$\vec{\gamma}_2 = (\gamma_{21} \;\; \gamma_{22}) \Longrightarrow \vec{\beta}_2 = (\gamma_{21} L^0_1 \;\; \gamma_{22} L^0_2)$$

Now What about the $\xi_{\lambda\mu}$???

Evidently from LT IX eqn. 1.8 (p. 294)

$$\left.\begin{aligned} \xi_{11} &= (\gamma_{11} \;\; \gamma_{12}) \cdot (\gamma_{11} L^0_1 \;\; \gamma_{12} L^0_2) \\ \xi_{12} &= (\gamma_{11} \;\; \gamma_{12}) \cdot (\gamma_{21} L^0_1 \;\; \gamma_{22} L^0_2) \\ \xi_{21} &= (\gamma_{21} \;\; \gamma_{22}) \cdot (\gamma_{11} L^0_1 \;\; \gamma_{12} L^0_2) \\ \xi_{22} &= (\gamma_{21} \;\; \gamma_{22}) \cdot (\gamma_{21} L^0_1 \;\; \gamma_{22} L^0_2) \end{aligned}\right\}$$

This is a $\boxed{2 \times 2}$ Matrix

$$\begin{pmatrix} \vec{\gamma}_1 \cdot \vec{\beta}_1 & \vec{\gamma}_1 \cdot \vec{\beta}_2 \\ \vec{\gamma}_2 \cdot \vec{\beta}_1 & \vec{\gamma}_2 \cdot \vec{\beta}_2 \end{pmatrix}$$

EXERCISE FOR STUDENT

Add another level $\lambda = 3$ such that $\vec{\gamma}_3 = (\gamma_{31} \;\; \gamma_{32})$

Show that $\xi_{\lambda\mu}$ is a 3×3 Matrix

etc, usw.

Let us actually write out the 2x2 matrix of the previous page.

$$\xi = \begin{pmatrix} \gamma_{11}^2 L^0_1 + \gamma_{12}^2 L^0_2 & \gamma_{11}\gamma_{21} L^0_1 + \gamma_{12}\gamma_{22} L^0_2 \\ \gamma_{21}\gamma_{11} L^0_1 + \gamma_{22}\gamma_{12} L^0_2 & \gamma_{21}^2 L^0_1 + \gamma_{22}^2 L^0_2 \end{pmatrix}$$

SYMMETRIC MATRIX

IMPORTANT : THESE ELEMENTS ARE SUMS OVER CHANNELS.

Thus, as pointed out on p.3, if we had MANY channels per level, even though we have only Two levels, we still get only a 2x2 matrix, but the "external" sums can be over as many channels (5, 10, 100) as we want.

*** From a computational point of view
We need only invert a matrix whose rank = No. of levels.
And sum (a relatively inexpensive CPU operation) over the many channels.

To Calculate $A = (e - E - \xi)^{-1}$

This is now straight forward

$$A = \left[\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} - \begin{pmatrix} E & 0 \\ 0 & E \end{pmatrix} - \begin{pmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \end{pmatrix}\right]^{-1}$$

$$A = \begin{pmatrix} E_1 - E - \xi_{11} & -\xi_{12} \\ -\xi_{21} & E_2 - E - \xi_{22} \end{pmatrix}^{-1}$$

equal

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Now we have to Invert This Matrix

EXERCISE FOR STUDENT: —

Complete soln. of this example where the 2nd exit channel is INELASTIC SCATTERING.

To make Matters Simple Here, and to compare with our previous calculation on the R-Matrix lecture 2 p. 9.

Let us re-write these eqns. for 1-level, 2-channels.

A is a 1×1 matrix.

$$A = \left(E_1 - E - \xi_{11}\right)^{-1} \quad \text{where } L^0 = \begin{pmatrix} S_1 - B + iP_1 & 0 \\ 0 & S_2 - B + iP_2 \end{pmatrix}$$

$$A = \left(E_1 - E - \gamma_{11}^2 L_1^0 - \gamma_{12}^2 L_2^0\right)^{-1} \text{ (from previous page)}$$

Hence $$A = \left(E_1 - E - \gamma_{11}^2(S_1 - B_1 + iP_1) - \gamma_{12}^2(S_2 - B_2 + iP_2)\right)^{-1}$$

$$A = \Big(\underbrace{E_1 - E - \gamma_{11}^2(S_1 - B_1) - \gamma_{12}^2(S_2 - B_2)}_{\substack{\text{Real part} \\ \Rightarrow \text{ Total Shift.}}} - i\underbrace{\left(P_1\gamma_{11}^2 + P_2\gamma_{12}^2\right)}_{\substack{\text{Imag part} \\ \Rightarrow \text{ total width}}}\Big)^{-1}$$

$$\boxed{\text{ENERGY DENOMINATOR}}$$

THUS TAKING THE INVERSE

$$A = \frac{\text{etc.}}{E_1 - E - \underbrace{\left[\gamma_{11}^2(S_1 - B) + \gamma_{12}^2(S_2 - B)\right]}_{\text{Total Shift.}} - \underset{\text{Total Width}}{\left[P_1\gamma_{11}^2 + P_2\gamma_{12}^2\right]}} \quad (\text{sum})$$

To understand eqn. IX 1.4 (Fundamental Level-Matrix Defn)

$$\boxed{(1-R'L')^{-1} = 1 + \sum_{\mu\nu} (\gamma_\mu \times \beta_\nu) A_{\mu\nu}}$$

This definition is quoted EVERYWHERE

This is the SAME matrix as in our development of the R-Matrix. IT is a Matrix IN CHANNELS

MATRIX IN CHANNELS

ELEMENT OF A-MATRIX (whatever they are ??)

Let us take a simple example to see how this equation works.

Suppose we have a problem with 2 Levels, 4 channels each level.

(Remember, there are 2 levels of the same J^π, ∴ they both have the same set of channels)

Thus $(1-R'L')^{-1} \Longrightarrow$ 4×4 matrix

$(\gamma_\mu \times \beta_\nu) \Longrightarrow$ 4×4 matrix

$A \Longrightarrow$ 2×2 matrix

In the summation above, we have 4 terms.

$$\sum_{\mu\nu} \Rightarrow (1,1) + (1,2) + (2,1) + (2,2)$$

Remember, the $\vec{\gamma}_\mu$ are γ-vectors (or row or col matrices) for each level μ in the group of J^π.

Columns $\vec{\beta}_\mu$ are also vectors associated with each level.

$\boxed{\text{We know NEITHER } \beta_\mu \text{ or } A_{\lambda\mu}}$

We have

$\gamma_1 = (\gamma_{11}\ \gamma_{12}\ \gamma_{13}\ \gamma_{14})$ $\qquad \beta_1 = (\beta_{11}\ \beta_{12}\ \beta_{13}\ \beta_{14})$ $\qquad$ Level 1

$\gamma_2 = (\gamma_{21}\ \gamma_{22}\ \gamma_{23}\ \gamma_{2})$ $\qquad \beta_2 = (\beta_{21}\ \beta_{22}\ \beta_{23}\ \beta_{24})$ $\qquad$ Level 2

Our Eqn. now becomes

$$(1-R'L')^{-1} = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} + (\gamma_1 \times \beta_1) A_{11} + (\gamma_1 \times \beta_2) A_{12} + (\gamma_2 \times \beta_1) A_{21} + (\gamma_2 \times \beta_2) A_{22}$$

4x4 matrices.

scalar

as an example, let us work out the second term.

$$\begin{pmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{14} \end{pmatrix} \begin{pmatrix} \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \end{pmatrix} = \begin{pmatrix} \gamma_{11}\beta_{21} & \gamma_{11}\beta_{22} & \gamma_{11}\beta_{23} & \gamma_{11}\beta_{24} \\ \gamma_{12}\beta_{21} & \gamma_{12}\beta_{22} & \cdots & \\ & & & \\ \cdots & & & \gamma_{14}\beta_{24} \end{pmatrix} A_{12}$$

MATRIX IN CHANNELS

Hence for Example.

$$\left[(1-R'L')^{-1} \right]_{ij} = 1 + \gamma_{1i}\beta_{1j} A_{11} + \gamma_{1i}\beta_{2j} A_{12} + \gamma_{2i}\beta_{1j} A_{22} + \gamma_{2i}\beta_{2j} A_{22}$$

MATRIX IN CHANNELS

$\gamma_2 \times \beta_1$

$$= \begin{pmatrix} \gamma_{11}\beta_{11} & \gamma_{11}\beta_{12} & \gamma_{13}\beta_{13} & \gamma_{14}\beta_{14} \\ \gamma_{12}\beta_{11} & \gamma_{12}\beta_{12} & \gamma_{12}\beta_{13} & \gamma_{12}\beta_{14} \\ \gamma_{13}\beta_{11} & \gamma_{13}\beta_{12} & \gamma_{13}\beta_{13} & \gamma_{13}\beta_{14} \\ \gamma_{14}\beta_{11} & \gamma_{14}\beta_{12} & \gamma_{14}\beta_{13} & \gamma_{14}\beta_{14} \end{pmatrix} A_{11} + \begin{pmatrix} \gamma_{11}\beta_{21} & \gamma_{11}\beta_{22} & \gamma_{11}\beta_{23} & \gamma_{11}\beta_{24} \\ \gamma_{12}\beta_{21} & \gamma_{12}\beta_{22} & \gamma_{12}\beta_{23} & \gamma_{12}\beta_{24} \\ \gamma_{13}\beta_{21} & \gamma_{13}\beta_{22} & \gamma_{13}\beta_{23} & \gamma_{13}\beta_{24} \\ \gamma_{14}\beta_{21} & \gamma_{14}\beta_{22} & \gamma_{14}\beta_{23} & \gamma_{14}\beta_{24} \end{pmatrix} A_{12} + \begin{pmatrix} \cancel{\gamma_{21}\beta_{12}} & \cancel{\gamma_{21}\beta_{21}} & \cancel{\gamma_{21}\beta_{23}} & \cancel{\gamma_{21}\beta_{24}} \\ \cancel{\gamma_{22}\beta_{11}} & \cancel{\gamma_{22}\beta_{22}} & \cancel{\gamma_{22}\beta_{23}} & \cancel{\gamma_{22}\beta_{24}} \\ \cancel{\gamma_{23}\beta_{12}} & \cancel{\gamma_{23}\beta_{22}} & \cancel{\gamma_{23}\gamma_{33}} & \cancel{\gamma_{23}\beta} \\ & & & \end{pmatrix}$$

$$+ \begin{pmatrix} \gamma_{21}\beta_{11} & \gamma_{21}\beta_{12} & \gamma_{21}\beta_{13} & \gamma_{21}\beta_{14} \\ \gamma_{22}\beta_{11} & \gamma_{22}\beta_{12} & \gamma_{22}\beta_{13} & \gamma_{22}\beta_{14} \\ \gamma_{23}\beta_{11} & \gamma_{23}\beta_{12} & \gamma_{23}\beta_{13} & \gamma_{23}\beta_{14} \\ \gamma_{24}\beta_{11} & \gamma_{24}\beta_{12} & \gamma_{24}\beta_{13} & \gamma_{24}\beta_{14} \end{pmatrix} A_{21} + \begin{pmatrix} \gamma_{21}\beta_{21} & \gamma_{21}\beta_{22} & \gamma_{21}\beta_{23} & \gamma_{21}\beta_{24} \\ \gamma_{22}\beta_{21} & \gamma_{22}\beta_{22} & \gamma_{22}\beta_{23} & \gamma_{22}\beta_{24} \\ \gamma_{23}\beta_{21} & \gamma_{23}\beta_{22} & \gamma_{23}\beta_{23} & \gamma_{23}\beta_{24} \\ \gamma_{24}\beta_{21} & \gamma_{24}\beta_{22} & \gamma_{24}\beta_{23} & \gamma_{24}\beta_{24} \end{pmatrix} A_{22}$$

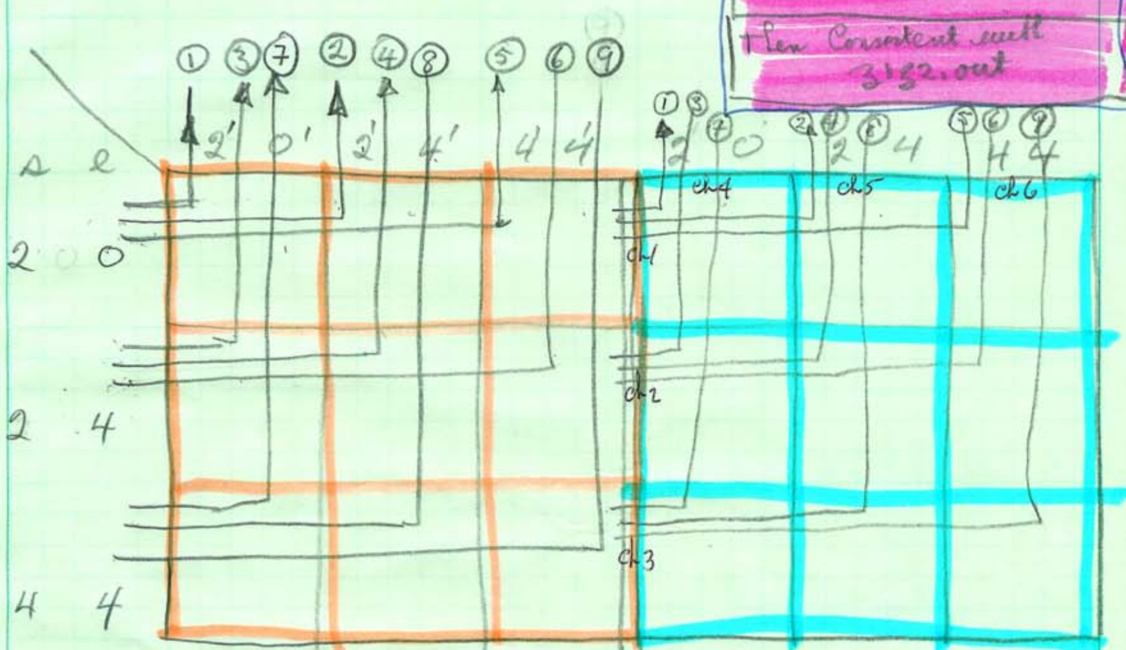
AZURE17.FOR MAY 03/03

SEE SECTION 18B
THEY ARE IDENTIFIED
AS SUCH (ir, k, m)

Reaction Pathways. wb.par.

Identifying the reaction pathways with the elements of the R-Matrix

ID FOR PATHWAYS
ID = ID (ir, k, m)
then Consistent with 3'32.out



wb.par

i	m	R	J	s	l	sp	lp	wch	wchp	ml	mh	z1	z2	ef	gout	k	nlb(ir,k)
1	1	1	1	2	0	2	0	1	1	1.000	21.000	1.000	11.000	0.000	0.601	1	1
2	2	1	1	2	0	2	4	1	2	1.000	21.000	1.000	11.000	0.000	0.601	1	2
3	3	1	1	2	4	2	0	2	1	1.000	21.000	1.000	11.000	0.000	0.601	1	3
4	4	1	1	2	4	2	4	2	2	1.000	21.000	1.000	11.000	0.000	0.601	1	4
W-order=====																	
5	1	1	1	2	0	4	4	1	3	1.000	21.000	1.000	11.000	0.000	0.601	2	1
6	2	1	1	2	4	4	4	2	3	1.000	21.000	1.000	11.000	0.000	0.601	2	2
W-order=====																	
7	1	1	1	4	4	2	0	3	1	1.000	21.000	1.000	11.000	0.000	0.601	3	1
8	2	1	1	4	4	2	4	3	2	1.000	21.000	1.000	11.000	0.000	0.601	3	2
W-order=====																	
9	1	1	1	4	4	4	4	3	3	1.000	21.000	1.000	11.000	0.000	0.601	4	1
W-order=====																	
1	1	2	1	2	0	2	0	1	4	1.000	21.000	1.000	11.000	0.260	0.609	1	1
2	2	2	1	2	0	2	4	1	5	1.000	21.000	1.000	11.000	0.260	0.609	1	2
3	3	2	1	2	4	2	0	2	4	1.000	21.000	1.000	11.000	0.260	0.609	1	3
4	4	2	1	2	4	2	4	2	5	1.000	21.000	1.000	11.000	0.260	0.609	1	4
W-order=====																	
5	1	2	1	2	0	4	4	1	6	1.000	21.000	1.000	11.000	0.260	0.609	2	1
6	2	2	1	2	4	4	4	2	6	1.000	21.000	1.000	11.000	0.260	0.609	2	2
W-order=====																	
7	1	2	1	4	4	2	0	3	4	1.000	21.000	1.000	11.000	0.260	0.609	3	1
8	2	2	1	4	4	2	4	3	5	1.000	21.000	1.000	11.000	0.260	0.609	3	2
W-order=====																	
9	1	2	1	4	4	4	4	3	6	1.000	21.000	1.000	11.000	0.260	0.609	4	1
W-order=====																	