LECTURE 4

The Differential Cross Section

In most of our previous work we RESTRICTED ourselves to a group of levels with SAME J^π.

However, we were aware that ALL J^π's contribute to the yield of a particular reaction.

We have seen this also from the fact the a PLANE WAVE $\Longrightarrow$ composition of INFINITE no. of ℓ-values.

e.g.

$$e^{ikz} = (4\pi)^{1/2} \sum j_\ell(kr)\left(i^\ell P_\ell(\cos\theta)\right)$$

This is an expansion of Plane wave in Legendre Polynomials.

QUESTION? How would you derive this eqn. ???

If this were a course in scattering theory, this would be an exercise for the student.

∴ IN OUR CALCULATIONS OF CROSS SECTIONS $\Longrightarrow$ INCLUDE ALL J^π's.

Start with eqn. 53 EV (p. 236) N.B

$$d\sigma_{\alpha' s' \nu', \alpha s \nu} = \frac{1}{k_\alpha^2} \left| A_{\alpha' s' \nu', \alpha s \nu}(\theta, \phi) \right|^2 d\Omega.$$

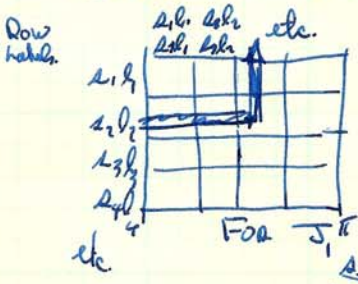
N.B. Presence of $\nu\nu' \Longrightarrow$ presence of ϕ

What is meant "PHYSICALLY" by this amplitude??

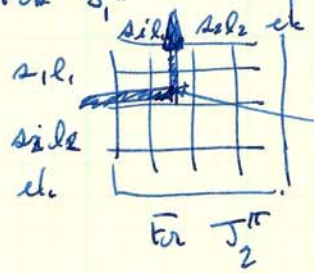
- Actually it can be viewed as the AMPLITUDE for a given "Reaction Pathway"

Before exploring this further, lets get rid of the $\nu\nu'$ indices. Thus we average over ν and sum over ν'

Remember our R-matrix [ignore the index $\nu\nu'$]



Here "Amplitudes" $A_{\alpha's',\alpha s}$ corresponds to a SUM of all the Reaction Pathways corresponding to $\alpha's', \alpha s$.



Remember

$$R_\lambda = \sum_\lambda' \frac{\gamma_\lambda \gamma_\lambda'}{E_\lambda - E}$$

ETC.

THUS:- From ALL the R^J-matrices we have to select the reaction paths for $\alpha's' \to \alpha s$

$$\left| F_{\alpha s, \alpha s'} \right|^2 = \left| A^1_{\alpha's', \alpha s} + A^2_{\alpha's', \alpha s} + \cdots \right|^2 .$$

From a Computing point of view, we have to

(i) List all the reaction pathways according to $\alpha's' \leftrightarrow \alpha s$ for all J^π (for all R^J-matrices)

(ii) Re-Group the reaction pathways in groups of common $\alpha s \to \alpha's'$

N.B. THIS IS DONE IN SECTION 20 OF AZURE

Note that in the R-matrix, each element is a $\sum_\lambda$ over the states of the same J^π, so that the contribution for all the levels in the group is contained in the ~~sum~~ internal sum.

We can SUPPRES $\nu\nu'$ AVERAGE OVER ν
SUM OVER ν'

$$d\sigma_{\alpha' s', \alpha s} = \frac{1}{2s+1} \sum_{\nu} \sum_{\nu'} d\sigma_{\alpha' s' \nu', \alpha s \nu}$$

THIS IS A COMPLICATED SUM.

(i) Sum over ν, ν' for given s, s'

(ii) For given s, s', sum over all l and l'-values associated with each s, s'

(iii) For given s, s' sum over all R^I-matrices (for all J^π's)

then. SUM $|\quad|^2$ ADDING COHERENTLY

So all interferences have to be included

(iv) Finally, Sum over all s', s INCOHERENTLY TO GET

$$d\sigma_{\alpha, \alpha'}$$

Consider the sum in (iii) above. $|A_{\alpha' l' s', \alpha l s}|^2$

First back to fundamentals.

How do you sum $[a_1 + a_2 + a_3 + \cdots]^2$???

$$\text{Sum} = \sum_i \sum_j a_i a_j = a_1 a_1 + a_1 a_2 + \cdots + a_1 a_n + a_2 a_1 + a_2 a_2 + \cdots + a_n a_n$$

Index set 1 — Index set 2.

GO TO LT p 292 JUST BELOW eqn. (2.3)

GO TO $\{ls\,lS\,JM\}$ SCHEME !!!

GO BACK TO P2 TO THE R-MATRIX DIAGRAMS

a_i a_f

Index Set 1 — entrance channel

$\{J, M, l, l', m'\}$ — exit channel

Index Set 2

$\{J_2 M_2 l_2 l_2' m_2'\}$

We have gone from the $\{\alpha s l J m\}$ system to the $\{\alpha s l J M\}$ system.

Reactn Pathway 1

Reactn Pathway 2

THUS $|SUM|^2 = \sum_{\substack{J, J_2 M, M_2 \\ l, l_2 l_1' l_2' \\ J J' m, m_1'}} \{\text{Pathway}_1\} \{\text{Pathway}_2\}$

(FOR GIVEN $\alpha \to \alpha'$)

THESE PRODUCTS INVOLVE ENTITIES WHICH ARE CHARACTERIZED BY ANG. MOM. QUANT. NOS.

SO EACH PRODUCT HAS TO CONSERVE ANG. MOM.

WHO IS WILLING TO DO THIS SUM.

ANSWER

LANE & THOMAS ; BLATT & BIEDENHARN.

RESULT

$$(2s+1)\frac{k_\alpha^2}{\pi} d\sigma_{\alpha' s', \alpha s} d\Omega_{\alpha 1} = (2s+1)\left|C_{\alpha'}(\vartheta_{\alpha 1})\right|^2 \delta_{\alpha' s', \alpha s}$$

$$+ B_L(\alpha' s', \alpha s) P_L(\cos\vartheta_{\alpha 1})$$

$$+ \frac{1}{(4\pi)^{1/2}} \sum_{Jl}^{I} (2J+1) \operatorname{Re}\left[i\, T_{\alpha' s' l', \alpha s l} \right.$$

$$\left. * C_{\alpha'}(\vartheta_{\alpha 1}) P_L(\cos\vartheta_{\alpha 1}) \right] \quad \text{EQN 2.6}$$

$$= \text{Coulomb Cross Section}$$

$$+ \text{Resonance Term}$$

$$+ \text{Interference Term.}$$

N.B. If this were an advanced graduate course, then

EXERCISE FOR STUDENT

(i) ~~Work~~ DERIVE IN DETAIL, including the

(ii) Show that Interference Term is Incorrect BL factors. and derive correct version

(this has been done by Claudio)

The code AZURE, translates (2.6) exactly (with IT correction) into fortran, the ~~no~~ incoherent sum over ss' done, and the final cross section calculated.

To do this we need

$$B_L(\alpha' s', \alpha s) = \frac{1}{4}(-)^{s-s'} \sum_{J_1 J_2 \ell_1 \ell_2 \ell_1' \ell_2'} \overline{Z}(\ell_1 J_1 \ell_2 J_2, sL) \times \overline{Z}(\ell_1' J_1 \ell_2 J_2, s'L)$$

$$* \left(T^{J_1}_{\alpha' s' \ell_1', \alpha s \ell_1}\right)\left(T^{J_2}_{\alpha' s' \ell_2', \alpha s \ell_2}\right)^* \qquad (2.7)$$

where $T^J_{\alpha' s' \ell', \alpha s \ell} = e^{2i\omega_{\alpha' s' \ell'}} \delta_{\alpha' s' \ell', \alpha s \ell} - U^J_{\alpha' s' \ell', \alpha s \ell}$ (2.3)

AND $\overline{Z}(\ell_1 J_1 \ell_2 J_2, sL) = (2\ell_1+1)^{1/2}(2\ell_2+1)^{1/2}(2J_1+1)^{1/2}(2J_2+1)^{1/2}$

$$* (\ell_1 \ell_2 00 | L0)\, W(\ell_1 J_1 \ell_2 J_2, sL)$$

Comments about the $B_\lambda(\alpha'\lambda', \alpha\lambda)$ Quantities

Let us write out the Z quantities fully.

$\vec{l}_1 + \vec{l}_2$ combine to form a single L

$$Z_1 = (l_1 J_1 l_2 J_2, sL) = (\ )^{1/2} - \cdots (\ )^{1/2} (l_1 l_2 00 | L0)(l_1 J_1 l_2 J_2, sL)$$ INCOMING CHANNELS

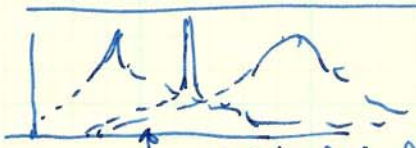
$$Z_2 = (l_1' J_1 l_2' J_2, sL) = (\ )^{1/2} - \cdots (\ )^{1/2} (l_1' l_2' 00 | L0)(l_1' J_1 l_2' J_2, sL)$$ OUTGOING CHANNELS.

again, for outgoing channel $\vec{l}_1' + \vec{l}_2'$ combine to form a single L. for $P_L(\cos\theta')$

NOTE:- Our sum is over ALL combinations of J^π, not just the members in a given J^π group.

COMMENT:— ALL LEVELS OVERLAP

∴ ALL LEVELS CONTRIBUTE TO ANY GIVEN REACTION



E_B = Contribution for ALL J's.

CANNOT DISTINGUISH WHICH CHANNEL CONTRIBUTES

∴ ALL CHANNELS "INTERFERE"

The Product of T- Matrices

$$\left(T^{J_1}_{\alpha_1'\lambda_1' l_1', \alpha_1 l_1}\right)\left(T^{J_2}_{\alpha_2'\lambda_2' l_2', \alpha_2 l_2}\right)$$

Reaction Pathway for J_1

Reaction Pathway for J_2.

NOTE: EXIT CHANNELS

- $\alpha's'l'$ refer to the exit channel associated with a given reaction pathway.
- If the formulation is correct ???
  Then the α' can be any exit channel including γ-rays !!
  all that is needed is that one includes the appropriate C-G and W coefficients.
  (LT note one exception :- low energy n-capture where γ-width >> neutron width).

~~Comp~~ γ-Ray Channels, in General

LT $\Rightarrow$ Ignore contribution to shift & total gamma

BOMKER $\Rightarrow$ " " " " " "

use $(E_\gamma)^{2L+1}$ in numerator