Shell Model for Heavy Deformed Nuclei

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Advances in nuclear shell model

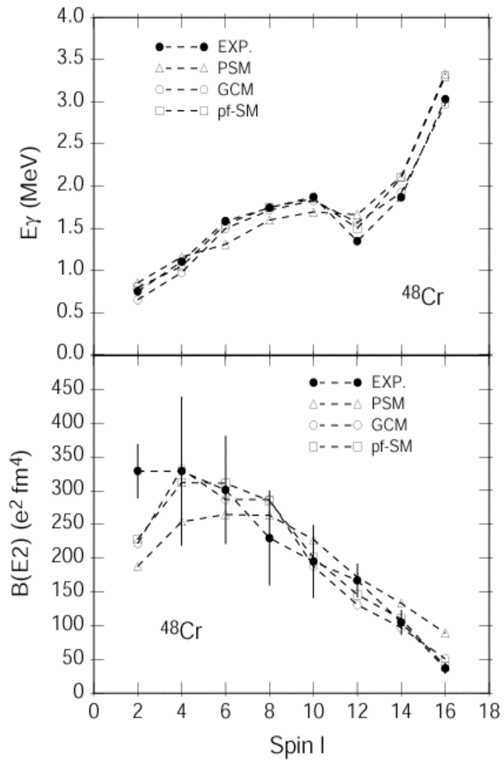
- Two major directions in developing shell model:
  - Take advantage of growing computer power and employ sophisticated diagonalization algorithms to improve shell model code.
  - Seek good truncation schemes to construct smaller bases that contain important correlations as much as possible.
- Projected shell model uses deformed basis and builds its bases using angular momentum projection. It belongs to the second category.
  - K. Hara and Y. Sun, Int. J. Mod. Phys. E4 (1995) 637
  - Y. Sun and C.-L. Wu, Phys. Rev. C68 (2003) 024315

Deformed basis vs spherical basis

- Most nuclei are deformed. To describe a deformed nucleus, a spherical shell model needs a huge configuration space.
- J.P. Elliott was the first to take the advantage of a deformed many-body basis and developed the SU(3) Shell Model.
- For heavy nuclei, the original Elliott SU(3) scheme is no longer valid. One can use generalized SU(3) schemes if such a symmetry exists. Or generally, one can start from a deformed basis and apply angular momentum projection numerically.

Deformed basis vs spherical basis

- Rotational spectrum in ^{48}Cr
  - Exp. data: Brandolini et al, NPA 642 (1998) 387
  - PSM and GCM: Hara, Sun and Mizusaki, PRL 83 (1999) 1922. Deformed basis with angular-momentum projection; Basis states ~ 50
  - pf-SM: Caurier et al, PRL 75 (1995) 2466. Conventional M-scheme spherical shell model; Basis states ~ 2 million



The projected shell model

- Take a set of deformed (quasi)particle states (e.g. solutions of HF, HFB or HF + BCS).
- Select configurations built from qp vacuum + a few qp states near the Fermi level.
- Project them onto good angular momentum (and if necessary, also parity, isospin, particle number) to form a basis in laboratory frame.
- If necessary, superimpose configurations belonging to different qp sets (the GCM-concept).
- Diagonalize a two-body Hamiltonian in this basis.

The projected shell model

- Wavefunction: $$\psi^I_M = \sum_\kappa f_\kappa \hat{P}^I_{MK_\kappa} \left|\phi_\kappa\right\rangle$$

  with the projector: $$\hat{P}^I_{MK} = \frac{2I+1}{8\pi^2}\int d\Omega\, D^I_{MK}(\Omega)\hat{D}(\Omega)$$

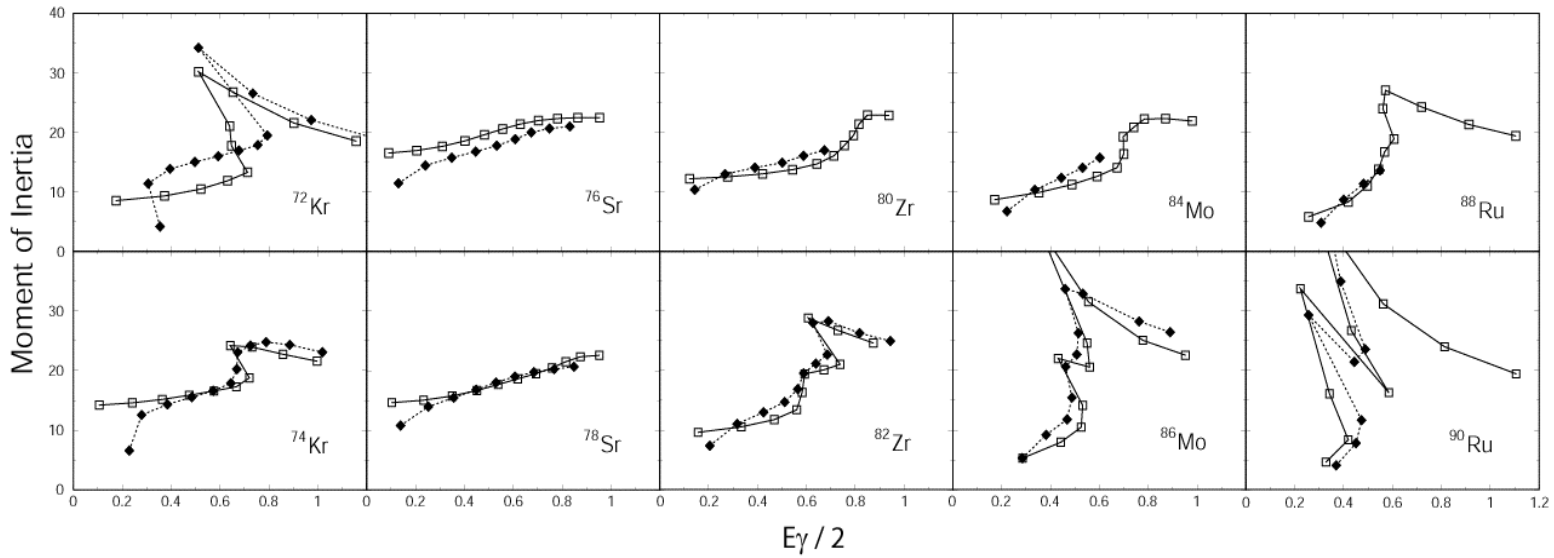
- The eigenvalue equation: $$\sum_{\kappa'}\left(H^I_{\kappa\kappa'} - EN^I_{\kappa\kappa'}\right)f_{\kappa'} = 0$$

  with matrix elements: $$H^I_{\kappa\kappa'} = \left\langle\phi_\kappa\right|\hat{H}\hat{P}^I_{KK'}\left|\phi_{\kappa'}\right\rangle \qquad N^I_{\kappa\kappa'} = \left\langle\phi_\kappa\right|\hat{P}^I_{KK'}\left|\phi_{\kappa'}\right\rangle$$

- The Hamiltonian is diagonalized in the projected basis $\left\{\hat{P}^I_{MK}\left|\phi_\kappa\right\rangle\right\}$

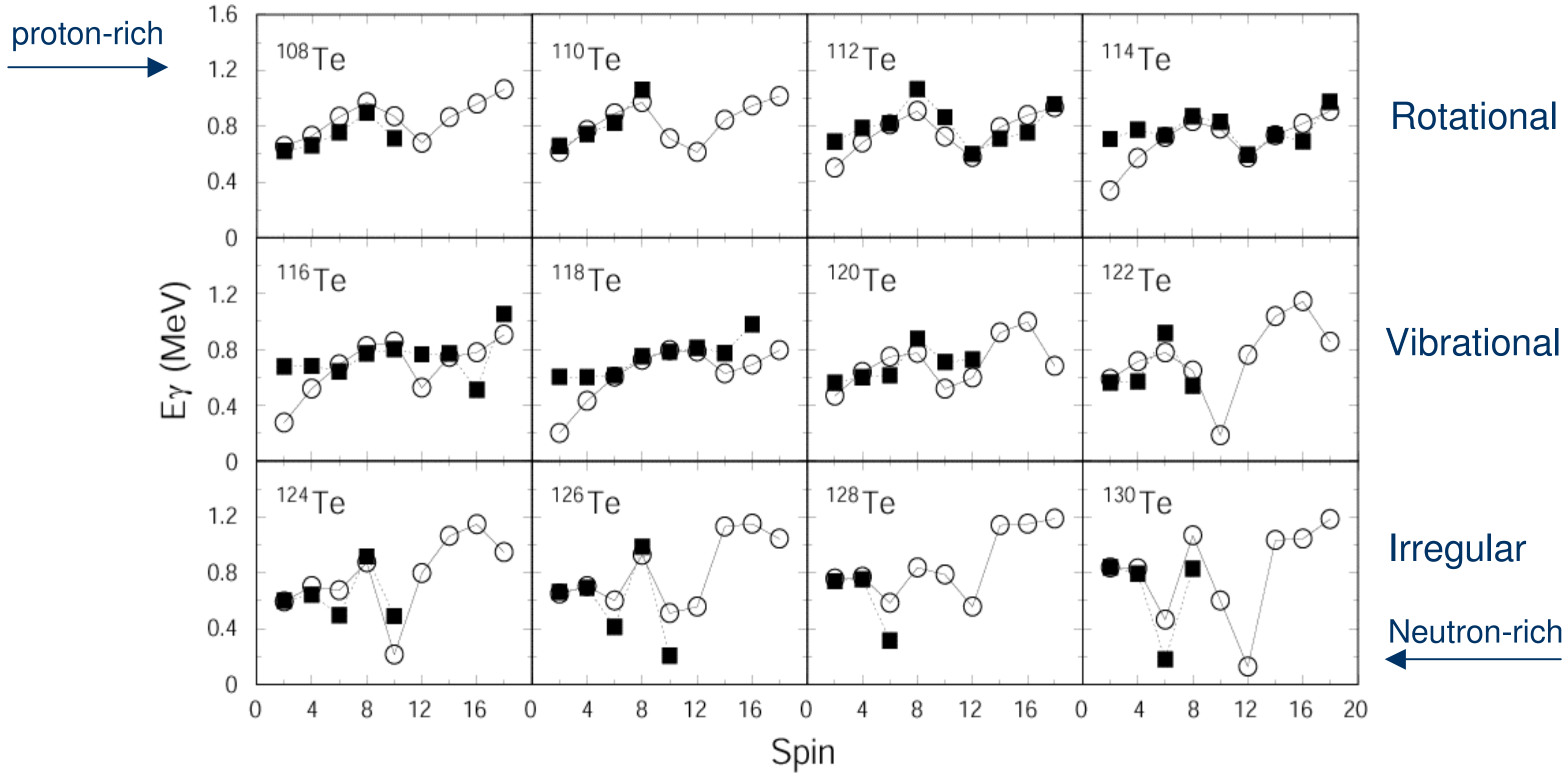
Examples of the PSM calculation

- Proton-rich nuclei near the N=Z line



Examples of the PSM calculation

- Neutron-rich nuclei extended to the ^{132}Sn region



Stellar weak interaction rates

- Recent spherical shell model calculations for weak interaction rates:
  - A. Brown, Prog. Part. Nucl. Phys. 47 (2001) 517
  - K. Langanke and G. Martinez-Pinedo, Rev. Mod. Phys. 75 (2003) 819

  Calculations were restricted for medium-light nuclei.
- It is a challenge to perform a shell-model calculation for heavy, deformed nuclei beyond the fp-shell and to use the resulting wavefunctions to calculate stellar weak interaction rates.

Stellar weak interaction rates

- Gamow-Teller or Fermi rate $B(GT/F) \sim \left\langle \psi_f \left| \hat{O}_{GT/F} \right| \psi_i \right\rangle^2$
- Wavefunction $\psi_M^I = \sum_\kappa f_\kappa \hat{P}_{MK_\kappa}^I \left| \phi_\kappa \right\rangle$
- E-e system $\left| \phi_e(\varepsilon_e) \right\rangle = \left\{ \left| \varepsilon_e \right\rangle, b_\nu^+ b_\nu^+ \left| \varepsilon_e \right\rangle, b_\pi^+ b_\pi^+ \left| \varepsilon_e \right\rangle, b_\nu^+ b_\nu^+ b_\pi^+ b_\pi^+ \left| \varepsilon_e \right\rangle, \cdots \right\}$
- O-o system $\left| \phi_o(\varepsilon_o) \right\rangle = \left\{ a_\nu^+ a_\pi^+ \left| \varepsilon_o \right\rangle, a_\nu^+ a_\nu^+ a_\nu^+ a_\pi^+ \left| \varepsilon_o \right\rangle, a_\nu^+ a_\pi^+ a_\pi^+ a_\pi^+ \left| \varepsilon_o \right\rangle, \cdots \right\}$
- Overlap matrix element

$$\left\langle \phi_o(\varepsilon_o) \right| \hat{O} \hat{P}_{K_a K_b}^I \left| \phi_e(\varepsilon_e) \right\rangle \sim \int d\Omega D_{K_a K_b}^I(\Omega) \left\langle \phi_o(\varepsilon_o) \right| \hat{O} \hat{R}(\Omega) \left| \phi_e(\varepsilon_e) \right\rangle$$

Summary

- Angular momentum projection is an efficient way to truncate shell model space to perform shell model calculations in a larger space. The method can be applied to any size of nuclear system without dimension problems. The results can usually be interpreted using simple physical terms.
- The projection technique has been well developed. Projected shell model is an established example. Using the projected shell model wavefunctions to calculate weak interaction rates is in progress.