Elastic Scattering of Spinless Particles by a Central Pot.

Generalization: add ℓ & Coulomb Potential

LANE & THOMAS – Sec. 4.

We have phase shifts, δ_ℓ, for each ℓ
and collision functions $U_\ell \equiv e^{2i\delta_\ell}$

also, we still have

$$\frac{d\sigma}{d\Omega} = \frac{1}{4k^2}\left|\Sigma_\ell (2\ell+1)(1-U_\ell)P_\ell(\cos\theta)\right|^2$$

$$\sigma \equiv \int \frac{d\sigma}{d\Omega}d\Omega = \frac{\pi}{k^2}\Sigma_\ell (2\ell+1)|1-U_\ell|^2$$

"Internally" we still have, at radius a

$$R_\ell \equiv \Sigma_\lambda \frac{\gamma_{\lambda\ell}^2}{E_{\lambda\ell}-E}$$

with $\gamma_{\lambda\ell}^2 = \frac{\hbar^2}{2ma}(u_{\lambda\ell}(a))^2$

where the internal resonances satisfy

$$a\frac{du_{\lambda\ell}}{dr}\Big|_{r=a} \Big/ u_{\lambda\ell}(a) = b \leftarrow \text{real.}$$

By the application of Greens theorem to the internal w.f. $u_\ell(r)$ and the resonances $u_{\lambda\ell}(r)$ we get again:

$$[r\,du_\ell(r)/u_\ell(r)\big|_{r=a} = \frac{1+bR_\ell}{R_\ell}$$

By matching logarithmic derivatives of external & internal w.f. we get again.

$$u_\ell = O_\ell^{-1}(1-R_\ell L_\ell)^{-1}(1-R_\ell L_\ell^*)I_\ell$$

which are still functions, not matrices.

But the addition of ℓ & Coulomb has altered all the external quantities: O_ℓ, I_ℓ, & L_ℓ

The external radial equation now is

$$u''(kr) - \left[\frac{\ell(\ell+1)}{r^2} + \frac{2m}{\hbar^2}\left(\frac{Z_1 Z_2 e^2}{r} - E\right)\right]u(kr) = 0$$

Now the asymptotic form of the regular (F_ℓ) and irregular (G_ℓ) solutions are

$$F_\ell \sim \sin\left(kr - \eta \log 2kr - \tfrac{1}{2}\ell\pi + \sigma_\ell\right)$$

$$G_\ell \sim \cos(\;\;"\;\;\;\;"\;\;\;\;"\;\;\;\;"\;\;)$$

$\sigma_\ell \equiv \arg\Gamma(1+\ell+i\eta)$ Coulomb phase shift.

$\eta \equiv \dfrac{Z_1 Z_2 e^2}{\hbar v}$ Coulomb parameter and we have

$$I_\ell = (G_\ell - iF_\ell)\,e^{i\omega_\ell}$$

$$O_\ell = (G + iF_\ell)\,e^{-i\omega_\ell}$$

with $\omega_\ell \equiv \sum_{n=1}^{\ell} \tan^{-1}(\eta/n)$

and therefore

$' = r\frac{\partial}{\partial r}$

$$L_\ell \equiv O_\ell'/O_\ell - b_\ell \equiv S_\ell + iP_\ell$$

we find.

$$P_\ell = \frac{kr}{F_\ell^2 + G_\ell^2}$$

$$S_\ell = -b_\ell + \frac{F_\ell F_\ell' + G_\ell G_\ell'}{F_\ell^2 + G_\ell^2}$$

$$O_\ell^{-1} I_\ell = e^{2i\Omega_\ell}$$

$$\Omega_\ell \equiv \underbrace{\omega_\ell}_{\text{Coulomb}} - \underbrace{\tan^{-1}(F_\ell/G_\ell)}_{\text{hard-sphere.}}$$

R-MATRIX & OTHER FRAMEWORKS

we look at:

$$U_\ell = O_\ell^{-1}(1 - R_\ell L_\ell)^{-1}(1 - R_\ell L_\ell^*) I_\ell$$

with

$$L_\ell \equiv S_\ell + i P_\ell$$

$$R_\ell \equiv \sum_\lambda \frac{\gamma_{\lambda\ell}^2}{E_{\lambda\ell} - E}$$

All these quantities will become channel matrices but here they are still channel functions.

KAPUR & PEIERLS (1937)

choose b_ℓ so that $L_\ell = 0$

remember: $S_\ell = -b_\ell + \dfrac{F_\ell F_\ell' + G_\ell G_\ell'}{F_\ell^2 + G_\ell^2}$

Advantages: i) resonances are "outgoing" waves
ii) no matrix inversion.

Disadvantages: i) b_ℓ is intrinsically energy-dependent
ii) b_ℓ is complex (completeness?)
iii) Interpretation of resonance parameters.

R-Matrix Framework

We need auxiliary quantities for each ℓ

a_ℓ = channel radius

b_ℓ = channel boundary condition

The choice of channel radii is obvious for a square-well ($a_\ell = R_0$) and otherwise related to physics.

CHOICE OF b_ℓ

In early work Wigner liked $b_\ell = 0$ for all ℓ. This is OK for s-wave neutrons but, in general, it forces the wrong harmonics on the system.

Rosenfeld & Humblet
Lane & Thomas.

"NATURAL" BOUNDARY CONDITIONS

$$b_\ell = Re[O_\ell' O_\ell^{-1}] = S_\ell(E)$$

Then "resonances" resemble outgoing waves as closely as permissible.
But b_ℓ is now energy-dependent
Only slow energy dependence.

PROBLEMS OF THE R-MATRIX FRAMEWORK.

1) Energy dependence of b_ℓ.

Solution: the energy dependence of the shift function is slow (exception for bound s-wave neutrons). Therefore choose b so that S_ℓ is zero in the energy interval of the analysis.
(we can use a Taylor-series expansion of S_ℓ which renormalizes widths).

2) Direct Reactions (including neutron capture) & Halo States.

3) Hard Sphere Phase Shifts and Penetration Factors.

Solution: For elastic scattering use optical model phase shifts

For penetration factors multiply by a "reflection factor".

$$P_\ell = \frac{ka_\ell}{\sqrt{F_\ell^2 + G_\ell^2}} \Rightarrow f \, P_\ell$$

$$f = \sqrt{2}\, \pi K_0 a_\ell \coth(\sqrt{2}\, \pi K_0 a_\ell)$$

REFLECTION FACTOR

$$f = \pi K a \coth(\pi K a)$$

wave number inside potential

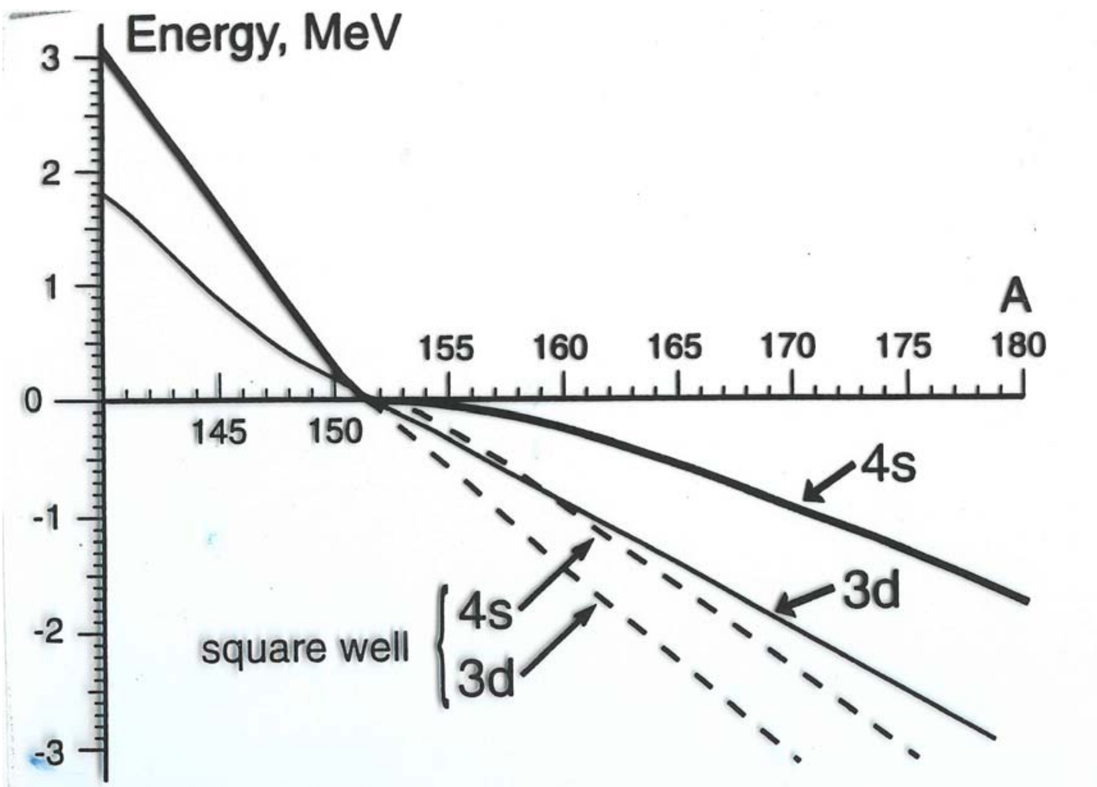
surface thikness.

D.C. Peaslee, Nucl. Phys. $\underline{3}$, 255 (1957)

For the standard optical model potential,

$V_0 \approx 50$ MeV, $a \approx 0.67$ fm.

$f \approx 3.3$



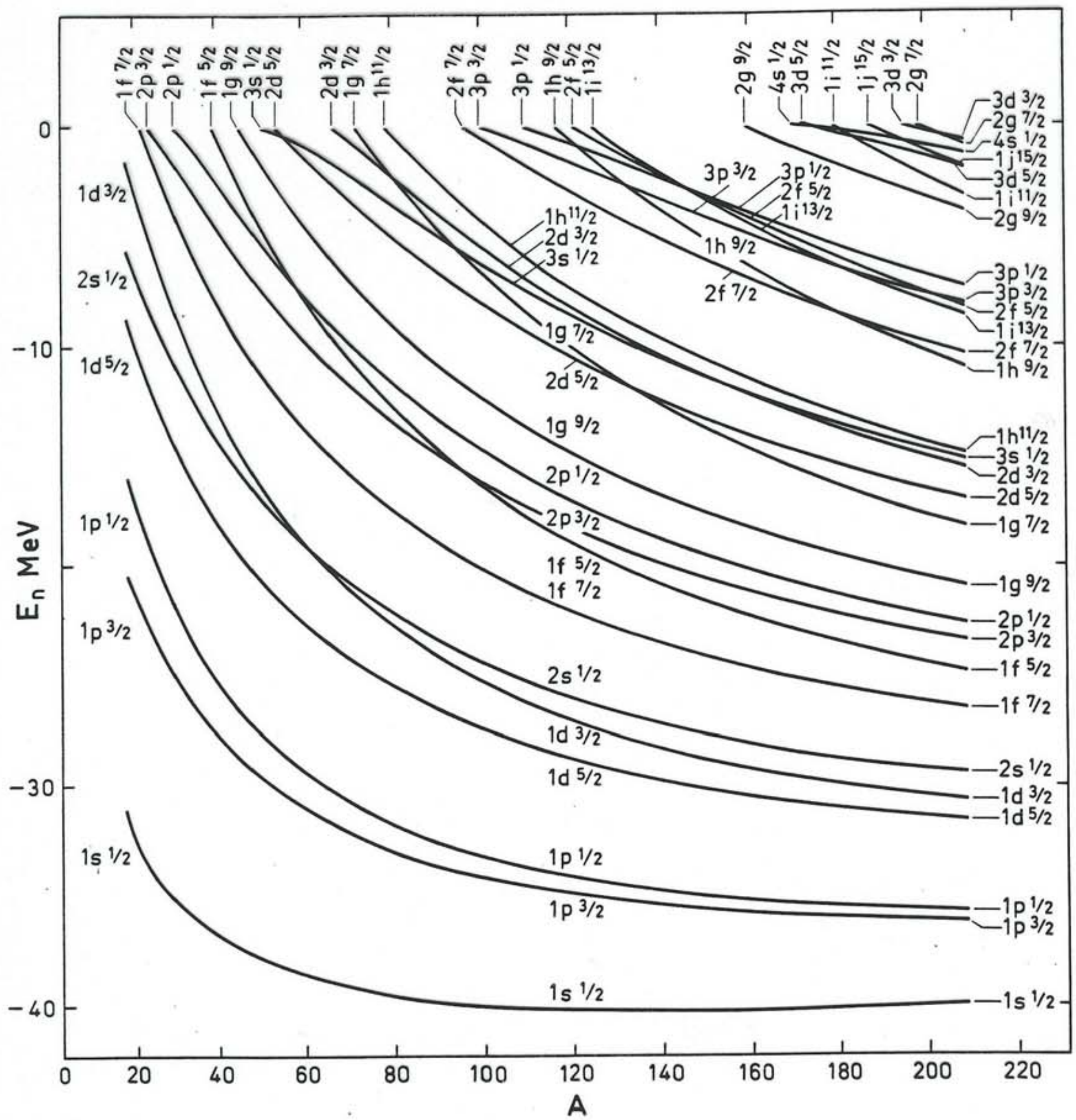


Figure 2-30 Energies of neutron orbits calculated by C. J. Veje (private communication).

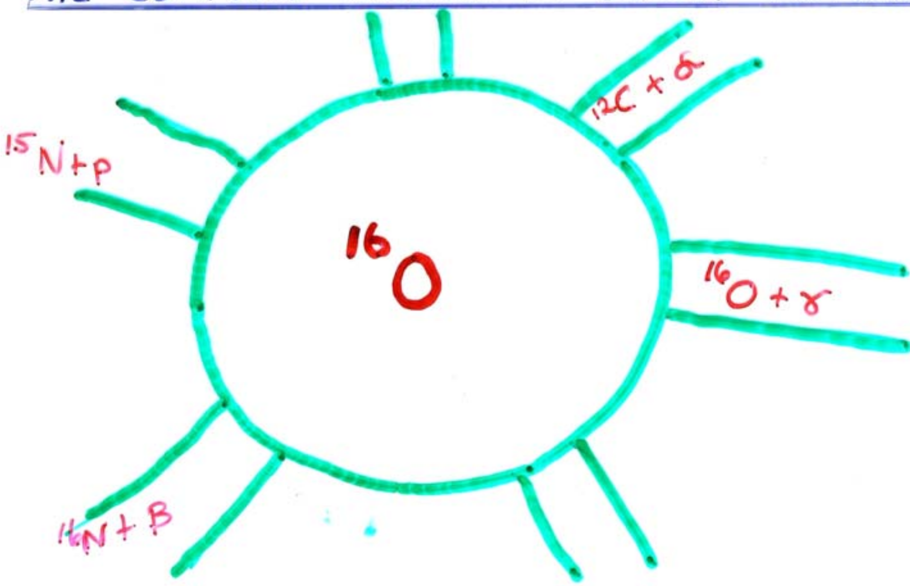
▼ surface thickness parameter a is taken to be A independent.

$$R = r_0 A^{1/3} \qquad r_0 = 1.27 \text{ fm}$$
$$a = 0.67 \text{ fm} \tag{2-181}$$

The potential strengths include a term depending on the neutron excess, in order to describe approximately the potential acting on a single neutron,

$$V = \left(-51 + 33\,\frac{N-Z}{A}\right) \quad \text{MeV} \tag{2-182}$$

THE COMPOUND NUCLEUS IN 3(N+Z) CONFIGURATION SPACE



IT LOOKS LIKE A PRIMITIVE SCOTTISH MUSICAL INSTRUMENT (BAGPIPES)

FOR AN UNDERSTANDING OF ASTROPHYSICAL REACTIONS WE NEED AN ACCURATE ANALYSIS OF THE SYSTEM ("BAG PLUS PIPES") IN THE SINGULAR CASE WHERE WAVES IN ANY OF THE "PIPES" HAVE GREAT TROUBLE IN REACHING THE "BAG"

ALSO, THE "BAG" HAS MANY "WOLF NOTES" (RESONANCES)

$E_x \sim 5\ \mathrm{MeV}$

$$n + A \longrightarrow (A+1)^* \longrightarrow n + A$$
$$\longrightarrow (A+1) + \gamma\text{'s}$$