LECTURE 4 NOTRE DAME

DEALING WITH THE COLLISION MATRIX

$$\mathcal{U} = (kr)^{1/2} O^{-1} (1-RL)^{-1} (1-RL^*) I (kr)^{-1/2}$$

To show the matrix components we write.

$$\mathcal{U}_{cc'} = (kr)_c^{1/2} O_c^{-1} \sum_{c''} [1-RL]^{-1}_{cc''} (\delta_{c''c'} - R_{c''c'} L^*_{c'}) I_{c'} (kr)^{-1/2}_{c'}$$

First we simplify a little.

$$(k_c r_c)^{1/2} O_c^{-1} I_{c'} (k_{c'} r_{c'})^{-1/2} = e^{i(\Omega_c + \Omega_{c'})} P_c^{1/2} P_{c'}^{-1/2}$$

where: $L_c \equiv S_c + i P_c$

$\Omega_c \equiv \omega_c - \tan^{-1}(F_c / G_c)$ see Vogt '62

$\omega_c \equiv \sum_{n=1}^{l_c} \tan^{-1}(\eta_c / n)$

then

$$\mathcal{U}_{cc'} = e^{i(\Omega_c + \Omega_{c'})} P_c^{1/2} P_{c'}^{-1/2} \sum_{c''} \underbrace{[1-RL]^{-1}_{cc''}}_{\text{we must invert this channel matrix}} (\delta_{c''c'} - R_{c''c'} L^*_{c'})$$

Remember $R_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_\lambda - E}$

Few-Channel, Many-Level Collision Matrix

FEW-LEVEL MANY-CHANNEL $\mathcal{U}$.

Using some matrix "magic" we can turn the cumbersome problem of inverting a channel matrix into, instead, inverting a level matrix.

$$U_{cc'} = e^{i(\Omega_c + \Omega_{c'})} \left\{ \delta_{cc'} + i \sum_{\lambda\lambda'} \Gamma_{\lambda c}^{1/2} \Gamma_{\lambda' c'}^{1/2} A_{\lambda\lambda'} \right\}$$

where $\Gamma_{\lambda c}^{1/2} \equiv (2P_c)^{1/2} \gamma_{\lambda c}$

$$(A^{-1})_{\lambda\lambda'} = (E_\lambda - E)\delta_{\lambda\lambda'} + \Delta_{\lambda\lambda'} - \left(\frac{i}{2}\right)\Gamma_{\lambda\lambda'}$$

If $(A^{-1})_{\lambda\lambda'}$ was a matrix diagonal in λ, λ' (which it isn't) then, in $U_{cc'}$ we would simply have a sum of Breit-Wigner amplitudes. $U_{cc'}$, above, is the few-channel, many-level collision matrix.

Proof of the Level Matrix, U. Thomas 1955

Let's assume that

$$[(1-RL)^{-1}(1-RL^*)]_{cc'} = \delta_{cc'} + \sum_{\lambda\lambda'} 2i\, P_{c'} \gamma_{\lambda c} \gamma_{\lambda' c'} A_{\lambda\lambda'}$$

and find $A_{\lambda\lambda'}$

We multiply both sides of this equation by $(1-RL)$, from the left and thus get

$$(1-RL^*)_{cc'} = \sum_{c''} (\delta_{cc''} - R_{cc''} L^*_{c''}) \delta_{c''c'}$$

$c \to c''$
$\lambda \to \lambda''$

$$+ \sum_{c''} \left(\delta_{cc''} - \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c''}}{E_\lambda - E}\right)\Big(\underbrace{2i\, P_{c''} \gamma_{\lambda'' c''} \gamma_{\lambda' c'} A_{\lambda''\lambda'}}_{\sum_{\lambda''\lambda'}}\Big)$$

$$\Downarrow$$

$$\delta_{cc'} - \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E} L^*_{c'}$$

$$= \delta_{cc'} - \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E} L_{c'} + \sum_{\lambda\lambda'} 2i\, P_{c'} \gamma_{\lambda c} \gamma_{\lambda' c'} A_{\lambda\lambda'}$$

$$- \sum_{\lambda\lambda'\lambda''c''} \frac{\gamma_{\lambda c}\gamma_{\lambda c''}}{E_\lambda - E} 2i\, P_{c'} \gamma_{\lambda'' c''} \gamma_{\lambda' c'} A_{\lambda''\lambda}$$

collecting terms

$$0 = \sum_{\lambda\lambda'} 2i\, P_{c'} \frac{\gamma_{\lambda c}\gamma_{\lambda' c'}}{E_\lambda - E}\Big[\delta_{\lambda\lambda'} - (E_\lambda - E) A_{\lambda\lambda'} + \overbrace{\xi_{\lambda\lambda''} A_{\lambda''\lambda'}}^{\sum_{\lambda''}}\Big]$$

with $\xi_{\lambda\lambda''} = \sum_{c''} \gamma_{\lambda c''} \gamma_{\lambda'' c''} L_{c''}$

$$= -\Delta_{\lambda\lambda'} + \tfrac{i}{2} \Gamma_{\lambda\lambda'}$$

Conclusion of proof:

The identity must hold for arbitrary $\gamma_{\lambda c}, \gamma_{\lambda' c'}$ and therefore the square bracket must be equal to zero, that is:

$$[A^{-1}]_{\lambda\lambda'} = (E_\lambda - E)\delta_{\lambda\lambda'} + \Delta_{\lambda\lambda'} - \frac{i}{2}\Gamma_{\lambda\lambda'}$$

where

$$\Delta_{\lambda\lambda'} \equiv \sum_c S_c \gamma_{\lambda c} \gamma_{\lambda' c}$$

$$\Gamma_{\lambda\lambda'} \equiv \sum_c 2P_c \gamma_{\lambda c} \gamma_{\lambda' c}$$

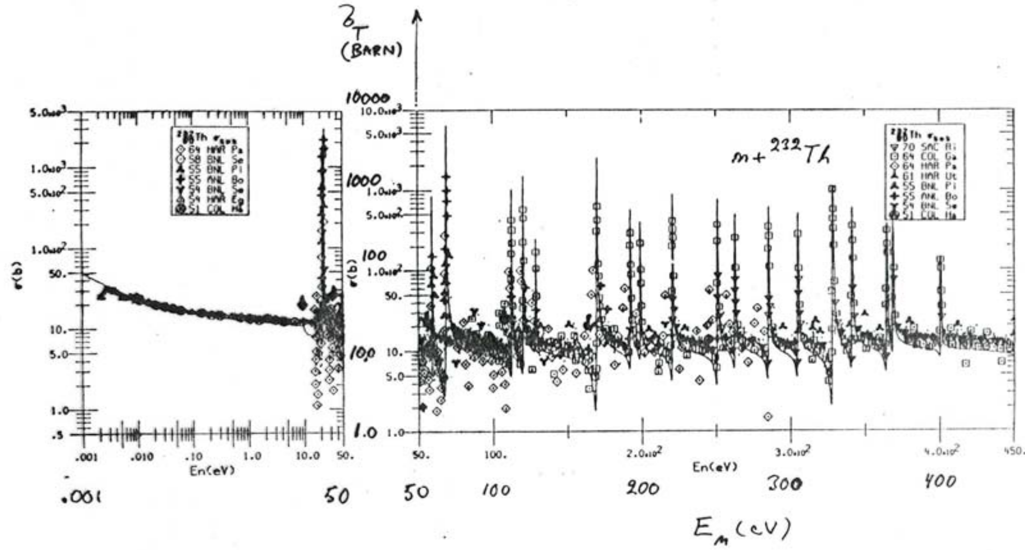
Can one ignore the off-diagonal terms?

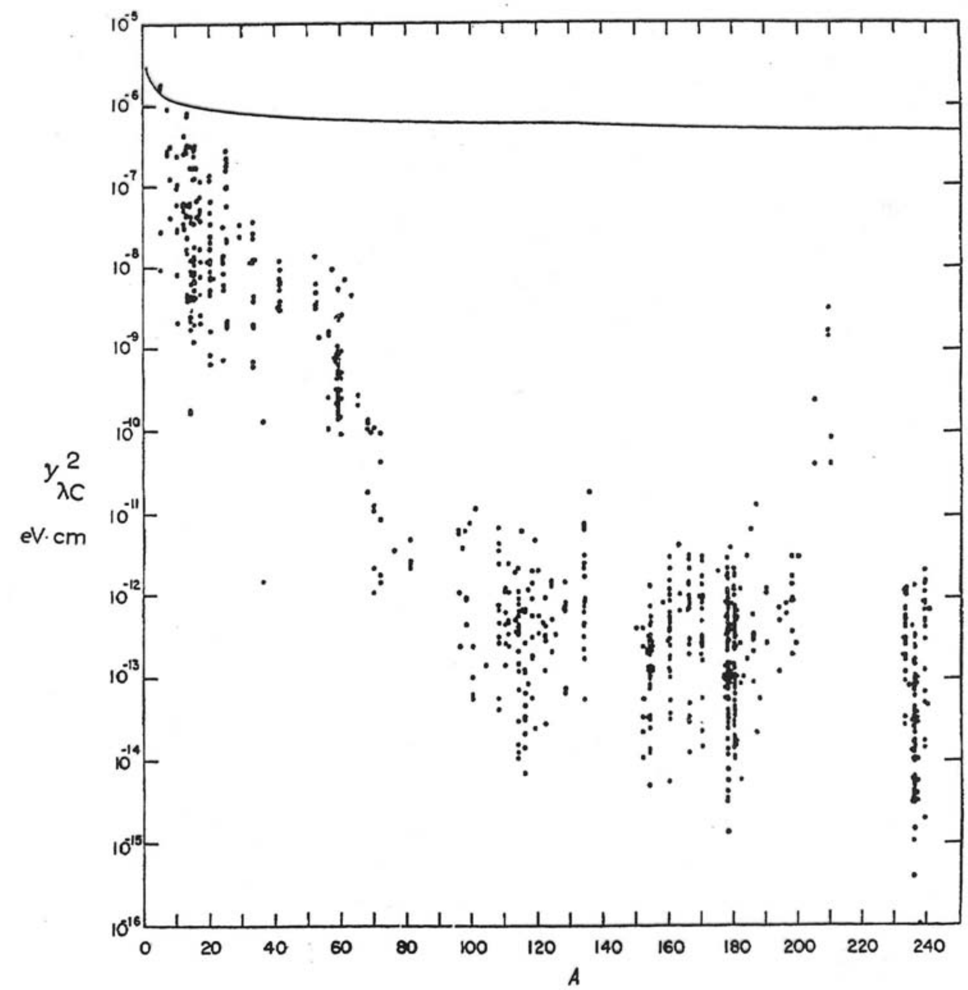
Answer: No ($U^{235}+n$ — Vogt 1956) etc.

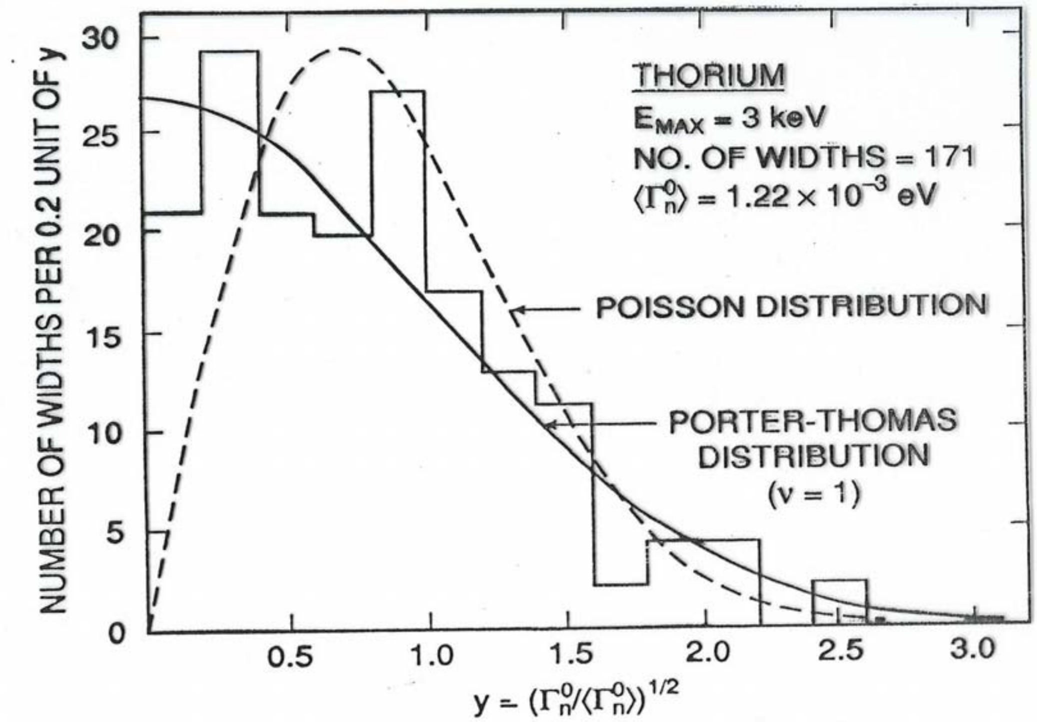
Thus $U_{cc'}$ is not simply a sum over Breit-Wigner amplitudes.

$E_x \sim 5\,MeV$

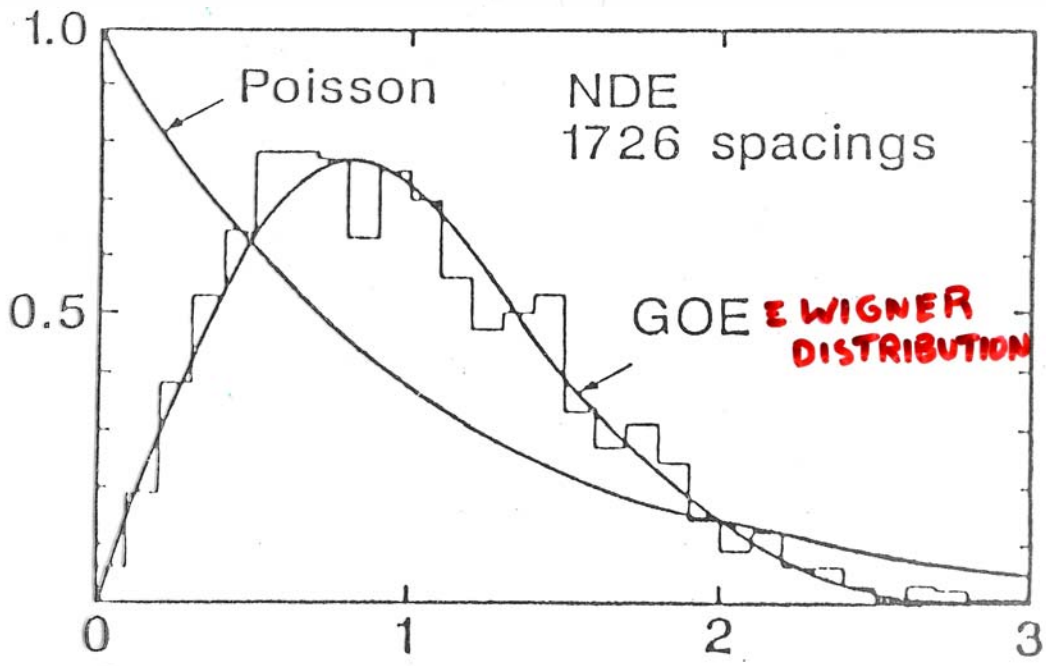
$$n + A \to (A+1)^* \to n + A$$
$$\to (A+1) + \gamma\text{'s}$$







$\sqrt{\Gamma_n} \sim |\text{Decay Amplitude}|$



Histogram for the nearest-neighbor spacing distribution for the nuclear data plotted versus the level spacing x in units of its mean value. The solid line labeled would apply to an integrable system, see Section 4.2. Taken from (123).

THE LEVEL MATRIX $(A^{-1})_{\lambda\lambda'}$

$$(A^{-1}) = \begin{pmatrix} E_1+\Delta_1-E-\frac{i}{2}\Gamma_1 & \Delta_{12}-\frac{i}{2}\Gamma_{12} & \Delta_{13}-\frac{i}{2}\Gamma_{13} \\ \Delta_{12}-\frac{i}{2}\Gamma_{12} & E_2+\Delta_2-E-\frac{i}{2}\Gamma_2 & \Delta_{23}-\frac{i}{2}\Gamma_{23} \\ \Delta_{13}-\frac{i}{2}\Gamma_{13} & \Delta_{23}-\frac{i}{2}\Gamma_{23} & E_3+\Delta_3-E-\frac{i}{2}\Gamma_3 \\ & & & \ddots \end{pmatrix}$$

s-wave neutron Capture

We take $\Delta_{\lambda\lambda'} \doteq 0$

Diagonal elements: $E_\lambda - E - \frac{i}{2}\sum_c \Gamma_{\lambda c}$

$$\Gamma_{\lambda c} = \Gamma_{\lambda n} + \Gamma_{\lambda\gamma_1} + \Gamma_{\lambda\gamma_2} + \Gamma_{\lambda\gamma_3} + \cdots$$

$$\approx \Gamma_{\lambda n} + 0.030\,\text{eV} - \Gamma_{\lambda\gamma}$$

Sometimes $\Gamma_{\lambda\gamma} < \Gamma_{\lambda n}$

Then the peak cross sections are all equal to $\frac{4\pi}{k^2}$

More often $\Gamma_{\lambda n} < \Gamma_{\lambda\gamma}$

then peak cross sections, $\frac{4\pi}{k^2}\frac{\Gamma_{\lambda n}}{\Gamma_{\lambda\gamma}}$, fluctuate

What about off-diagonal elements of (A^{-1})?

for example $-\frac{i}{2}\Gamma_{12}$

for neutron capture

$$\Gamma_{12} = \Gamma_{1n}^{1/2}\Gamma_{2n}^{1/2} + \underbrace{\Gamma_{1\gamma_1}^{1/2}\Gamma_{2\gamma_1}^{1/2} + \Gamma_{1\gamma_2}^{1/2}\Gamma_{2\gamma_2}^{1/2} + \Gamma_{1\gamma_3}^{1/2}\Gamma_{2\gamma_3}^{1/2} + \cdots}_{\approx 0}$$

$$= \Gamma_{1n}^{1/2}\Gamma_{2n}^{1/2}$$

Special Case of
neutron fission cross sections
of U^{235}, U^{233}, Pu^{239}