Behavior of Nuclear Reaction Networks

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A little about me

- I was born in New York but grew up in Tempe, Arizona
- Undergrad at Rice University (Houston, TX)
- Ph. D. University of Chicago (advisor: David Schramm)
- Postdoc at Lawrence Livermore National Laboratory
- At Clemson since 1991

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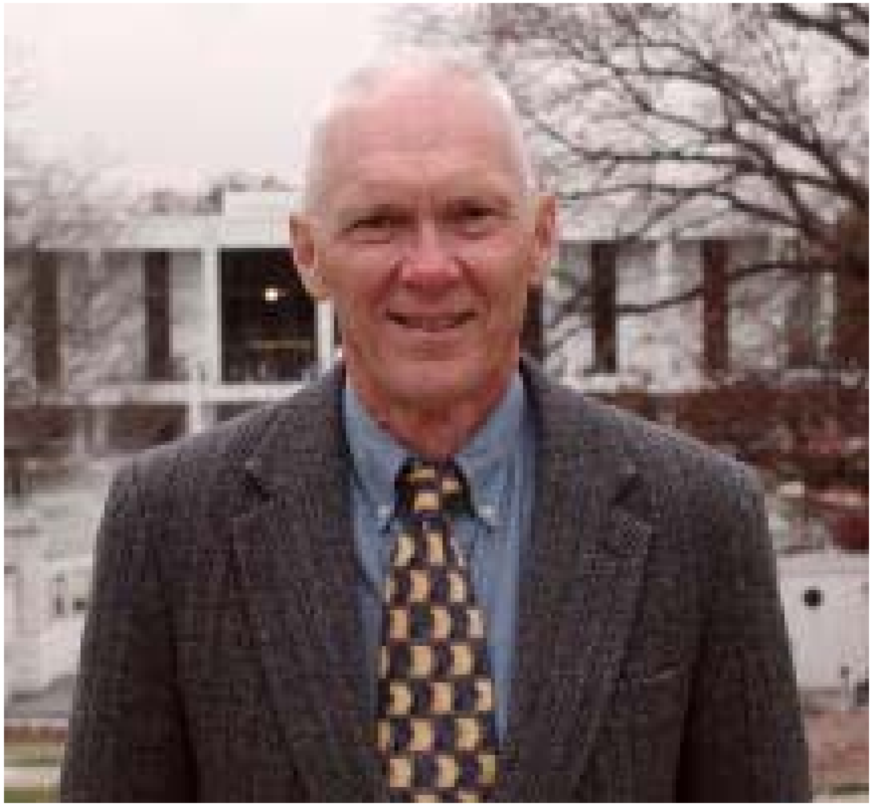
Our Group



Don Clayton

Dieter Hartmann

Mark Leising

Jeremy King

Bruce Rafert

Goals

- Understand basic behaviors of nucleosynthesis networks
- Gain some knowledge of online tools for nuclear astrophysics

Basic behaviors of Nuclear Networks

- Steady states
- Equilibria
- Both involve time derivatives going to zero!

Steady State

"Flow in" = "flow out"

$$\frac{dN}{dt} = P - \lambda N$$

$$N(t) = N(0)e^{-\lambda t} + \frac{P}{\lambda}(1 - e^{-\lambda t}) \xrightarrow[t\to\infty]{} \frac{P}{\lambda}$$

http://nucleo.ces.clemson.edu

Equilibrium

Balancing act

“Push” = “pull” or “flow out = flow back in” http://nucleo.ces.clemson.edu/home/online_tools/newton/0.1

Steady State

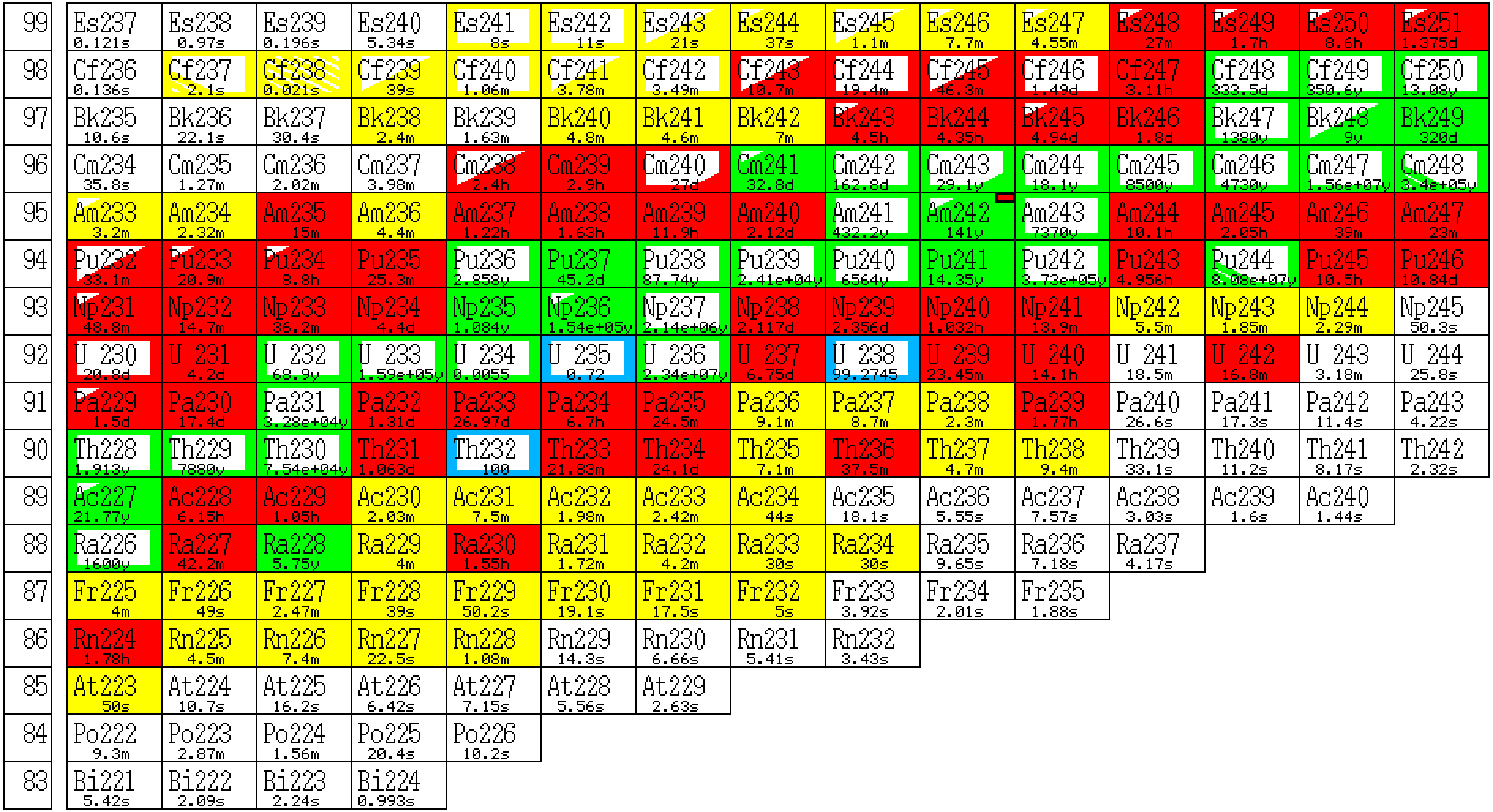
- Nuclear decay chains (analytic solution by Bateman in 1910 and used by Seeger et al. 1965 for the r process):

$$n_{26}(t) = N_o e^{-\lambda_{26} t}, \quad \frac{d n_Z(t)}{dt} = +\lambda_{Z-1} n_{Z-1}(t) - \lambda_Z n_Z(t); Z = 27, 28, \ldots,$$

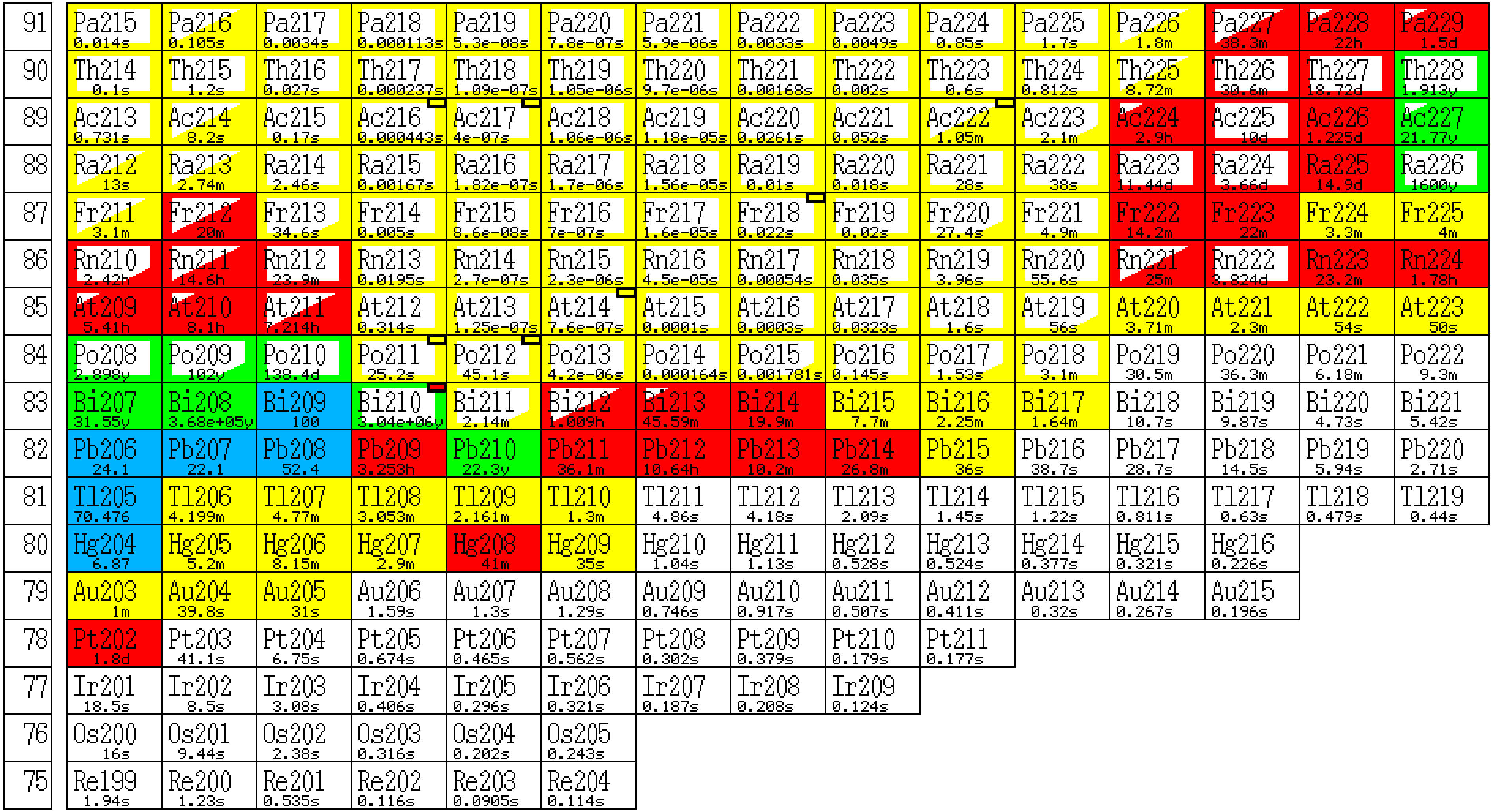
$$n_Z(t) = N_o \sum_{i=26}^{Z} \left\{ \frac{\lambda_i}{\lambda_Z} \left[\prod_{\substack{j=26 \\ j \neq i}}^{Z} \frac{\lambda_j}{(\lambda_j - \lambda_i)} \right] e^{-\lambda_i t} \right\}.$$

Alpha decay chains

- ^{238}U decays to ^{206}Pb (8 alpha decays)
- ^{235}U decays to ^{207}Pb (7 alpha decays)
- ^{232}Th decays to ^{208}Pb (6 alpha decays)



99	Es237 0.121s	Es238 0.97s	Es239 0.196s	Es240 5.34s	Es241 8s	Es242 11s	Es243 21s	Es244 37s	Es245 1.1m	Es246 7.7m	Es247 4.55m	Es248 27m	Es249 1.7h	Es250 8.6h	Es251 1.375d
98	Cf236 0.136s	Cf237 2.1s	Cf238 0.021s	Cf239 39s	Cf240 1.06m	Cf241 3.78m	Cf242 3.49m	Cf243 10.7m	Cf244 19.4m	Cf245 46.3m	Cf246 1.49d	Cf247 3.11h	Cf248 333.5d	Cf249 350.6y	Cf250 13.08y
97	Bk235 10.6s	Bk236 22.1s	Bk237 30.4s	Bk238 2.4m	Bk239 1.63m	Bk240 4.8m	Bk241 4.6m	Bk242 7m	Bk243 4.5h	Bk244 4.35h	Bk245 4.94d	Bk246 1.8d	Bk247 1380y	Bk248 9y	Bk249 320d
96	Cm234 35.8s	Cm235 1.27m	Cm236 2.02m	Cm237 3.98m	Cm238 2.4h	Cm239 2.9h	Cm240 27d	Cm241 32.8d	Cm242 162.8d	Cm243 29.1y	Cm244 18.1y	Cm245 8500y	Cm246 4730y	Cm247 1.56e+07y	Cm248 3.4e+05y
95	Am233 3.2m	Am234 2.32m	Am235 15m	Am236 4.4m	Am237 1.22h	Am238 1.63h	Am239 11.9h	Am240 2.12d	Am241 432.2y	Am242 141y	Am243 7370y	Am244 10.1h	Am245 2.05h	Am246 39m	Am247 23m
94	Pu232 33.1m	Pu233 20.9m	Pu234 8.8h	Pu235 25.3m	Pu236 2.858y	Pu237 45.2d	Pu238 87.74y	Pu239 2.41e+04y	Pu240 6564y	Pu241 14.35y	Pu242 3.73e+05y	Pu243 4.956h	Pu244 8.08e+07y	Pu245 10.5h	Pu246 10.84d
93	Np231 48.8m	Np232 14.7m	Np233 36.2m	Np234 4.4d	Np235 1.084y	Np236 1.54e+05y	Np237 2.14e+06y	Np238 2.117d	Np239 2.356d	Np240 1.032h	Np241 13.9m	Np242 5.5m	Np243 1.85m	Np244 2.29m	Np245 50.3s
92	U 230 20.8d	U 231 4.2d	U 232 68.9y	U 233 1.59e+05y	U 234 0.0055	U 235 0.72	U 236 2.34e+07y	U 237 6.75d	U 238 99.2745	U 239 23.45m	U 240 14.1h	U 241 18.5m	U 242 16.8m	U 243 3.18m	U 244 25.8s
91	Pa229 1.5d	Pa230 17.4d	Pa231 3.28e+04y	Pa232 1.31d	Pa233 26.97d	Pa234 6.7h	Pa235 24.5m	Pa236 9.1m	Pa237 8.7m	Pa238 2.3m	Pa239 1.77h	Pa240 26.6s	Pa241 17.3s	Pa242 11.4s	Pa243 4.22s
90	Th228 1.913y	Th229 7880y	Th230 7.54e+04y	Th231 1.063d	Th232 100	Th233 21.83m	Th234 24.1d	Th235 7.1m	Th236 37.5m	Th237 4.7m	Th238 9.4m	Th239 33.1s	Th240 11.2s	Th241 8.17s	Th242 2.32s
89	Ac227 21.77y	Ac228 6.15h	Ac229 1.05h	Ac230 2.03m	Ac231 7.5m	Ac232 1.98m	Ac233 2.42m	Ac234 44s	Ac235 18.1s	Ac236 5.55s	Ac237 7.57s	Ac238 3.03s	Ac239 1.6s	Ac240 1.44s	
88	Ra226 1600y	Ra227 42.2m	Ra228 5.75y	Ra229 4m	Ra230 1.55h	Ra231 1.72m	Ra232 4.2m	Ra233 30s	Ra234 30s	Ra235 9.65s	Ra236 7.18s	Ra237 4.17s			
87	Fr225 4m	Fr226 49s	Fr227 2.47m	Fr228 39s	Fr229 50.2s	Fr230 19.1s	Fr231 17.5s	Fr232 5s	Fr233 3.92s	Fr234 2.01s	Fr235 1.88s				
86	Rn224 1.78h	Rn225 4.5m	Rn226 7.4m	Rn227 22.5s	Rn228 1.08m	Rn229 14.3s	Rn230 6.66s	Rn231 5.41s	Rn232 3.43s						
85	At223 50s	At224 10.7s	At225 16.2s	At226 6.42s	At227 7.15s	At228 5.56s	At229 2.63s								
84	Po222 9.3m	Po223 2.87m	Po224 1.56m	Po225 20.4s	Po226 10.2s										
83	Bi221 5.42s	Bi222 2.09s	Bi223 2.24s	Bi224 0.993s											



91	Pa215 0.014s	Pa216 0.105s	Pa217 0.0034s	Pa218 0.000113s	Pa219 5.3e-08s	Pa220 7.8e-07s	Pa221 5.9e-06s	Pa222 0.0033s	Pa223 0.0049s	Pa224 0.85s	Pa225 1.7s	Pa226 1.8m	Pa227 38.3m	Pa228 22h	Pa229 1.5d
90	Th214 0.1s	Th215 1.2s	Th216 0.027s	Th217 0.000237s	Th218 1.09e-07s	Th219 1.05e-06s	Th220 9.7e-06s	Th221 0.00168s	Th222 0.002s	Th223 0.6s	Th224 0.812s	Th225 8.72m	Th226 30.6m	Th227 18.72d	Th228 1.913y
89	Ac213 0.731s	Ac214 8.2s	Ac215 0.17s	Ac216 0.000443s	Ac217 4e-07s	Ac218 1.06e-06s	Ac219 1.18e-05s	Ac220 0.0261s	Ac221 0.052s	Ac222 1.05m	Ac223 2.1m	Ac224 2.9h	Ac225 10d	Ac226 1.225d	Ac227 21.77y
88	Ra212 13s	Ra213 2.74m	Ra214 2.46s	Ra215 0.00167s	Ra216 1.82e-07s	Ra217 1.7e-06s	Ra218 1.56e-05s	Ra219 0.01s	Ra220 0.018s	Ra221 28s	Ra222 38s	Ra223 11.44d	Ra224 3.66d	Ra225 14.9d	Ra226 1600y
87	Fr211 3.1m	Fr212 20m	Fr213 34.6s	Fr214 0.005s	Fr215 8.6e-08s	Fr216 7e-07s	Fr217 1.6e-05s	Fr218 0.022s	Fr219 0.02s	Fr220 27.4s	Fr221 4.9m	Fr222 14.2m	Fr223 22m	Fr224 3.3m	Fr225 4m
86	Rn210 2.42h	Rn211 14.6h	Rn212 23.9m	Rn213 0.0195s	Rn214 2.7e-07s	Rn215 2.3e-06s	Rn216 4.5e-05s	Rn217 0.00054s	Rn218 0.035s	Rn219 3.96s	Rn220 55.6s	Rn221 25m	Rn222 3.824d	Rn223 23.2m	Rn224 1.78h
85	At209 5.41h	At210 8.1h	At211 7.214h	At212 0.314s	At213 1.25e-07s	At214 7.6e-07s	At215 0.0001s	At216 0.0003s	At217 0.0323s	At218 1.6s	At219 56s	At220 3.71m	At221 2.3m	At222 54s	At223 50s
84	Po208 2.898y	Po209 102y	Po210 138.4d	Po211 25.2s	Po212 45.1s	Po213 4.2e-06s	Po214 0.000164s	Po215 0.001781s	Po216 0.145s	Po217 1.53s	Po218 3.1m	Po219 30.5m	Po220 36.3m	Po221 6.18m	Po222 9.3m
83	Bi207 31.55y	Bi208 3.68e+05y	Bi209 100	Bi210 3.04e+06y	Bi211 2.14m	Bi212 1.009h	Bi213 45.59m	Bi214 19.9m	Bi215 7.7m	Bi216 2.25m	Bi217 1.64m	Bi218 10.7s	Bi219 9.87s	Bi220 4.73s	Bi221 5.42s
82	Pb206 24.1	Pb207 22.1	Pb208 52.4	Pb209 3.253h	Pb210 22.3y	Pb211 36.1m	Pb212 10.64h	Pb213 10.2m	Pb214 26.8m	Pb215 36s	Pb216 38.7s	Pb217 28.7s	Pb218 14.5s	Pb219 5.94s	Pb220 2.71s
81	Tl205 70.476	Tl206 4.199m	Tl207 4.77m	Tl208 3.053m	Tl209 2.161m	Tl210 1.3m	Tl211 4.86s	Tl212 4.18s	Tl213 2.09s	Tl214 1.45s	Tl215 1.22s	Tl216 0.811s	Tl217 0.63s	Tl218 0.479s	Tl219 0.44s
80	Hg204 6.87	Hg205 5.2m	Hg206 8.15m	Hg207 2.9m	Hg208 41m	Hg209 35s	Hg210 1.04s	Hg211 1.13s	Hg212 0.528s	Hg213 0.524s	Hg214 0.377s	Hg215 0.321s	Hg216 0.226s		
79	Au203 1m	Au204 39.8s	Au205 31s	Au206 1.59s	Au207 1.3s	Au208 1.29s	Au209 0.746s	Au210 0.917s	Au211 0.507s	Au212 0.411s	Au213 0.32s	Au214 0.267s	Au215 0.196s		
78	Pt202 1.8d	Pt203 41.1s	Pt204 6.75s	Pt205 0.674s	Pt206 0.465s	Pt207 0.562s	Pt208 0.302s	Pt209 0.379s	Pt210 0.179s	Pt211 0.177s					
77	Ir201 18.5s	Ir202 8.5s	Ir203 3.08s	Ir204 0.406s	Ir205 0.296s	Ir206 0.321s	Ir207 0.187s	Ir208 0.208s	Ir209 0.124s						
76	Os200 16s	Os201 9.44s	Os202 2.38s	Os203 0.316s	Os204 0.202s	Os205 0.243s									
75	Re199 1.94s	Re200 1.23s	Re201 0.535s	Re202 0.116s	Re203 0.0905s	Re204 0.114s									

$$\frac{dY_{234}}{dt} = -\lambda_{234} Y_{234} + \lambda_{238} Y_{238}$$

$$\frac{dY}{dt} = AY$$

$$\Rightarrow Y(t) = e^{At} Y(0)$$

Finite difference solution

$$\frac{\Delta Y}{\Delta t} = A(Y + \Delta Y)$$

$$so$$

$$\left(\frac{1}{\Delta t} - A\right)\Delta Y = AY$$

http://nucleo.ces.clemson.edu/home/online_tools/nuclear_decay http://www.ces.clemson.edu/~mbradle/JINA/alpha_beta.html

A steady state

$$X(^{238}U) = 4.975135\times10^{-1} \Rightarrow Y(^{238}U) = X(^{238}U)/238 = 0.00209039$$

$$X(^{234}Th) = 7.223465\times10^{-12} \Rightarrow Y(^{234}Th) = X(^{234}Th)/234 = 3.08695\times10^{-14}$$

$$\lambda_{\alpha,238} = 4.916093\times10^{-18}\,s^{-1}$$

$$\lambda_{\beta,234} = 3.328853\times10^{-7}\,s^{-1}$$

$$\alpha \quad flow \quad in = 1.02766\times10^{-20}$$

$$\beta \quad flow \quad out = 1.02760\times10^{-20}$$

4.4 Gyr Old Rocks!

Jack Hills, Western Australia

Zircon

Zircons like U but not Pb!

How old is the rock?

$$Y(^{238}U) = Y_0(^{238}U)e^{-\lambda_{238}t}$$

$$Y(^{206}Pb) = Y_0(^{206}Pb) + Y_0(^{238}U) \times (1 - e^{-\lambda_{238}t})$$

$$\Rightarrow Y(^{206}Pb) = Y_0(^{206}Pb) + Y(^{238}U) \times (e^{\lambda_{238}t} - 1)$$

Tasks for today

- Study the alpha decay chains for ^{232}Th, ^{235}U, and ^{238}U and check for steady states.
- Consider the decay back to stability of ^{150}Sn (try for different times).
- You may find useful the online chart of the nuclides: http://wwwndc.tokai.jaeri.go.jp/CN03/