Neutron Rich Nuclei in Heaven and Earth

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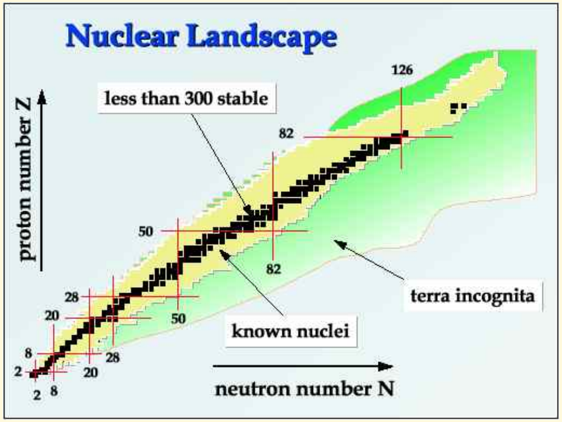
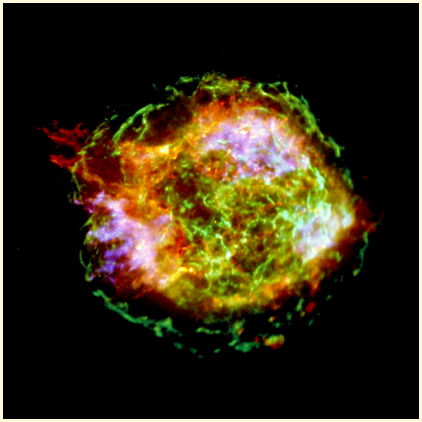




Cassiopeia A: Chandra 08/23/04

Workshop on Nuclear Incompressibility and Equation of State
University of Notre Dame – July 14-15, 2005

Outline

The Nuclear Matter Equation of State:

- The “Holy Grail” of Nuclear Physics
- Incompressibility: a fundamental parameter of the EoS
- Constraints from Nuclear Compressional Modes
- The role of the Symmetry Energy
- Constraints from Isovector Modes
- Constraints from Heavy-Ion Physics
- Constraints from Neutron-Star Physics

Equation of State I: Generalities

The Bethe-Weizsäcker (BW) Mass Formula:

$$E(Z,N) = a_{\rm vol}A + a_{\rm surf}A^{2/3} + a_{\rm coul}Z^2/A^{1/3} + a_{\rm symm}(N-Z)^2/A + \ldots$$

- Parameters extracted from a fit to thousands of known nuclear masses
- Hidden behind its success is the saturation of the nuclear force
- BW constrains the above parameters at (or near) saturation density:
  $\rho_0 \simeq 0.15\ {\rm fm}^{-3}$: $a_{\rm vol} \simeq -16$ MeV, $a_{\rm symm} \simeq +30$ MeV, ...

BW offers little on the density dependence of the parameters!

Taking the thermodynamic limit: Infinite Nuclear Matter

- Turn off the long-range Coulomb force
- Let Z, N and V go to infinity with ratios remaining finite:
  $\rho = A/V$, $\rho_p = Z/V$, $\rho_n = N/V$, $Y_p = Z/A$, $b = \delta = (N-Z)/A$, ...
- Only surviving terms in the thermodynamic limit:
  $E(Z,N)/A = a_{\rm vol} + a_{\rm symm}b^2$

Goal: Study the density dependence of the equation of state (EoS)!

Symmetric *vs* Asymmetric Matter

- Expand the total energy per nucleon around $b=0$:

$$E(\rho;b)/A = \underbrace{E(\rho;b=0)/A}_{\text{Symmetric Matter}} + \underbrace{b\left(\frac{\partial E/A}{\partial b}\right)_{b=0}}_{0} + \underbrace{b^2\,\frac{1}{2}\left(\frac{\partial^2 E/A}{\partial b^2}\right)_{b=0}}_{\text{Symmetry Energy}} + \ldots$$

Note: Pure neutron matter ≈ Symmetric Matter + Symmetry Energy!

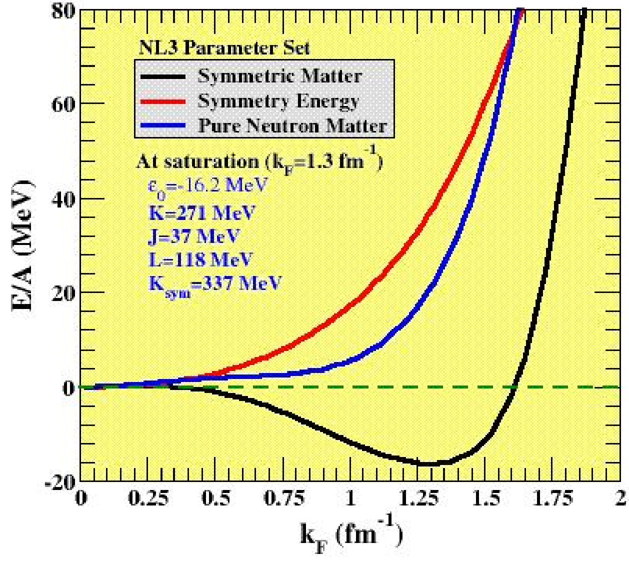
Equation of State II: Parametrization

Density dependence of the EoS: Characterization

- Expand the EoS around saturation density: $\xi \equiv (k_{\rm F} - k_{\rm F}^{0})/k_{\rm F}^{0}$

$$E/A(k_{\rm F}, b=0) = \epsilon_0 + \frac{1}{2}K\xi^2 + \ldots$$

$$\mathcal{S}/\mathcal{A}(k_{\rm F}) = J + L\xi + \frac{1}{2}K_{\rm sym}\xi^2 + \ldots$$



Density Functional, Kohn-Sham, MF Theory

Improving the Bethe-Weizsäcker (BW) Mass Formula:

$$\mathcal{L}_{\rm int} = g_{\rm s}\bar\psi\psi\phi - g_{\rm v}\bar\psi\gamma^{\mu}\psi V_{\mu} - \frac{g_{\rho}}{2}\bar\psi\gamma^{\mu}\boldsymbol{\tau}\cdot\mathbf{b}_{\mu}\psi - e\bar\psi\gamma^{\mu}\tau_{p}\psi A_{\mu}$$
$$-\frac{\kappa}{3!}(g_{\rm s}\phi)^{3} - \frac{\lambda}{4!}(g_{\rm s}\phi)^{4} + \Lambda_{\rm v}(g_{\rm v}^{2}V^{\mu}V_{\mu})(g_{\rho}^{2}b^{\mu}b_{\mu}) + \frac{\zeta}{4!}g_{\rm v}^{4}(V_{\mu}V^{\mu})^{2}$$

- Parameters fitted to a large body of ground-state properties
  (mostly binding energies and charge radii of many nuclei)
- Ground-state observables computed at the mean-field level
- Formalism is **NOT** Hartree (Hartree-Fock) theory
- Parameters of the model encode correlations that go beyond two-body
  (short, long, and pairing correlations in an average way)

Resulting model unlikely to describe correctly NN physics!

Correlating Model Parameters to the Physics:

Parameters	Constrained by
$g_{\rm s}, g_{\rm v}$	Ground state properties of finite nuclei
g_{ρ}	Ground state properties of heavy nuclei
κ, λ	Isoscalar giant monopole resonance
$\Lambda_{\rm v}$	Neutron radius of heavy nuclei
ζ	Neutron star structure

- Existent observables insufficient to constrain all parameters
- Determination of neutron radii of neutron-rich nuclei presses!
- Simultaneous mass-radius measurement of neutron stars presses!

Crucial measurements in Heaven and Earth on the horizon!

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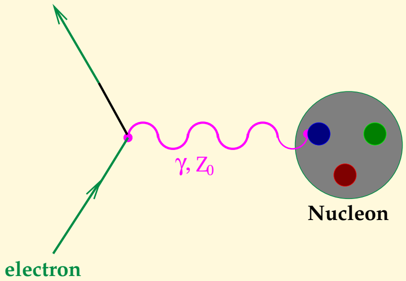
Experimental Extraction of $R_n - R_p$

JLAB Experiment 00-003 (03-011) [Michaels, Souder, Urciuoli]:

- Parity Violating Asymmetry in elastic $e-$Pb scattering
- Electroweak (as opposed to hadronic) probe of neutron density
- Weak-vector boson Z^0 couples strongly to neutrons
- A clean and accurate measurement of the neutron radius

1% or 0.05 fm measurement of the neutron radius of ^{208}Pb



Particle	EM coupling	Weak-Vector coupling
up-quark	$+2/3$	$+1-4\sin^2\theta_{\rm w}(+2/3) \simeq +1/3$
down-quark	$-1/3$	$-1-4\sin^2\theta_{\rm w}(-1/3) \simeq -2/3$
proton	$+1$	$+1-4\sin^2\theta_{\rm w} \simeq 0$
neutron	0	-1

$$g_{\rm v}^{f} = 2T_z^{f} - 4\sin^2\theta_{\rm w}Q^{f}\,, \quad \sin^2\theta_{\rm w} \approx 0.231 \simeq 1/4$$

Accurately Calibrated Models

Relativistic *vs* NonRelativistic: Model Dependence

Model	$k_{\rm F}^{0}$ ($\rm fm^{-1}$)	ϵ_0 (MeV)	K (MeV)	J (MeV)	L (MeV)
SkX	1.32	−16.1	271	31.1	33.2
SGII	1.33	−15.6	215	26.8	37.6
SKM*	1.33	−15.8	217	30.0	45.8
NL3	1.30	−16.2	271	37.4	118.5
TM1	1.29	−16.3	281	37.9	114.0

- 5% discrepancy in the binding energy and density at saturation
- 25% discrepancy in the value of the compression modulus
- 250% discrepancy in the slope of the symmetry energy!

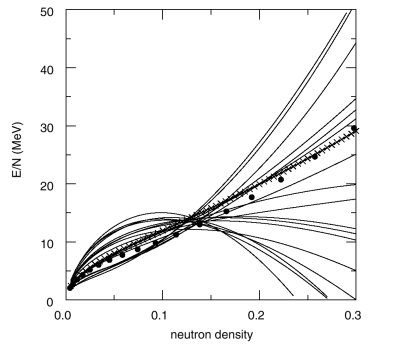
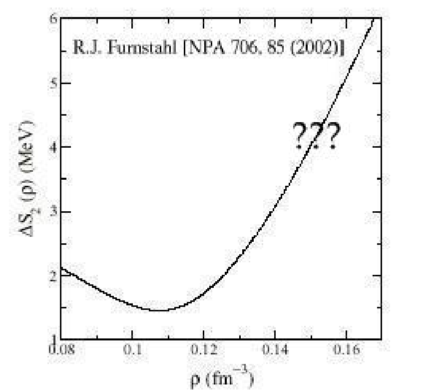


FIG. 2. The neutron EOS for 18 Skyrme parameter sets. The filled circles are the Friedman-Pandharipande (FP) variational calculations and the crosses are SkX. The neutron density is in units of neutron/fm^3.



Symmetry Energy:

- Constrained to within 1-2 MeV — slightly below saturation density
- Density dependence unconstrained by available nuclear data

The "Garg" Challenge

Question:

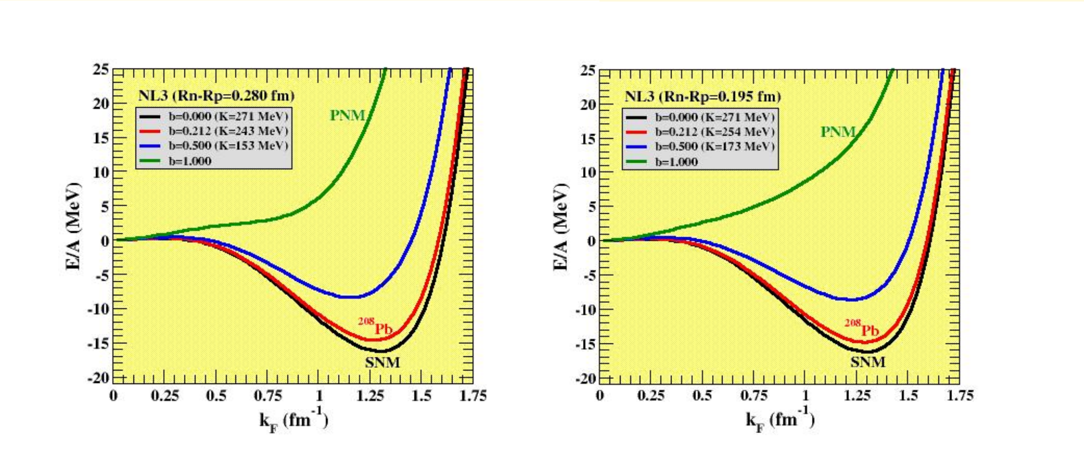
How can Relativistic and NonRel models reproduce the ISGMR in ^{208}Pb — while predicting compression moduli that differ by 25%?

Answer:

Could the discrepancy in the values of K, L be related?

- ^{208}Pb is a neutron-rich $(b = 0.212)$ nucleus
- ISGMR in ^{208}Pb measures the compression modulus of asymmetric matter

Sensitive to the density dependence of the symmetry energy!



The curvature of the EoS of asymmetric nuclear matter depends sensitively on the density dependence of the symmetry energy!

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When Two Wrongs Make a Right ...

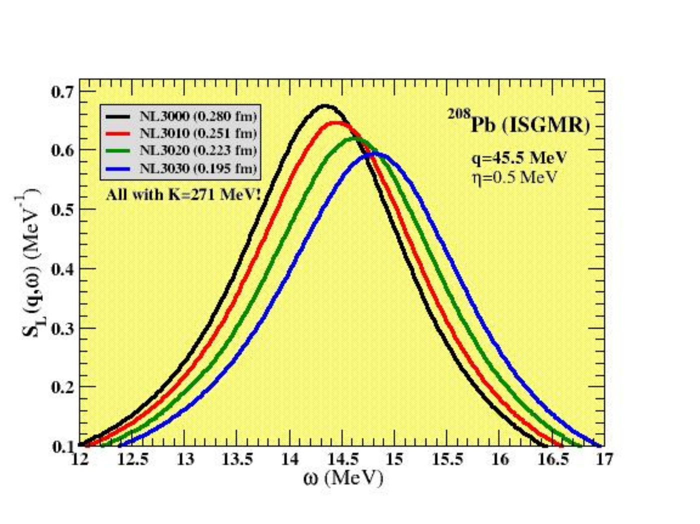
(NL3 and the ISGMR in ^{208}Pb)

Conjecture:

While enormously successful in the description of ground state properties of a variety of nuclei, NL3 predicts a compression modulus and a symmetry energy that are too stiff.

Consequences:

- Large K, L cancel each other in the ISGMR in ^{208}Pb ($b = 0.212$)
  NL3 reproduces the location of the ISGMR in ^{208}Pb
- Large K prevails in the ISGMR in ^{90}Zr ($b = 0.111$)
  NL3 overestimates the location of the ISGMR in ^{90}Zr
- Large L prevails in the IVGDR in ^{208}Pb in the ISGMR in ^{208}Pb
  NL3 underestimates the location of the IVGDR in ^{208}Pb



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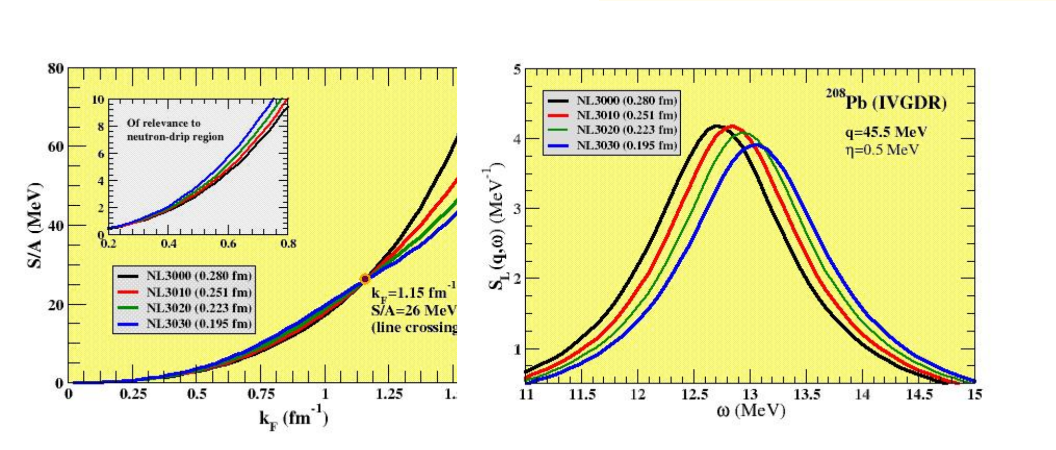
Testing the Conjecture: the IVGDR in ^{208}Pb

The Isovector Giant Dipole Resonance: The Quintessential Nuclear Mode:

- Surface oscillation of neutrons against protons
- Sensitive to the density dependence of the symmetry energy at low densities



The stiffer the symmetry energy, the lower the peak position of the IVGDR mode

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The FSUGold Parametrization*

(An accurately calibrated relativistic parameter set)

The Program:

- Input binding energy and charge radii of doubly magic nuclei
  Solve in self-consistent mean-field approximation
- Compute the linear response of the mean-field ground state
  Solve in self-consistent MF+RPA approximation
- Without any further adjustment, predict neutron-star structure
  Solve Tolman-Oppenheimer-Volkoff equations

Only physics that neutron stars are sensitive to is the equation of state of neutron-rich matter ...

Nucleus	Observable	Experiment	NL3	FSUGold
^{90}Zr	B/A (MeV)	8.71	8.69	8.68
	$R_{\rm ch}$ (fm)	4.26	4.26	4.25
	$R_n - R_p$ (fm)	—	0.11	0.09
	ISGMR (MeV)	17.89 ± 0.20	18.62	17.98
^{208}Pb	B/A (MeV)	7.87	7.88	7.89
	$R_{\rm ch}$ (fm)	5.50	5.51	5.52
	$R_n - R_p$ (fm)	—	0.28	0.21
	ISGMR (MeV)	14.17 ± 0.28	14.32	14.04
	IVGDR (MeV)	13.30 ± 0.10	12.70	13.07

$*$ **Disclaimer: Gold is referred to the color — not the metal!**

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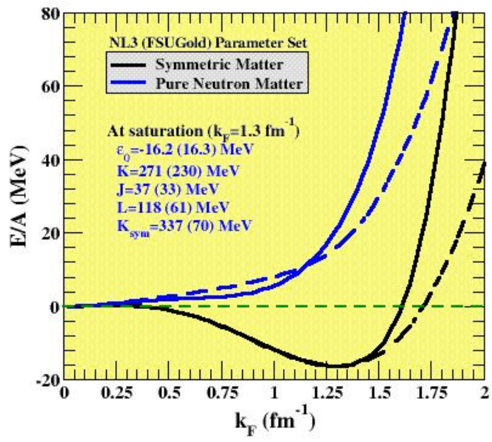
Answer to the "Garg" Challenge

Main lessons learned:

- NL3 enormously successful in the description of ground state properties
  Yet existent ground-state data unable to constrain K, L
- ISGMR in ^{208}Pb sensitive to both K and L
  "Unfortunate" cancellation: *when two wrongs make a right …*
- Accurately calibrated models must be further constrained:
  ISGMR in ^{90}Zr (K) and IVGDR in ^{208}Pb (L)

Model	$k_{\rm F}^{0}$ (fm^{-1})	ϵ_0 (MeV)	K (MeV)	J (MeV)	L (MeV)
SkX	1.32	−16.1	271	31.1	33.2
SGII	1.33	−15.6	215	26.8	37.6
SKM*	1.33	−15.8	217	30.0	45.8
NL3	1.30	−16.2	271	37.4	118.5
FSUGold	1.30	−16.3	230	32.6	60.5



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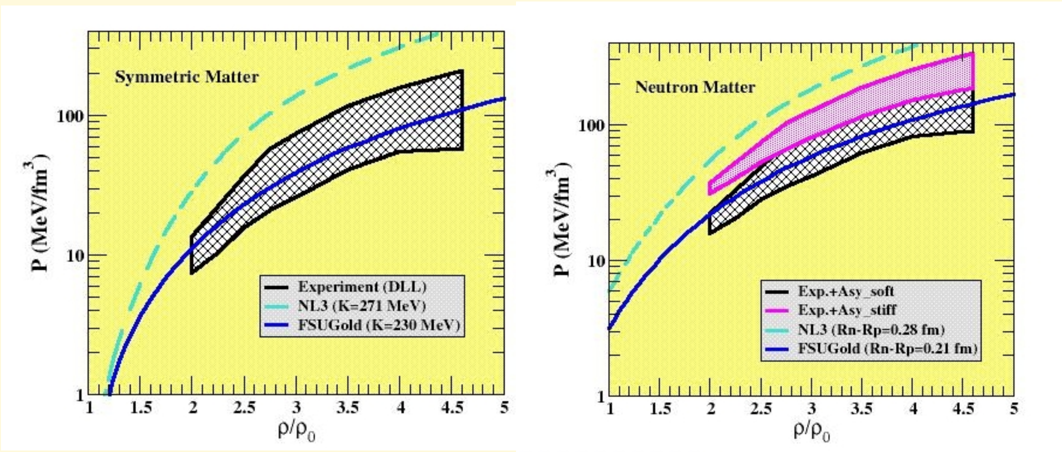
High Densities in Earth

[Danielewicz, Lacey, and Lynch – Science **298**, 1592 (2002)]

Nuclear Collisions: Constraints and Predictions

- Sole earthly tool available to compress nuclear matter
- Compressions up to several (five) times nuclear saturation density
- Imprint of the EoS left in the flow and fragment distribution



FSUGold provides a reliable extrapolation to high density

Food for thought: $\mathcal{S}/A(\mathrm{NL3}) \simeq 37(\rho/\rho_0)^{0.98}$ *vs* $\mathcal{S}/A(\mathrm{FSU}) \simeq 33(\rho/\rho_0)^{0.64}$

High Densities in Heaven

Neutron Stars: Constraints and Predictions

- Sole heavenly tool available to compress nuclear matter
- Compressions up to several (ten) times nuclear saturation density
- Imprint of the EoS left in limiting mass, radius, cooling history, ...

Observable	NL3	FSUGold
ρ_c (fm^{-3})	0.052	0.076
R (km)	15.05	12.66
$M_{\rm max}(M_\odot)$	2.78	1.72
$\rho_{\rm Urca}$ (fm^{-3})	0.21	0.47
$M_{\rm Urca}(M_\odot)$	0.84	1.30
$\Delta M_{\rm Urca}$	0.38	0.06


NASA News Release

National Aeronautics and Space Administration

Marshall Space Flight Center - Huntsville, Ala. 35812
http://www.msfc.nasa.gov/news

For Release: April 10, 2002

Release: 02-082

Cosmic X-rays reveal evidence for new form of matter

NASA's Chandra X-ray Observatory has found two stars - one too small, one too cold - that reveal cracks in our understanding of the structure of matter. These discoveries open a new window on nuclear physics, offering a link between the vast cosmos and its tiniest constituents.

Chandra?s observations of RXJ1856.5-3754 and 3C58 suggest that the matter in these stars is even denser than nuclear matter found on Earth. This raises the possibility these stars are composed of pure quarks or contain crystals of sub-nuclear particles that normally have only a fleeting existence following high-energy collisions.

By combining Chandra and Hubble Space Telescope data, astronomers found that RXJ 1856 radiates like a solid body with a temperature of 1.2 million degrees Fahrenheit (700,000 degrees Celsius) and has a diameter of about 7 miles (11.3 kilometers). This size is too small to reconcile with standard models for neutron stars - until now the most extreme form of matter known.

Some Questions and Answers:

- Is the pulsar in 3C58 an exotic (quark?) star?

  Not necesarilly if $M_* > 1.3\ M_\odot$
- Is the limiting mass of a neutron star $M_{\rm max} \simeq 1.72\ M_\odot$?

  Maybe not; recent report suggests $M(PSRJ0751+1807) = 2.1^{+0.4}_{-0.5}$

Fascinating times lie ahead!

Conclusions and Outlook

Fascinating times lie ahead!

- A new accurately calibrated relativistic model *FSUGold* has been fitted to the binding energies and charge radii of a variety of magic nuclei — and constrained by a few nuclear collective modes.
- Without any further adjustment of parameters *FSUGold* successfully reproduces the EoS of high-density symmetric matter extracted from an analysis of heavy-ion experiments.
- Without any further adjustment of parameters *FSUGold* predicts various neutron-star observables that will be tested in the near future.