Behavior of Nuclear Reaction Networks

Brad Meyer
Clemson University

Comments from yesterday's work

- Mail list
- Beta-decay rates for ^{150}Sn calculation: experimental where available, otherwise theoretical rates from Peter Moller
- Picture of decay chains

Steady states

$$\frac{dY_{238U}}{dt} = -\lambda_{238U,\alpha} Y_{238U}$$

$$\Rightarrow Y_{238U} = Y_{238U}(0) e^{-\lambda_{238U,\alpha} t}$$

$$\frac{dY_{234Th}}{dt} = -\lambda_{234Th,\beta} Y_{234Th} + \lambda_{238U,\alpha} Y_{238U} \xrightarrow[t >> 1/\lambda_{234Th,\beta}]{} 0$$

$$\Rightarrow \frac{Y_{234Th}}{Y_{238U}} = \frac{\lambda_{238U,\alpha}}{\lambda_{234Th,\beta}}$$

Explicit differentiation

$$\frac{dy}{dt} = -y \Rightarrow y(t) = y(0)e^{-t}$$

$$\frac{\Delta y}{\Delta t} = \frac{y(t+\Delta t) - y(t)}{\Delta t} = -y(t)$$

so

$$y(t+\Delta t) = y(t) - y(t) \times \Delta t \xrightarrow[\Delta t \to \infty]{} -\infty$$

Implicit differentiation

$$\frac{dy}{dt} = -y \Rightarrow y(t) = y(0)e^{-t}$$

$$\frac{\Delta y}{\Delta t} = \frac{y(t+\Delta t) - y(t)}{\Delta t} = -y(t+\Delta t)$$

so

$$y(t+\Delta t) = \frac{y(t)}{1+\Delta t} \xrightarrow[\Delta t \to \infty]{} 0$$

For the ^{238}U decay chain

$$\frac{dY_{234}}{dt} = -\lambda_{234} Y_{234} + \lambda_{238} Y_{238}$$

$$\left(\frac{1}{\Delta t} + \lambda_{234}\right)\Delta Y_{234} - \lambda_{238}\Delta Y_{238} = -\lambda_{234} Y_{234} + \lambda_{238} Y_{238}$$

and

$$\frac{dY_{238}}{dt} = -\lambda_{238} Y_{238}$$

$$\left(\frac{1}{\Delta t} + \lambda_{238}\right)\Delta Y_{238} = -\lambda_{238} Y_{238}$$

Simple non-linear problem: p+n<->d

$$\frac{dn_p}{dt} = -\langle \sigma v \rangle n_p n_n + \lambda_\gamma n_d - \frac{n_p}{\tau}$$

where

$$\frac{1}{\tau} = -\frac{1}{\rho}\frac{d\rho}{dt}$$

Define

$$n_i = \rho N_A Y_i$$

so

$$\frac{dn_i}{dt} = \frac{d\rho}{dt} N_A Y_i + \rho N_A \frac{dY_i}{dt} = \frac{1}{\rho}\frac{d\rho}{dt}\rho N_A Y_i + \rho N_A \frac{dY_i}{dt}$$

$$= -\frac{n_i}{\tau} + \rho N_A \frac{dY_i}{dt}$$

$$\rho N_A \frac{dY_p}{dt} = -\langle \sigma v \rangle \rho^2 N_A{}^2 Y_p Y_n + \lambda_\gamma \rho N_A Y_d$$

thus

$$\frac{dY_p}{dt} = -N_A \langle \sigma v \rangle \rho Y_p Y_n - \lambda_\gamma Y_d$$

Collecting all equations for this particular reaction

$$\frac{dY_p}{dt} = -N_A \langle \sigma v \rangle \rho Y_p Y_n + \lambda_\gamma Y_d$$

$$\frac{dY_n}{dt} = -N_A \langle \sigma v \rangle \rho Y_p Y_n + \lambda_\gamma Y_d$$

$$\frac{dY_d}{dt} = N_A \langle \sigma v \rangle \rho Y_p Y_n - \lambda_\gamma Y_d$$

Finite Differencing

$$\frac{\Delta Y_p}{\Delta t} = -N_A \langle \sigma v \rangle \rho (Y_p + \Delta Y_p)(Y_n + \Delta Y_n) + \lambda_\gamma (Y_d + \Delta Y_d)$$

so

$$\left(\frac{1}{\Delta t} + N_A \langle \sigma v \rangle \rho Y_n \right) \Delta Y_p + N_A \langle \sigma v \rangle \rho Y_p \Delta Y_n - \lambda_\gamma \Delta Y_d =$$

$$- N_A \langle \sigma v \rangle \rho Y_p Y_n + \lambda_\gamma Y_d$$

Final Matrix Problem

$$Solve \quad A \cdot \Delta Y = b$$

Viewing the output: FITS

http://heasarc.gsfc.nasa.gov/lheasoft/ftools/fv/fv_download.html

The alpha decay fits file

http://www.ces.clemson.edu/~mbradle/JINA/

Choose alpha_decay_u238.fits

The beta decay fits file

http://www.ces.clemson.edu/~mbradle/JINA/

Choose beta_decay_sn150.fits

A simple hydrogen burning example

- Start with solar abundances:

http://nucleo.ces.clemson.edu/pages/solar_abundances

T_9=0.015

density = 150 g/cc

The fits file

http://www.ces.clemson.edu/~mbradle/JINA/

Choose hburn1.fits

Y_e

$$Y_e = \sum_i Z_i Y_i$$

Another hydrogen burning example

- Start with solar abundances:

http://nucleo.ces.clemson.edu/pages/solar_abundances

T_9=0.040

density = 10 g/cc

The fits file

http://www.ces.clemson.edu/~mbradle/JINA/

Choose hburn2.fits

Tasks for today

- Solve for ΔY_{234} and ΔY_{238} in slide 6 algebraically. Consider the behavior of the solutions for large Δt
- Download and install fv, the FITS viewer
- Study the alpha decay chain FITS file (alpha_decay_u238.fits). Graph Y(A) vs. A for several time records.
- Study the beta decay chain FITS file (beta_decay_sn150.fits). Graph Y(Z) vs. Z for several time records. Graph the change in Y_e with time.
- Study the hydrogen burning FITS file 1 (hburn1.fits). Graph an abundance such as ^{14}N versus time. Do the same for hydrogen burning FITS file 2 (hburn2.fits). Compute and graph the change of Y_e with time.