

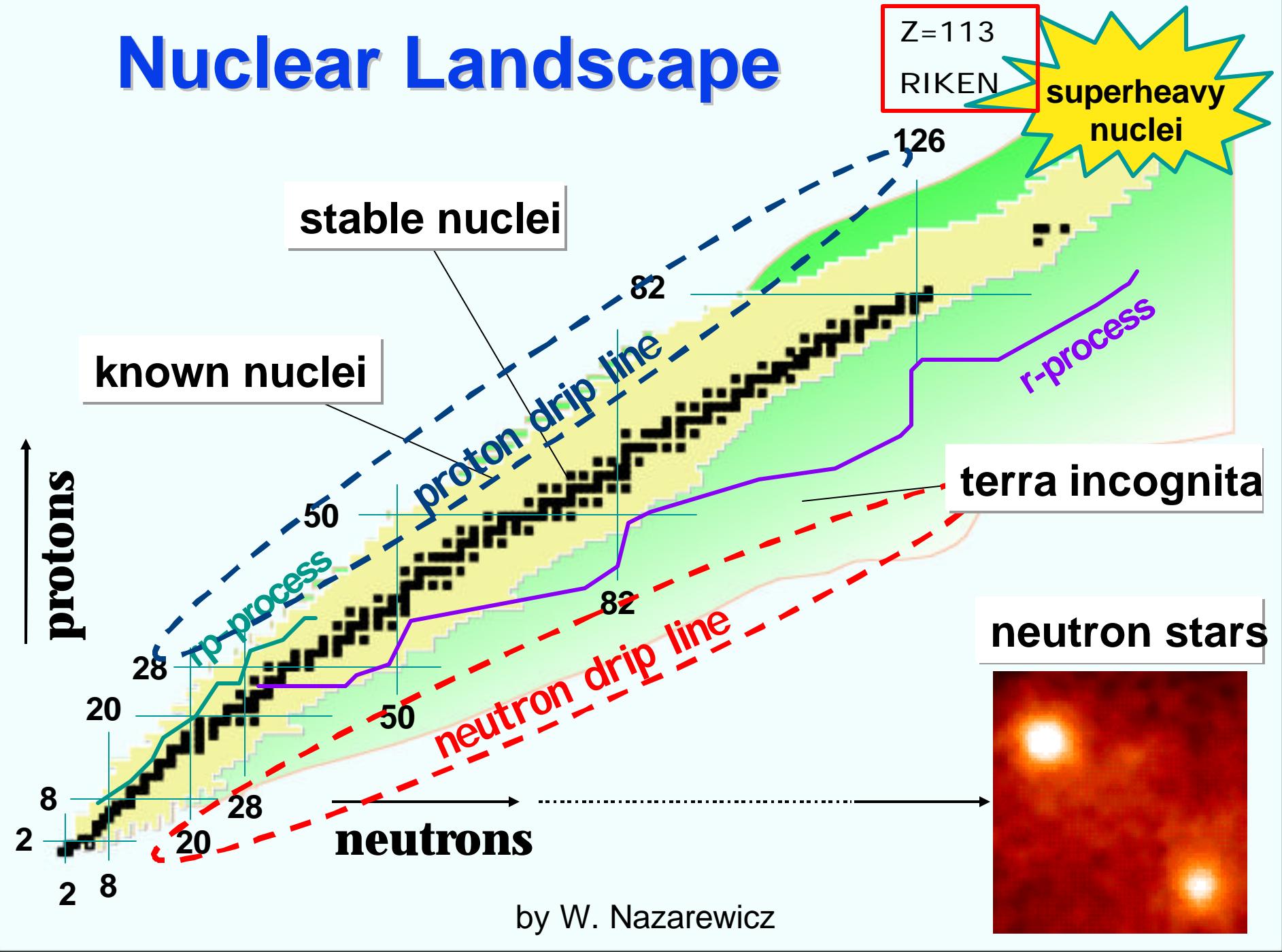
Giant resonances, neutron skin thickness and EOS in asymmetric nuclear matter

Workshop on “Nuclear Incompressibility and EOS”
Notre Dame USA ,July 14-15, 2005

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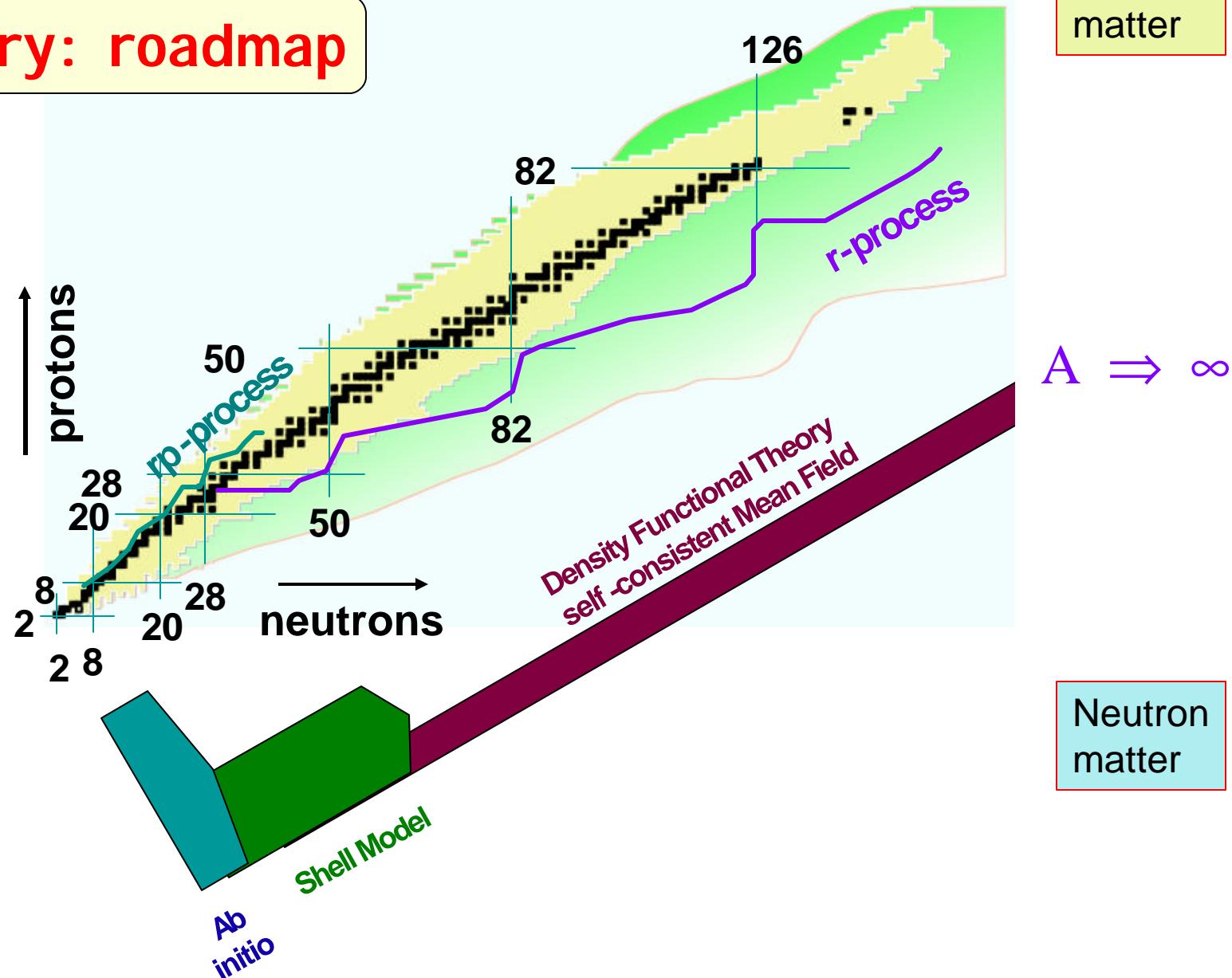
1. Introduction
2. Mean field models: SHF and RMF
3. EOS and incompressibility, IS giant resonances
4. Pressure of neutron matter, neutron skin thickness and symmetry energy coefficient
5. Summary

Nuclear Landscape



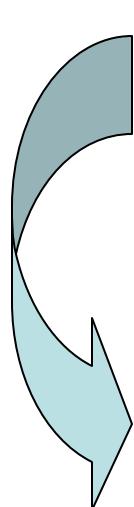
Theory: roadmap

Nuclear matter



Neutron matter

Nuclear Matter EOS



Isoscalar Monopole Giant
Resonances

Isoscalar Compressional Dipole
Resonances

Incompressibility K

How much ?

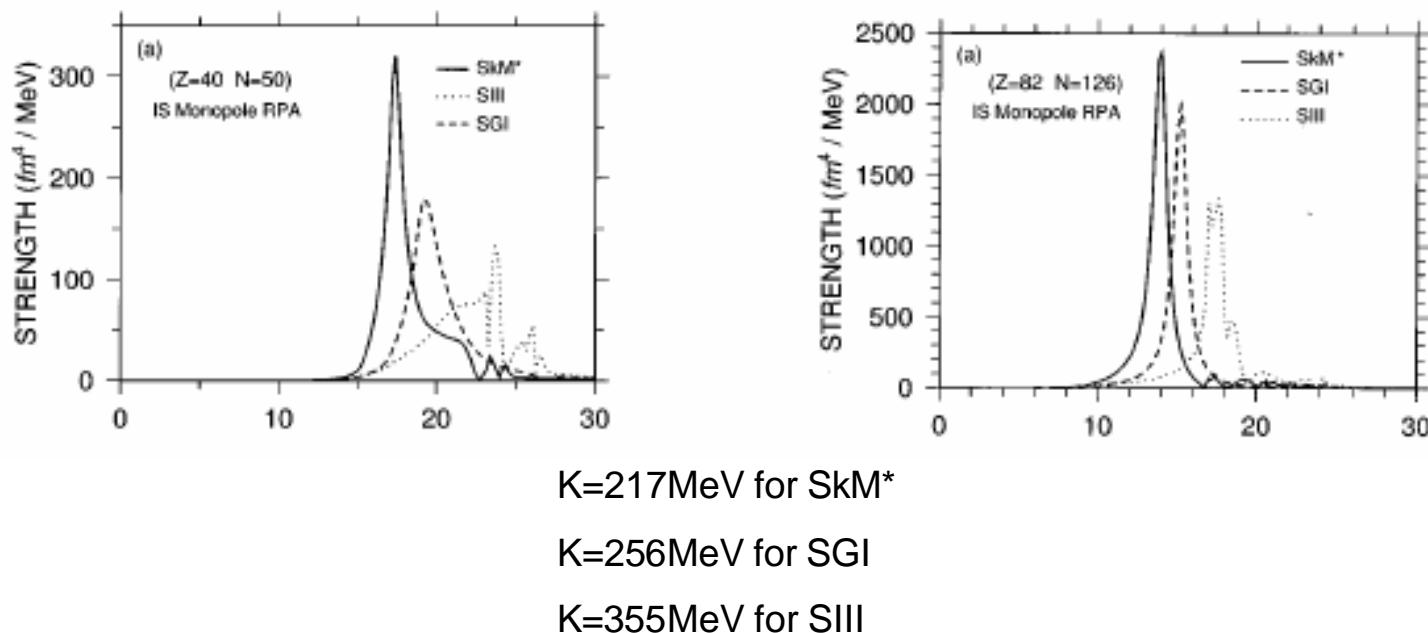
Self consistent HF+RPA calculations

Self consistent RMF+RPA (TDHF) calculations

(a,a) experiment

TABLE II. Peak energies, widths, and EWSR fractions for the ISGMR and ISGDR. The errors in fitting the $L=0$ and $L=1$ strengths with the Breit-Wigner functions are included.

	ISGMR			LE ISGDR			HE ISGDR		
	E_{ISGMR} (MeV)	Γ (MeV)	EWSR (%)	E_{ISGDR} (MeV)	Γ (MeV)	EWSR (%)	E_{ISGDR} (MeV)	Γ (MeV)	EWSR (%)
^{90}Zr	16.6 ± 0.1	4.9 ± 0.2	101 ± 3	17.8 ± 0.5	3.7 ± 1.2	7.9 ± 2.9	26.9 ± 0.7	12.0 ± 1.5	67 ± 8
^{116}Sn	15.4 ± 0.1	5.5 ± 0.3	95 ± 4	15.6 ± 0.5	2.3 ± 1.0	4.9 ± 2.2	25.4 ± 0.5	15.7 ± 2.3	68 ± 9
^{144}Sm [21]	$15.3_{-0.12}^{+0.11}$	$3.70_{-0.63}^{+0.12}$	84_{-25}^{+4}	14.2 ± 0.2	4.8 ± 0.8	23_{-10}^{+4}	$25.0_{-0.3}^{+1.7}$	19.9 ± 1.4	91_{-17}^{+25}
^{208}Pb	13.4 ± 0.2	4.0 ± 0.4	104 ± 9	13.0 ± 0.1	1.1 ± 0.4	7.0 ± 0.4	22.7 ± 0.2	11.9 ± 0.4	111 ± 6



PHYSICAL REVIEW C 69, 051301(R) (2004)

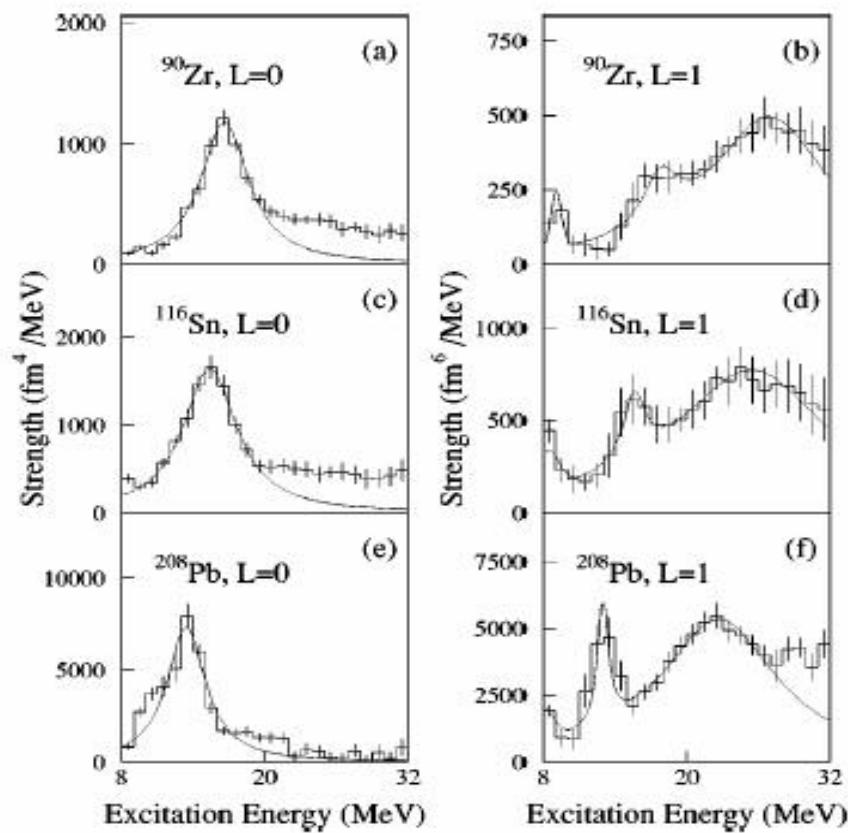


FIG. 3. Experimentally obtained strength distributions of the ISGMR and the ISGDR in ^{90}Zr , ^{116}Sn , and ^{208}Pb . The error bars are determined by changing the $E1$ strength within $\chi^{\text{total}} \leq \chi^{\text{total}}_{\min} + 1$ in the MDA. In the case of the ISGMR, the results of fitting with a Breit-Wigner function are indicated. In the ISGDR's case, the results of fitting with two Breit-Wigner functions for the region of $E_x > 10$ MeV, and a Gaussian function for the peak at $E_x < 10$ MeV are included.

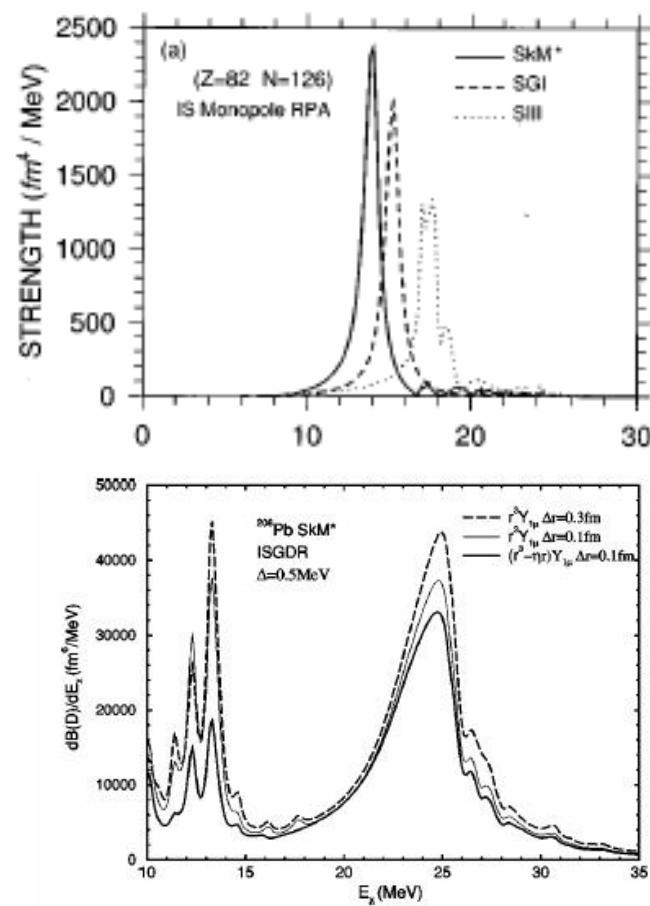
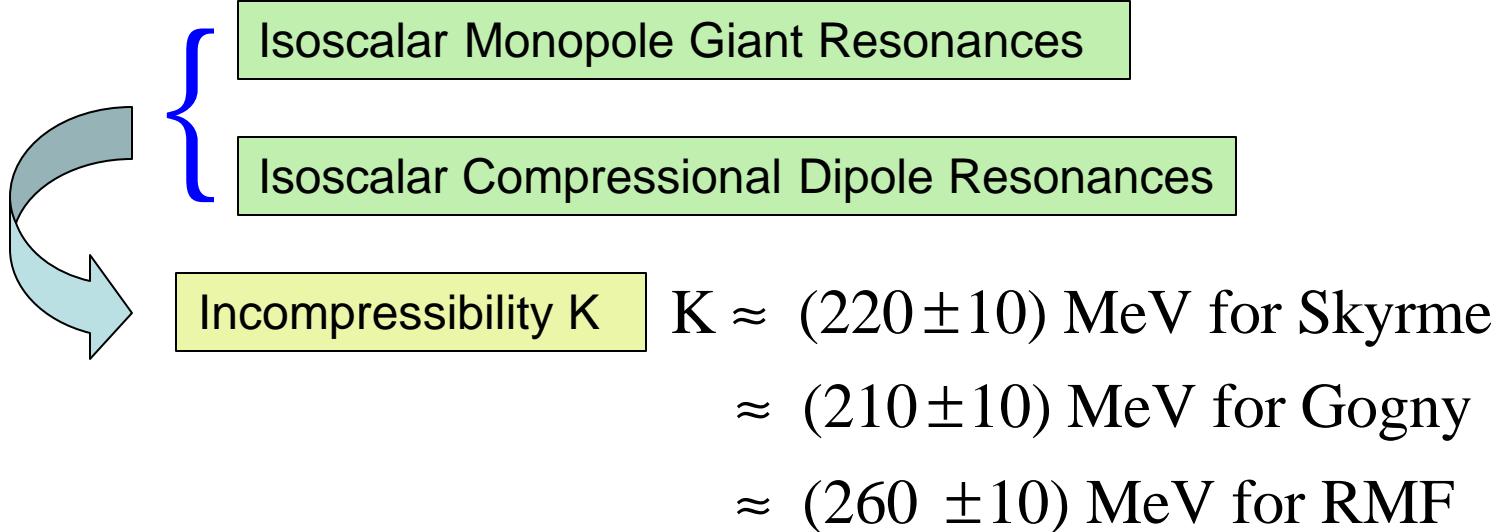


FIG. 1. RPA response functions to the IS dipole operators (1) and (2) as a function of excitation energy, which are obtained from the self-consistent HF plus RPA calculations solved in coordinate space. A radial mesh $\Delta r = 0.1$ fm is used in the HF calculation, while the results obtained by using $\Delta r = 0.1$ and 0.3 fm in the RPA calculation are compared. We show the calculated response functions, which are smeared out using the width of 0.5 MeV.

Nuclear Matter EOS



What can we learn about neutron EOS from nuclear physics?

Neutron surface thickness

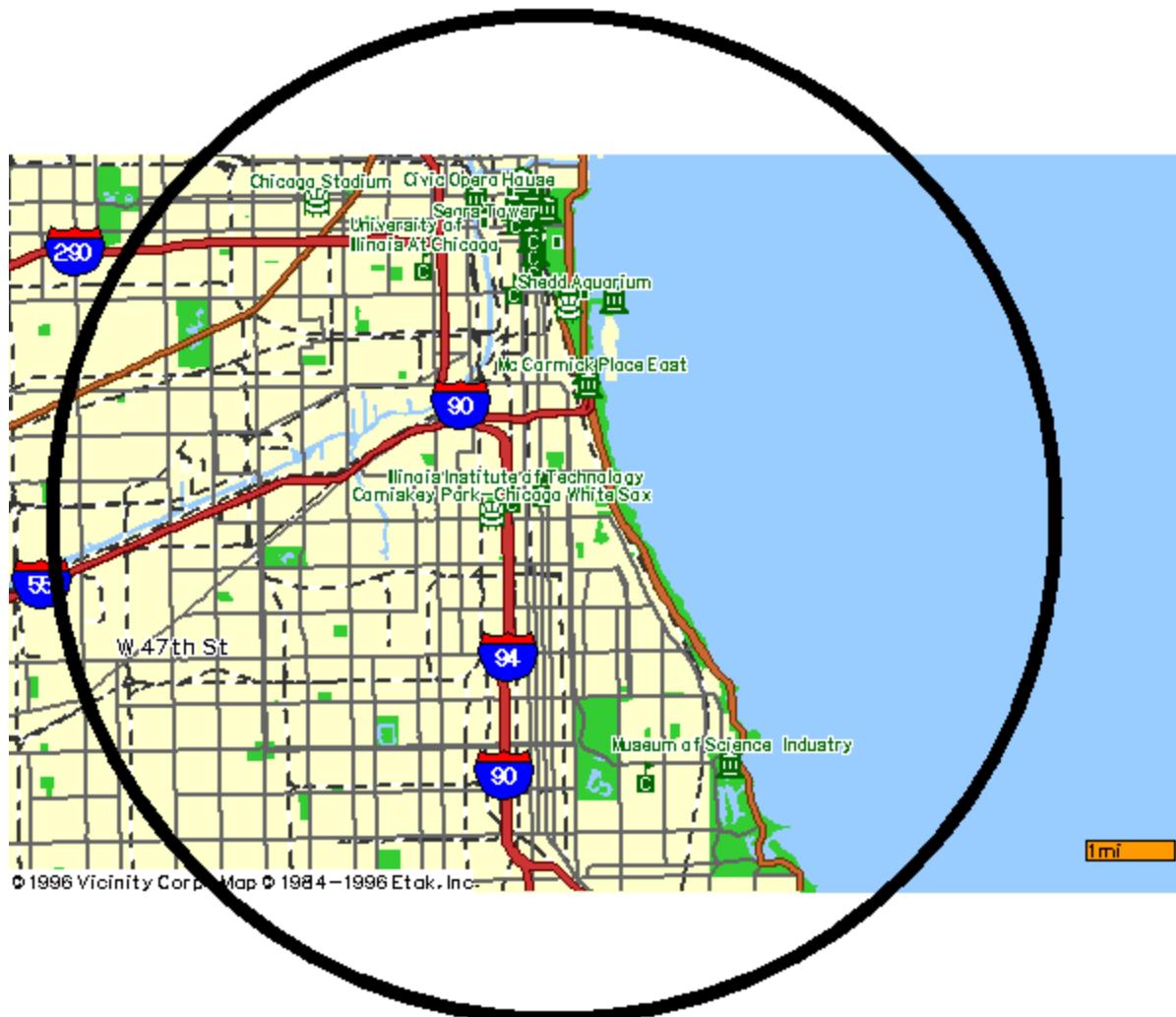
Size ~10fm

Pressure of neutron EOS

Neutron star ~10km

size difference ~ 10^{18}

Neutron star vs. Chicago

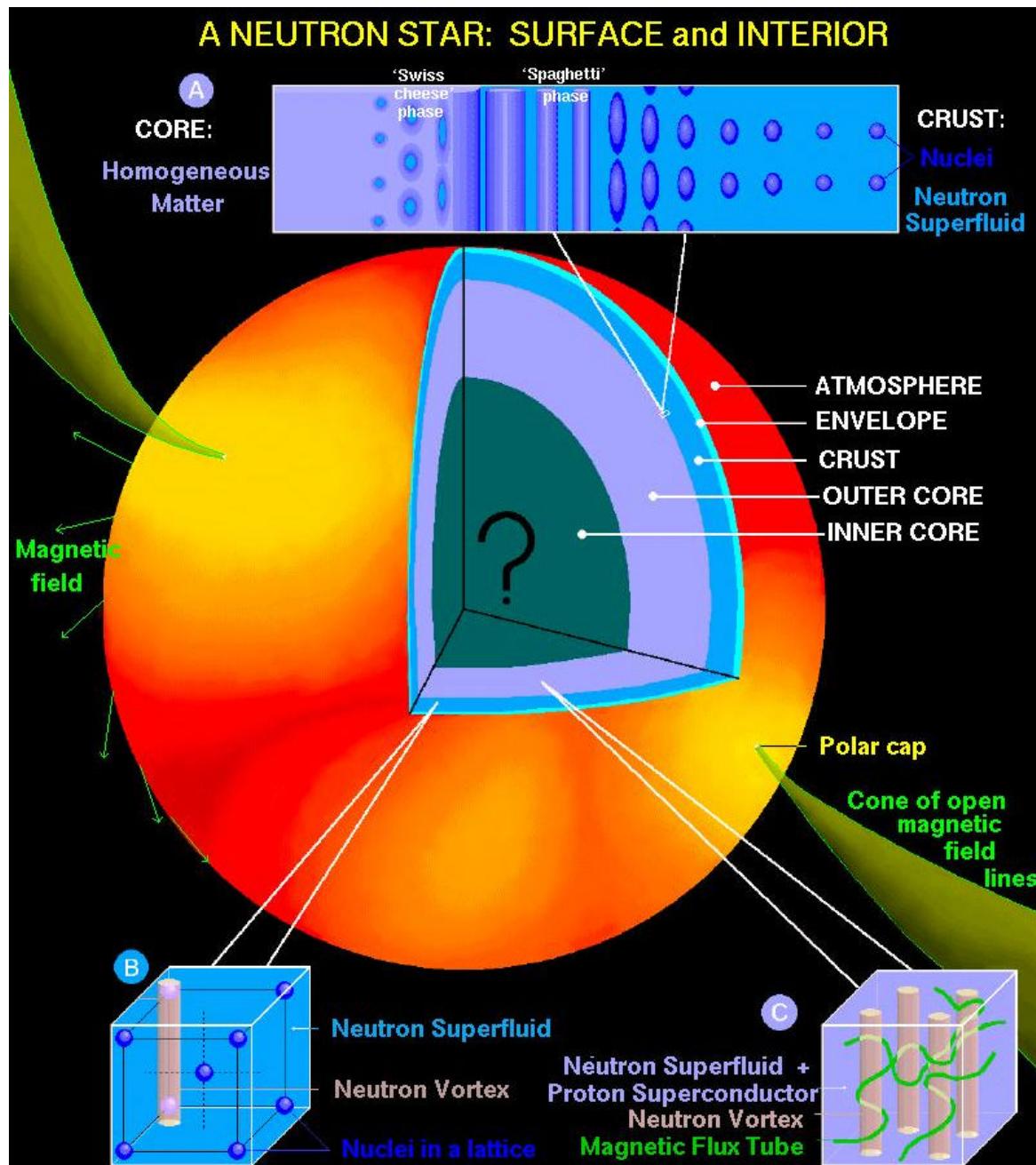


Nucleus
R=10fm



Mass= $1.4 M_{\text{sun}}$, Radius=10 km
Spin rate up to 38,000 rpm
Density~ 10^{14} g/cc, Magnetic field~ 10^{12} Gauss

<http://www.astro.umd.edu/~miller/nstar.html>



Mean Field Model (Skyrme HF model)

Skyrme force V_{Sky} is an effective zero-range force with density-dependent and momentum-dependent terms [11],

$$\begin{aligned}
 V_{\text{Sky}}(\vec{r}_1, \vec{r}_2) = & t_0(1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) + \frac{1}{2} t_1(1 + x_1 P_\sigma) \{ \vec{k}'^2 \delta \\
 & \times (\vec{r}_1 - \vec{r}_2) + \delta(\vec{r}_1 - \vec{r}_2) \vec{k}'^2 \} + t_2(1 + x_2 P_\sigma) \vec{k}' \cdot \delta \\
 & \times (\vec{r}_1 - \vec{r}_2) \vec{k} + \frac{1}{6} t_3(1 + x_3 P_\sigma) \rho^\alpha(\vec{r}) \delta(\vec{r}_1 - \vec{r}_2) \\
 & + iW(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k}' \times \delta(\vec{r}_1 - \vec{r}_2) \vec{k}, \tag{1}
 \end{aligned}$$

where $\vec{k} = (\vec{\nabla}_1 - \vec{\nabla}_2)/(2i)$ acting on the right and $\vec{k}' = -(\vec{\nabla}_1 - \vec{\nabla}_2)/(2i)$ acting on the left are the relative momentum operators, P_σ is the spin exchange operator, $\vec{\sigma}$ is the Pauli spin matrix, and $\vec{r} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$.

Energy density functional of Skyrme interaction

$$\begin{aligned}
\mathcal{H}(\rho_n, \rho_p) = & \frac{\hbar^2}{2m}(\tau_n + \tau_p) + \frac{1}{4}t_0(1-x_0)(\rho_n^2 + \rho_p^2) + t_0\left(1 + \frac{1}{2}x_0\right)\rho_n \rho_p + \frac{1}{12}t_3\left(1 + \frac{1}{2}x_3\right)\rho^{a+2} - \frac{1}{12}t_3\left(\frac{1}{2} + x_3\right)\rho^a(\rho_n^2 + \rho_p^2) \\
& + \frac{1}{8}[t_1(1-x_1) + 3t_2(1+x_2)](\rho_n\tau_n + \rho_p\tau_p) + \frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right](\rho_n\tau_p + \rho_p\tau_n) - \frac{3}{32}[t_1(1-x_1) \\
& - t_2(1+x_2)](\rho_n\nabla^2\rho_n + \rho_p\nabla^2\rho_p) - \frac{1}{16}\left[3t_1\left(1 + \frac{1}{2}x_1\right) - t_2\left(1 + \frac{1}{2}x_2\right)\right](\rho_n\nabla^2\rho_p + \rho_p\nabla^2\rho_n) - \frac{1}{2}W(\rho\vec{\nabla} \cdot \vec{J} + \rho_n\vec{\nabla} \cdot \vec{J}_n \\
& + \rho_p\vec{\nabla} \cdot \vec{J}_p) + \mathcal{H}_{\text{Coul}} - \frac{1}{16}(t_1x_1 + t_2x_2)\vec{J}^2 + \frac{1}{16}(t_1 - t_2)(\vec{J}_n^2 + \vec{J}_p^2),
\end{aligned}$$

where $\rho_n(\rho_p)$ is the density of neutrons (protons) and $\rho = \rho_n + \rho_p$, while $\tau_n(\tau_p)$ and $\vec{J}_n(\vec{J}_p)$ are the kinetic energy and the spin-orbit densities of neutrons (protons), respectively. The

Lagrangian of RMF

$$\begin{aligned}
\mathcal{L} = & \bar{\psi} \left(i\gamma^\mu \partial_\mu - M - g_\sigma \sigma - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\tau} \vec{b}_\mu \right. \\
& \left. - e \gamma^\mu \frac{1 - \tau_3}{2} A_\mu \right) \psi + \frac{1}{2} (\partial_\mu \partial^\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{b}_\mu \vec{b}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \vec{G}^{\mu\nu} \\
& - \frac{1}{4} H_{\mu\nu} H^{\mu\nu}, \tag{4}
\end{aligned}$$

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{b}_\nu - \partial_\nu \vec{b}_\mu$, $H_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and σ , ω_μ , \vec{b}_μ , and A_μ are the σ , ω , ρ mesons, and the electromagnetic field, respectively. The quantities g_σ , g_ω , and g_ρ are the coupling constants between nucleons and σ , ω , and ρ mesons, respectively, while $e^2/4\pi=1/137$ is the

The quantity $U(\sigma)$ is the nonlinear potential of σ

$$U(\sigma) = \frac{1}{3}g_1\sigma^3 + \frac{1}{4}g_2\sigma^4, \quad (5)$$

where g_1 and g_2 are parameters of the potential. The Dirac equation for nucleons and the Klein-Gordon equations for mesons are derived by the classical variational principle with time-reversal symmetry and charge conservation,

$$\begin{aligned} & \left[-i\vec{\alpha} \cdot \vec{\nabla} + \beta M^* + g_\omega \omega(\vec{r}) + g_\rho \tau_3 b(\vec{r}) + e \frac{1 - \tau_3}{2} A(\vec{r}) \right] \psi_i(\vec{r}) \\ &= \epsilon_i \psi_i(\vec{r}), \end{aligned} \quad (6)$$

$$(-\nabla^2 + m_\sigma^2)\sigma(\vec{r}) = -g_\sigma \rho_s(\vec{r}) - g_1 \sigma^2(\vec{r}) - g_2 \sigma^3(\vec{r}),$$

$$(-\nabla^2 + m_\omega^2)\omega(\vec{r}) = g_\omega \rho_B(\vec{r}),$$

$$(-\nabla^2 + m_\rho^2)b(\vec{r}) = g_\rho \rho_3(\vec{r}),$$

$$-\nabla^2 A(\vec{r}) = e \rho_p(\vec{r}), \quad (7)$$

where $\vec{\alpha}$ and β are defined by

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad \text{and} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

where $\vec{\sigma}$ is the Pauli matrix and I is the 2×2 unit matrix,

BCS model with surface pairing

$$V_{\text{pair}} = \frac{V_0}{2} (1 - P_\sigma) \left(1 - \frac{\rho(\vec{r}_1)}{\rho_c} \right) \delta(\vec{r}_1 - \vec{r}_2),$$

Skyrme HF and RMF both are equally successful to describe ground state properties of many nuclei in the mass table.

Pressure and Incompressibility

$$K = \frac{d^2}{dr^2} \left(\frac{H}{r} \right)_{r=r_{nm}}$$

for nuclear matter

$$P = \rho_n^2 \frac{d}{d\rho_n} \left(\frac{\mathcal{H}}{\rho_n} \right)$$

for neutron matter

Hamiltonian density of RMF

$$\begin{aligned} \mathcal{H}_{\text{RMF}} = & \frac{2}{(2\pi)^3} \left[\int_0^{k_F p} + \int_0^{k_F n} \right] (k^2 + M^2)^{1/2} d^3 k + g_\omega \omega \rho_B \\ & - \frac{1}{2} m_\omega^2 \omega^2 + U(\sigma) + \frac{1}{2} m_\sigma^2 \sigma^2 + g_\rho b \rho_3 - \frac{1}{2} m_\rho^2 b^2, \quad (9) \end{aligned}$$

Symmetry Energy coefficient

$$a_{\text{sym}} = \frac{1}{2} \lim_{I \rightarrow 0} \frac{\partial^2}{\partial I^2} \left(\frac{\mathcal{H}}{\rho} \right)$$

SHF model

$$\begin{aligned} a_{\text{sym}} = & \frac{\hbar^2}{6m} \left(\frac{3\pi^2}{2} \right) \rho^{2/3} - \frac{1}{8} t_0 (1 + 2x_0) \rho - \frac{1}{48} t_3 (1 + 2x_3) \rho^{a+1} \\ & - \frac{1}{24} \left(\frac{3\pi^2}{2} \right)^{2/3} \{3t_1 x_1 - t_2 (4 + 5x_2)\} \rho^{5/3}, \end{aligned} \quad (\text{A3})$$

RMF

$$a_{\text{sym}} = \frac{3\pi^2 A}{16g_\sigma k_F^3} \left[3(m_\sigma^2 \phi + g_1 \phi^2 + g_2 \phi^3) - \frac{M^*}{g_\sigma} (m_\sigma^2 + 2g_1 \phi + 3g_2 \phi^2) \right] + \frac{g_\rho^2 k_F^3}{3\pi^2 m_\rho^2} + \frac{1}{24} \left[-\frac{k_F^4}{B^3} + \frac{5k_F^2}{B} + \frac{9M^* A}{B} \right], \quad (\text{A4})$$

where A and B are given by

$$A = \left[\frac{2g_\sigma k_F}{\pi^2} \left(B - \frac{M^{*2}}{B} \right) + \frac{3}{M^*} (m_\sigma^2 \phi + g_1 \phi^2 + g_2 \phi^3) - \frac{1}{g_\sigma} (m_\sigma^2 + 2g_1 \phi + 3g_2 \phi^2) \right]^{-1} \left[-\frac{2k_F^5 M^*}{9B^3} \right], \quad (\text{A5})$$

$$B = \sqrt{k_F^2 + M^{*2}}. \quad (\text{A6})$$

Isovector properties of energy density functional

$$L = 3\rho_{\text{nm}} \frac{de_\delta(\rho)}{d\rho_{\text{nm}}}. \quad (\text{A9})$$

The isovector density e_δ can be expressed as a Taylor expansion around ρ_{nm} :

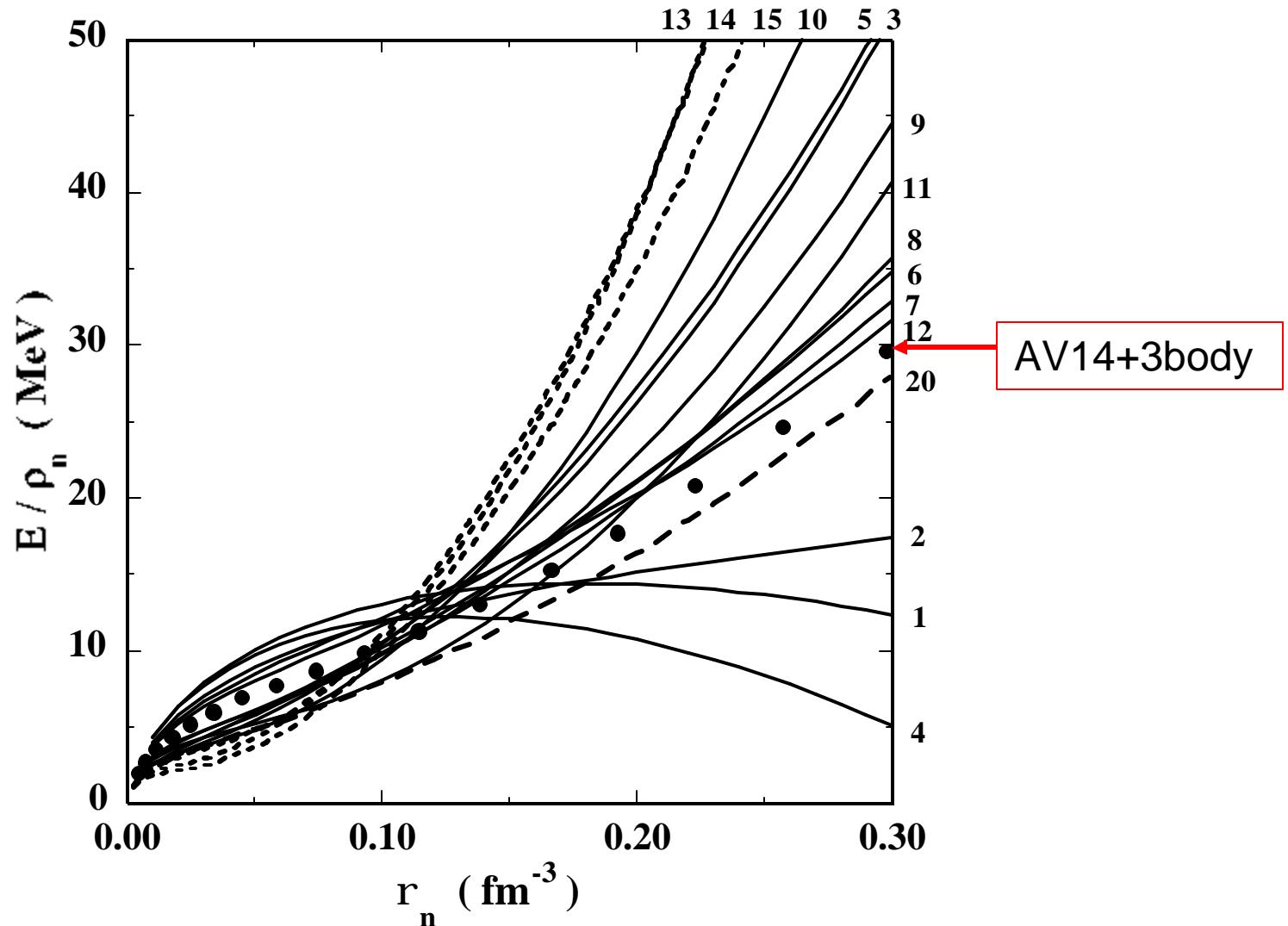
$$e_\delta(\rho) = J + \frac{L}{3} \frac{\rho - \rho_{\text{nm}}}{\rho_{\text{nm}}} + \frac{K_{\text{sym}}}{18} \left(\frac{\rho - \rho_{\text{nm}}}{\rho_{\text{nm}}} \right)^2. \quad (\text{A10})$$

The quantity K_{sym} is defined by

$$K_{\text{sym}} = 9\rho_{\text{nm}}^2 \frac{d^2 e_\delta(\rho)}{d\rho_{\text{nm}}^2}. \quad (\text{A11})$$

The surface symmetry energy e_{ss} is evaluated to be

$$e_{ss} = -\frac{2a}{r_{\text{nm}}} \left(L - \frac{1}{12} K_{\text{sym}} \right) + 2\varepsilon_s(0) \frac{L}{K} \quad (\text{A12})$$



interactions: 1 for SI, 2 for SIII, 3 for SIV, 4 for SVI, 5 for Skya, 6 for SkM, 7 for SkM^* , 8 for SLy4, 9 for MSkA, 10 for SkI3, 11 for SkI4, 12 for SkX, 13 for NLSH, 14 for NL3, 15 for NLC, 16 for SII, 17 for SV, 18 for Skyb, 19 for SGI, 20 for SGII, 21 for SLy10, 22 for NL1, and 23 for NL2.

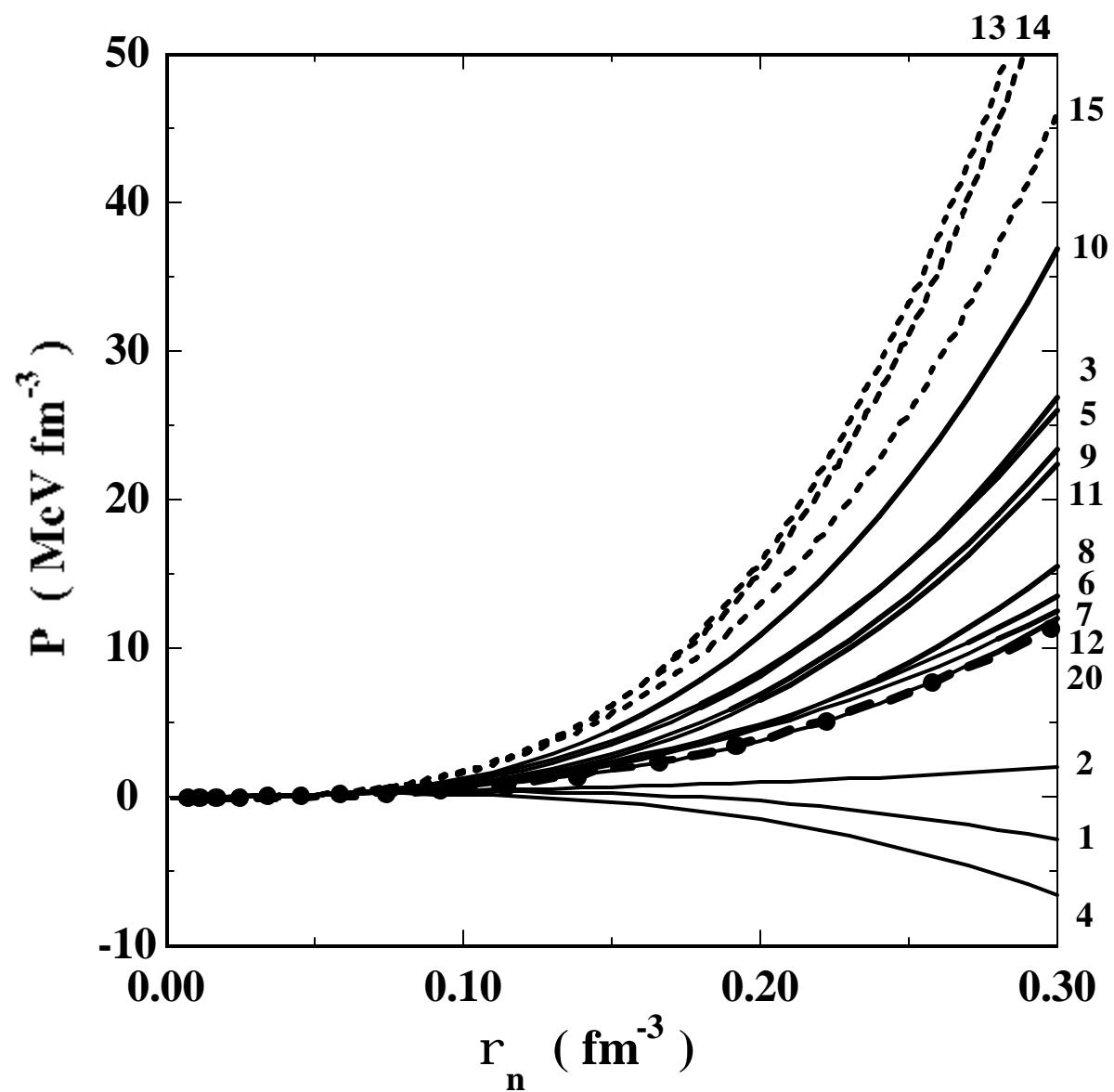


Fig. 4

Neutron skin thickness

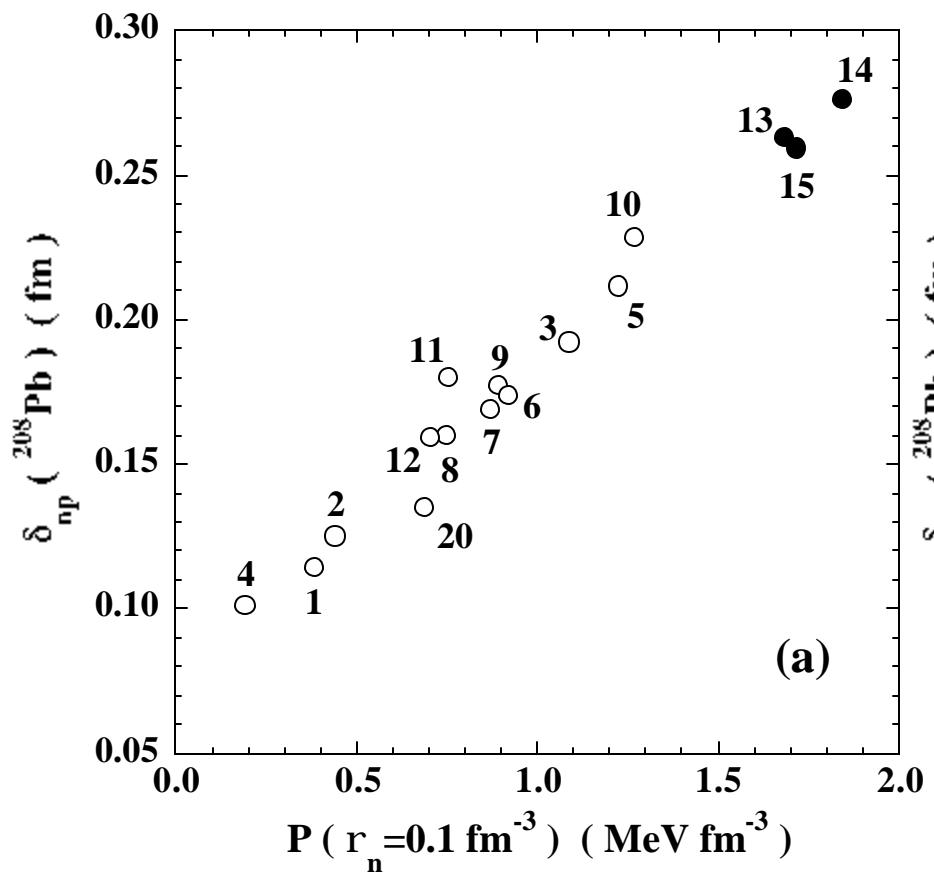
$$\delta_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p}$$

Charge radius $r_c = (5.503 \pm 0.002) \text{ fm}$

Neutron radius

GDR : $d_{np} = (0.19 \pm 0.09) \text{ fm}$

(p,p') : $d_{np} = (0.17 \pm 0.20) \text{ fm}$



(a)

Fig. 5

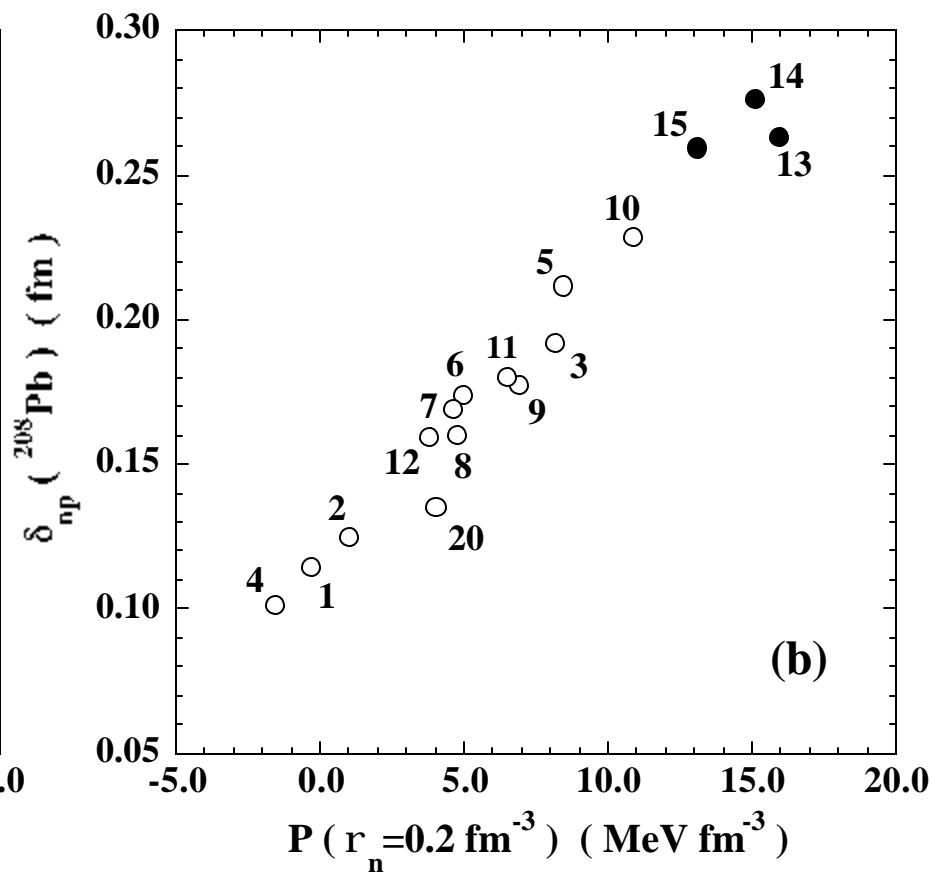
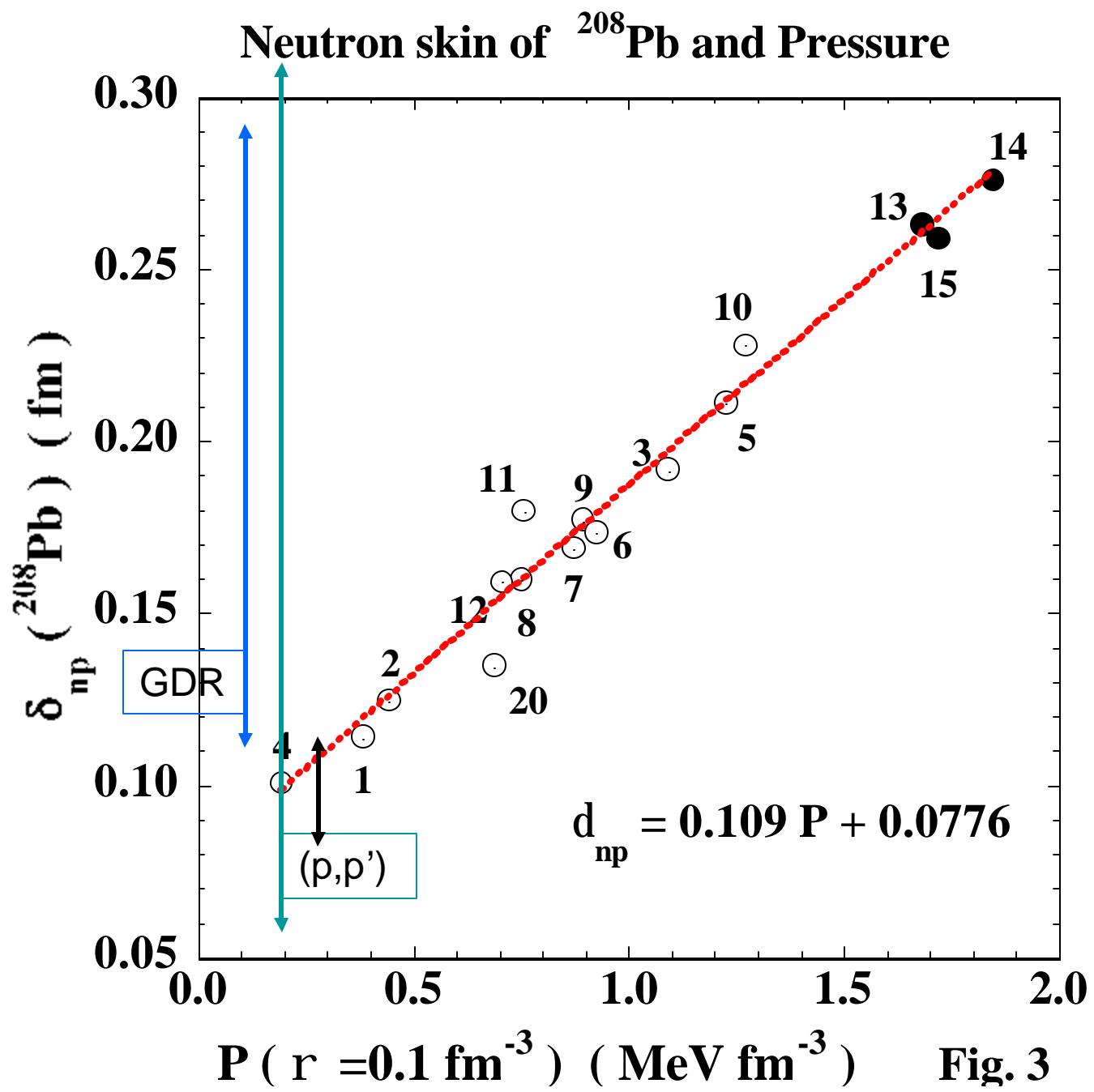
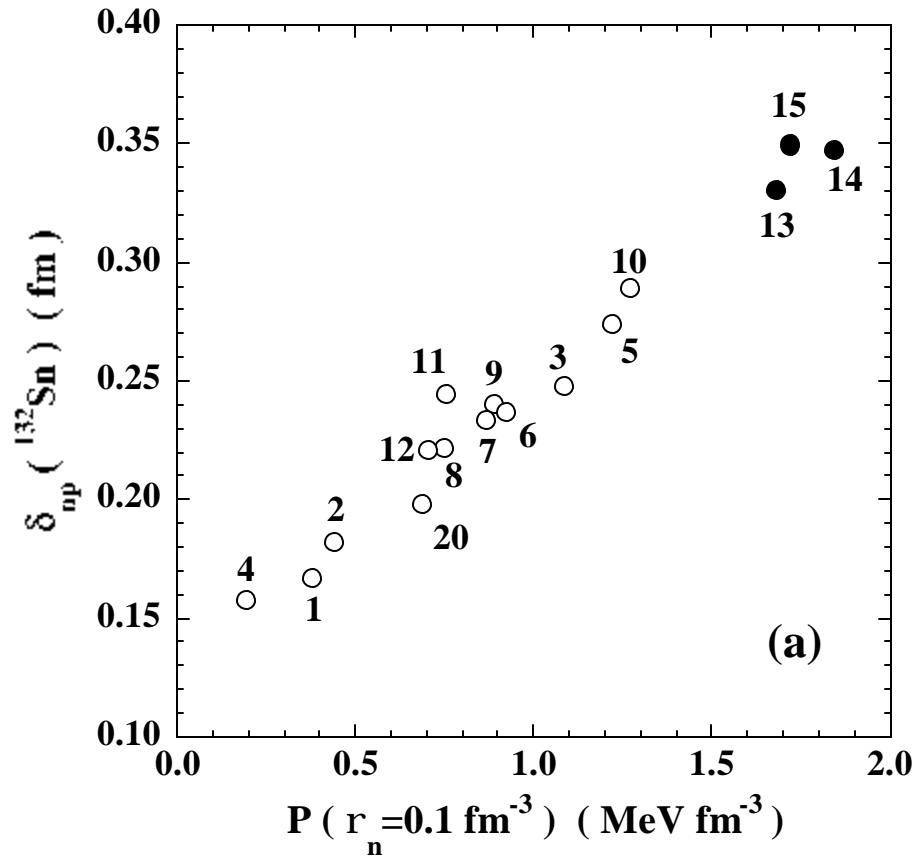
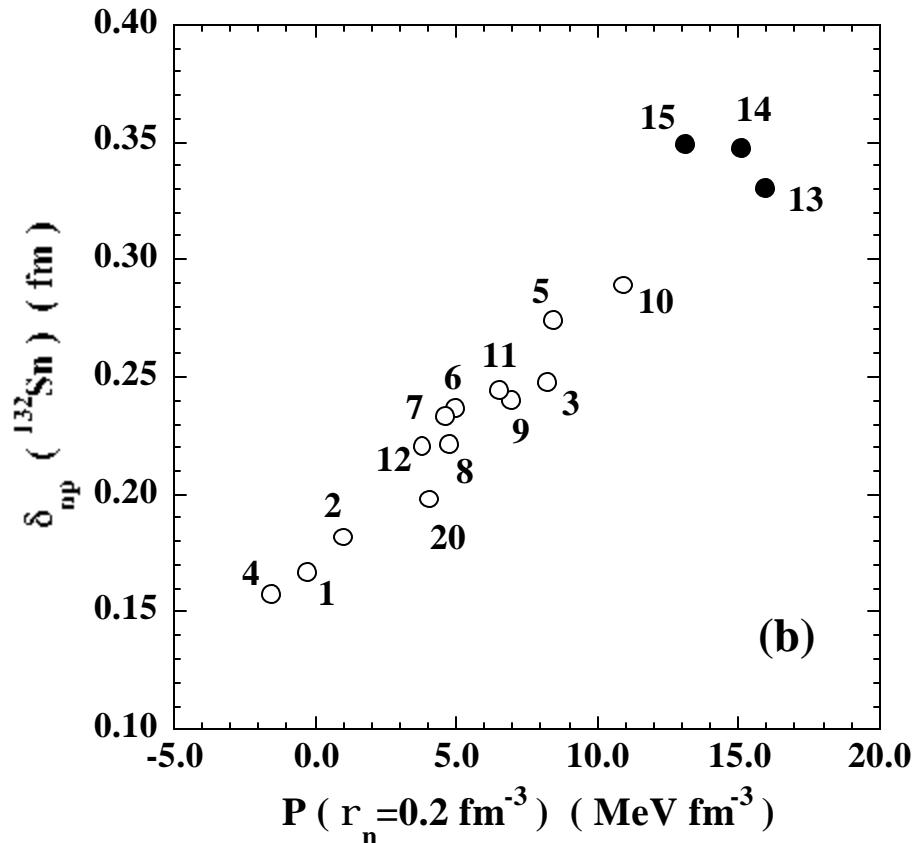


Fig. 5





(a)

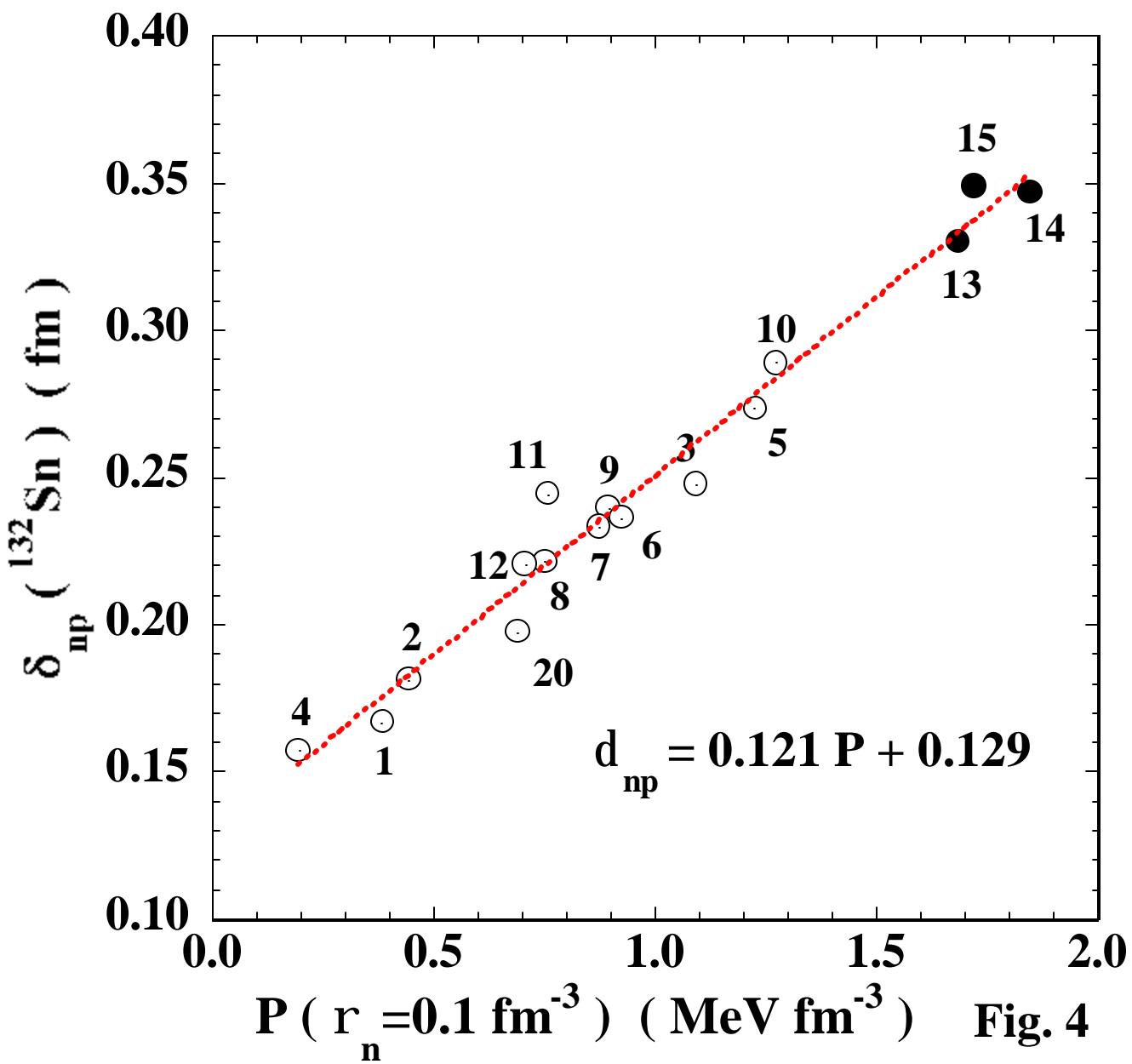


(b)

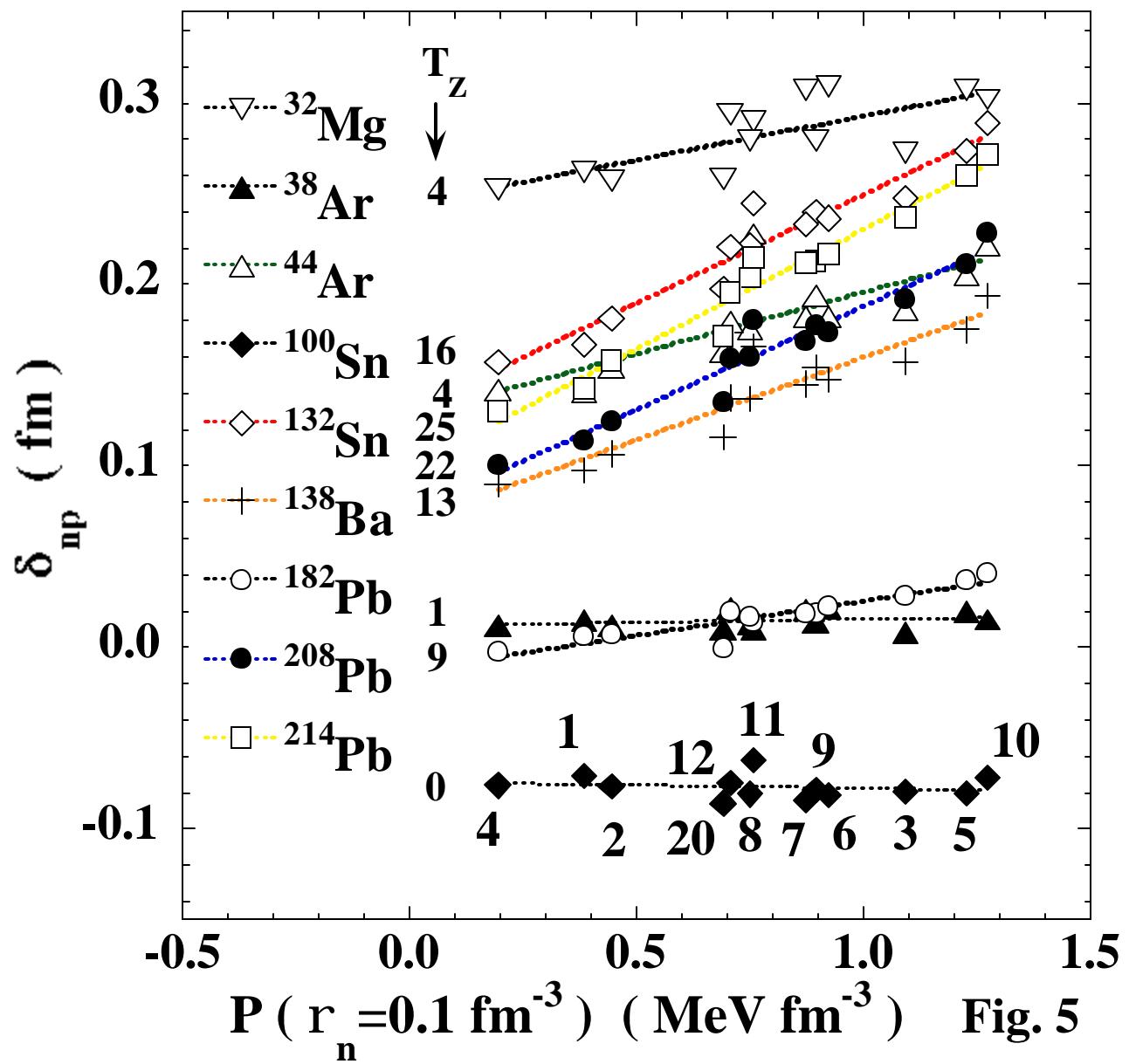
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Fig. 6

Neutron skin of ^{132}Sn and Pressure



Neutron skin and pressure



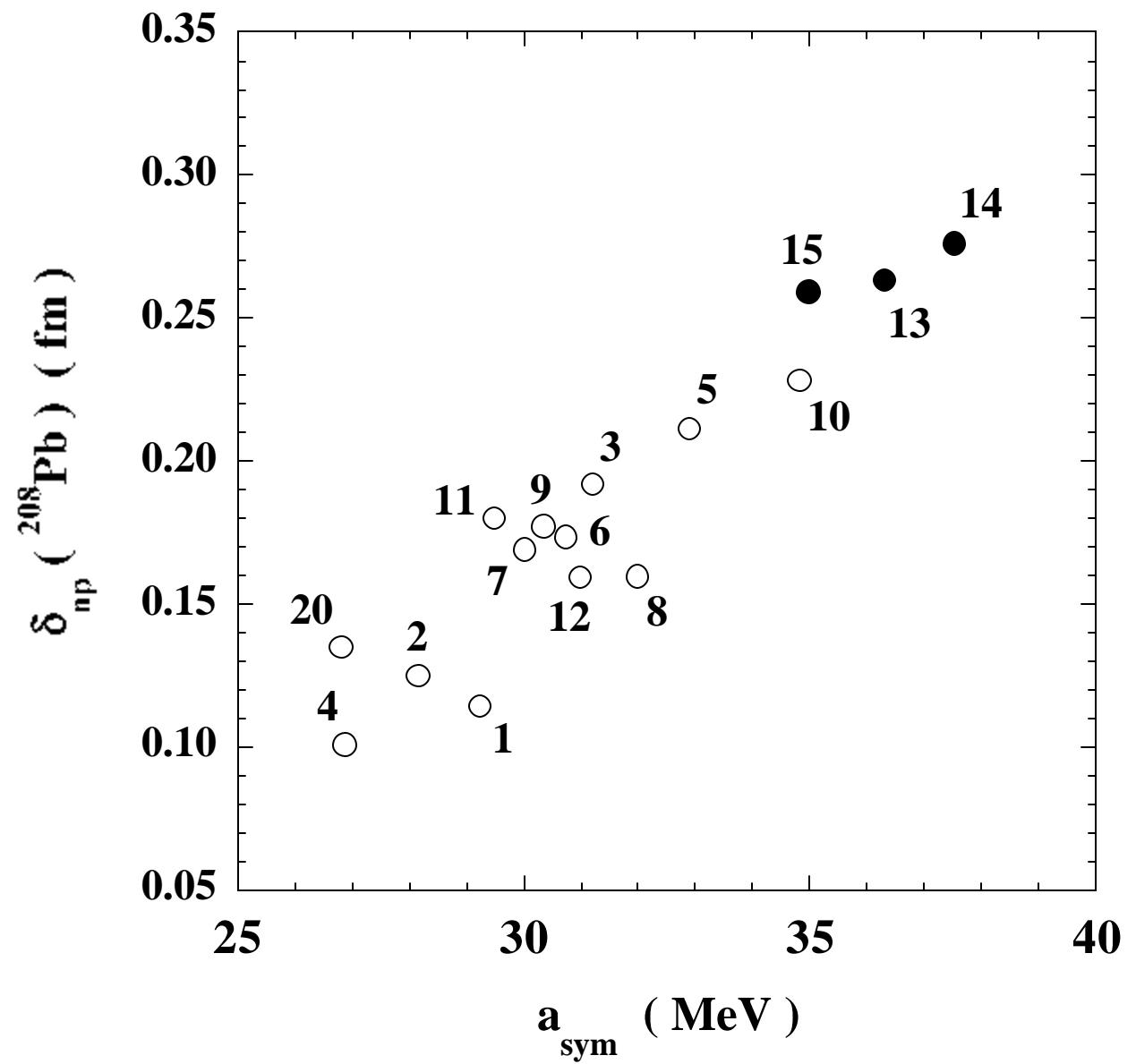


Fig. 8

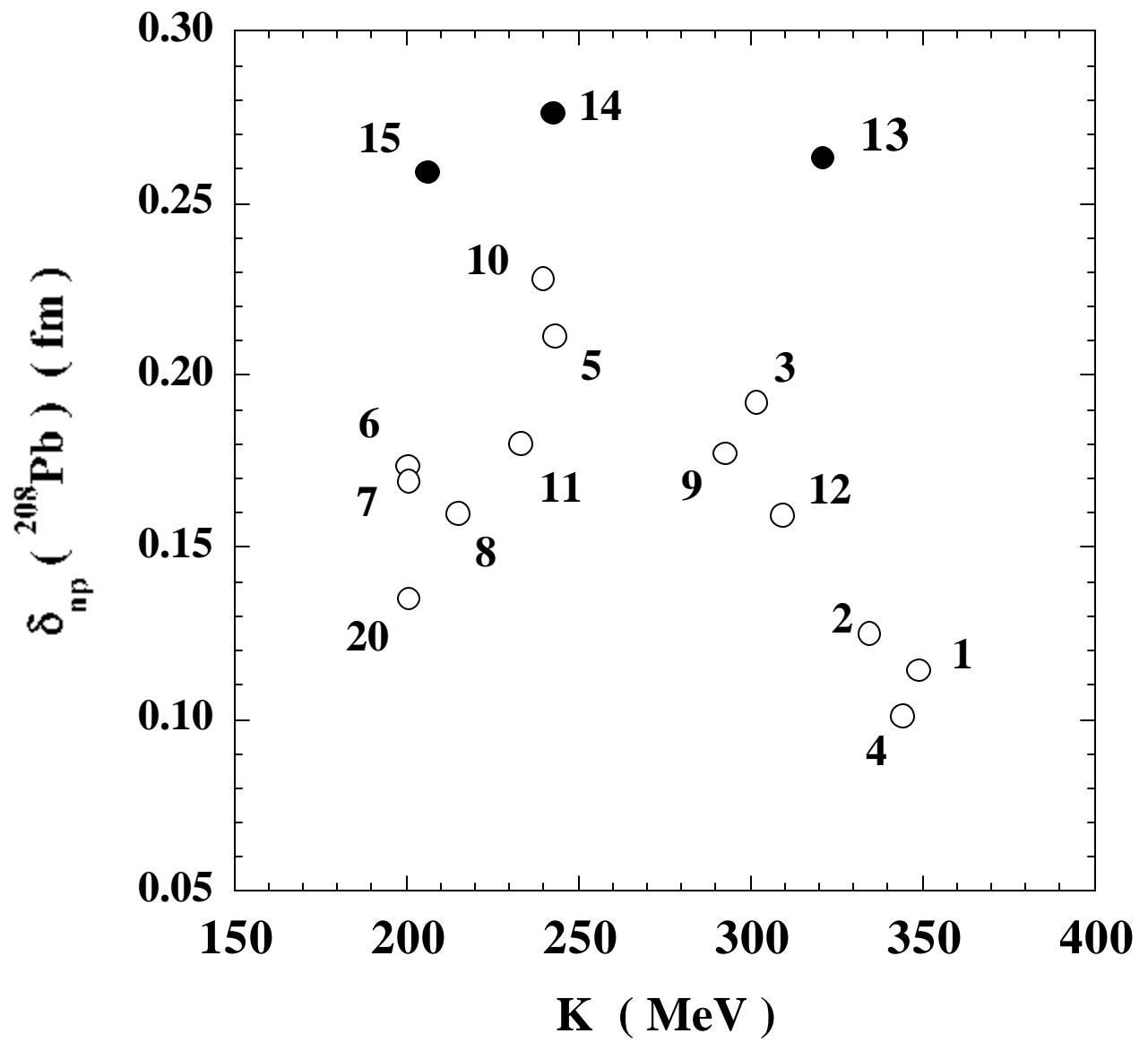


Fig. 9

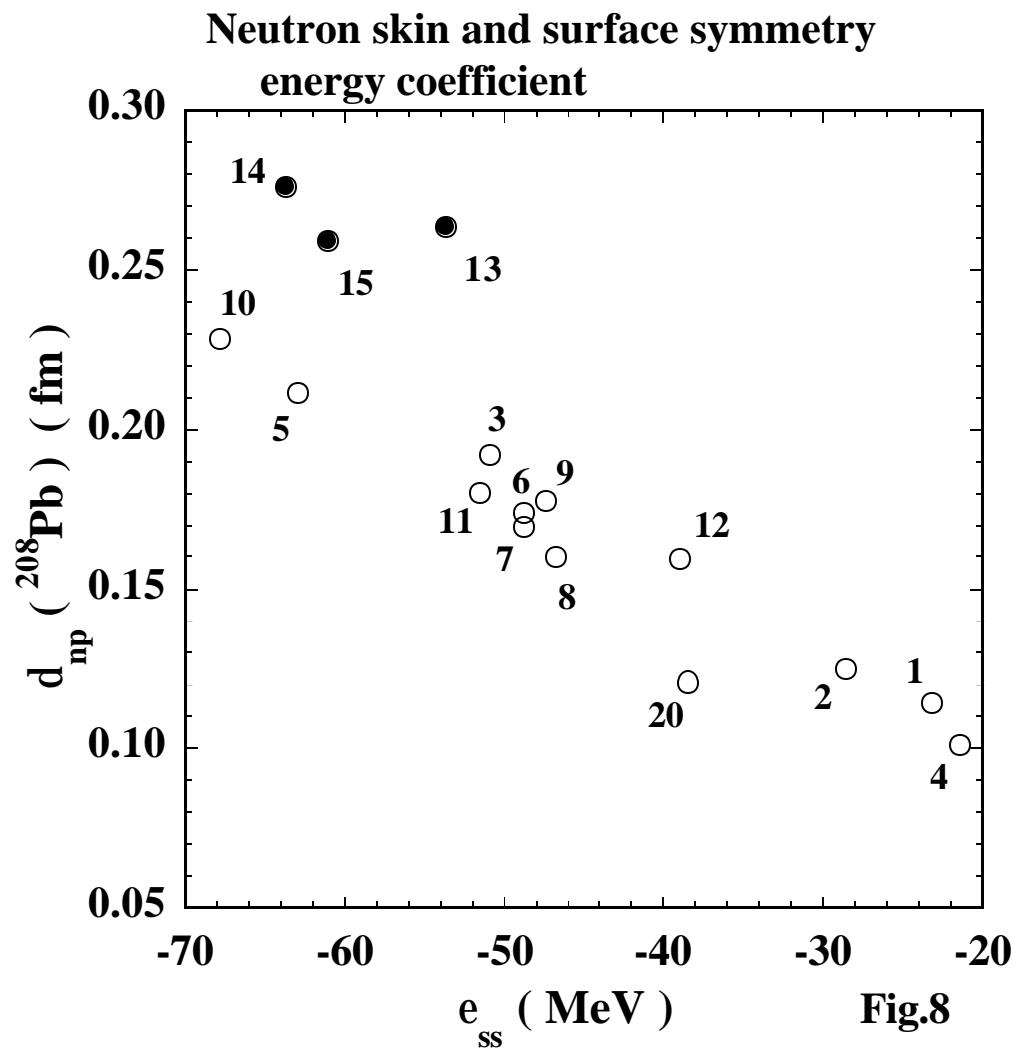
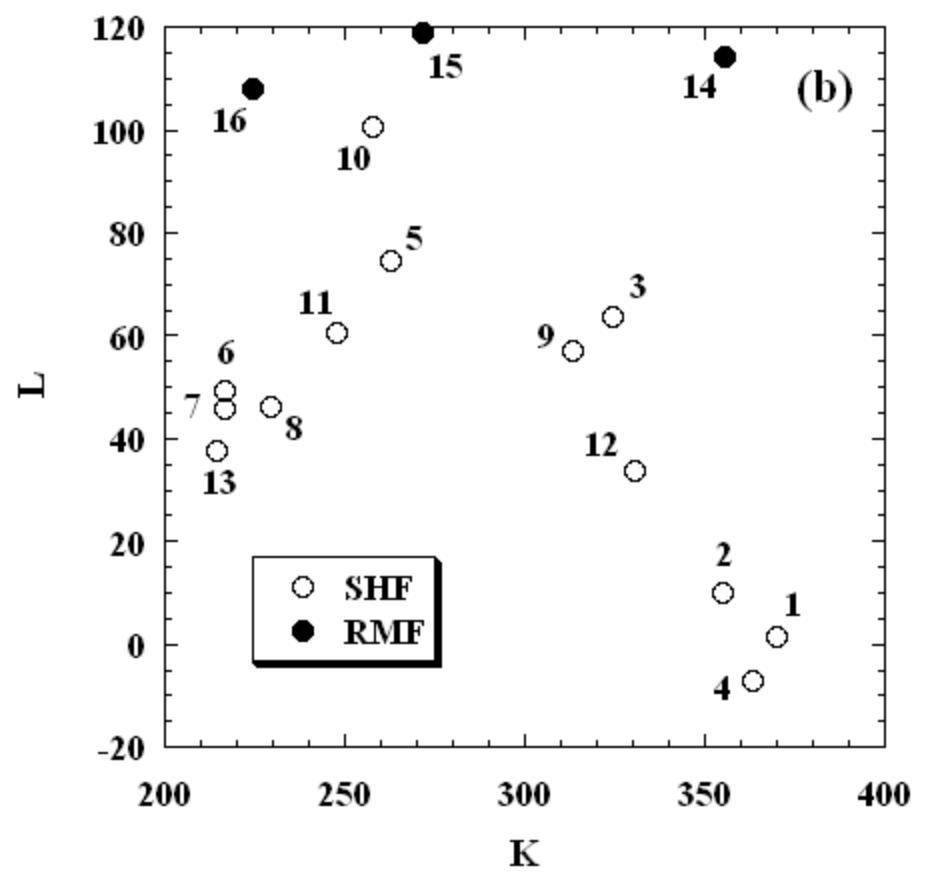
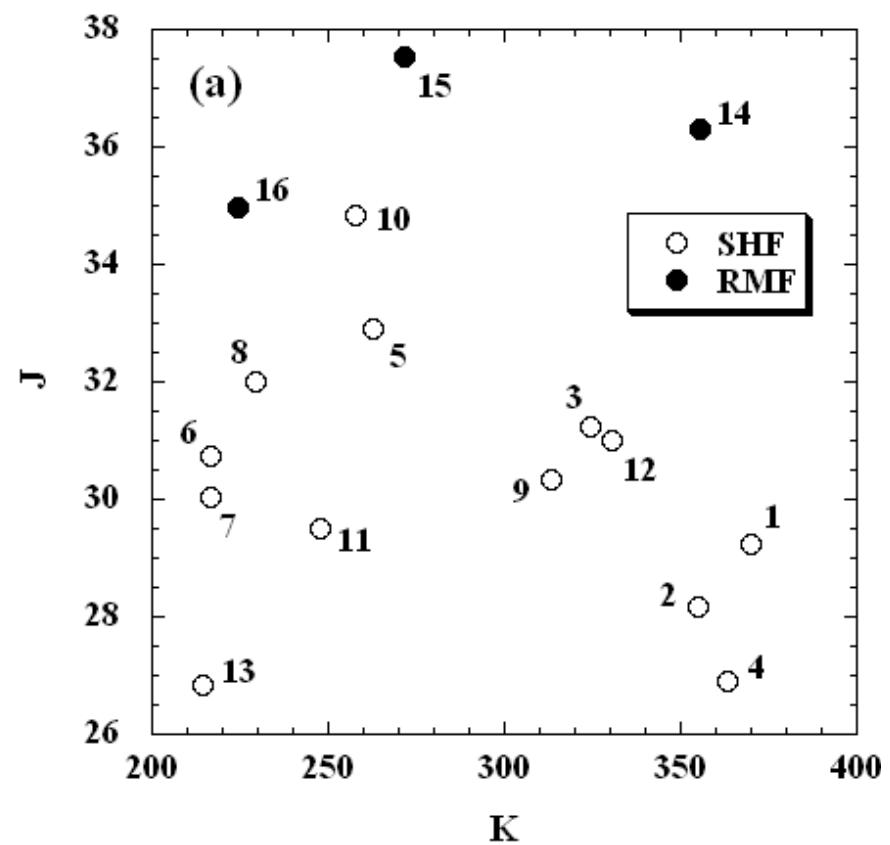
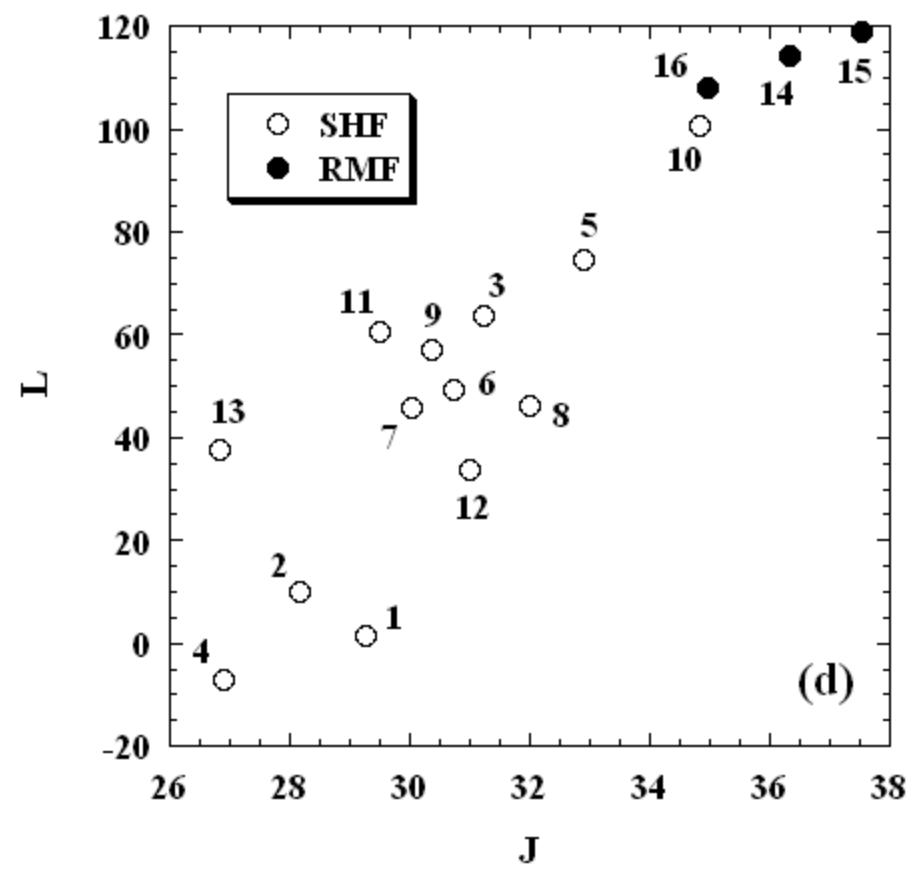
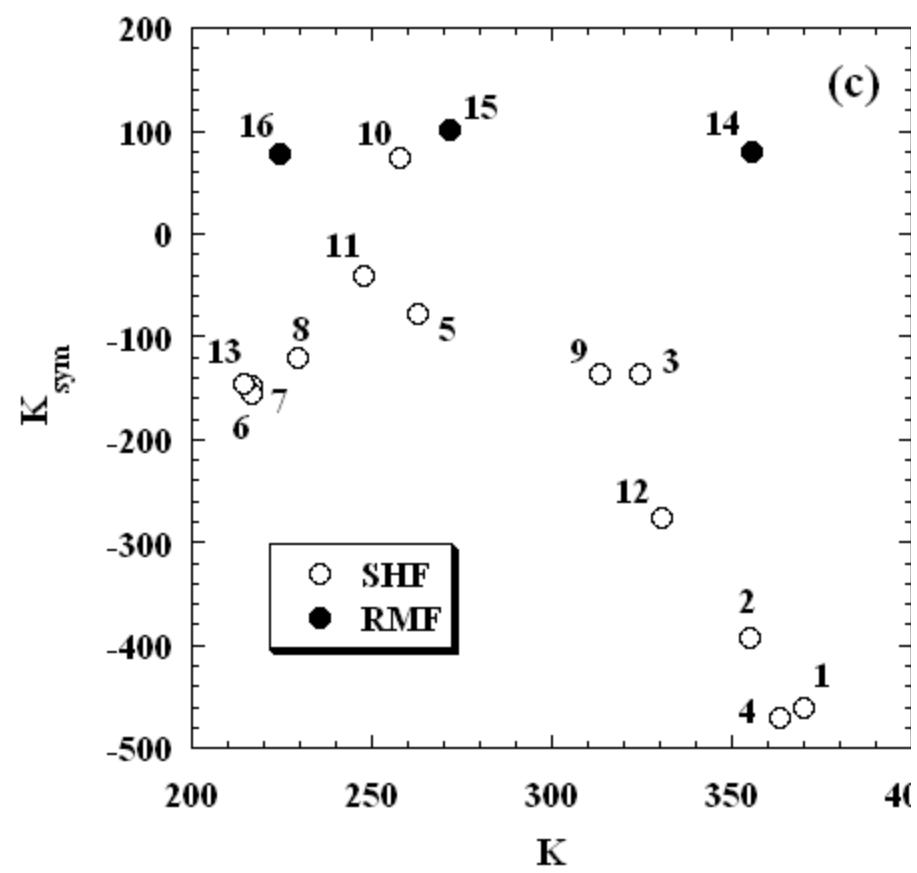
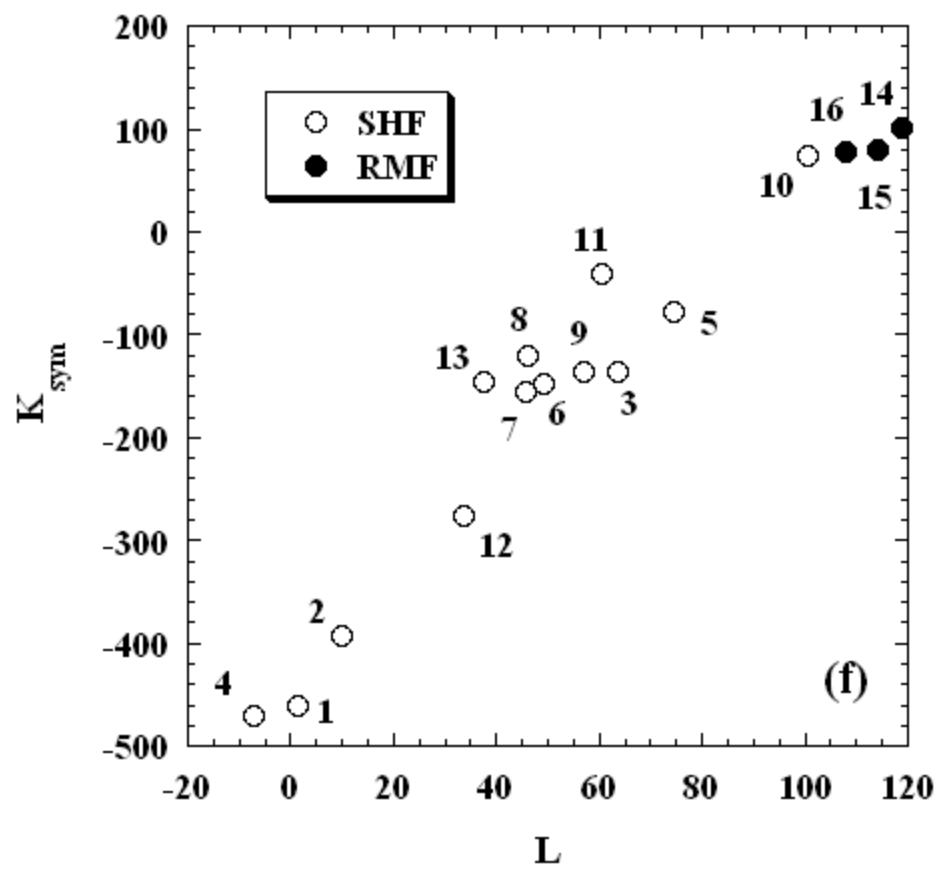
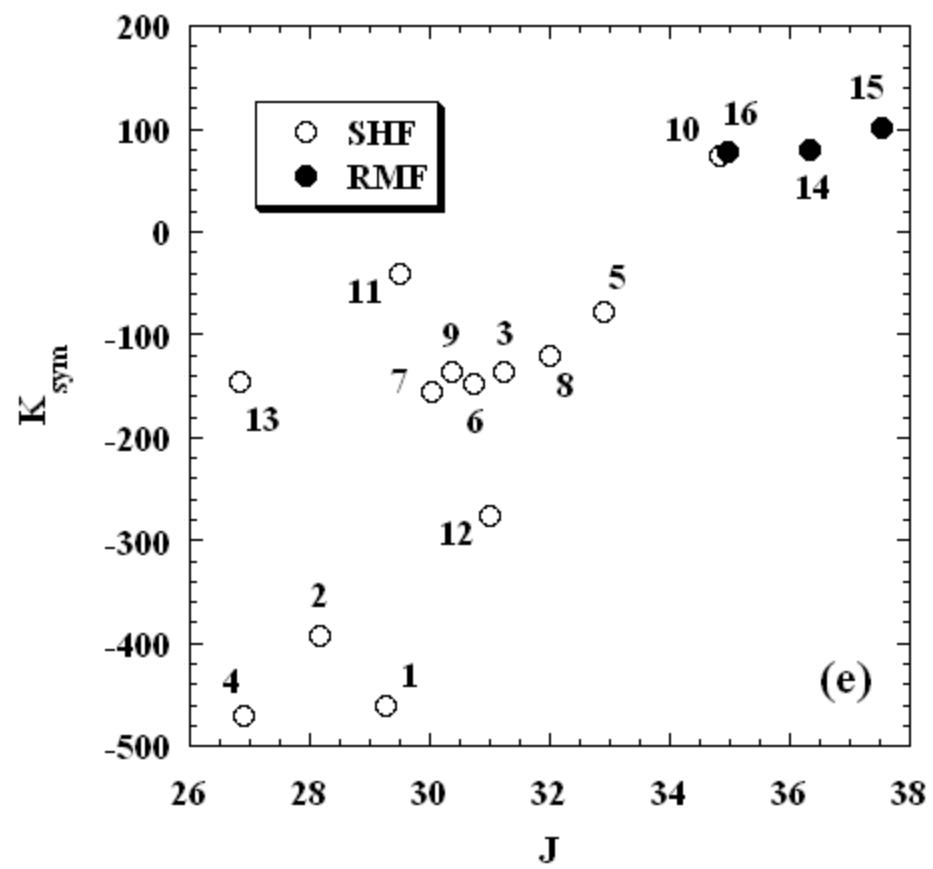


Fig.8

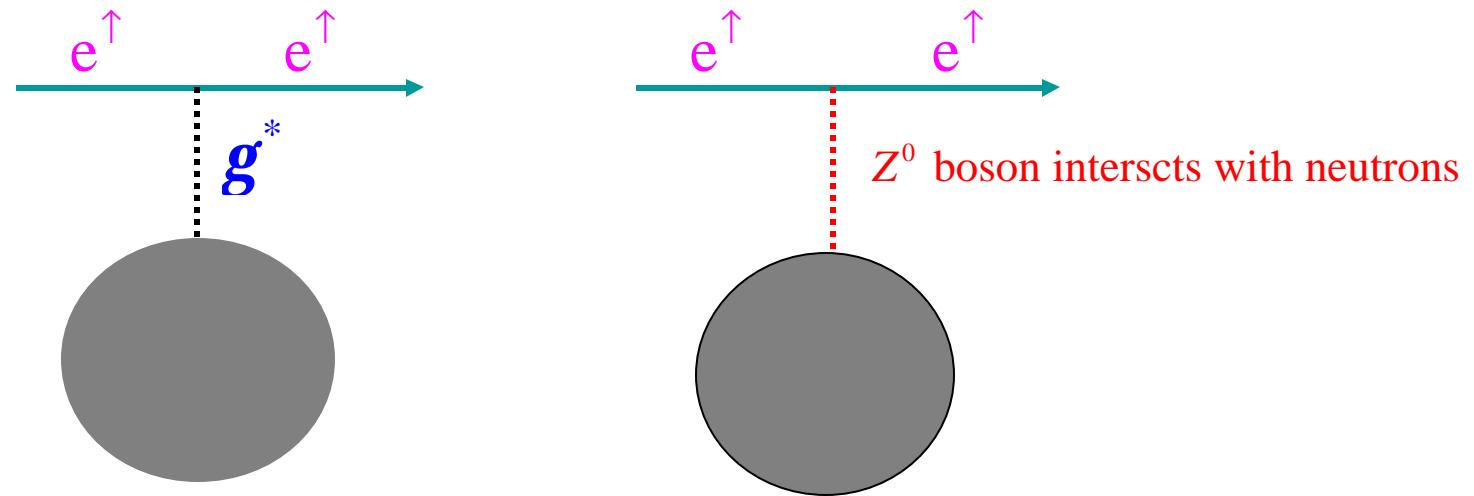






Future experiments

Polarized electron scattering (Jafferson laboratory)



More precise (p,p') experiments (RCNP)

Summary

1. Nuclear incompressibility K is converging empirically $K \sim 220\text{MeV}$
2. A clear correlation between neutron skin thickness and neutron matter EOS
3. Neutron skin thickness is large in neutron-rich unstable nuclei, but the correlation is weak.
4. There is also a clear correlation between the neutron skin thickness and the symmetry energy coefficient.
5. The pressure of RMF is higher than that of SHF in general.
6. The empirical neutron skin thickness gives a critical information both on the neutron EOS and mean field models.

S. Yoshida and HS, Phys. Rev. C69, 024318 (2004)

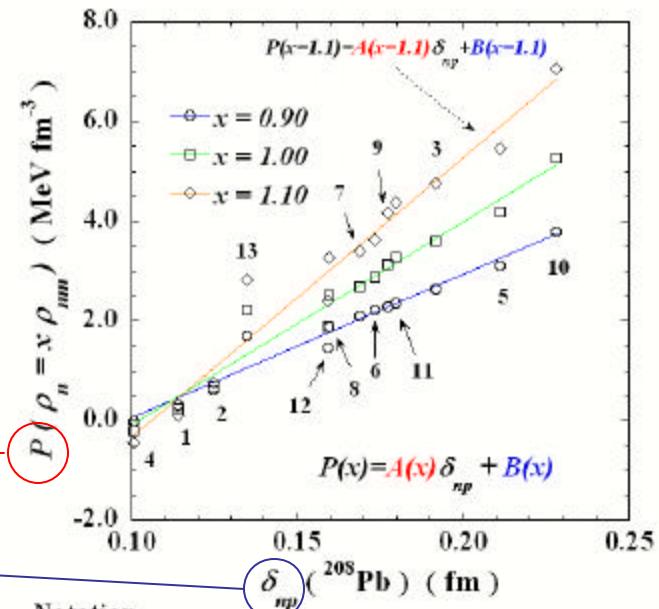
There is a clear linear correlation between the pressure P of neutron matter and the neutron skin thickness d_{np} of ^{208}Pb in SHF at each $x (= r_n / r_{nm})$ neighborhood of $x = 1.0$.

$$P(x) = A(x)d_{np} + B(x)$$

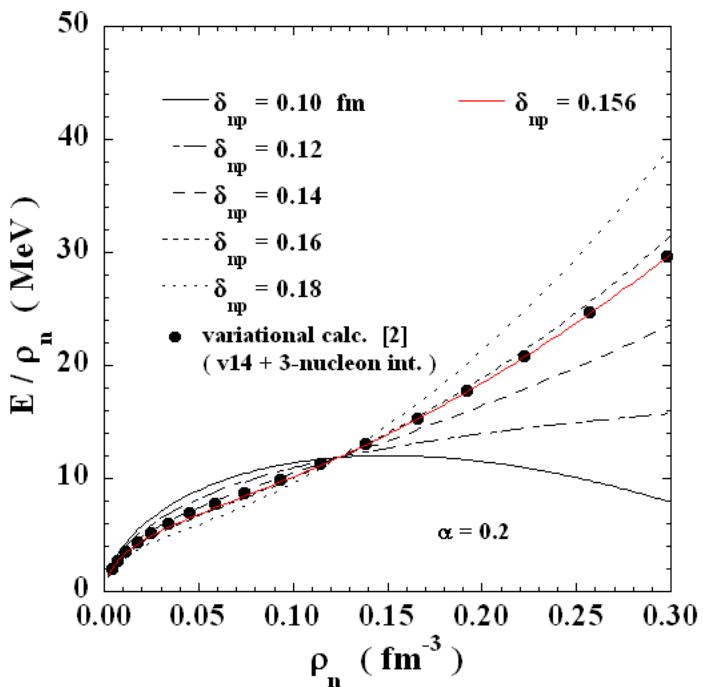
? Assumption

$A(x)$ and $B(x)$ are quadratic functions at $x = 1.0$.

$$P(x) \rightarrow P_{\text{quad}}(x) = (a_1 x^2 + a_2 x + a_3) d_{np} + (b_1 x^2 + b_2 x + b_3)$$



By the method of least squares, coefficients of quadratic functions are obtained. Furthermore by assumption that $P_{quad}(x)$ and the pressure $P_{SHF}(x)$ of neutron matter by Skyrme Hartree Fock model satisfy the three conditions, the EOS of neutron matter is represented by a simple equation which is independent on the incompressibility analytically. ($r_{nm} = 0.16 \text{ fm}^{-3}$, $E_0 = 16 \text{ MeV}$)



Summary

- ? The value of neutron skin thickness of ^{208}Pb plays a very important role in the determination of not only nuclear matter properties but also EOS of neutron matter.
- ? In the assumption that the pressure of neutron matter is given by the quadratic function of x at neighborhood of $x=1.0$, the Skyrme Hartree Fock model seems to suggest that the value of neutron skin thickness of ^{208}Pb is **0.156 fm**.

[2] B.Friedman and V.R.Pandharipande,
Nucl.Phys.A361,502(1981)