LECTURE 5 - NOTRE DAME

SINGLE PARTICLE STATES & SUM RULE LIMITS

First:
The machinery (Clebsch-Gordanerie) connecting actual cross-sections to Collision Matrix components.

Note interferences and special properties such as unitarity.

SINGLE PARTICLE STATES

Remember $H\Psi = E\Psi$

$H X_\lambda = E_\lambda X_\lambda$

with $r_c \frac{dX_\lambda}{dr_c} = b_c X_\lambda \Big|_{s_c}$.

with $\Psi = \sum_\lambda C_\lambda X_\lambda$

$C_\lambda \equiv \int_v X_\lambda^* \Psi \, dv$

Here $H = \sum_i T_i + \sum_{i,j} v_{ij}$

ANGULAR MOMENTUM RECOUPLIN

channel spin $\vec{S} = \vec{I} + \vec{i}$

$\vec{J} = \vec{l} + \vec{S}$

When we have a resonance of gi
J^π we know the system's total
angular momentum & parity. We
must prepare the w.f. of the beam
& target to reflect this.

Initial Quantum Numbers i, m_i, I, m_I, l, m_l
(but not J)

we take appropriate combinations of
these initial states to create states
of good J, M_J.

Clebsch-Gordan

$$\psi_{s,m_s} = \sum_{m_I, m_i} (I m_I i m_i | s m_s) \psi_{I m_I} \psi_{i m_i}$$

$$\psi_{J,m_J} = \sum_{m_l, m_s} (l m_l s m_s | J m_J) \psi_{s m_s} \psi_{l m_l}$$

Thus in the construction of states of good
J, m_J we have products of C-G coefficients

differential cross section

$$d\sigma_{\alpha' s';\alpha s} = \frac{(k_\alpha)^{-2}}{(2s+1)} \sum_{L=0}^{\infty} B_L(\alpha' s';\alpha s) P_L(\cos\theta) d\Omega \tag{56}$$

where

$$B_L(\alpha' s';\alpha s) = \frac{(-)^{s'-s}}{4} \sum_{J_1} \sum_{J_2} \sum_{l_1} \sum_{l_2} \sum_{l'_1} \sum_{l'_2}$$
$$\times i^{l_1-l_2-L} Z(l_1 J_1 l_2 J_2, sL)\, i^{l'_1-l'_2-L} Z(l'_1 J_1 l'_2 J_2, s' L) \tag{57}$$
$$\times \text{R.P.}[(\delta_{\alpha\alpha'}\delta_{l_1 l'_1}\delta_{ss'} - U_{\alpha' s' l'_1;\alpha s l_1}{}^{J_1})^* (\delta_{\alpha' \alpha}\delta_{s' s}\delta_{l_2 l'_2} - U_{\alpha' s' l'_2;\alpha s l_2}{}^{J_2})].$$

The differential cross section (56) may be integrated over all angles to obtain the well known result

angle-integrated cross sect

$$\sigma_{\alpha' s';\alpha s} = \frac{4\pi}{(2s+1)k_\alpha{}^2} B_0(\alpha' s';\alpha s) \tag{58}$$
$$= \frac{\pi}{(2s+1)k_\alpha{}^2} \sum_{J=0}^{\infty} \sum_{l=|J-s|}^{J+s} \sum_{l'=|J-s'|}^{J+s'} (2J+1)|\delta_{\alpha'\alpha}\delta_{s's}\delta_{l'l} - U_{\alpha' s' l';\alpha s l}{}^{J}|^2$$

$$\sigma_{\alpha' s' l';\alpha s l} = \frac{\pi}{(2s+1)k_\alpha{}^2} \sum_{J=0}^{\infty} (2J+1)|\delta_{\alpha'\alpha}\delta_{s's}\delta_{l'l} - U_{\alpha' s' l';\alpha s l}{}^{J}|^2. \tag{59}$$

If we sum the integrated cross section, (59), for a given α, s, and l over all possible outgoing waves, and average over the channel spins of the incoming wave, we obtain the total cross section

Total σ

$$\sigma_T(\alpha l) \equiv \sum_{s,\alpha',s',l'} \frac{(2s+1)}{(2I+1)(2i+1)} \sigma_{\alpha' s' l';\alpha s l} \tag{60}$$
$$= \frac{\pi}{(2I+1)(2i+1)k_\alpha{}^2} \sum_{J=0}^{\infty} (2J+1) 2\,\text{R.P.}[1 - U_{\alpha,s,l;\alpha s l}{}^{J}]$$

where R.P. means the real part of the square bracket. The second step on the right side of (60) follows from the fact that

$$\sum_{\alpha',s',l'} U_{\alpha' s' l';\alpha s l}{}^{J} U_{\alpha' s' l';\alpha s l}{}^{J*} = 1 \tag{61}$$

UNITARITY

THE OPTICAL MODEL

Ignoring antisymmetry let's assume that nucleon A interacts with the other A-1 nucleons only through the mean field, $\bar{V}_A$

$$\bar{V}_A \equiv \sum_{\gamma=1}^{A-1} v_{A\gamma}$$ ← average over all γ-coordinates

then

$$H = \underbrace{\sum_{i=1}^{A} T_i + \sum_{i,\gamma}^{A-1} + \bar{V}_A}_{H_0} + \underbrace{\left[\sum_{\gamma=1}^{A-1} v_{A\gamma} - \bar{V}_A\right]}_{\text{residual interaction } H_R}$$

H_0 has eigenstates $\psi_\alpha u_A$

where u_A is the radial w.f. of nucleon A.

dropping subscript A

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr_A^2} + \bar{V}_A u = Eu$$

if we add a boundary condition at the channel radius for r_A

$$r_A \, du_p/dr_A = b u_p \Big|_{r_A = R_A}$$

then we have a discrete set of radial states.

using our channel wave functions, ψ_c we then have a complete set of wave functions, $\psi_c u_p$ in terms of which we can expand any w.f. of the whole system, including the actual resonance states.

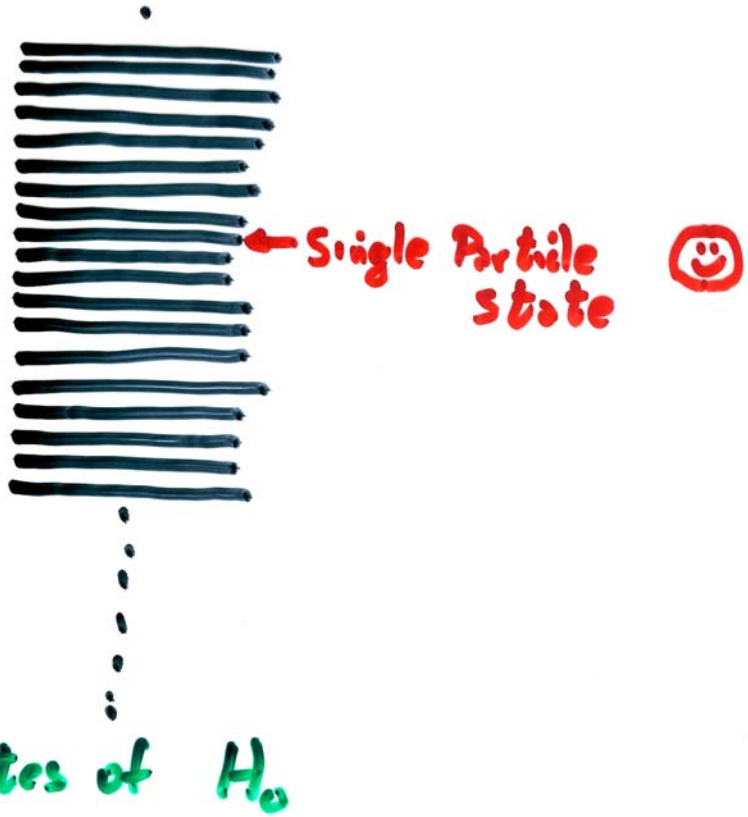
$$X_\lambda = \sum_{c,p} C_{\lambda;cp} \psi_c u_p$$

where c runs over all the channels for nucleon emission. Since all the states are properly normalized we have.

$$\sum_\lambda C_{\lambda;cp} C_{\lambda;c'p'} = \delta_{cc'} \delta_{pp'}$$

$$\sum_{cp} C_{\lambda;cp} C_{\lambda';cp} = \delta_{\lambda\lambda'}$$

Before we discuss the impact of the residual interaction we note the single-particle reduced width limits which come from these sum rules.



Only very, very few of the many eigenstates of H_o are single-particle states accessed by a nucleon beam

Sum Rule Limits

we have

$$\gamma_{\lambda c}^2 = \frac{\hbar^2}{2mR_c}\left[\sum_p u_l(R_c) C_{\lambda;cp}\right]^2$$

If the vestiges of the single-particle model remain then, for each $\gamma_{\lambda c}^2$, one value of p dominates. We can drop the sum over p.

$$\gamma_{\lambda c}^2 = \frac{\hbar^2}{2mR_c} u_p^2(R_c)\, C_{\lambda;cp}^2$$

If $\bar{V}$ is a square well then

$$\gamma_{\lambda c}^2 = \frac{\hbar^2}{mR_c^2} C_{\lambda;cp}^2$$

If we have a diffuse edge $\bar{V}$ then

$$\gamma_{\lambda c}^2 = \frac{\hbar^2}{mR_c^2} f\, C_{\lambda;cp}^2$$

$\frac{\hbar^2}{mR_c^2}$: single-particle reduced width

f: reflection factor

$C_{\lambda;cp}^2$: Spectroscopic Factor. (maximum = 1)

This is the sum-rule limit for experimental reduced widths.

CONFIGURATION MIXING.

If we could neglect the residual interaction we would still have the millions of compound states, χ_λ, but an incoming nucleon would excite only those rare compound states (spaced tens of MeV apart) which are eigenstates of H_0 with the target in its ground state. These rare states would have huge reduced widths while all the others would have zero reduced width.

The residual interaction mixes all the states together. All the spectroscopic factors become small (randomly small in a heavy nucleus) we can then enquire about the average value, per unit energy interval of the spectroscopic factor — that is the strength function

THE STRENGTH FUNCTION

Averaging cross sections over resonances yields the same cross sections as one finds with the optical model.

$$\sigma_{resonance} = \sigma_{optical}$$

and this is the justification of the optical model:

strength function: $$\frac{\langle C^2_{\lambda j(cp)} \rangle}{D} = \frac{W}{\pi} \sum_p \frac{1}{(E_p - E)^2 + W^2}$$

optical model: $$\overline{V} = -V_0 - iW$$

W is often referred to as the "spreading width" giving the energy interval into which the residual interaction spreads the single-particle states.

The spreading is small if it is much less than the the spacing (several tens of MeV) of the single particle levels and, if so, the shell model is valid

we note that the X_λ resonant states will, in general, be very complicated many-body states which, for resonance reactions, are parameterized by their reduced widths. Because of the complications the reduced widths, for heavy nuclei, are usually very small.

Show nucleon reduced widths versus A.

If we take the ratio of the average reduced widths divided by the average level-spacing, $\frac{\langle \gamma_{\lambda c}^2 \rangle}{D} \equiv s(E)$

does it display any gross structure as a function of the energy (or of A)?

If gross structure can be shown then nucleons might move in single-particle orbits — and the shell model might be valid.

Show s-wave neutron strength function

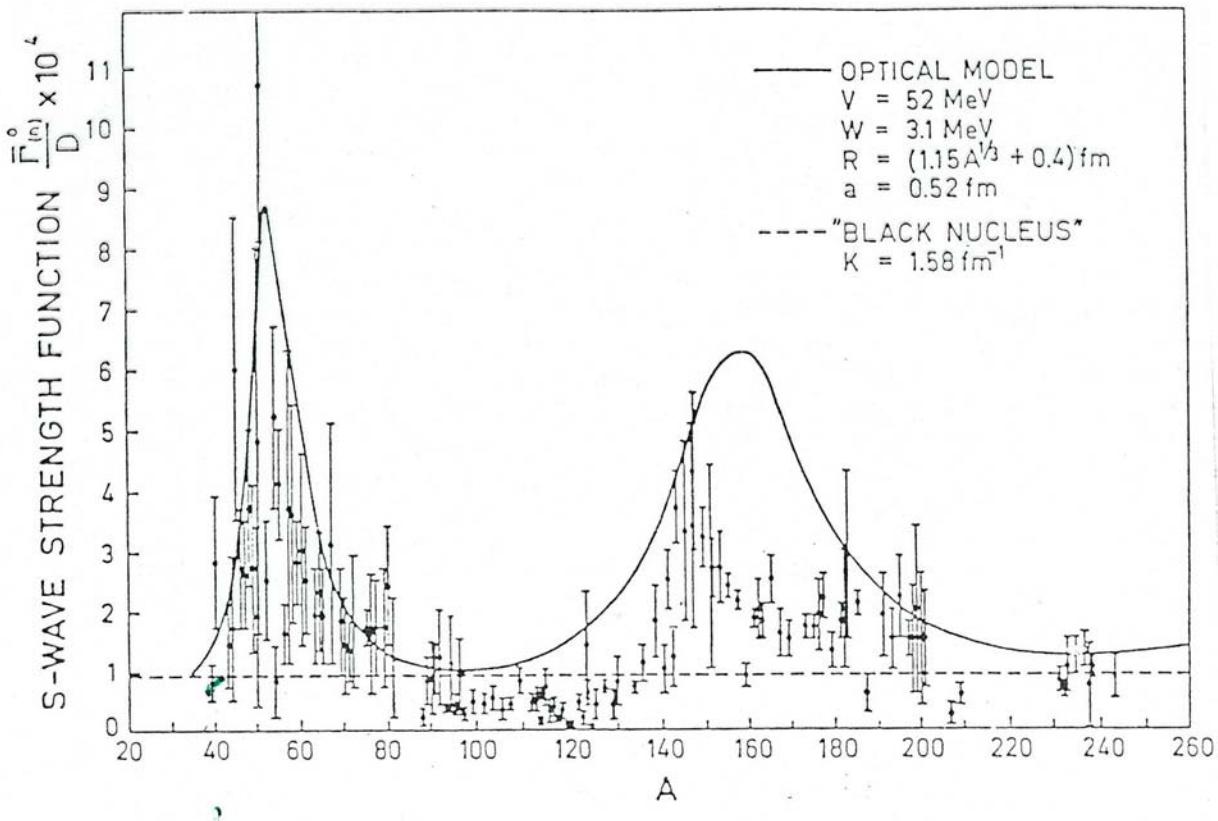


Figure 3

The s-wave neutron strength function versus mass number.

OTHER SPREADING WIDTHS

We could have chosen H_o to correspond to other possible single-particle motions. For example

$\alpha + {}^{12}C$

or $\quad {}^{12}C + {}^{12}C$

[Because of the Pauli Principle the mean fields would be very non-local.

Sometimes we get rather pure "cluster" states for which good physics tells us that the spreading width (Γ) might be small.

In general heavy-ion single-particle states are spread severly.