Behavior of Nuclear Reaction Networks

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Answer to Binding Energy Problem

$$B(^{62}Ni) = B(28{,}62) = 28 * 7.289 + (62 - 28) * 8.071 - (\text{-}66.7450)$$
$$= 545.251 \quad MeV$$

$$B(^{62}Ni) / A = 545.251 / 62 = 8.79437 \quad \text{MeV/nucleon}$$

$$B(^{56}Ni) = B(28{,}56) = 28 * 7.289 + (56 - 28) * 8.071 - (\text{-}53.9010)$$
$$= 483.981 \quad MeV$$

$$B(^{62}Ni) / A = 483.981 / 56 = 8.64252 \quad \text{MeV/nucleon}$$

NSE Calculator

http://nucleo.ces.clemson.edu/pages/nse/0.1

Formal Development

$$df = -sdT - PdV + \sum_i \mu_i dY_i$$

Consider

$$dT = dV = 0$$

$$\Rightarrow df = \sum_i \mu_i dY_i$$

For equilibrium

$$df = 0$$

$$so$$

$$\sum_i \mu_i dY_i = 0$$

But we have constraints

$$1)\sum_i X_i = \sum_i A_i Y_i = 1$$

$$2)\sum_i Z_i Y_i = Y_e$$

Detour (a simple minimization problem)

- Minimize the function $f(x,y)=x^2+y^2$

$$df = 2x\,dx + 2y\,dy = 0$$

$$arbitrary \quad dx, dy$$

$$so$$

$$x = 0, y = 0$$

A more interesting example

- Minimize the function $f(x,y)=x^2+y^2$ subject to the constraint that $y=-x+b$, b a constant

$$df = 2xdx + 2ydy = 0$$

but now

$$dy = -dx$$

$$df = 2xdx - 2ydx = (2x - 2y)dx = 0$$

$$\Rightarrow x = y$$

so

$$y = -y + b \Rightarrow y = b/2$$

A better way: Lagrange multipliers

- Minimize the function f(x,y)=x^2+y^2 subject to the constraint that y=-x+b, b a constant

$$
\begin{aligned}
&\textit{Define}\\
&f = x^2 + y^2, g = y + x - b\\
&\textit{Find}\\
&d(f - \lambda g) = 0\\
&\Rightarrow (2x - \lambda)dx + (2y - \lambda)dy = 0\\
&\textit{so}\\
&x = \lambda / 2, y = \lambda / 2\\
&y = -x + b \Rightarrow \lambda / 2 = -\lambda / 2 + b \Rightarrow \lambda = b\\
&\Rightarrow x = y = b / 2
\end{aligned}
$$

Our problem

$$Minimize$$

$$f$$

$$subject \quad to$$

$$1)\sum_i A_i Y_i = 1$$

$$and$$

$$2)\sum_i Z_i Y_i = Y_e$$

Define

$$g = \sum_i A_i Y_i - 1$$

and

$$h = \sum_i Z_i Y_i - Y_e$$

then

$$d(f - \lambda_1 g - \lambda_2 h) = 0$$

so

$$\sum_i \left(\mu_i - \lambda_1 A_i - \lambda_2 Z_i\right) dY_i = 0$$

$$Neutrons:$$

$$\mu_n - \lambda_1 = 0 \Rightarrow \lambda_1 = \mu_n$$

$$\mathrm{Pr}\,otons:$$

$$\mu_p - \lambda_1 - \lambda_2 = 0 \Rightarrow \lambda_2 = \mu_p - \mu_n$$

$$Others:$$

$$\mu_i - \lambda_1 A_i - \lambda_2 Z_i = 0 \Rightarrow \mu_i = A_i \mu_n + Z_i(\mu_p - \mu_n)$$

$$\Rightarrow \mu_i = Z_i \mu_p + N_i \mu_n$$

General Nucleosynthesis Network

$$\mu(Z, A) = Z\mu_p + (A - Z)\mu_n$$

$$N = A - Z$$

$$\mu(Z, A) = Z\mu_p + N\mu_n$$

Chemical potentials

- Consider nuclei as Maxwell-Boltzmann particles:

$$\mu(Z,A) = m(Z,A)c^2 + kT\ln\left[\frac{\rho N_A Y(Z,A)}{G(Z,A)}\left(\frac{h^2}{2\pi m(Z,A)kT}\right)^{3/2}\right]$$

The Nuclear Data Tool

http://nucleo.ces.clemson.edu/home/online_tools/nuclear_data/0.1

QSE: Quasi-statistical equilibrium

$$1)\sum_{i} A_i Y_i = 1$$

$$2)\sum_{i} Z_i Y_i = Y_e$$

$$3)\sum_{i, Z_i \geq 6} Y_i = Y_h$$

Key results

$$R_i \equiv \frac{Y_i}{Y_i^{NSE}}$$

$$QSE:$$

$$R_\alpha = R_p^2 R_n^2$$

$$and$$

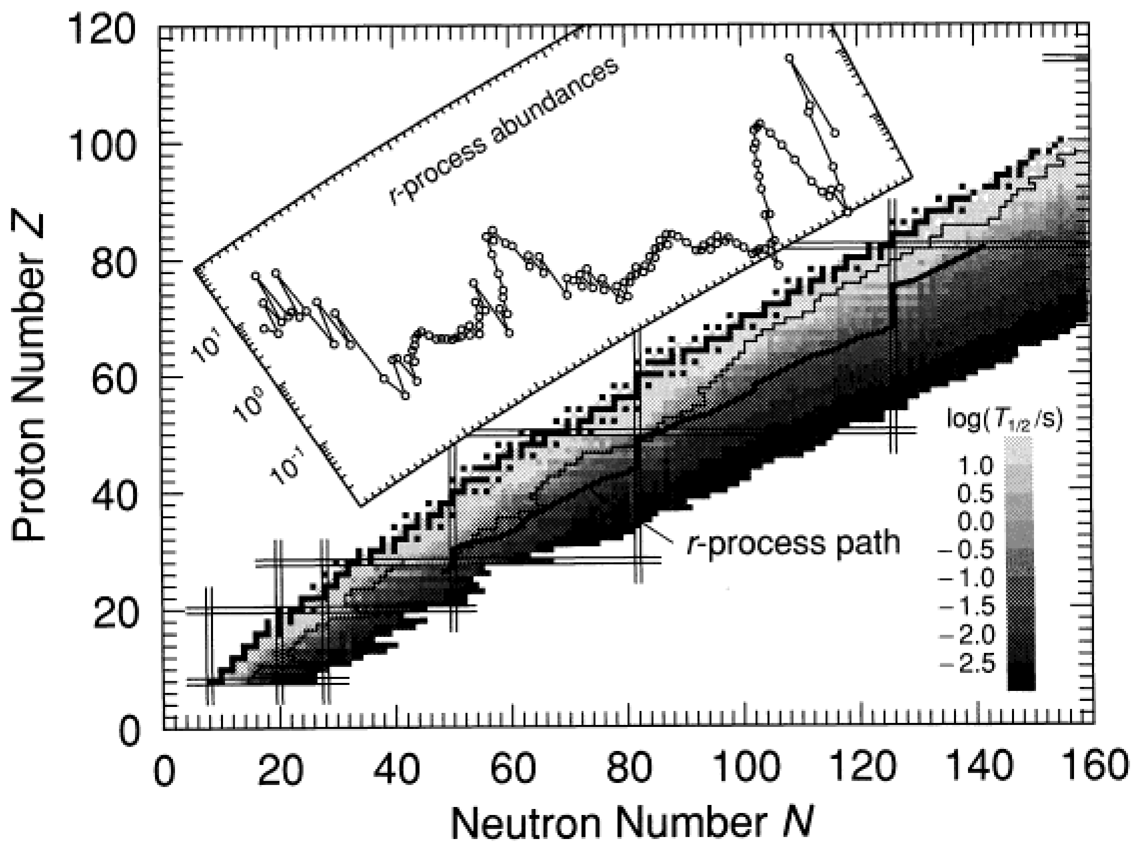
$$R_i = e^{\mu / kT} R_p^2 R_n^2$$

$$for$$

$$Z_i \geq 6$$

The Silicon-Burning Movies

R Process



r-process abundances
10^1
10^0
10^{-1}
r-process path
$\log(T_{1/2}/s)$
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160

The R-Process Movies

Hierarchy of Statistical Equilibria in Nucleosynthesis

(0) Equilibrium with nonconstant nucleon number

- -

(1) NSE with weak equilibrium

(2) NSE with fixed Ye

(3) QSE (equilibrium with fixed Ye and Yh)

(4) Two QSE clusters (equilibrium with fixed Ye, Yh1, and Yh2)

(5) Three QSE clusters (equilibrium with fixed Ye, Yh1, Yh2, and Yh3)

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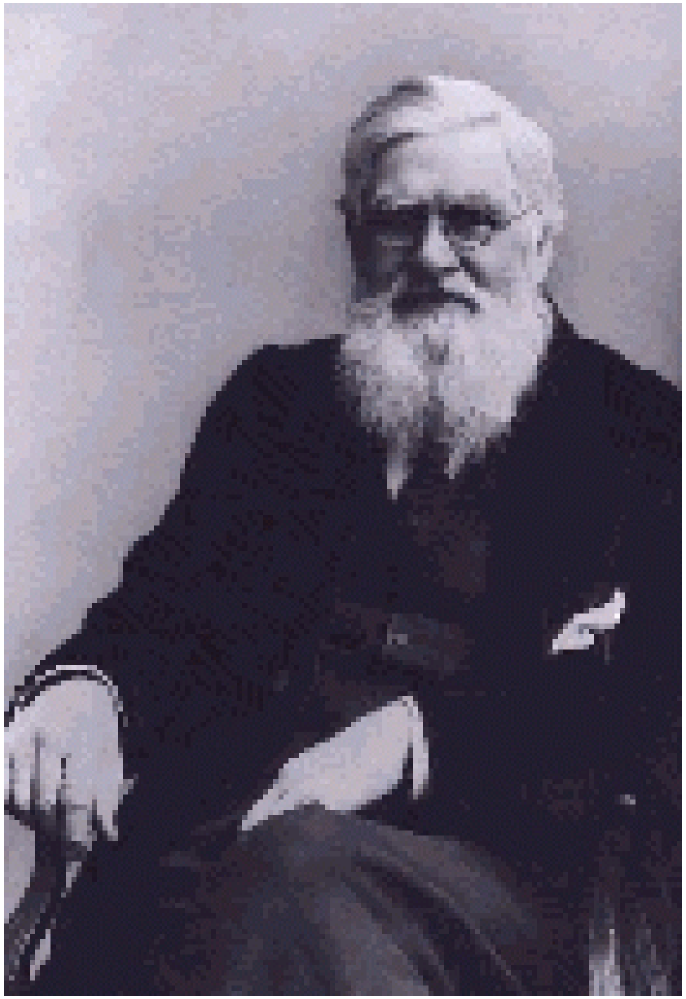
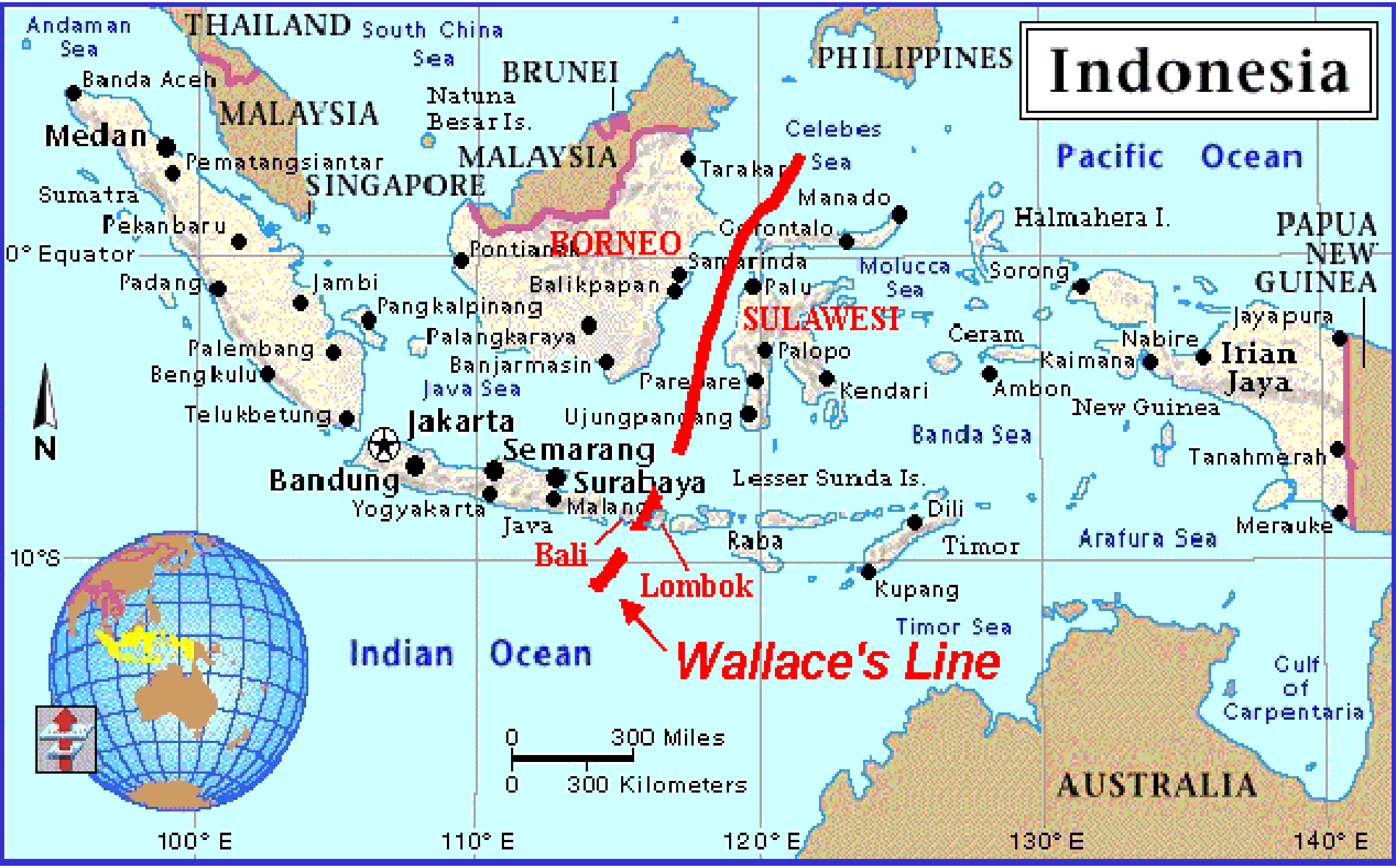
More constraints on system ↓

Physically possible equilibria in nucleosynthesis (constant nucleon number) ↓

More disorder in system ↑

Alfred Russel Wallace




Indonesia
Andaman Sea
THAILAND
South China Sea
BRUNEI
PHILIPPINES
Banda Aceh
MALAYSIA
Natuna Besar Is.
Celebes Sea
Pacific Ocean
Medan
Pematangsiantar
MALAYSIA
SINGAPORE
Tarakan
Sumatra
Manado
Halmahera I.
PAPUA NEW GUINEA
Pekanbaru
Gorontalo
Pontianak
BORNEO
0° Equator
Samarinda
Molucca Sea
Sorong
Padang
Jambi
Balikpapan
Palu
Pangkalpinang
SULAWESI
Jayapura
Palangkaraya
Ceram
Nabire
Palembang
Palopo
Irian Jaya
Banjarmasin
Kaimana
Bengkulu
Java Sea
Parepare
Kendari
Ambon
New Guinea
Telukbetung
Jakarta
Ujungpandang
N
Banda Sea
Semarang
Tanahmerah
Bandung
Surabaya
Lesser Sunda Is.
Yogyakarta
Malang
Dili
Java
Raba
Timor
Arafura Sea
Merauke
10°S
Bali
Lombok
Kupang
Timor Sea
Indian Ocean
Wallace's Line
Gulf of Carpentaria
0 300 Miles
0 300 Kilometers
AUSTRALIA
100° E
110° E
120° E
130° E
140° E

(n,gamma)-(gamma,n) equilibrium

- Nuclei in equilibrium under exchange of neutrons
- Beta decay: $$\lambda_Z Y_Z = \sum_A \lambda(Z,A) Y(Z,A)$$

where

$$Y_Z = \sum_A Y(Z,A)$$

and

then

$$\frac{dY_Z}{dt} = \lambda_{Z-1} Y_{Z-1} - \lambda_Z Y_Z$$

Tasks for today

- NSE
  - Compute the NSE abundances from the expressions in slides 13 and 14
  - Determine at what time record in alpha-rich.fits (fits file from yesterday) that the network abundances fall out of NSE
- QSE
  - Confirm the expressions in slide 17 by minimizing the free energy f subject to the constraints in slide 16
  - Confirm that the network abundances in alpha-rich.fits satisfy these relations after they fall out of NSE

Tasks for Today (cont.)

- R Process
  - Set up the constraint equations for the (n,gamma)-(gamma,n) phase of the r-process freezeout
  - Confirm (n,gamma)-(gamma,n) equilibrium in the r-process fits file rprocess.fits.
  - Confirm beta-decay steady state for a time record during the r-process phase.