Density Dependence of Energy in Symmetric and Asymmetric Matter, from Structure and Reactions

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Outline

Boltzmann Eq & Energy Functional

Ptcle phase space distributions f follow the Boltzmann eq:

$$\frac{\partial f}{\partial t} + \underbrace{\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}}}_{\text{velocity}} \frac{\partial f}{\partial \mathbf{r}} - \underbrace{\frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{r}}}_{\text{force}} \frac{\partial f}{\partial \mathbf{p}} = \int d\mathbf{p}_2 \int d\Omega' \, v_{12} \frac{d\sigma}{d\Omega'}$$

$$\times \Big(\underbrace{(1-f_1)(1-f_2) f_1' \, f_2'}_{\text{gain}} - \underbrace{(1-f_1')(1-f_2') f_1 \, f_2}_{\text{loss}} \Big)$$

where single-particle energy ϵ :

$$\epsilon(\mathbf{p}) = \frac{\delta E}{\delta f(\mathbf{p})}$$

and energy functional: $$E = E_{vol} + E_{gr} + E_{sym} + E_{Coul}$$

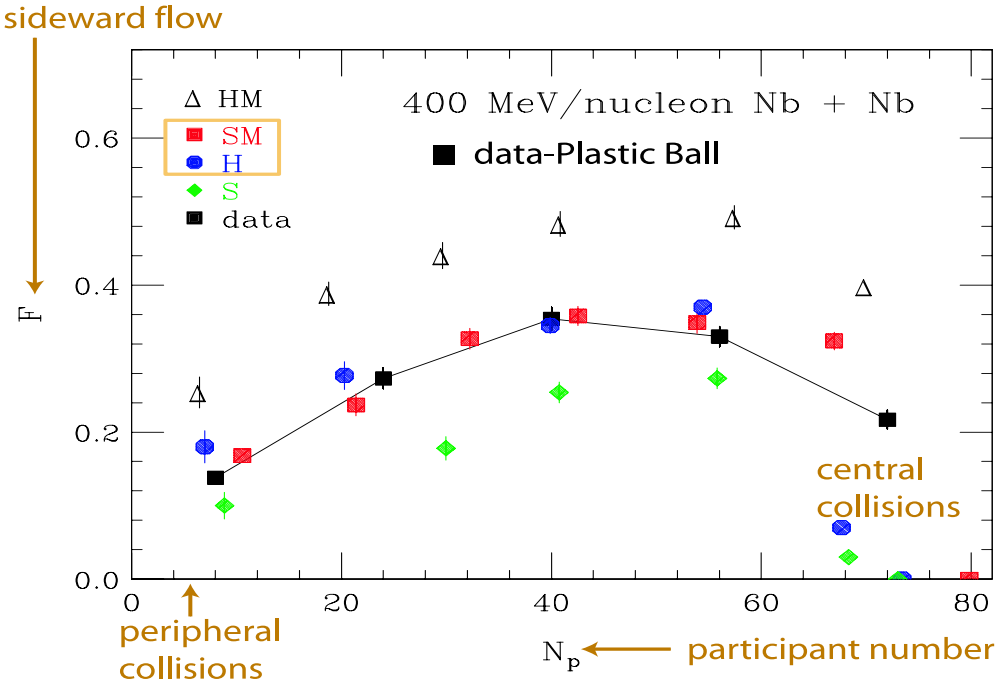
with $$E_{gr} = \frac{a_{gr}}{\rho_0} \int d\mathbf{r} \, (\nabla \rho)^2$$

Practice

Transport:

Individual data sets can commonly be explained using different sets of assumptions!

- ▷ Successful, yields insight, flexible!
- ▷ Uncertainties: $\epsilon(p,\rho)$, $\frac{d\sigma}{d\Omega} \Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{free}$?
  $\longrightarrow$ difficulty & opportunity
  EOS, transport cfs
- ▷ Need for dedicated observables...



Pan & P.D. PRL70(93)2063

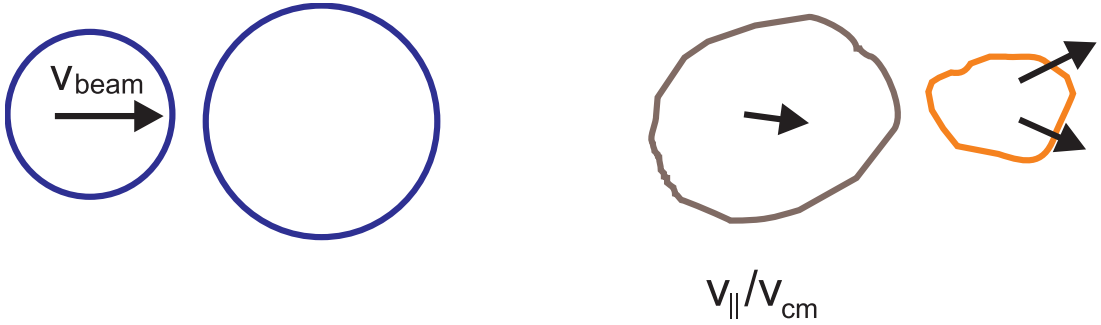
SM - strong dependence of ϵ on p

H - strong dependence of ϵ on ρ

Stopping: Linear Momentum Transfer

Stopping observables could constrain elementary in-medium cross-sections??

Linear mo transfer:
Mass asymmetric reactions ($b \sim 0$) examined in lab frame



Velocity component along the beam of the most massive fragment determined and its average compared to the cm velocity.

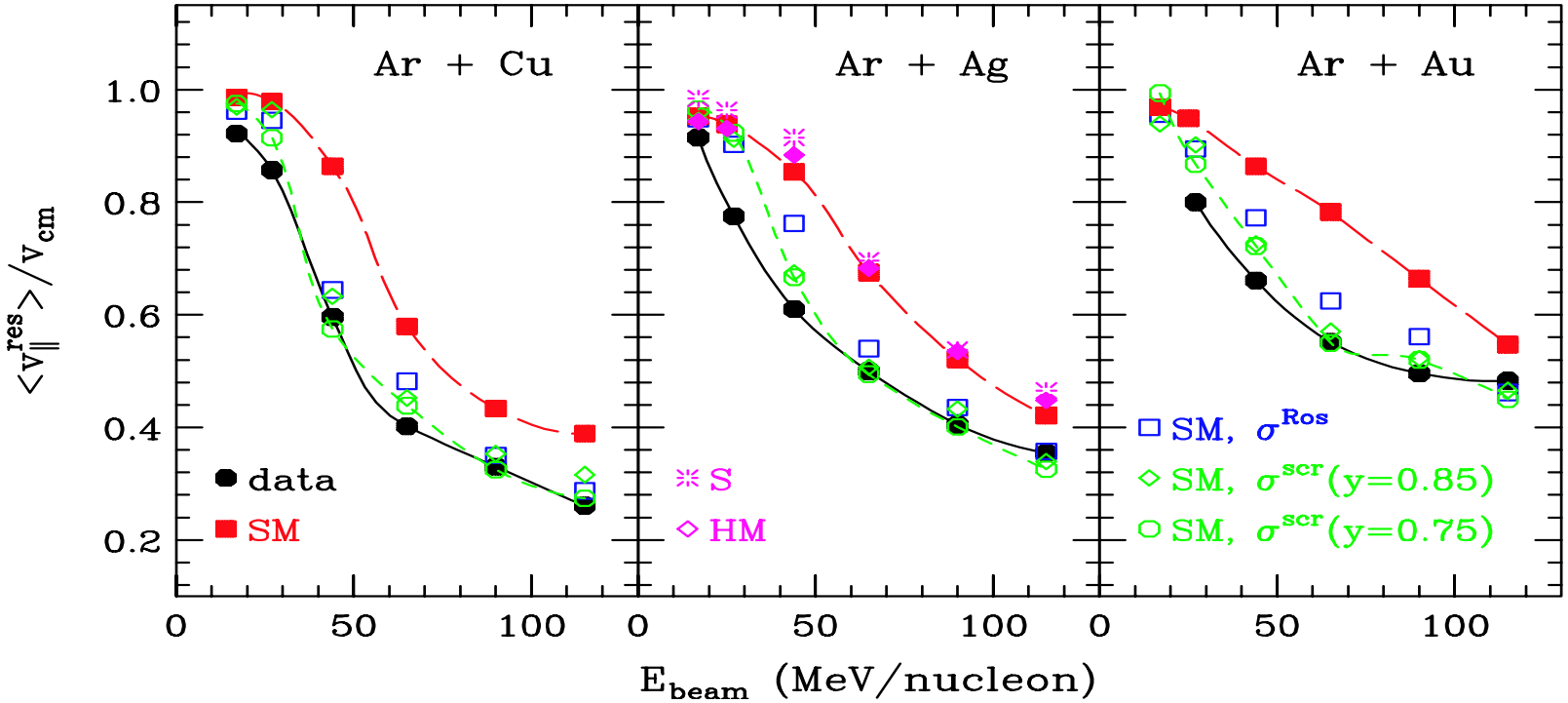
Limits:

Little stopping:	$\langle v_{\|\|} \rangle \sim 0$	small c.s.?
Large stopping, fusion:	$\langle v_{\|\|} \rangle \simeq v_{cm}$	large c.s.?

In-Medium Cross-Section Reduction

Data: Conlin *et al.* PRC57(98)R1032

Ar + Cu, Ag, Au High multiplicity events $\langle b \rangle \sim b_{max}/4$



With free cross-sections (red/pink) transport overestimates stopping data (black) irrespectively of mean field!

Different parameterizations of *reduced cross sections*, yielding *the same viscosity*, describe the data.

EOS and Flow Anisotropies

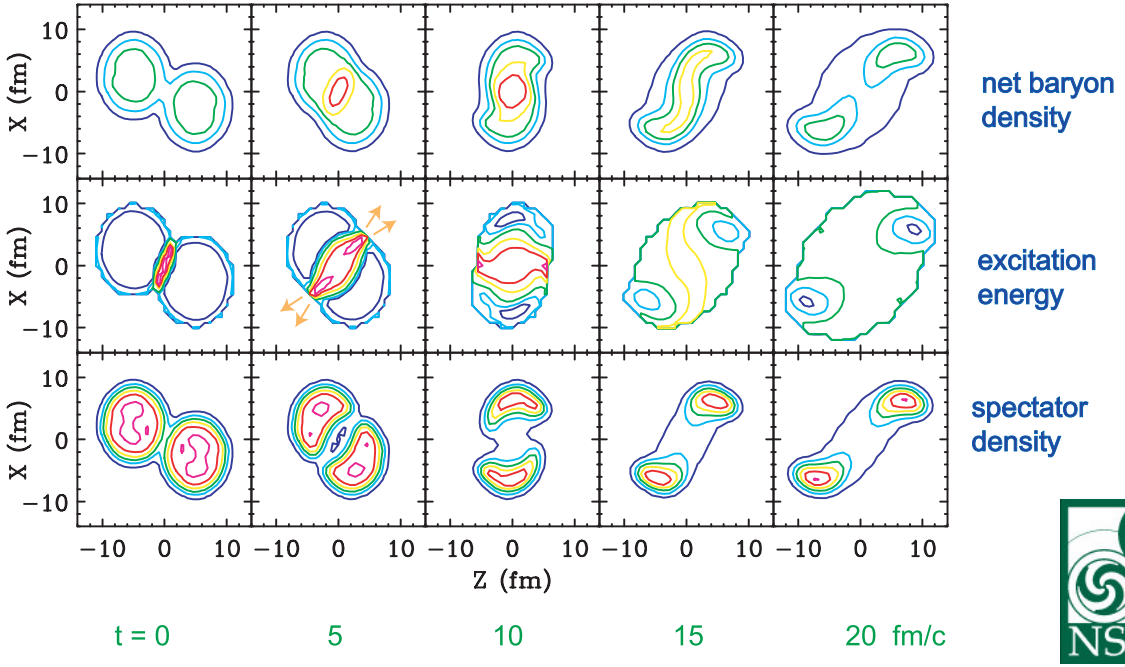
EOS assessed through reaction plane anisotropies characterizing particle collective motion.

Hydro? Euler eq. in $\vec{v} = 0$ frame: $\boxed{m_N \, \rho \, \frac{\partial}{\partial t} \, \vec{v} = -\vec{\nabla} p}$

where p - pressure. From features of v, knowing Δt, we may learn about p in relation to ρ. Δt fixed by spectator motion.

For high p, expansion rapid and much affected by spectators.

For low p, expansion sluggish and completes after spectators gone.



Sideward Flow Systematics

Deflection of forwards and backwards moving particles away from the beam axis, within the reaction plane.

Au + Au Flow Excitation Function

Note: *K* used as a label

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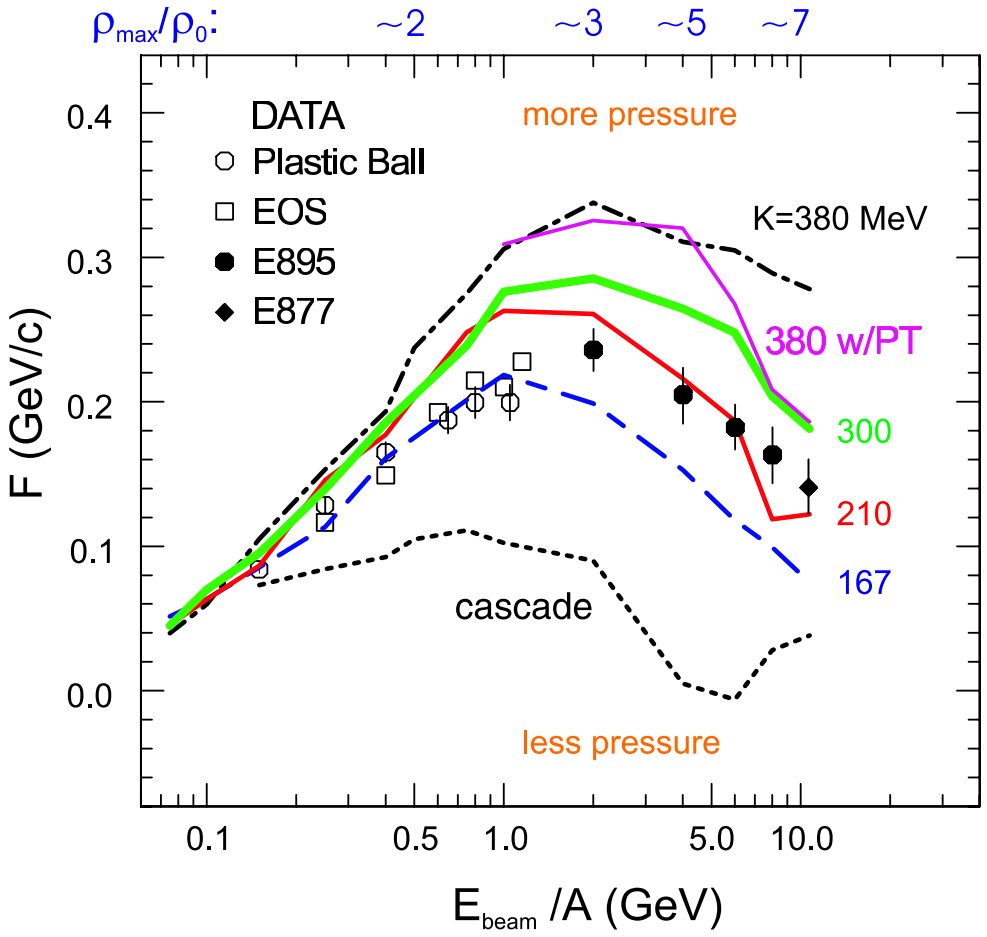
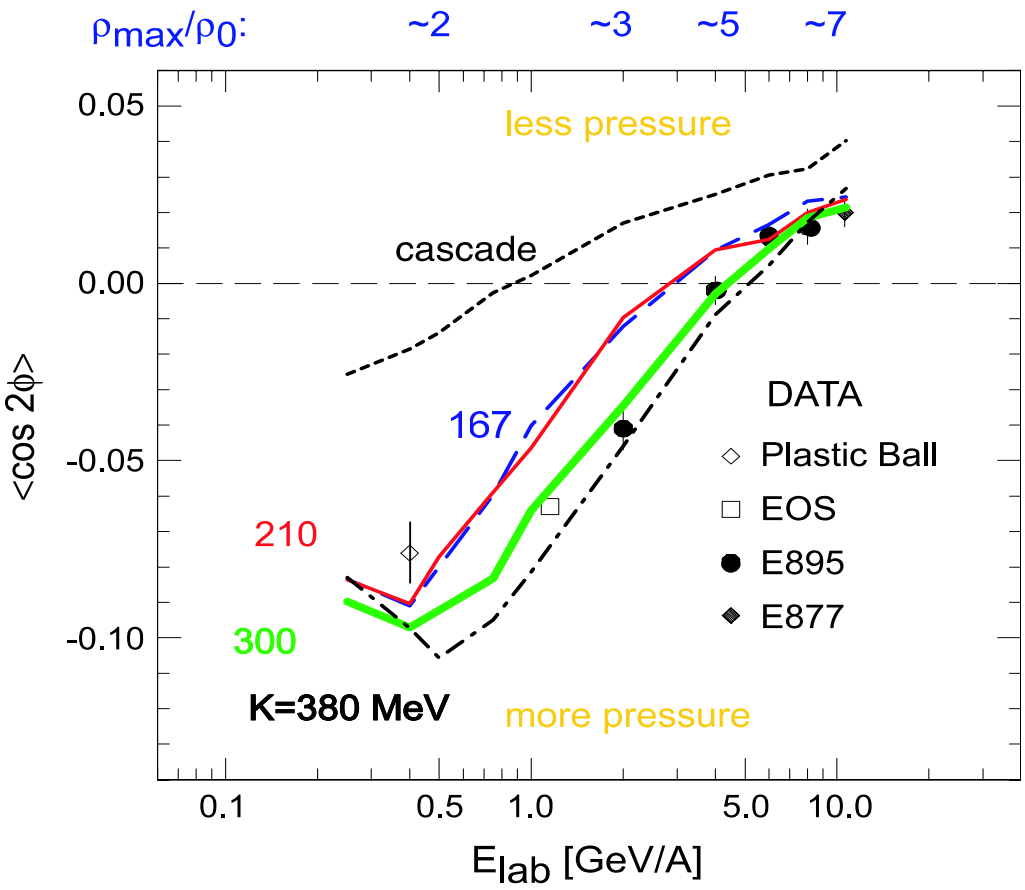
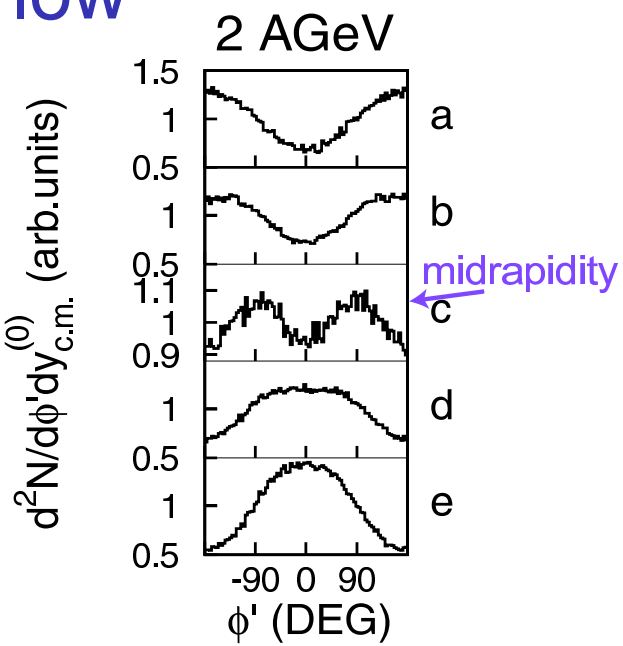
The sideward-flow observable results from dynamics that spans a density range varying with the incident energy.



2nd-Order or Elliptic Flow

Another anisotropy, studied at midrapidity: $v_2 = \langle \cos 2\phi \rangle$, where ϕ is azimuthal angle relative to reaction plane.





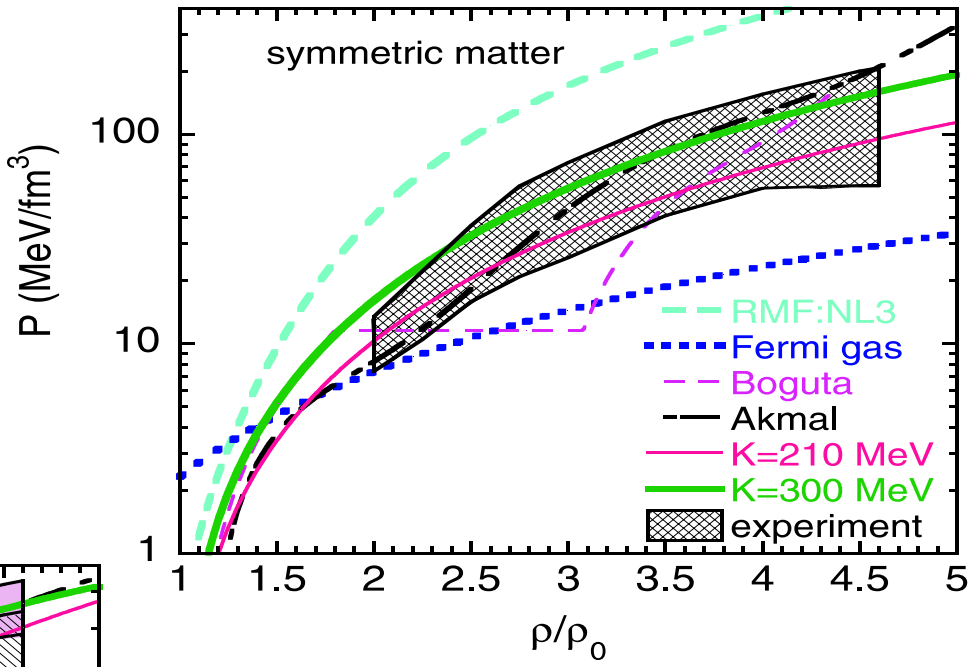
Au+Au v_2 Excitation Function

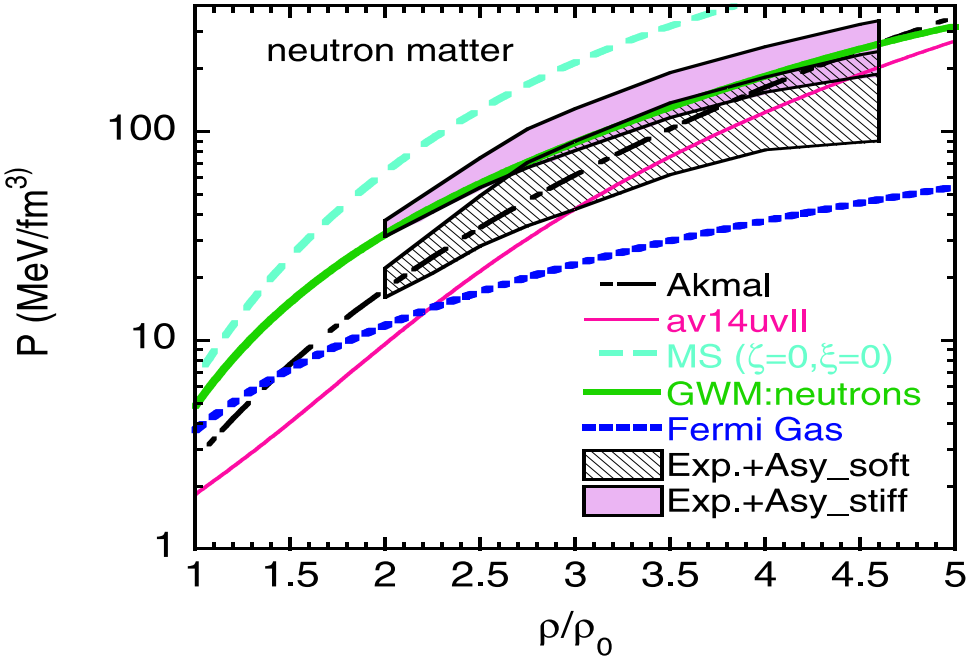
Constraints on EOS

Au+Au flow anisotropies: $\rho \simeq (2-4.6)\rho_0$.

No one EOS yields both flows right. Discrepancies: inaccuracy of theory.

Most extreme models for EOS can be eliminated.





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Neutron Matter:

Uncertainty in symmetry energy

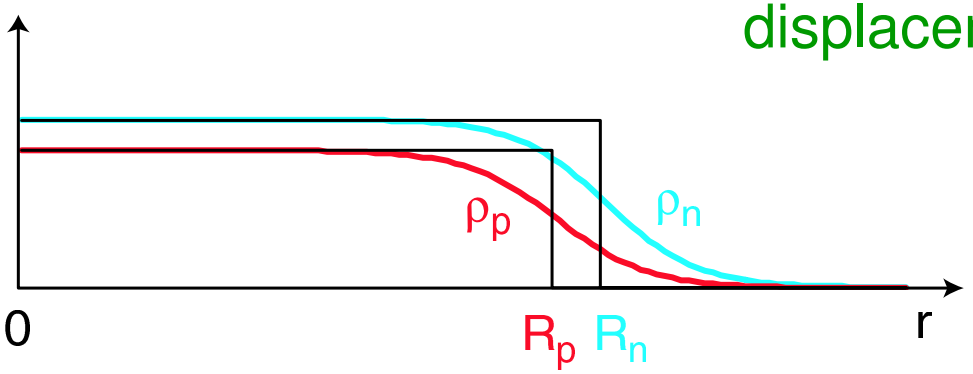
Symmetry Energy in Binding Formula

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(N-Z)^2}{A}$$

Asymmetry term of volume character only.

Formula intrinsically inconsistent! Surface tension $\sigma = \frac{\partial E}{\partial S} = \frac{a_S}{4\pi r_0^2 A^{2/3}}$ is work per unit area in creating surface. Must be done because nucleons at surface less bound. But! If a nucleus more asymmetric, nucleons generally less bound and tension should drop.

Consequences: 1. Surface energy must depend on asymmetry!
2. Surface must carry asymmetry (possible through relative displacement of n and p surfaces).



Symmetry Energy in Binding Formula

Under charge symmetry:

$$E_S = a_S\, A^{2/3} + a_A^S\, \frac{(N_S - Z_S)^2}{A^{2/3}} \qquad E_V = a_V\, A + a_A^V\, \frac{(N_V - Z_V)^2}{A}$$

$$E = E_S + E_V \qquad N - Z = (N_S - Z_S) + (N_V - Z_V)$$

Minimization of E with respect to the asymmetry partition: case of coupled capacitors for asymmetry, with the result

$$E = E_0 + E_A = E_0 + \frac{(N-Z)^2}{\underbrace{\frac{A}{a_A^V}}_{\text{volume capacitance}} + \underbrace{\frac{A^{2/3}}{a_A^S}}_{\text{surface capacitance}}}$$

Modified energy formula:

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \underbrace{\frac{a_A^V}{1 + \frac{a_A^V}{a_A^S}\, A^{-1/3}}}_{a_A(A)}\, \frac{(N-Z)^2}{A}$$

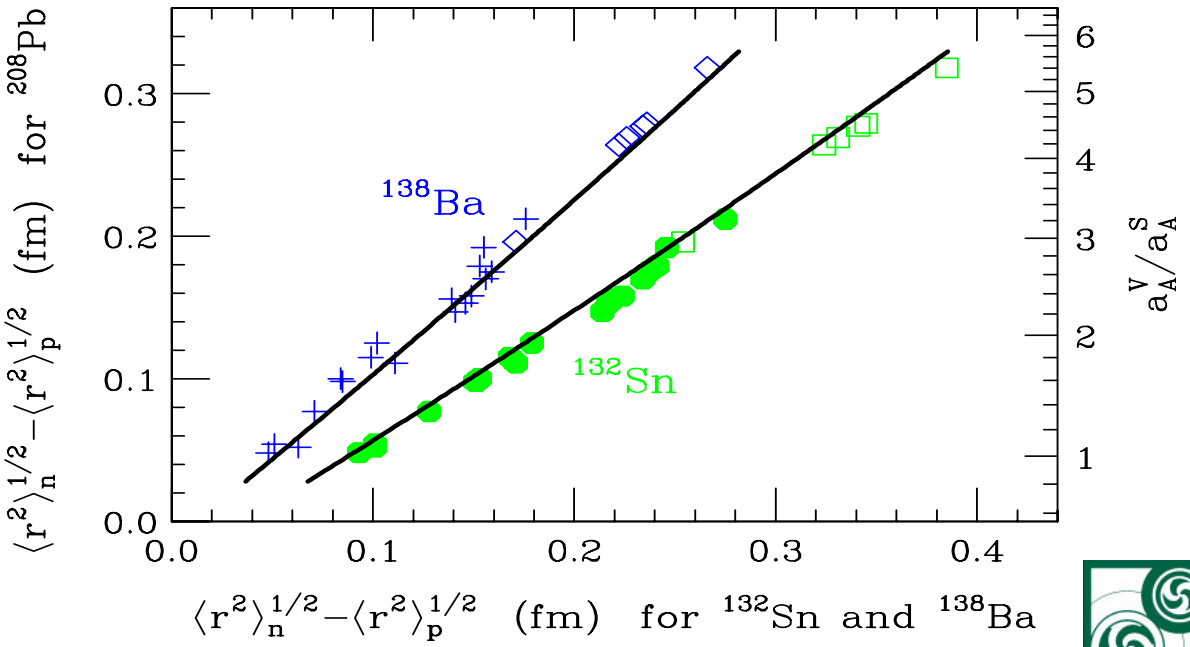
Skin Size

Surface-to-volume asymmetry:

$$\frac{N_S - Z_S}{N_V - Z_V} = \frac{C_S}{C_V} = \frac{A^{2/3}/a_A^S}{A/a_A^V} = \frac{a_A^V}{a_A^S} A^{-1/3}$$

$$\Rightarrow \quad \frac{\langle r^2\rangle_n^{1/2} - \langle r^2\rangle_p^{1/2}}{\langle r^2\rangle^{1/2}} = \frac{A}{6NZ} \frac{N-Z}{1+\frac{a_A^S}{a_A^V}} A^{1/3} + \text{Coulomb corr}$$

The formula (lines) compared to a multitude of nonrelativistic and relativistic mean-field calcs by Typel and Brown PRC64(01)027302 (symbols)

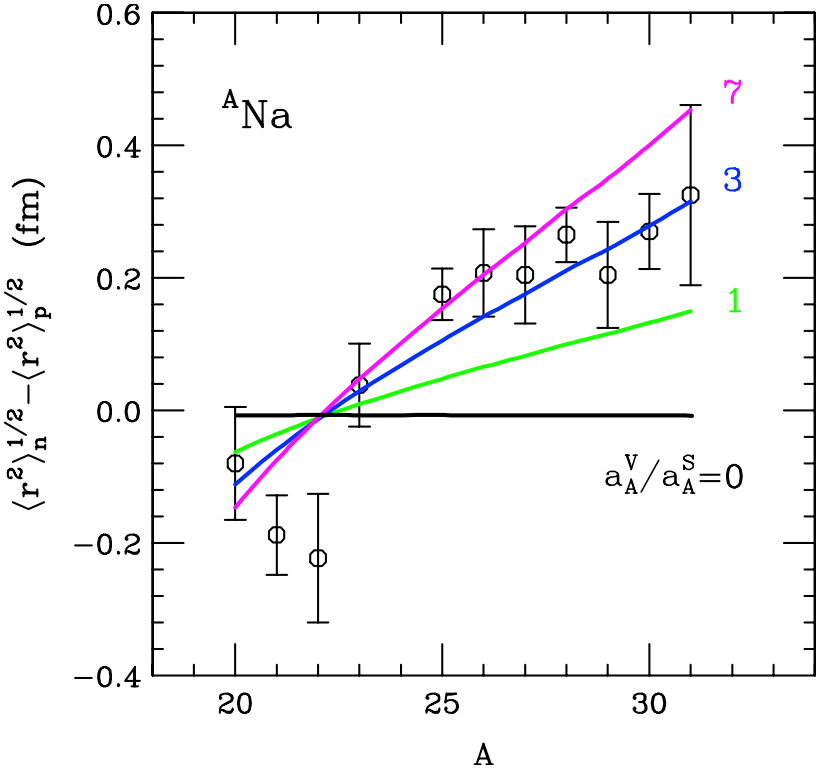


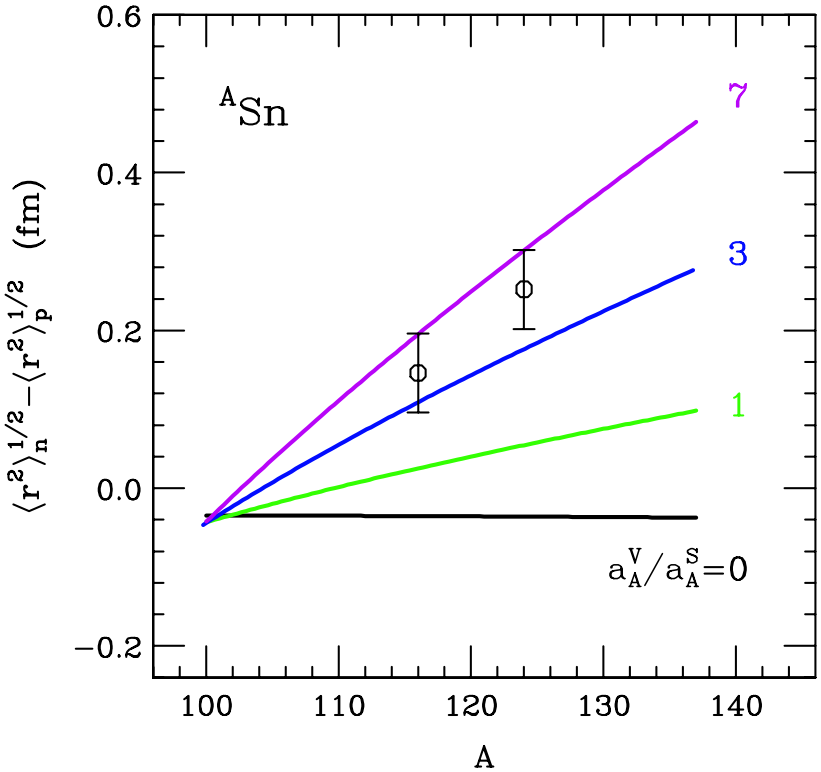
Accuracy, in reproducing microscopic theory, of ∼0.01 fm ?!

⇒ next data

Skin Size Analysis

Systematic of n-p skin sizes for different Na isotopes by Suzuki et al., PRL75(95)3241 + other data





difference between the rms n and p radii vs A: $a_A^V/a_A^S \sim 3$

Excitation of Isobaric Analog States

Detailed conclusions on the symmetry-energy parameters from *global mass fits*, using the mass formula, are complicated by details in other $(N-Z)$-sensitive terms: Coulomb (e.g. diffuseness), Wigner and pairing.

Charge invariance + isobaric analog states save the day?

$$E_A = a_A(A)\,\frac{(N-Z)^2}{A} = 4\,a_A(A)\,\frac{T_z^2}{A} \rightarrow 4\,a_A\frac{T^2}{A} = 4\,a_A\frac{T(T+1)}{A}$$

In the same nucleus:

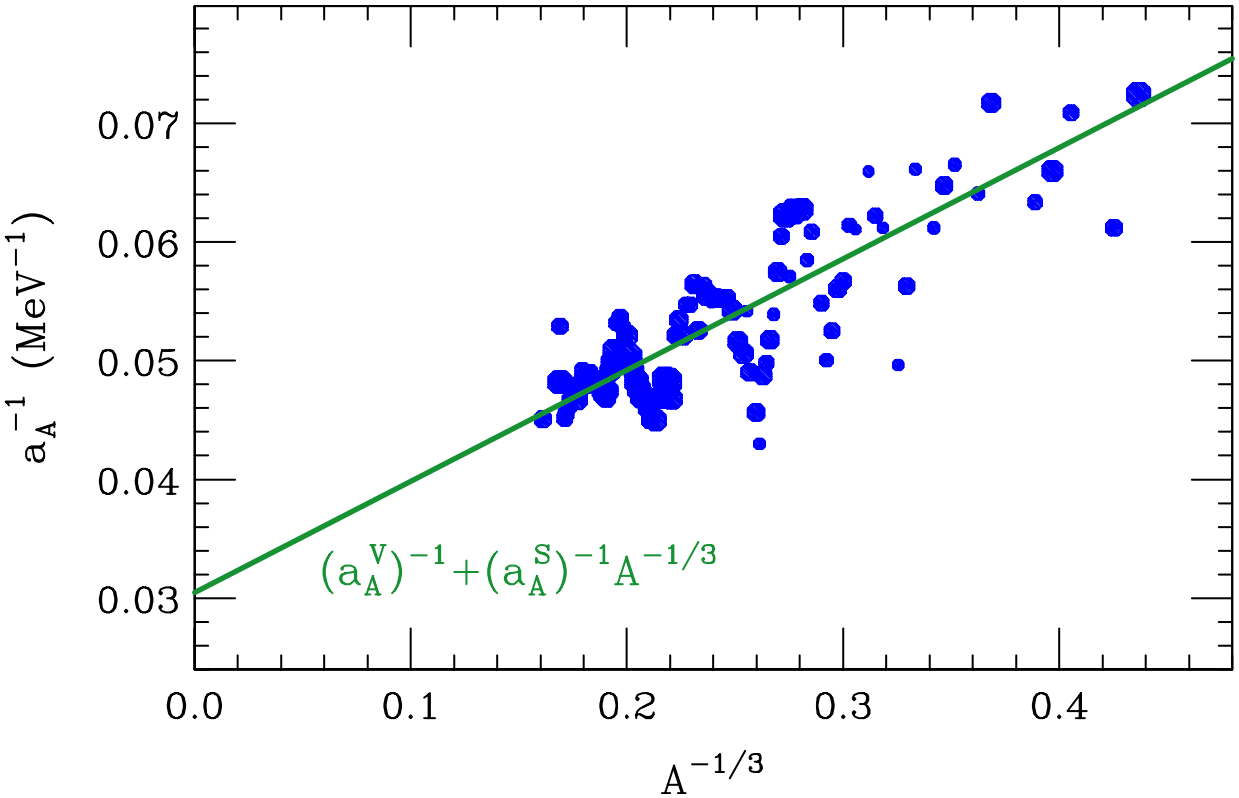
$$E_2(T_2) - E_1(T_1) = \frac{4\,a_A}{A}\left\{T_2(T_2+1) - T_1(T_1+1)\right\}$$

$$a_A^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E} \qquad \overset{?}{=} (a_A^V)^{-1} + (a_A^S)^{-1}\,A^{-1/3}$$

Analysis of IAS

Highest available ground-state IAS used for each A:

$$a_A^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E}$$

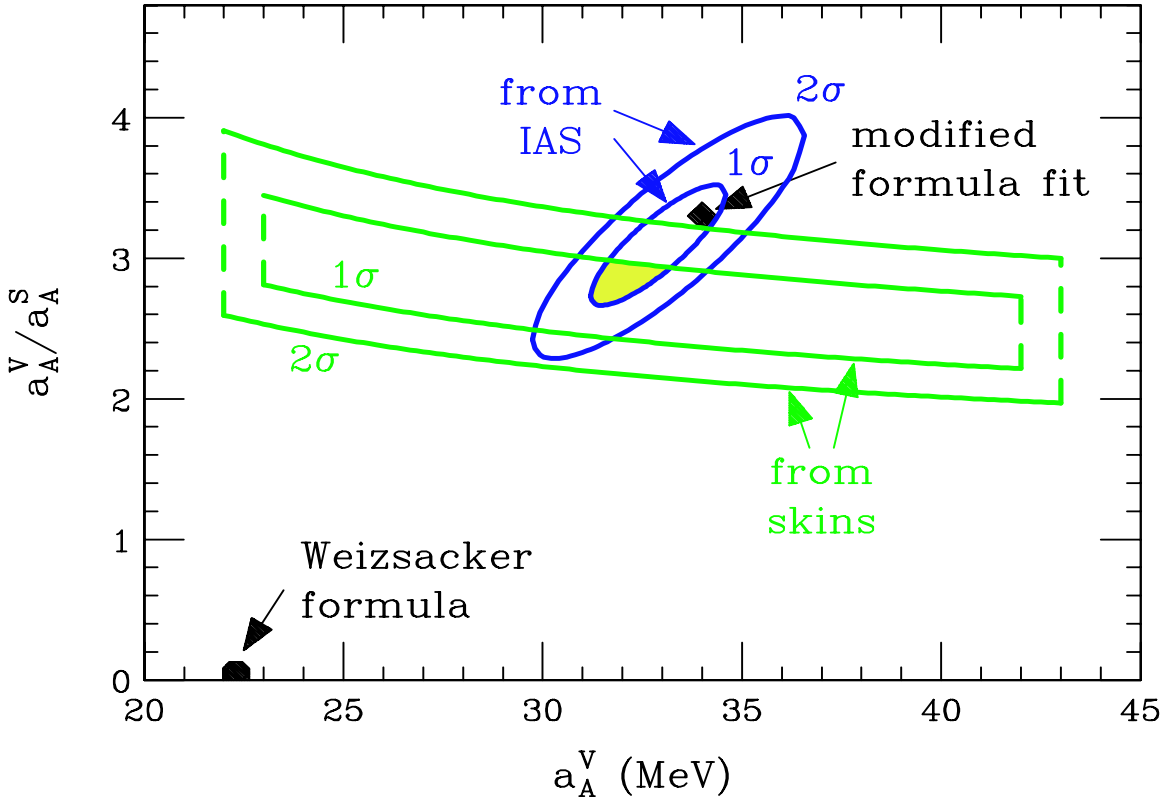


$a_A^V \sim 33\,\mathrm{MeV}$

$a_A^V / a_A^S \sim 3.2$

IAS + Skins

Fit combination:



Conclusions: $31\,\text{MeV} \lesssim a_A^V \lesssim 33\,\text{MeV}$ $2.7 \lesssim a_A^V/a_A^S \lesssim 3.0$

In $S(\rho) \simeq a_A^V \left(\frac{\rho}{\rho_0}\right)^\gamma$: $\gamma = (0.55 - 0.79)$

Conclusions

- Central collisions: many competing physical effects
- Some uncertainties narrowed using dedicated observables
- Pressure constrained at densities $\rho = (2 - 4.5)\rho_0$. Most extreme EOS eliminated.
- Structure: access to $\rho \leq \rho_0$.
- Macroscopic consistency puts surface symmetry energy into binding formula, with volume and surface symmetry energies combining as energies of coupled capacitors.
- Extension implies surface asymmetry skins and weakening of the symmetry term for light nuclei.
- Skins restrict ratio of symmetry coefficients; charge invariance allows to study symmetry term in one nucleus.
- Skin/IAS fits: $31\,\text{MeV} \lesssim a_A^V \lesssim 33\,\text{MeV}$ and $2.7 \lesssim a_A^V/a_A^S \lesssim 3.0^*$.