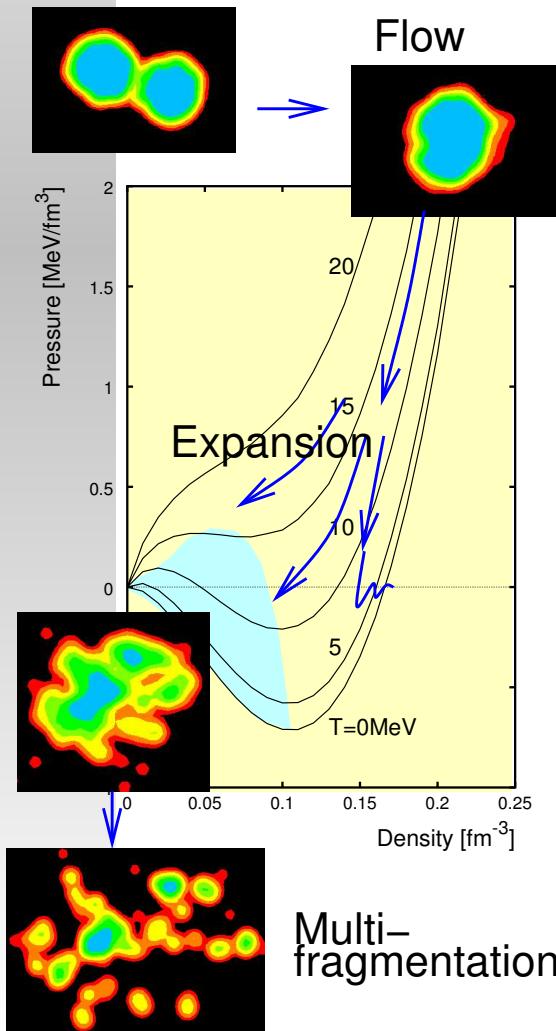


Effect of symmetry energy in fragment isospin composition

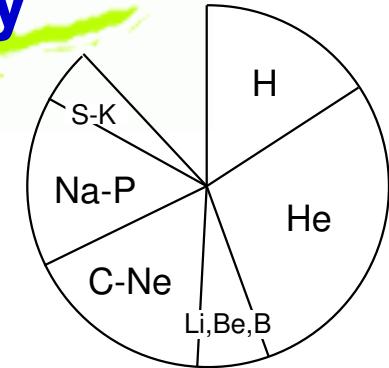
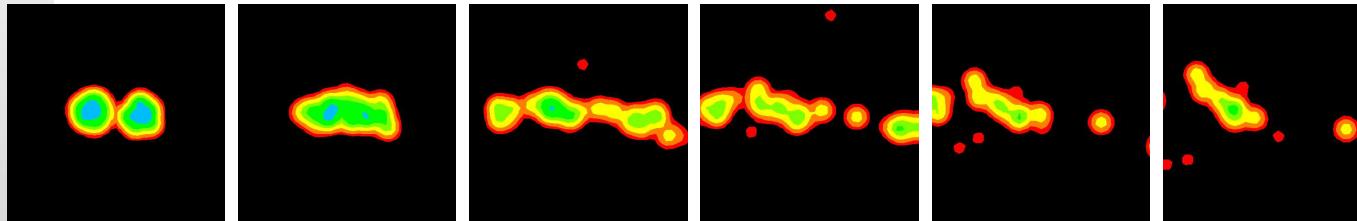
Akira Ono (Tohoku University, currently at MSU)



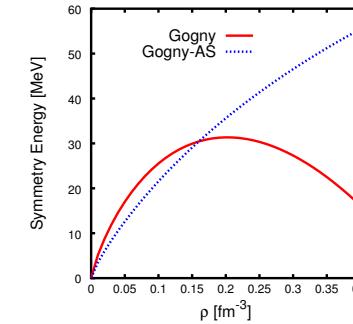
- How do dynamical collisions reflect EOS? Equilibrium?
 - High and low densities
 - Symmetry energy
- How are fragments formed? Dynamical models for fragmentation.
 - Based on mean field approximation.
 - Quantum many-body dynamics in very highly excited systems.

Antisymmetrized Molecular Dynamics

Fragmentation and Symmetry Energy



Fragments \Leftrightarrow Symmetry energy



- Low-density EOS of uniform nuclear matter?

$$E(\rho, \delta, T)/A = E(\rho, 0, T)/A + C(\rho, T)\delta^2 + \dots$$

- Symmetry energies of isolated excited nuclei?

$$-\text{BE}(N, Z) = a_v A + a_s A^{2/3} + [c_v + c_s A^{-1/3}] (N - Z)^2 / A + \dots$$

- Complicated effects through dynamics?

What is the effect of the secondary decay of excited fragments?

Conclusion

Fragment isospin composition reflects the symmetry energy $C_{\text{sym}}(\rho)$ for uniform nuclear matter at a reduced density $\rho \sim \frac{1}{2}\rho_0$.

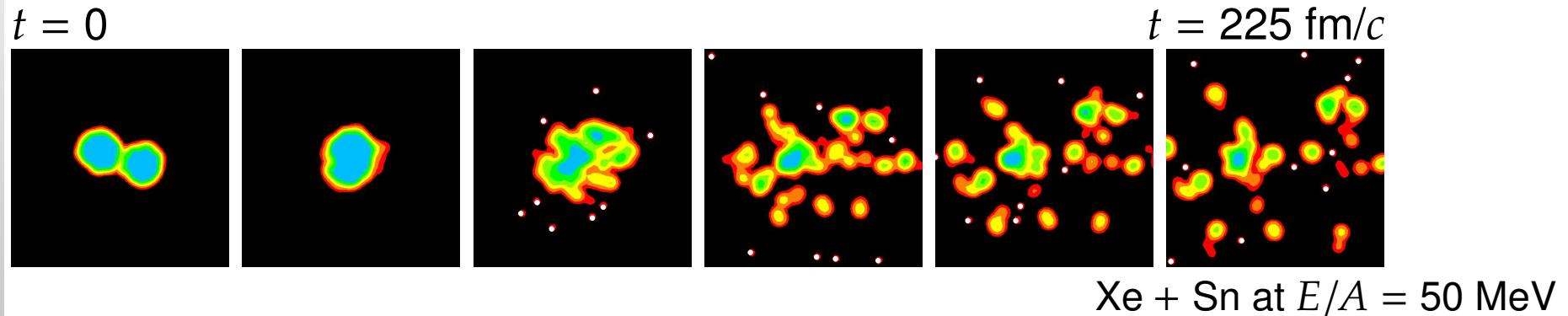
This conclusion is based on the AMD simulations:

- Dependence on the symmetry energy term in the effective interaction (Gogny and Gogny-AS).
- Dependence on the fragment size.

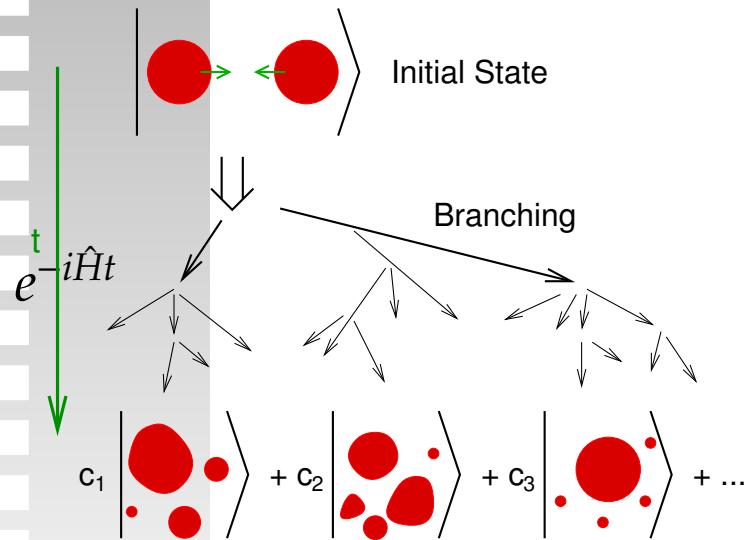
The above conclusion is for primary fragments at $t = 300 \text{ fm}/c$. The effect of secondary decay should be taken into account in order to compare the calculation with the data.

Antisymmetrized Molecular Dynamics

To calculate the time evolution of reactions up to 200 fm/c.



AMD wave function for each branch



$$|\Phi(Z)\rangle = \det_{ij} \left[\exp \left\{ -\nu \left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}} \right)^2 \right\} \chi_{\alpha_i}(j) \right]$$

$$\mathbf{Z}_i = \sqrt{\nu} \mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}} \mathbf{K}_i$$

ν : Width Parameter = 0.16 fm $^{-2}$

χ_{α_i} : Spin-Isospin States = $p \uparrow, p \downarrow, n \uparrow, n \downarrow$

Total state: $|\Psi(t)\rangle\langle\Psi(t)| = \int |\Phi(Z)\rangle\langle\Phi(Z)| w(Z, t) dZ$

Summary of AMD Equation of Motion

AMD wave function

$$|\Phi(\mathbf{Z})\rangle = \det_{ij} \left[\exp \left\{ -\nu (\mathbf{r}_i - \mathbf{Z}_j / \sqrt{\nu})^2 \right\} \chi_{\alpha_j}(i) \right]$$

Stochastic equation of motion for the centroids $Z = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_A\}$

$$\frac{d}{dt} \mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + (\text{NN coll}) + \Delta \mathbf{Z}_i(t) + \mu(\mathbf{Z}_i, \mathcal{H}')$$

- Deterministic term $\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$ \Leftarrow Effective Force
- Stochastic two-nucleon collision process. \Leftarrow In-medium σ_{NN}
- Fluctuation $\Delta \mathbf{Z}_i(t)$ \Leftarrow Mean Field \Leftarrow Effective Force
- Dissipation term $\mu(\mathbf{Z}_i, \mathcal{H}')$ \Leftarrow to compensate the energy violation by fluctuation.

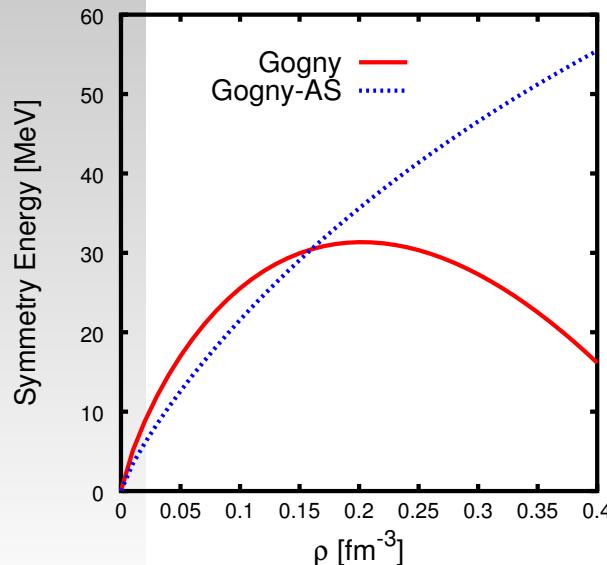
Effective forces and symmetry energies

Gogny-type Forces

$$v_{ij} = \sum_{k=1}^2 (W_k + B_k P_\sigma - H_k P_\tau - M_k P_\sigma P_\tau) e^{-(\mathbf{r}_i - \mathbf{r}_j)^2/a_k^2} + t_\rho (1 + x P_\sigma) \rho(\mathbf{r}_i)^\sigma \delta(\mathbf{r}_i - \mathbf{r}_j) + (1 - x) t_3 \rho_0^\sigma P_\sigma \delta(\mathbf{r}_i - \mathbf{r}_j)$$

EOS and symmetry energy of uniform nuclear matter

$$E(\rho_n, \rho_p, T)/A = E_0(\rho, T)/A + C(\rho, T) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + \dots$$

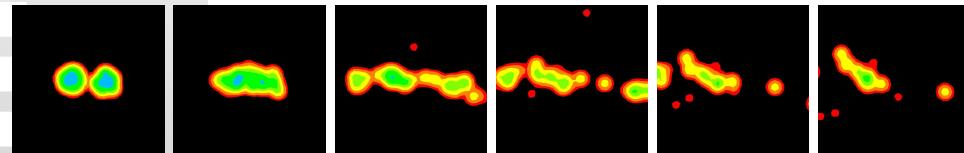


Symmetry energy $C(\rho, 0)$ at $T = 0$

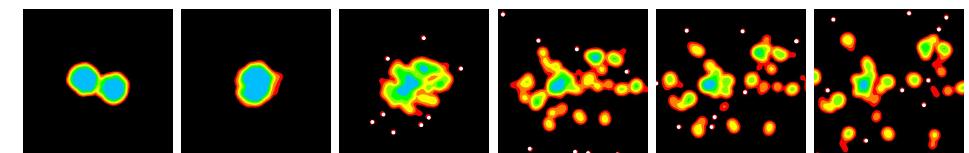
- $x = 1$: Gogny Force
- $x = -\frac{1}{2}$: Gogny-AS Force

AMD results for fragmentation

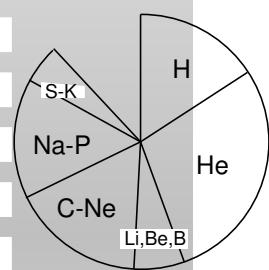
$^{40}\text{Ca} + ^{40}\text{Ca}$ at 35 MeV/u, $b = 0$



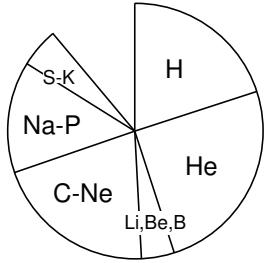
$\text{Xe} + \text{Sn}$ at 50 MeV/u, $0 \leq b \leq 4$ fm



Experiment



AMD

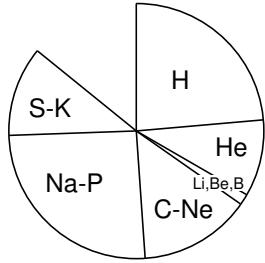


(Gogny force)

Soft EOS,

p -dep U

AMD



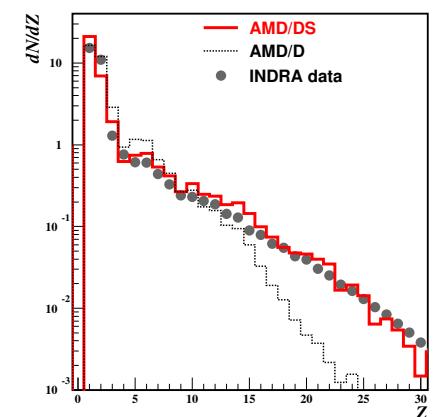
(SKG2 force)

Stiff EOS,

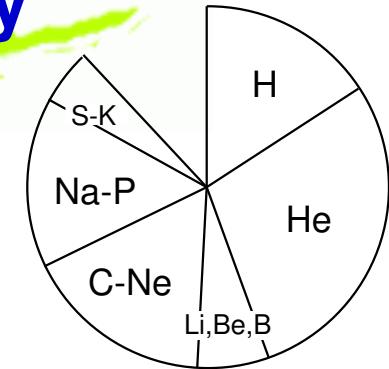
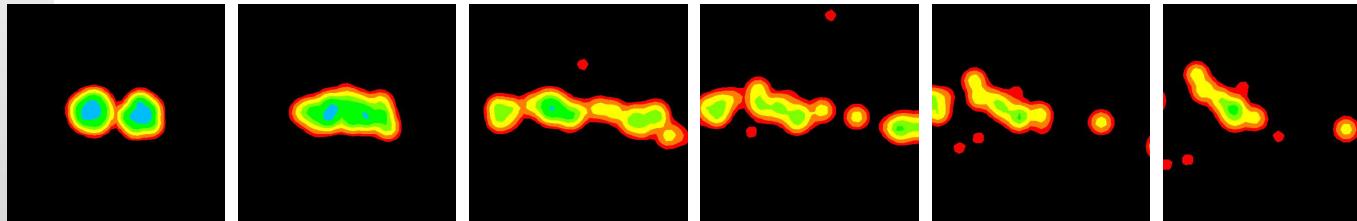
p -indep U

AMD with $\tau \rightarrow 0$.

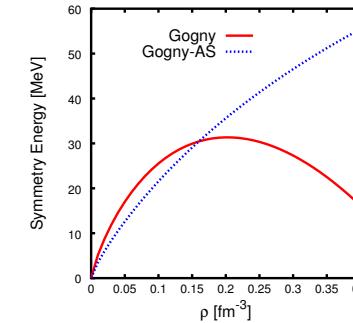
- AMD-V ($\tau = 0$)
- AMD/DS ($\tau = \tau_{NN}$)



Fragmentation and Symmetry Energy



Fragments \Leftrightarrow Symmetry energy



- Low-density EOS of uniform nuclear matter?

$$E(\rho, \delta, T)/A = E(\rho, 0, T)/A + C(\rho, T)\delta^2 + \dots$$

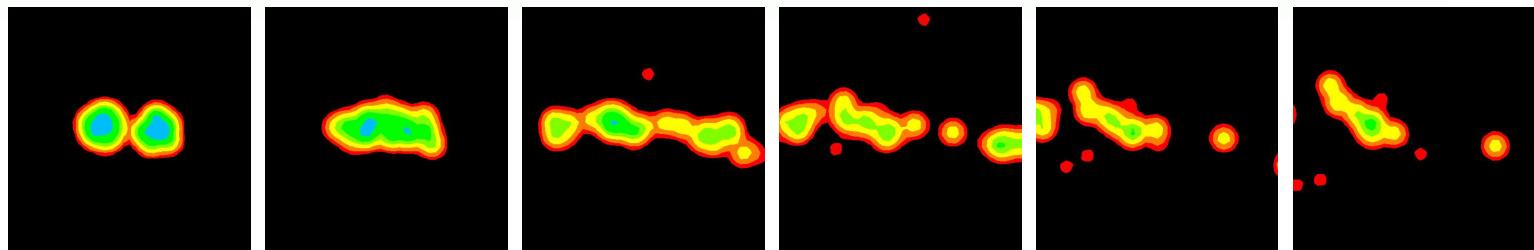
- Symmetry energies of isolated excited nuclei?

$$-\text{BE}(N, Z) = a_v A + a_s A^{2/3} + [c_v + c_s A^{-1/3}] (N - Z)^2 / A + \dots$$

- Complicated effects through dynamics?

What is the effect of the secondary decay of excited fragments?

Isoscaling in dynamical collisions



AMD, $t = 300 \text{ fm}/c$

Isoscaling

— Fragment yeilds from two systems

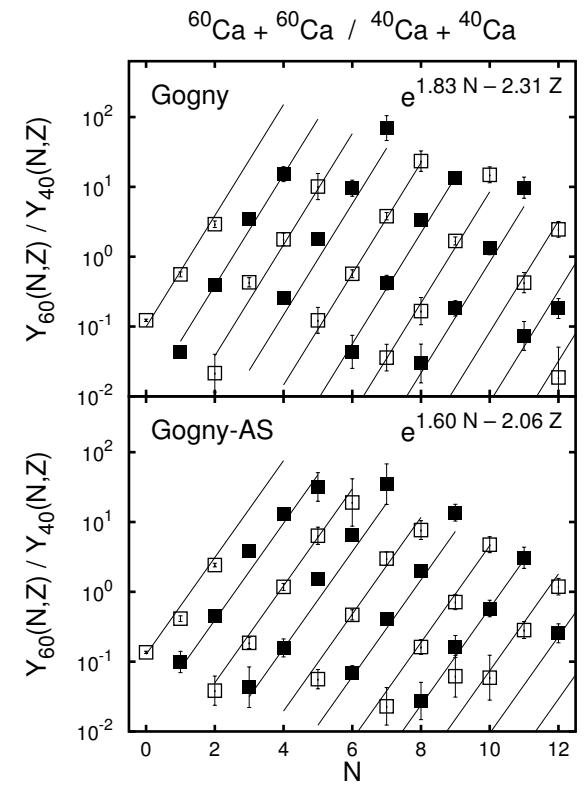
1: ${}^{40}\text{Ca} + {}^{40}\text{Ca}$ at 35 MeV/u

2: ${}^{60}\text{Ca} + {}^{60}\text{Ca}$ at 35 MeV/u

$$\frac{Y_2(N, Z)}{Y_1(N, Z)} \propto e^{\alpha N + \beta Z}$$

Observed in experiments, statistical calculations, and dynamical calculations.

The fragment isospin composition is largely governed by a statistical law (?)



Three characteristic quantities of $\gamma(N, Z)$

Isotope yeilds $\gamma_i(N, Z)$, i : Reaction system

- Isoscaling parameter: α

$$\alpha = \alpha_{ij} = \alpha_i - \alpha_j : \quad \frac{\gamma_i(N, Z)}{\gamma_j(N, Z)} = e^{\alpha N + \beta Z}$$

- Mean fragment isospin asymmetry: $(Z/A)_i^2$

$$(Z/A)_i^2 = \left(\frac{Z}{\bar{A}_i(Z)} \right)^2, \quad \bar{A}_i(Z) = \frac{\sum_N (N+Z) \gamma_i(N, Z)}{\sum_N \gamma_i(N, Z)}$$

- The width of isostope distribution: $\zeta(Z) \propto 1/(\text{width})^2$

$$\gamma_i(N, Z) = \exp[-K(N, Z) + \alpha_i N + \beta_i Z] \quad \Leftrightarrow \text{Isoscaling}$$

$$K(N, Z) = \xi(Z)N + \eta(Z) + \zeta(Z) \frac{(N-Z)^2}{N+Z}$$

A relation among these three quantities (without any more assumptions):

$$\alpha_{ij} = 4\zeta(Z) \times \left[(Z/A)_j^2 - (Z/A)_i^2 \right]$$

An equilibrium relation

For the grandcanonical ensemble of fragments within a freeze-out volume,

$$Y_i(N, Z) \propto \exp\left[-\frac{G_{\text{nuc}}(N, Z, T, P)}{T} + \frac{\mu_{ni}}{T}N + \frac{\mu_{pi}}{T}Z\right]$$

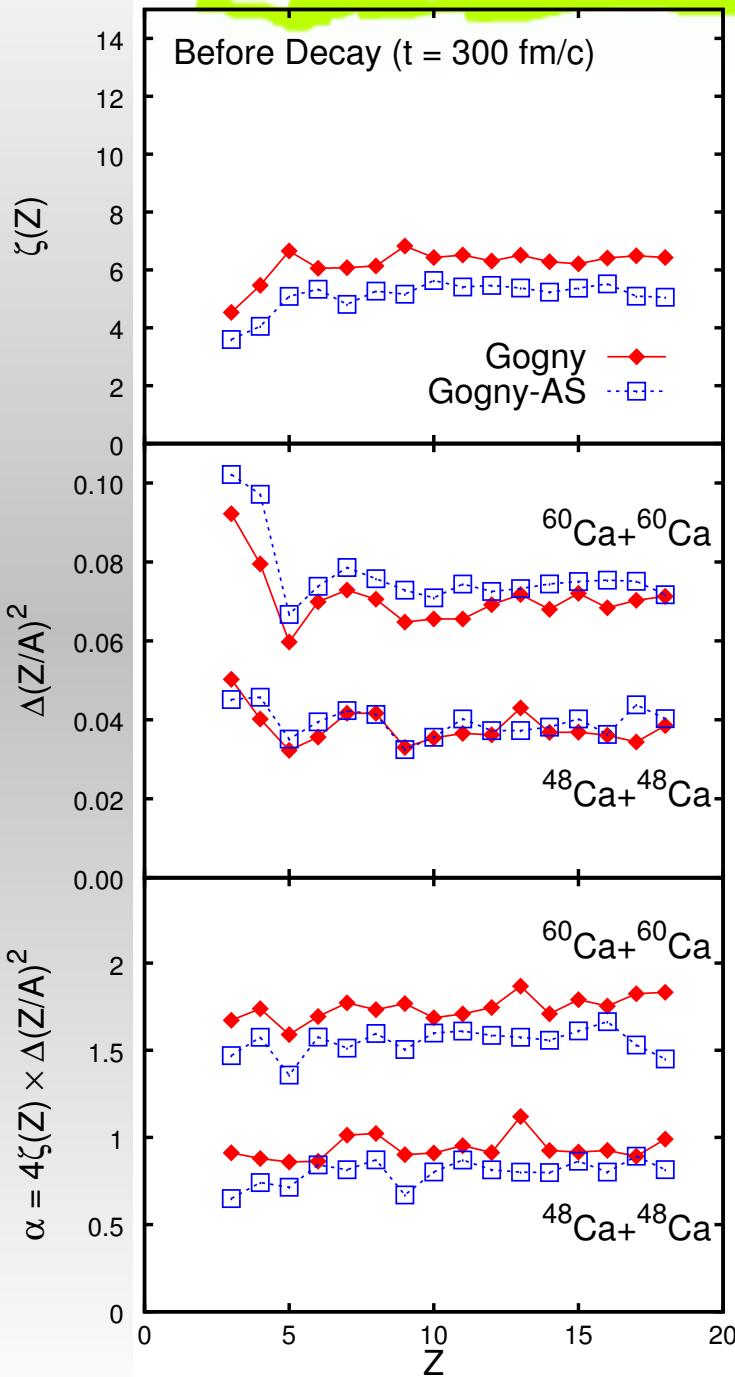
$$G_{\text{nuc}}(N, Z) = \dots + C_{\text{sym}} \frac{(N - Z)^2}{N + Z} + \dots$$

$$\Rightarrow \zeta = \frac{C_{\text{sym}}}{T}$$

$$\text{Equilibrium relation: } \alpha_{ij} = 4 \frac{C_{\text{sym}}}{T} \left[(Z/A)_j^2 - (Z/A)_i^2 \right]$$

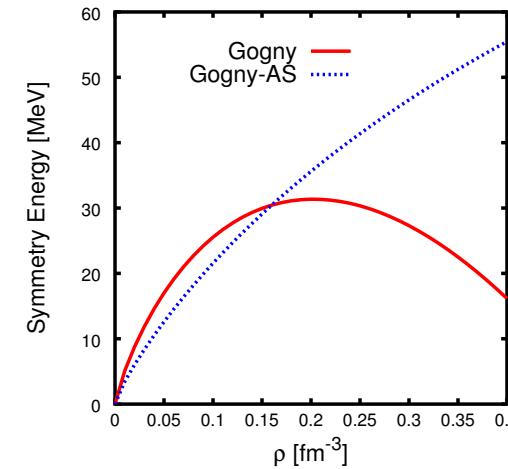
However, in general cases, ζ may not be directly related to C_{sym} . We should check whether $\zeta(Z)$ behaves like C_{sym}/T .

Fragment isospin composition at $t = 300 \text{ fm}/c$

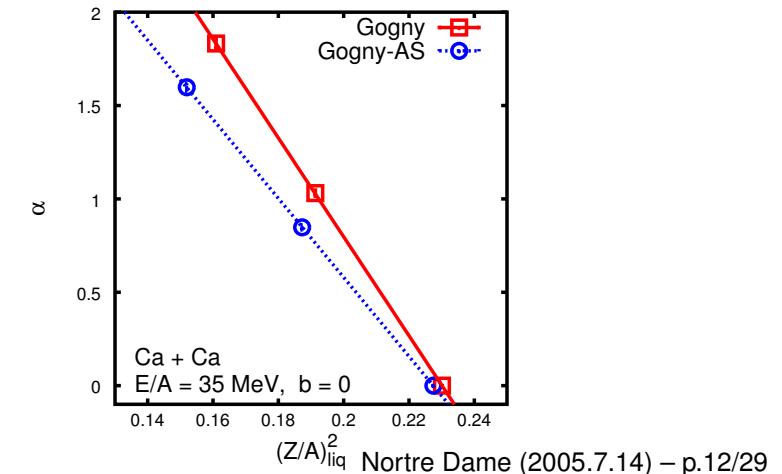


$^{60}\text{Ca} + ^{60}\text{Ca}, ^{48}\text{Ca} + ^{48}\text{Ca}, ^{40}\text{Ca} + ^{40}\text{Ca}$
 $b = 0, E/A = 35 \text{ MeV}, t = 300 \text{ fm}/c$

Gogny force and Gogny-AS force

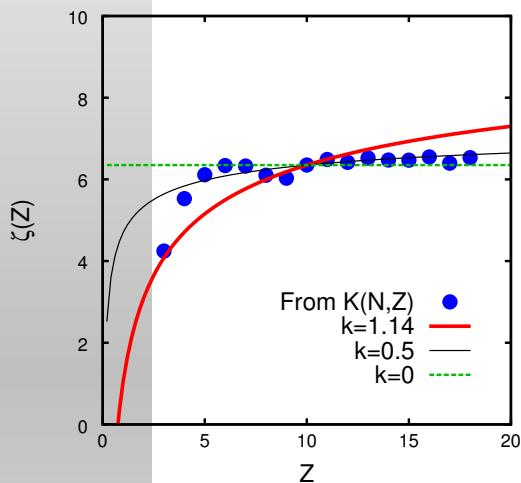


Liquid: Sum of fragments with $A > 4$.



What we can learn from $\zeta(Z)$ at $t = 300 \text{ fm}/c$

Z -dependence of $\zeta(Z)$



$\zeta(Z)$ and C_{sym}

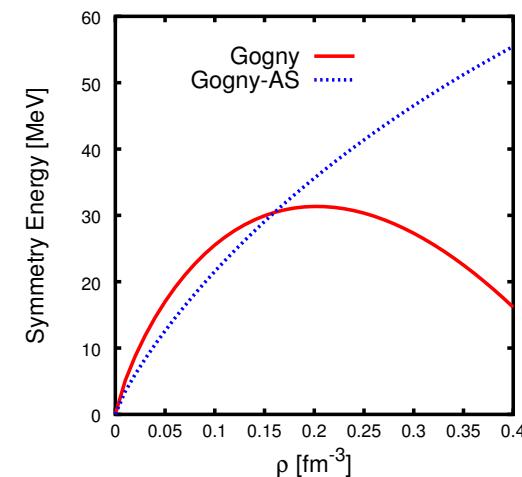
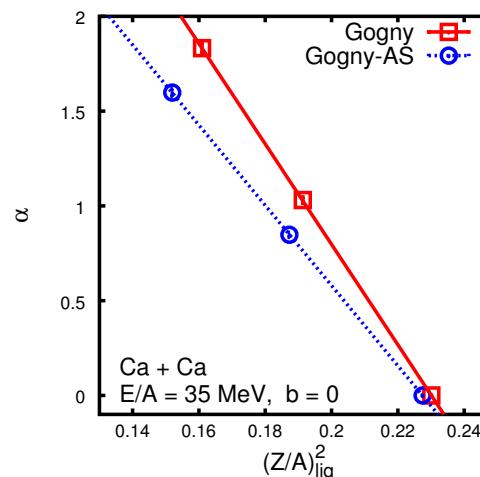
= Volume + Surface

$$\propto 1 - k(2Z)^{-1/3}$$

Very weak Z -dependence.



$$\text{Equilibrium: } Y_{NZ} \sim \exp\left[-\frac{C_{\text{sym}}}{T} \frac{(N-Z)^2}{N+Z}\right] \Rightarrow \zeta = \frac{C_{\text{sym}}}{T}$$



● $4\zeta(\text{Gogny}) = 26.5$

● $4\zeta(\text{Gogny-AS}) = 21.2$

\Leftrightarrow

● $\rho \sim 0.08 \text{ fm}^{-3}$

● $T \sim 3.4 \text{ MeV}$

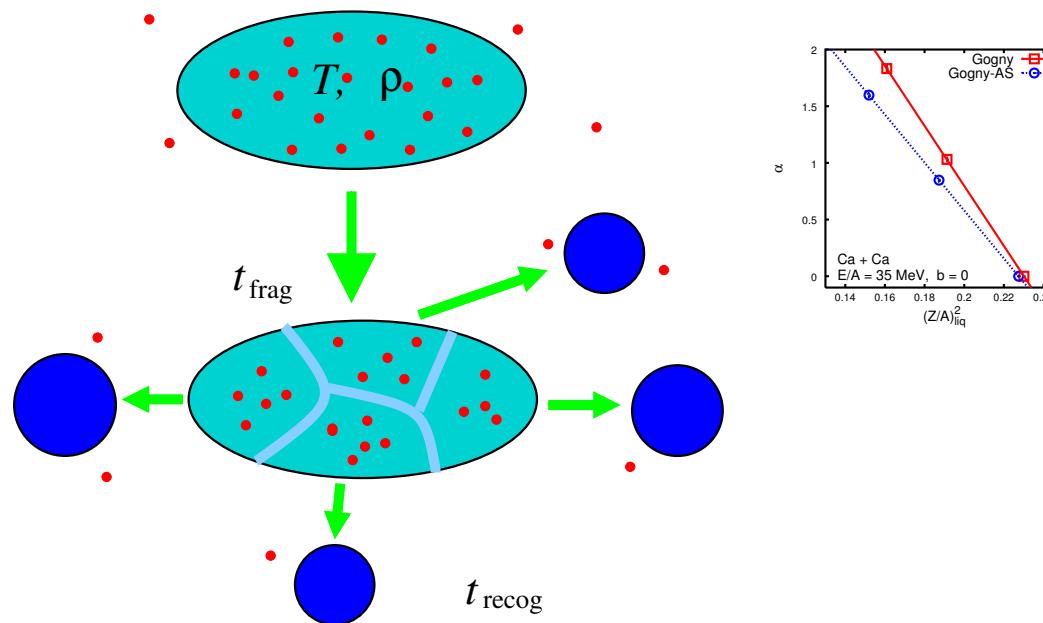
Ono, Danielewicz, Friedman, Lynch, Tsang,

PRC **68**, 051601(R) (2003); PRC **70**, 041604(R) (2004).

A mechanism suggested by AMD simulations

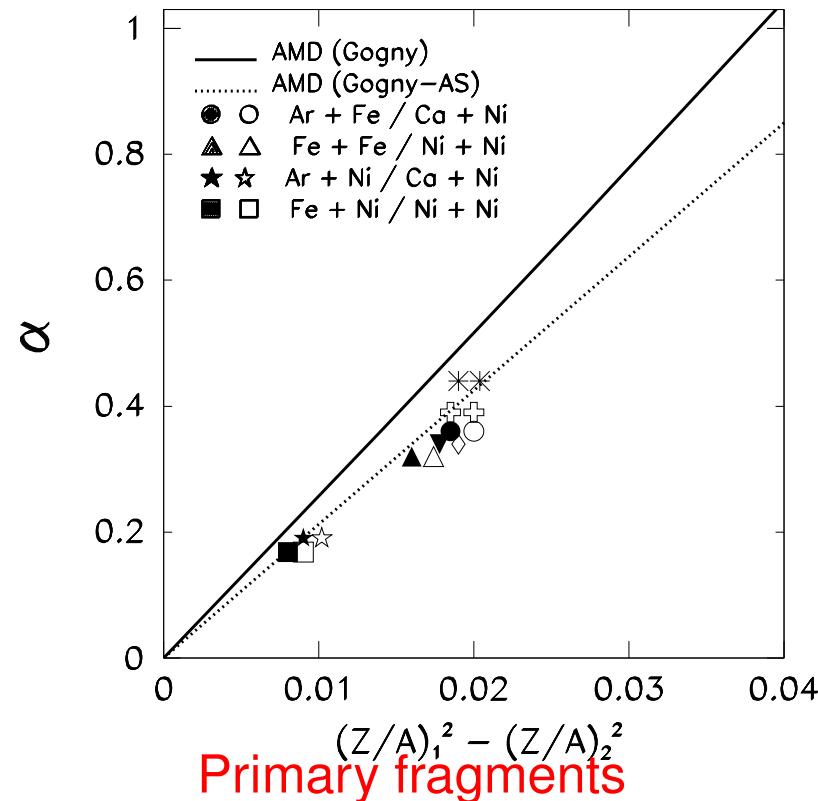
The primary fragments (recognized at $t = 300 \text{ fm}/c$) still remember some information of low-density uniform nuclear matter from which they were formed.

Especially, the isospin composition of primary fragments is governed by $C_{\text{sym}}(\rho \sim \frac{1}{2}\rho_0)/T$, where $C_{\text{sym}}(\rho \sim \frac{1}{2}\rho_0)$ is the symmetry energy of uniform nuclear matter at a reduced density.



Comparison with data (maybe incomplete)

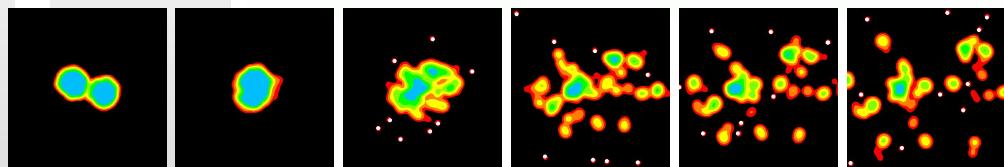
Shetty, Yenello et al., PRC70(2004)011601(R), nucl-ex/0505011.



To be confirmed:

- The change of α by the secondary decay of primary fragments.
- The relation among $(Z/A)_{\text{total}}^2 \Leftrightarrow (Z/A)_{\text{primary}}^2 \Leftrightarrow (Z/A)_{\text{final}}^2$.
Dependences on reaction system, impact parameter, ...

Sequential secondary decay



AMD \Rightarrow Fragments at $t = 300 \text{ fm}/c$ (for example)

Each fragment: $N, Z, E_{\text{internal}}$ or $E^*, \langle \mathbf{L} \rangle_{\text{internal}}, \mathbf{P}$

Sequential decay

$$1(N_1, Z_1, E_1^*, J_1) \rightarrow 2(N_2, Z_2, E_2^*, J_2) + 3(N_3, Z_3, E_3^*, J_3)$$

Statistical decay assumption

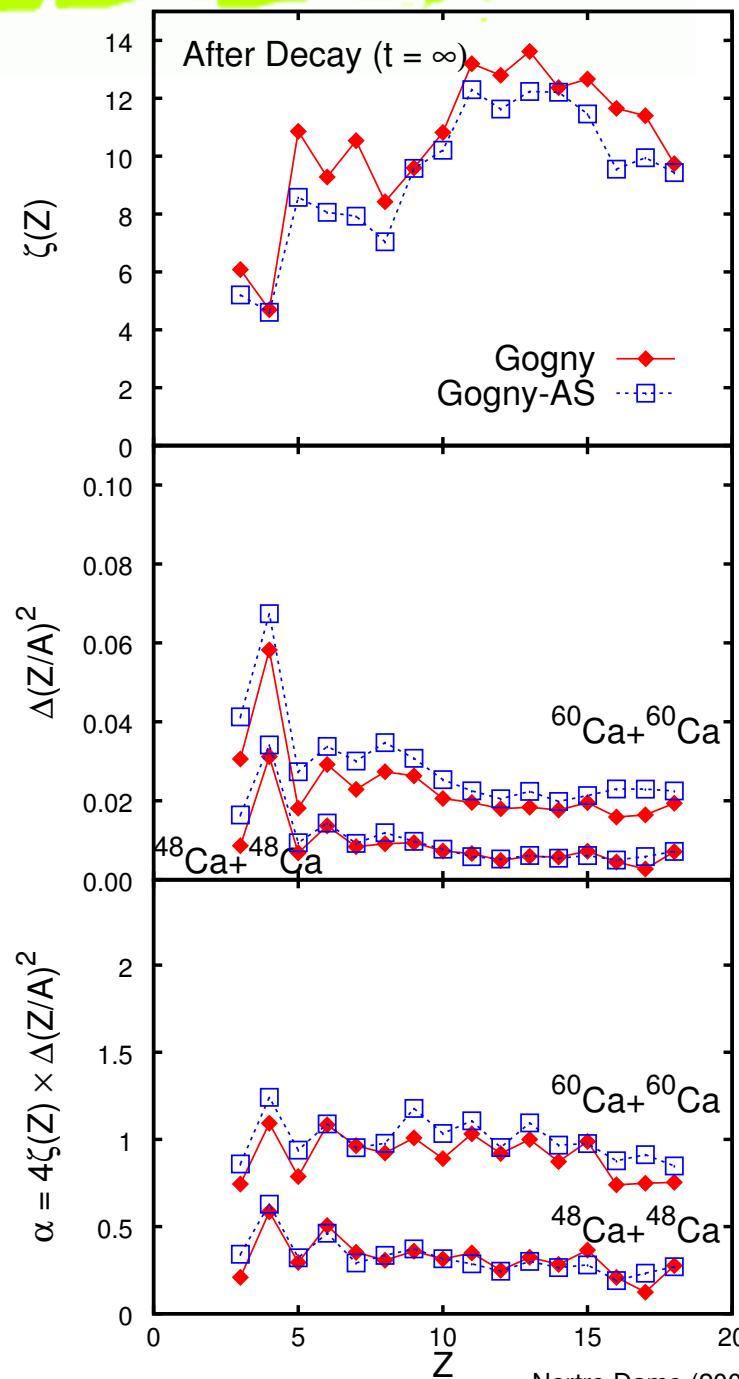
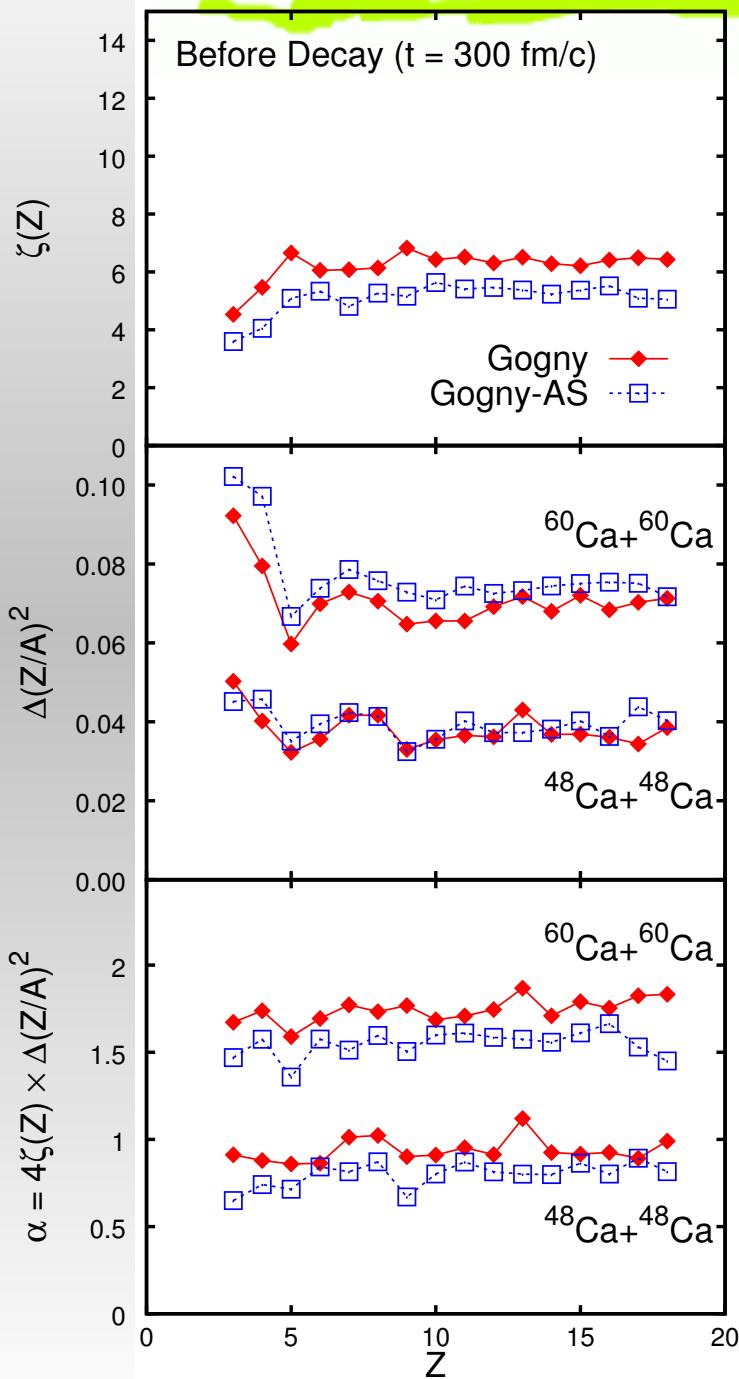
$$\Gamma_{1 \rightarrow 2+3} = \frac{\rho_2(E_2^*, J_2) \rho_3(E_3^*, J_3)}{2\pi\rho_1(E_1^*, J_1)} \sum_{J_{23}-|J_2-J_3|}^{J_2+J_3} \sum_{L=|J_1-J_{23}|}^{J_1+J_{23}} T_{2+3 \rightarrow 1}(\epsilon, L)$$

$\rho_i(E^*, J)$: Nuclear level densities

$T_{2+3 \rightarrow 1}(\epsilon, L)$: Fusion Probability at $\epsilon = E_1^* - E_2^* - E_3^* - Q_{123}$

\Rightarrow Final fragment yeilds $Y(N, Z)$

Effect of secondary decay: $t = 300 \text{ fm}/c \Rightarrow t = \infty$



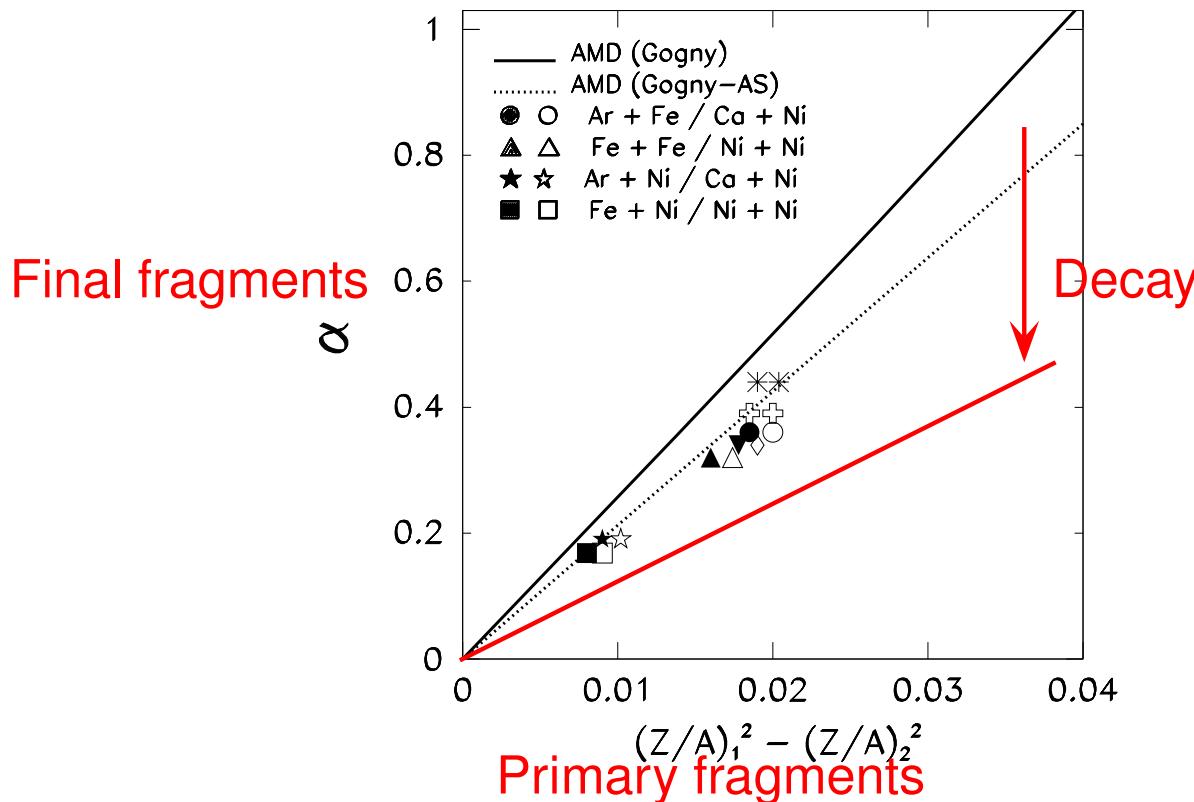
Summary of effect of secondary decay

	Primary fragments		Final fragments
$\zeta(Z)$	Gogny > Gogny-AS	↗	Gogny > Gogny-AS
$\Delta(Z/A)^2$	Gogny < Gogny-AS	↘	Gogny < Gogny-AS
α	Gogny > Gogny-AS	→	Gogny ? Gogny-AS

- The absolute values of $\zeta(Z)$ and $\Delta(Z/A)^2$ change as expected.
- The symmetry energy effect in primary fragments remains in $\zeta(Z)$ and $\Delta(Z/A)^2$ after secondary decay.
- The prediction of α of the final fragments is difficult. We need more precise description of the secondary decay and/or dynamical stages of reaction.

Comparison with data (maybe incomplete)

Shetty, Yenello et al., PRC70(2004)011601(R), nucl-ex/0505011.



To be confirmed:

- The change of α by the secondary decay of primary fragments.
- The relation among $(Z/A)_{\text{total}}^2 \Leftrightarrow (Z/A)_{\text{primary}}^2 \Leftrightarrow (Z/A)_{\text{final}}^2$.
Dependences on reaction system, impact parameter, ...

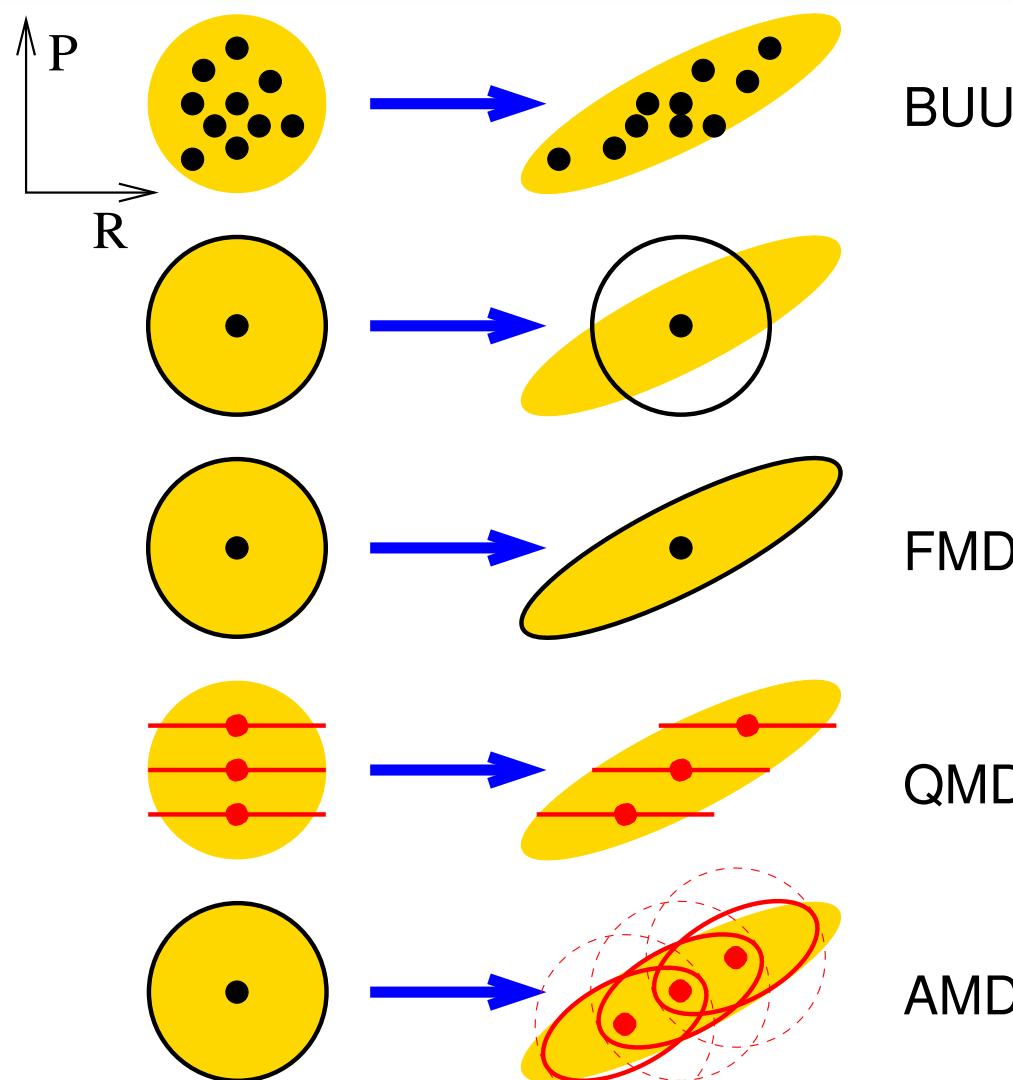
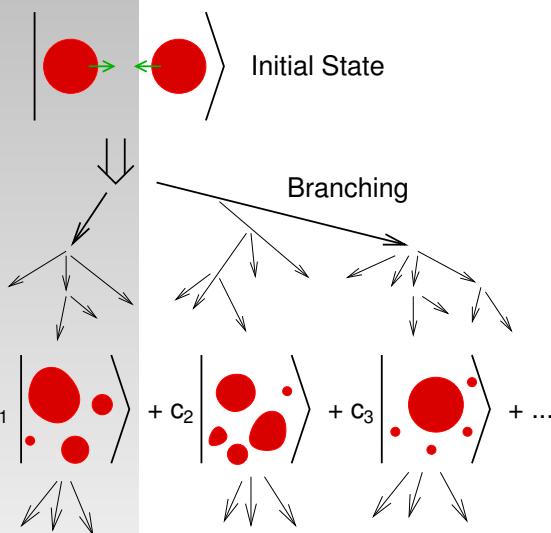
Summary

- AMD simulations suggest how the symmetry energy is reflected in the primary fragments recognized at $t = 300 \text{ fm}/c$:
The symmetry energy $C_{\text{sym}}(\rho \sim \frac{1}{2}\rho_0)/T$ of low-density uniform nuclear matter is reflected in the isospin composition of primary fragments.
- Secondary decay of fragments does not wash out the symmetry energy effects in the primary fragments.
However, to get a conclusion about $C_{\text{sym}}(\rho)/T$ from the experimental data, we need more precise understanding of the secondary decay and/or dynamical stages of reaction.

Various treatments of wave packet width

The simplest example:
Free motion of a nucleon
from a Gaussian packet

$$\Delta x \Delta p \geq \frac{1}{2} \hbar$$



Langevin-like equation of motion

Equation of motion for the wave packet centroids

$$\frac{d}{dt} \mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\} + \text{NN-Collision} + \Delta \mathbf{Z}_i(t) + \mu(\mathbf{Z}_i, \mathcal{H}')$$

If \mathbf{Z}_i were canonical variables for simplicity,

$$\{\mathbf{Z}_i, \mathcal{H}\} = \frac{1}{i\hbar} \frac{\partial \mathcal{H}}{\partial \mathbf{Z}_i^*}$$

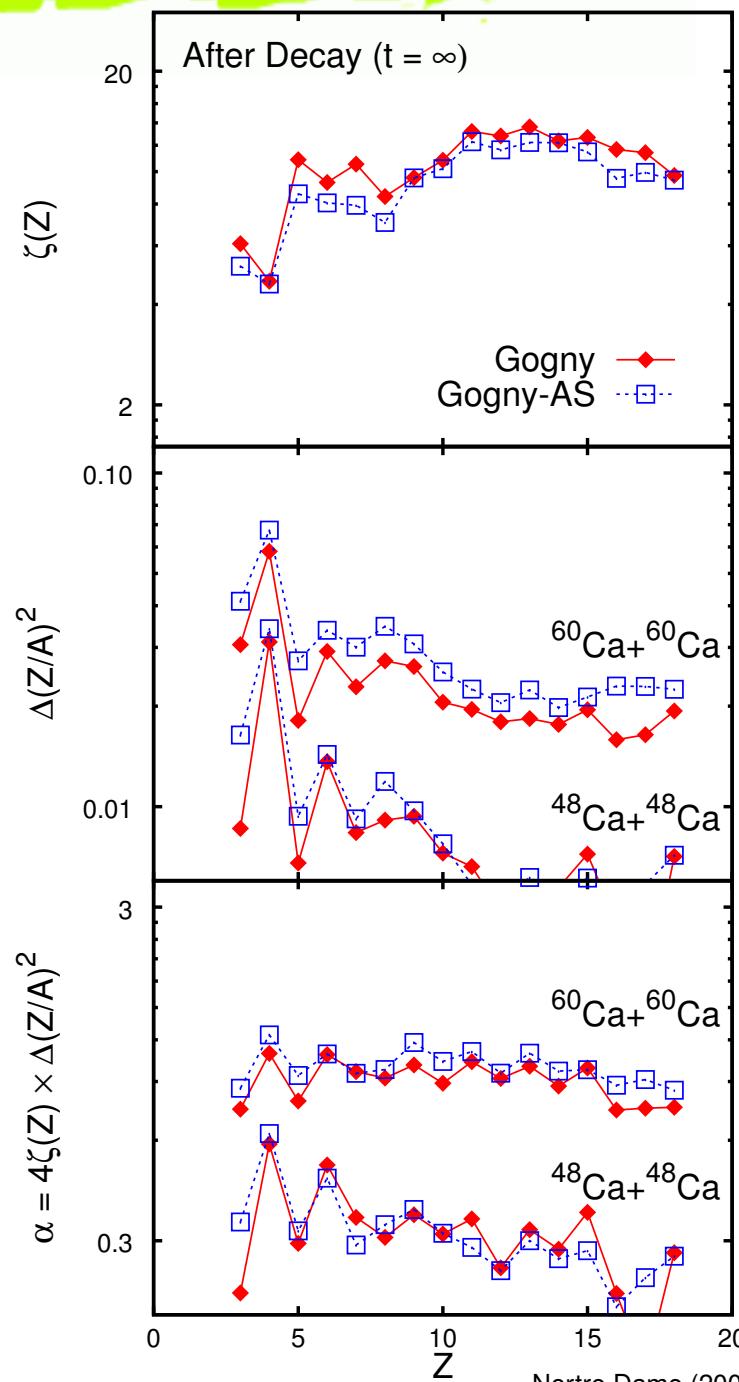
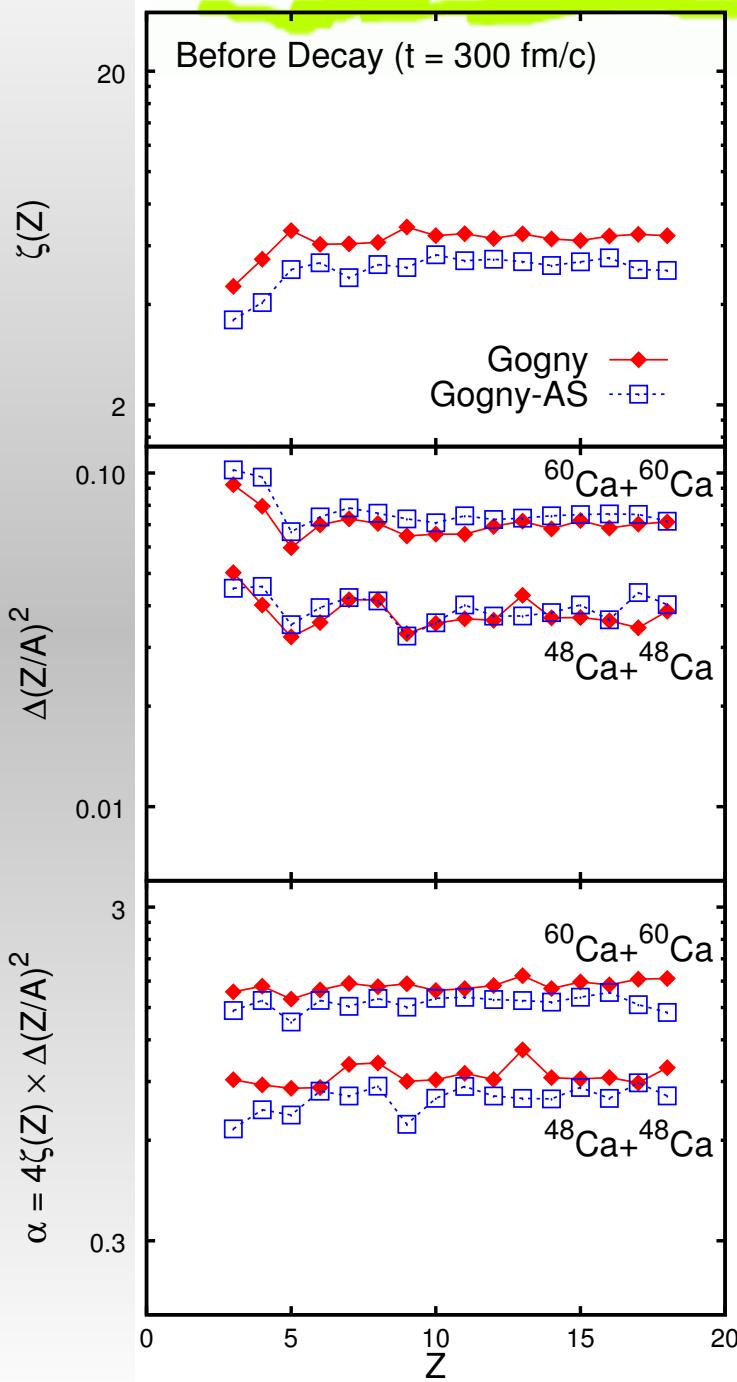
$$\overline{\Delta Z_{ia}(t)} = 0, \quad \overline{\Delta Z_{ia}(t)\Delta Z_{jb}(t)} = D_{iab}(t)\delta_{ij}\delta(t - t')$$

$$(\mathbf{Z}_i, \mathcal{H}') = \frac{1}{\hbar} \frac{\partial \mathcal{H}'}{\partial \mathbf{Z}_i^*}, \quad \mathcal{H}' = \mathcal{H} + \sum_m \beta_m Q_m$$

- μ is determined by the total energy conservation.
- Lagrange multipliers β_m are determined so that Q_m are not changed by the $(\mathbf{Z}_i, \mathcal{H}')$ term.

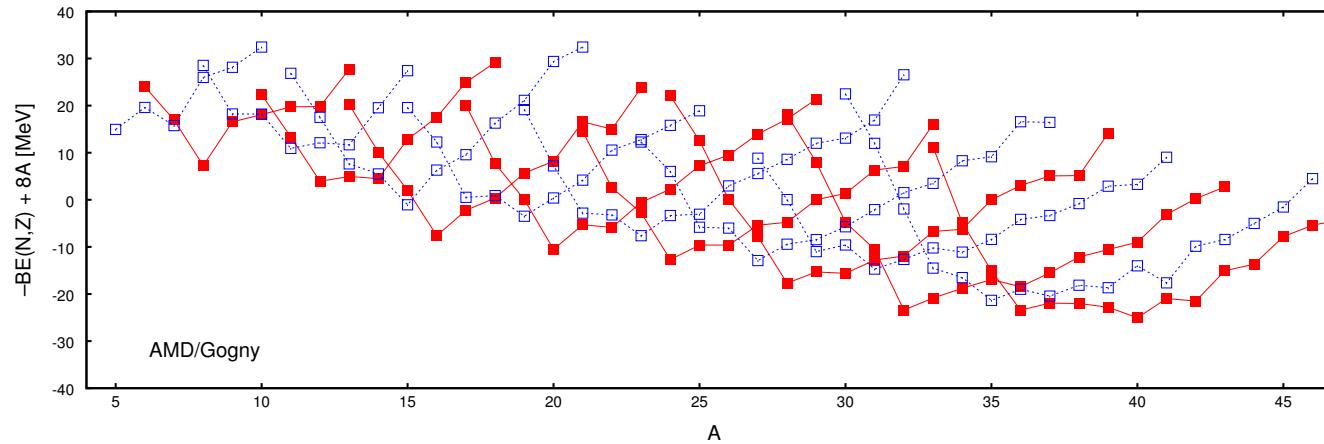
$$\{Q_m\} = \left\{ \left\langle \sum_i \mathbf{r}_i \right\rangle, \left\langle \sum_i \mathbf{p}_i \right\rangle, \left\langle \sum_i \mathbf{r}_i \times \mathbf{p}_i \right\rangle, \left\langle \sum_i r_{i\sigma} r_{i\tau} \right\rangle, \left\langle \sum_i p_{i\sigma} p_{i\tau} \right\rangle \right\} \quad \sigma, \tau = x, y, z$$

Effect of secondary decay: $t = 300 \text{ fm}/c \Rightarrow t = \infty$



Binding energies of ground state nuclei

$$-\text{BE}(N, Z) + 8A \text{ MeV} \Leftarrow \text{AMD/Gogny}$$



The AMD ground state masses are fitted by a liquid-drop formula:

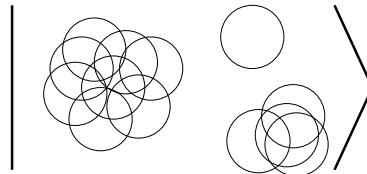
$$-\text{BE}(N, Z) = a_v A + a_s A^{2/3} + a_c Z^2 A^{-1/3} + [c_v + c_s A^{-1/3}] \frac{(N - Z)^2}{A} + \{\pm, 0\} \delta / A^{-1/2}$$
$$a_v = -14.6, \quad a_s = 14.9, \quad c_v = 30.9, \quad c_s = -35.2, \quad a_c = 0.65, \quad \delta = 10.1 \quad \text{in MeV}$$

Development of AMD

AMD wave function

~~ Structure

$$|\Phi(\mathbf{Z})\rangle = \det_{ij} \left[\exp \left\{ -\nu (\mathbf{r}_i - \mathbf{Z}_j / \sqrt{\nu})^2 \right\} \chi_{\alpha_j}(i) \right]$$



-  Time Dependent Variational Principle for $\{\mathbf{Z}_1(t), \dots, \mathbf{Z}_A(t)\}$

$$\delta \int dt \frac{\langle \Phi(\mathbf{Z}) | (i\hbar \frac{d}{dt} - H) | \Phi(\mathbf{Z}) \rangle}{\langle \Phi(\mathbf{Z}) | \Phi(\mathbf{Z}) \rangle} = 0 \quad \Rightarrow \quad \frac{d\mathbf{Z}_i}{dt} = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$$

$\sim\sim$ Dynamical widths $\nu_i(t)$ in FMD (Feldmeier) $\sim\sim$ TDHF

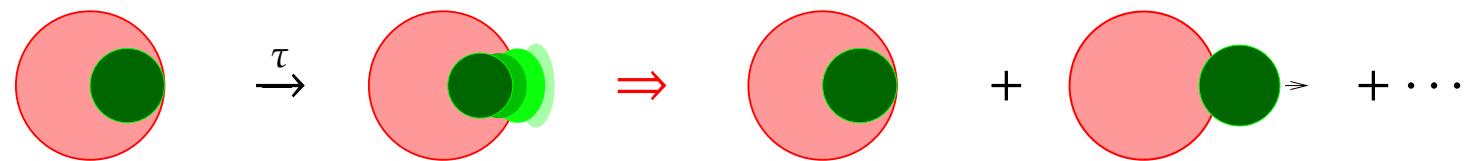
- + Two-nucleon collisions as a stochastic process $\sim\sim$ QMD, BUU
 - PRL68(1992), Prog.Theor.Phys.87(1992): Fragmentation of light projectiles
 - + Fluctuation to the wave packet centroids \Leftarrow Spreading of wave packets.
AMD-V: PRC53(1996), PLB422(1998) : Multifragmentation in Ca + Ca
 - More similarity to mean field models, for the short-time one-body dynamics.
PRC66(2002): Multifragmentation in Xe + Sn

Mean field + Quantum branching

At each time step t_0 , for each wave packet k, \dots

Mean field propagation for $t_0 \rightarrow t_0 + \tau$ + Branching at $t_0 + \tau$ τ : Coherence time

$$|Z_k\rangle\langle Z_k| \xrightarrow[\text{Mean field}]{\longrightarrow} |\psi_k\rangle\langle\psi_k| \xrightarrow[\text{Branching}]{\longrightarrow} \int |z\rangle\langle z| w_k(z) dz \quad \text{for } k = 1, \dots, A$$



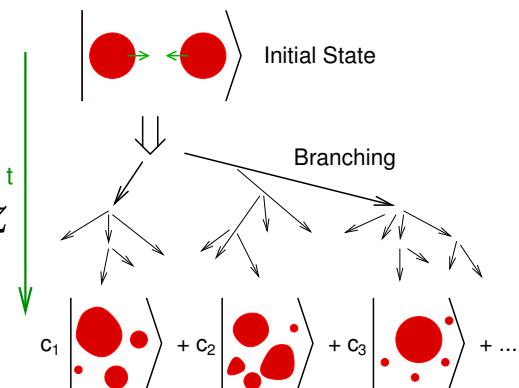
$$i\hbar \frac{d}{dt} |\psi_k(t)\rangle = h^{\text{HF}} |\psi_k(t)\rangle$$

$$\frac{\partial f_k}{\partial t} = -\frac{\partial h^{\text{HF}}}{\partial p} \cdot \frac{\partial f_k}{\partial r} + \frac{\partial h^{\text{HF}}}{\partial r} \cdot \frac{\partial f_k}{\partial p}$$

$$|\Phi(Z)\rangle\langle\Phi(Z)| \quad |\Psi\rangle\langle\Psi| \xrightarrow[\text{Branching}]{\longrightarrow} \int |\Phi(z)\rangle\langle\Phi(z)| w(z) dz$$

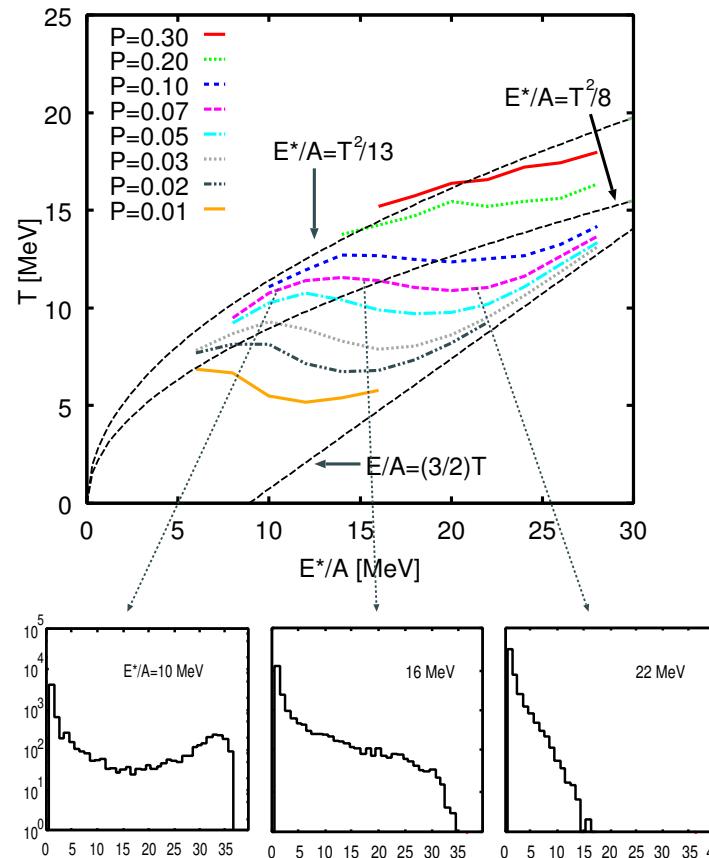
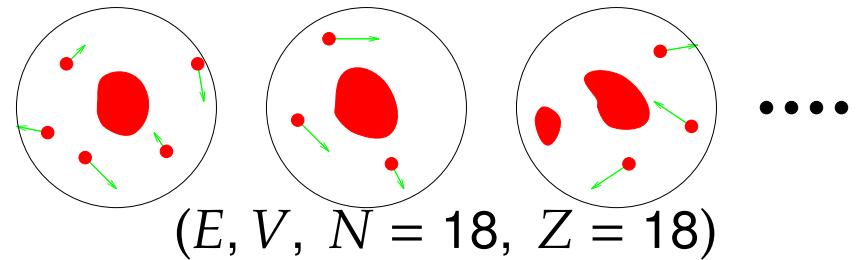
- \bullet $\tau \rightarrow 0$

- \bullet $\tau = \tau_{\text{NN-coll}}$



Caloric curve by AMD

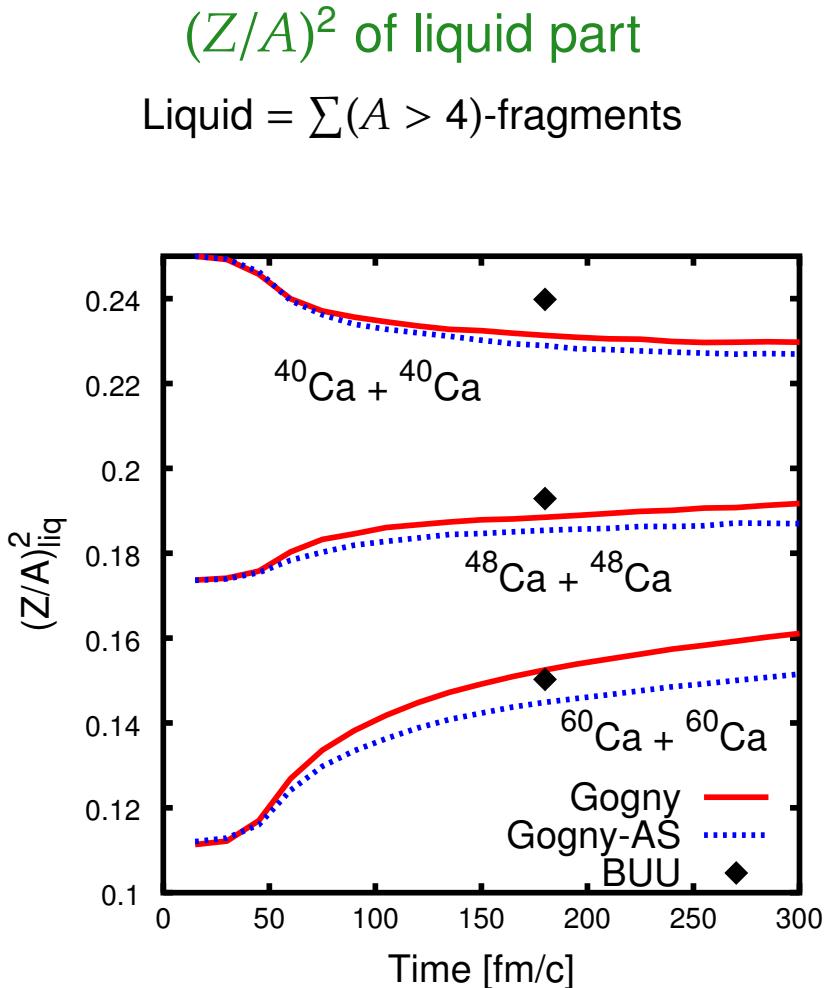
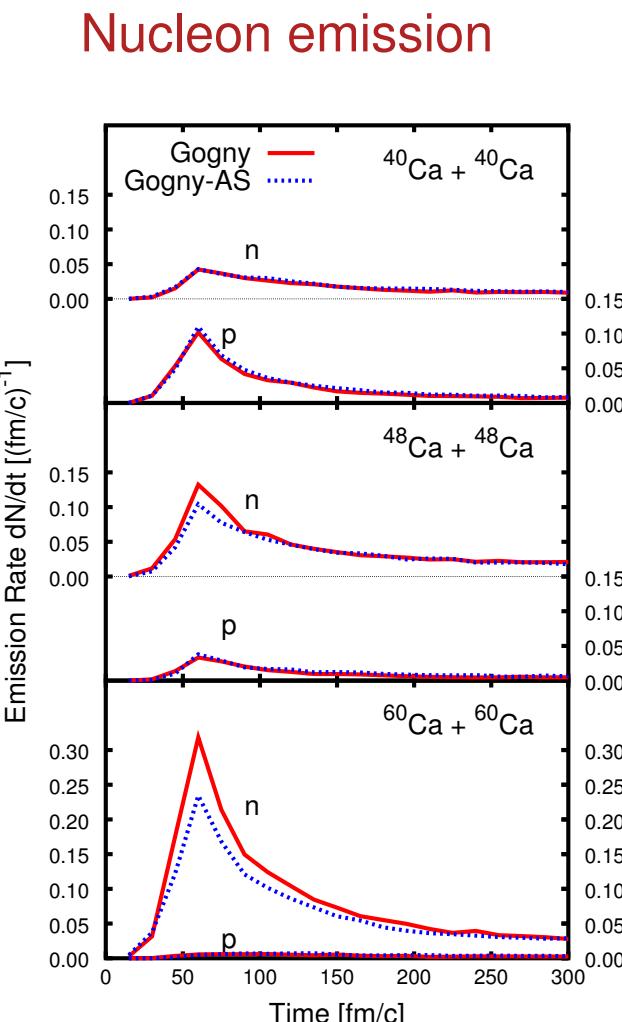
Microcanonical ensemble \Leftarrow Long-time solution of AMD



Fractionation/Distillation in collisions

In neutron-rich systems ($N^{\text{tot}} > Z^{\text{tot}}$),

- Gas part (nucleons): $(N/Z)_{\text{gas}} > N^{\text{tot}}/Z^{\text{tot}}$
- Liquid part (fragments): $(N/Z)_{\text{liq}} < N^{\text{tot}}/Z^{\text{tot}}$



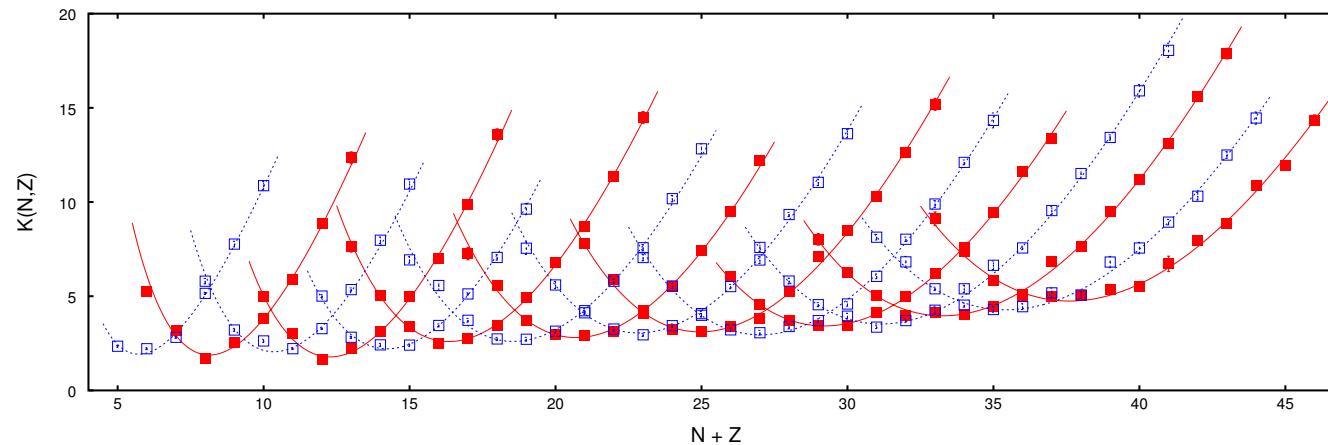
How to get the width $\zeta(Z)$ from $Y_i(N, Z)$

$Y_i(N, Z)$ from many systems

- $i = 1: {}^{40}\text{Ca} + {}^{40}\text{Ca}$
- $i = 2: {}^{48}\text{Ca} + {}^{48}\text{Ca}$
- $i = 3: {}^{60}\text{Ca} + {}^{60}\text{Ca}$
- $i = 4: {}^{46}\text{Fe} + {}^{46}\text{Fe}$

By employing isoscaling,

$$\left. \begin{aligned} & Y_1(N, Z) \\ & \approx Y_2(N, Z) e^{-\alpha_2 N - \beta_2 Z} \\ & \approx Y_3(N, Z) e^{-\alpha_3 N - \beta_3 Z} \\ & \approx Y_4(N, Z) e^{-\alpha_4 N - \beta_4 Z} \end{aligned} \right\} \equiv e^{-K(N, Z)}$$



$$K(N, Z) = \xi(Z)N + \eta(Z) + \zeta(Z) \frac{(N - Z)^2}{N + Z}$$