The Nuclear Level Density Continuum + Deformation + Asymmtery

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+ help from others at WU, ANL & IU

$\omega(E^*,\delta,Q)$

Experiments - Simulations - Theory

A. Some results

B. What are we talking about?

C. Two models for treating the continuum

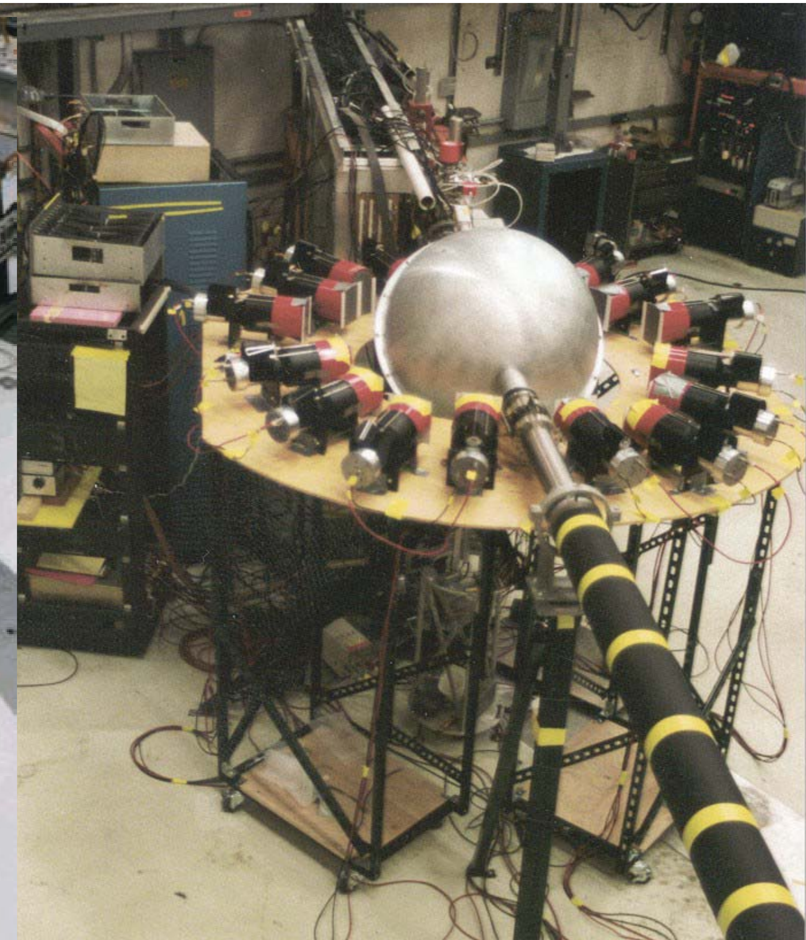
A. Some Results: 3 Experiments => 1 paper

PRC 67,044611 (2003)

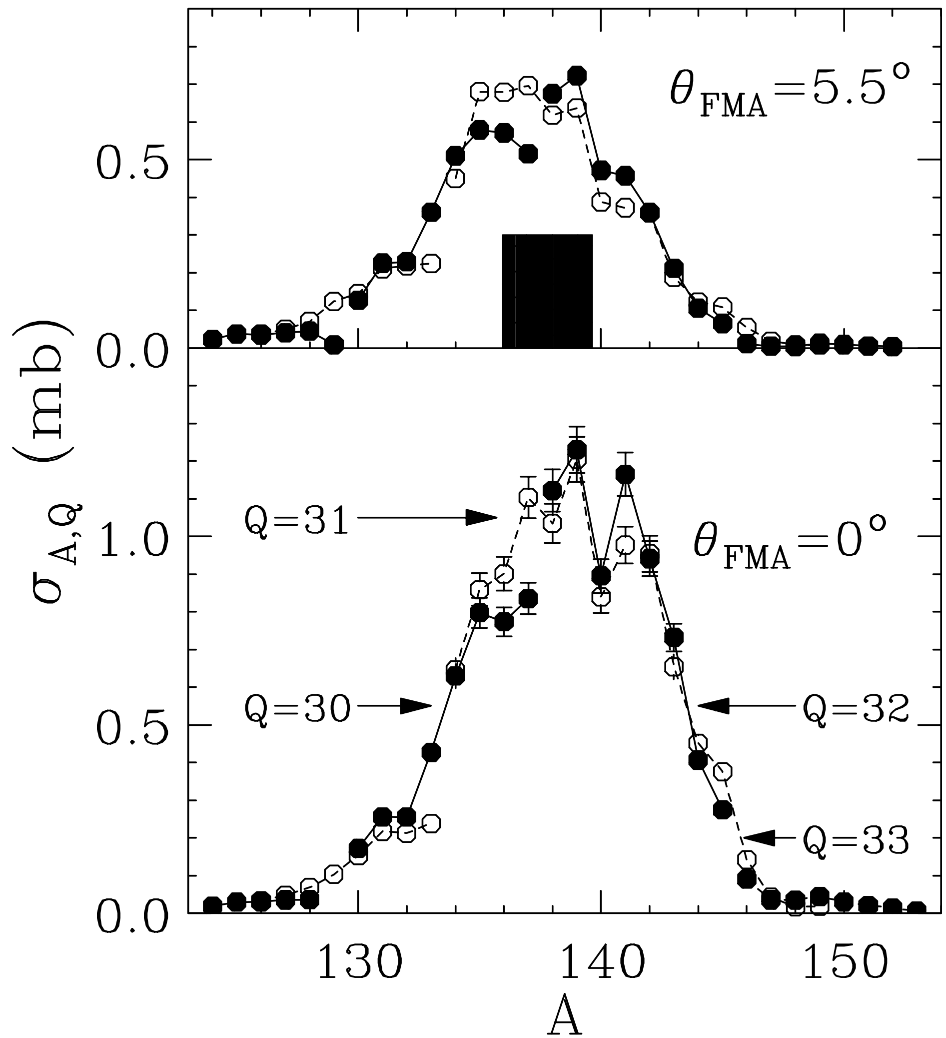
E/A(MeV) = 5,6,7,8,9 $^{60}Ni + ^{92,100}Mo$ => fusion

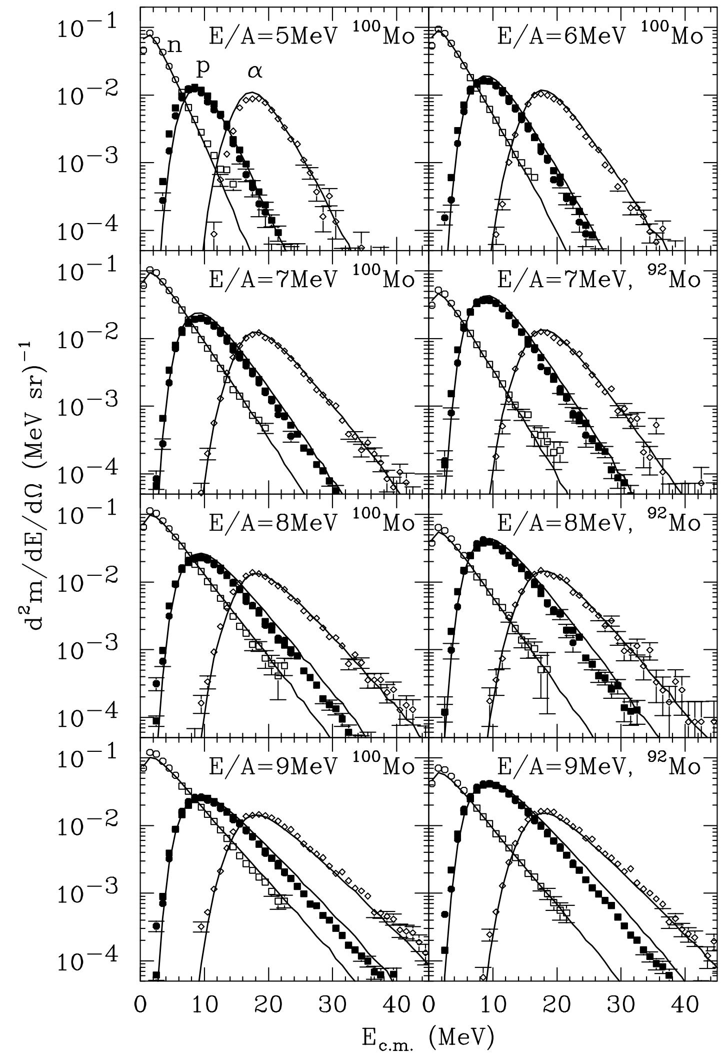
Measured: a) (N,Z) residues, b) particle Multiplicities, c) Spectra

Tools : a) FMA, b) Residue ToF, c) μBall + Si + n detectors

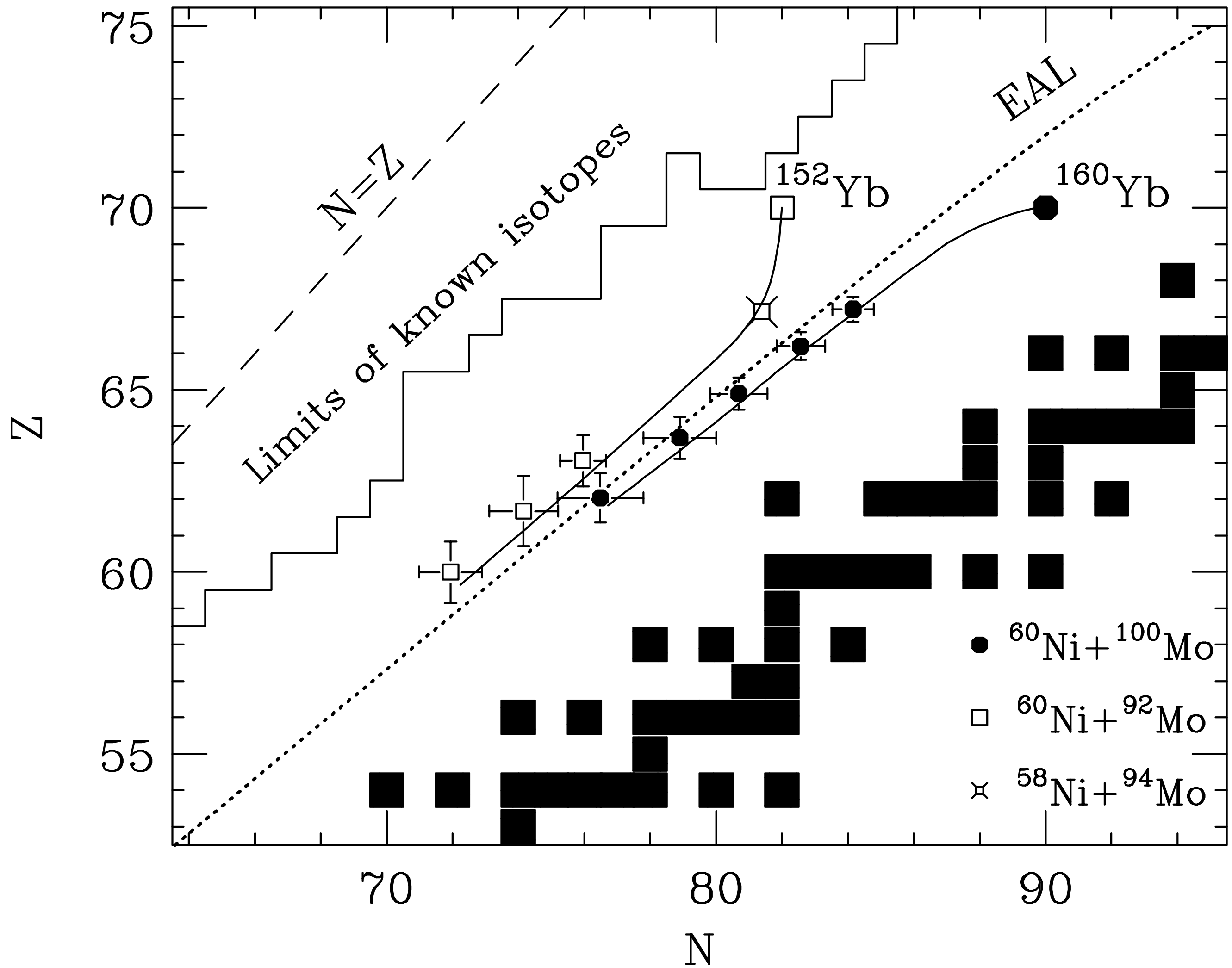


Residue (N,Z) distributions & Spectra & M(n,p,d,t,α)

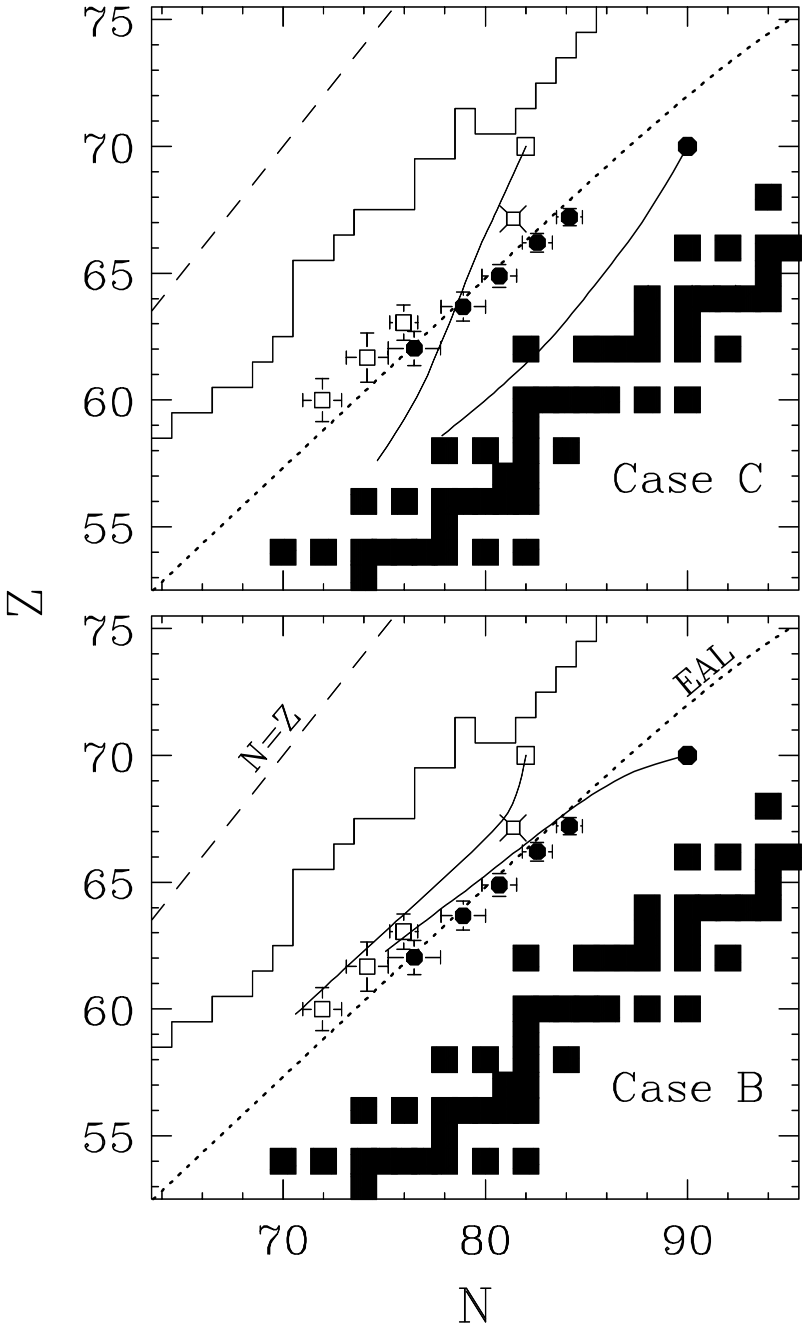


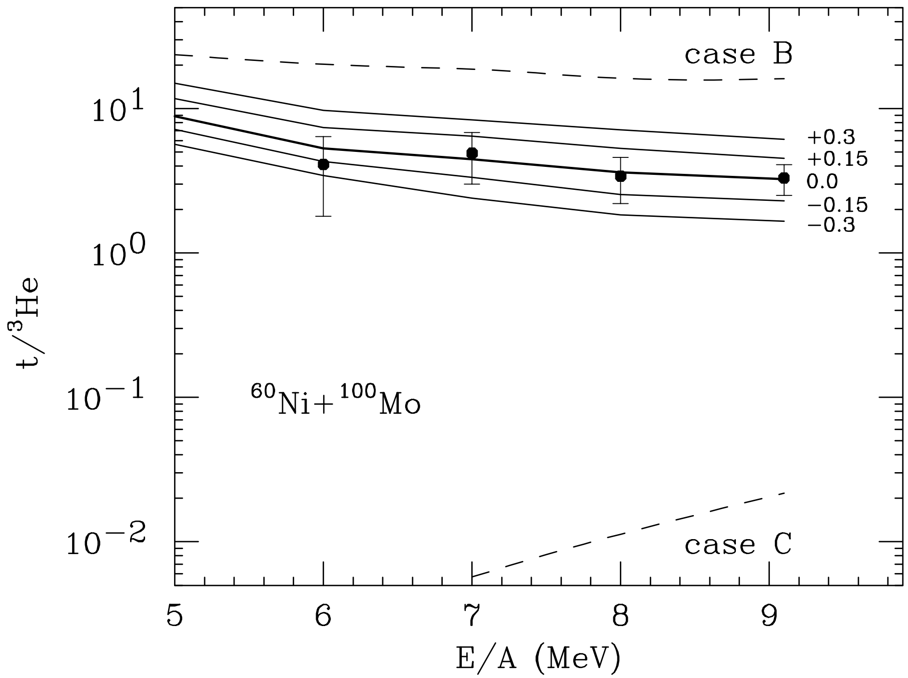


Final residue (N,Z) "fixes" Evaporation Attractor locus

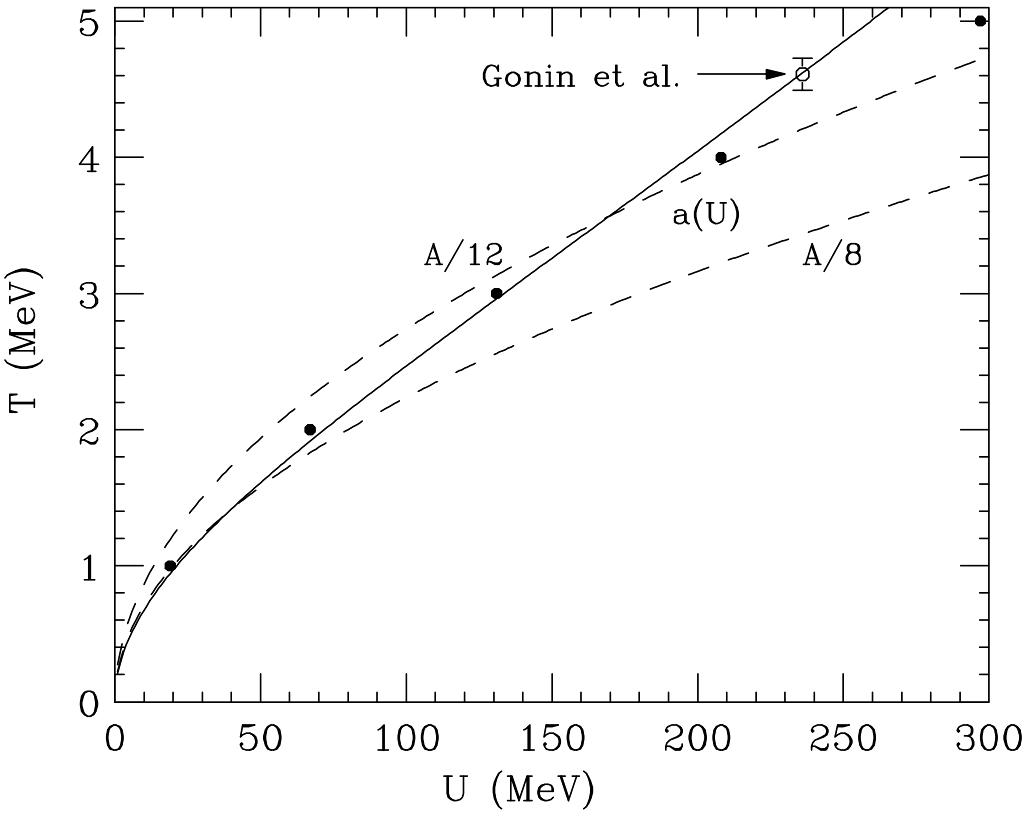


Grimes et al. model for: $\omega(E^*,\delta)$
"C" – WRONG
"B" – Better but still..





From the spectra we know a(A,U).



Dashed: simple FG expression

Solid: best fit to spectra

Dots: Shlomo & Natowitz

From another work we expect T to level off due to Expansion and Momentum dependence of the interaction (m_k).

What you are seeing here is the energy dependence (m_w).

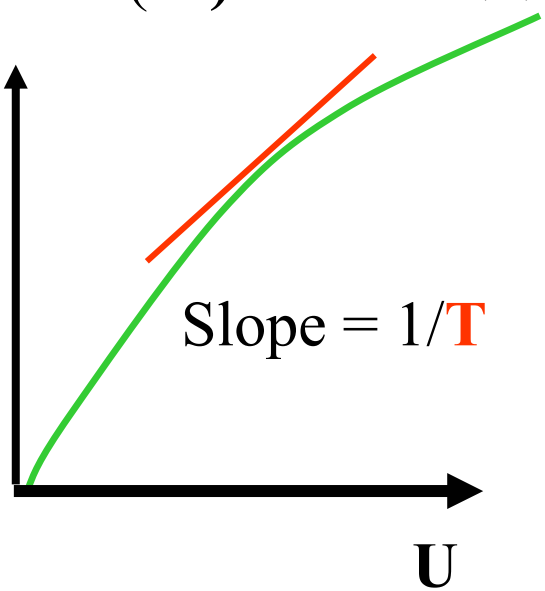
B. What are we talking about?
The density of states ω : The "go of it"

$dU = TdS \quad - \quad pdV \qquad \Rightarrow \; T = (\partial U / \partial S)_v$ or $1/T = (\partial S / \partial U)_v$

Thermal Newtonian (v means no "work")

(sectors)

S ~ Ln (**ω**) where **ω(U)** = # options at internal energy U (measure S in units of K_B)



In the non-interacting single particle model:

ω(U) = the # of ways to load the A particles into the solutions of the Q.M.'s (SP) problem (with total E*= U).

g(ε)[energy^{-1}] = number of "single-particle" solutions to the QM's problem/unit energy

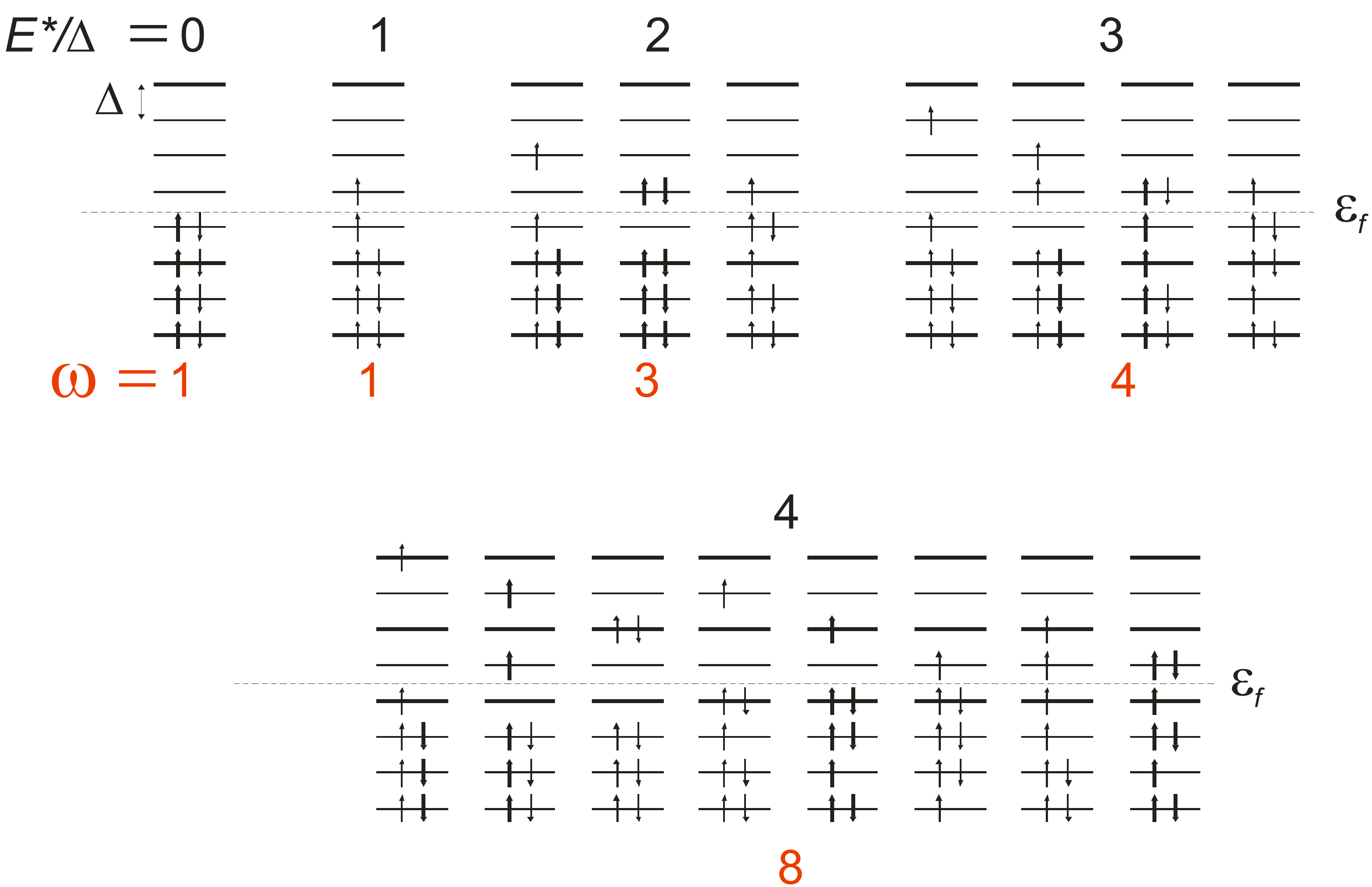
For a simple Fermi Gas: $U = aT^2$ and $S = 2(aU)^{0.5}$ where a α **g**

The density of states ω and entropy $S = K_b \ln \omega$.

Imagine a set of equally spaced dual occupancy levels (ignore spin interaction)

$\omega(E^*)$ = number of multiparticle configurations/energy

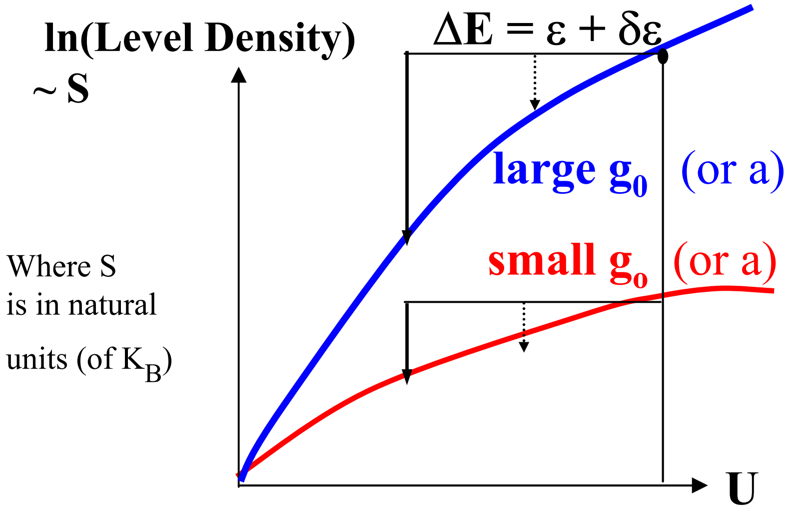
$g(\varepsilon)$ = number of "single" particle energy levels/energy $=1/\Delta$



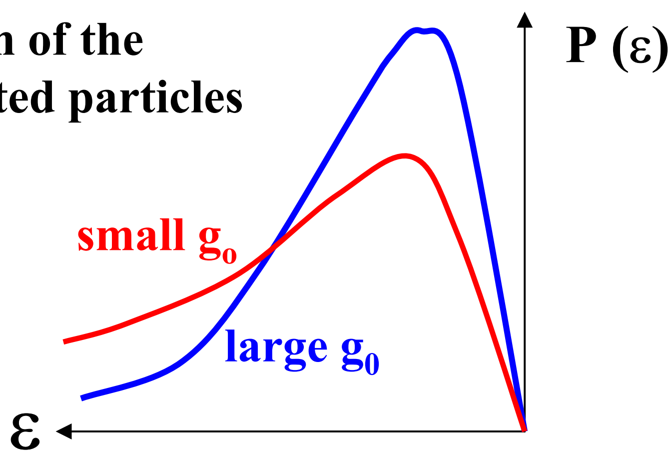
Determining $\omega(E^*)$ using evaporation spectra

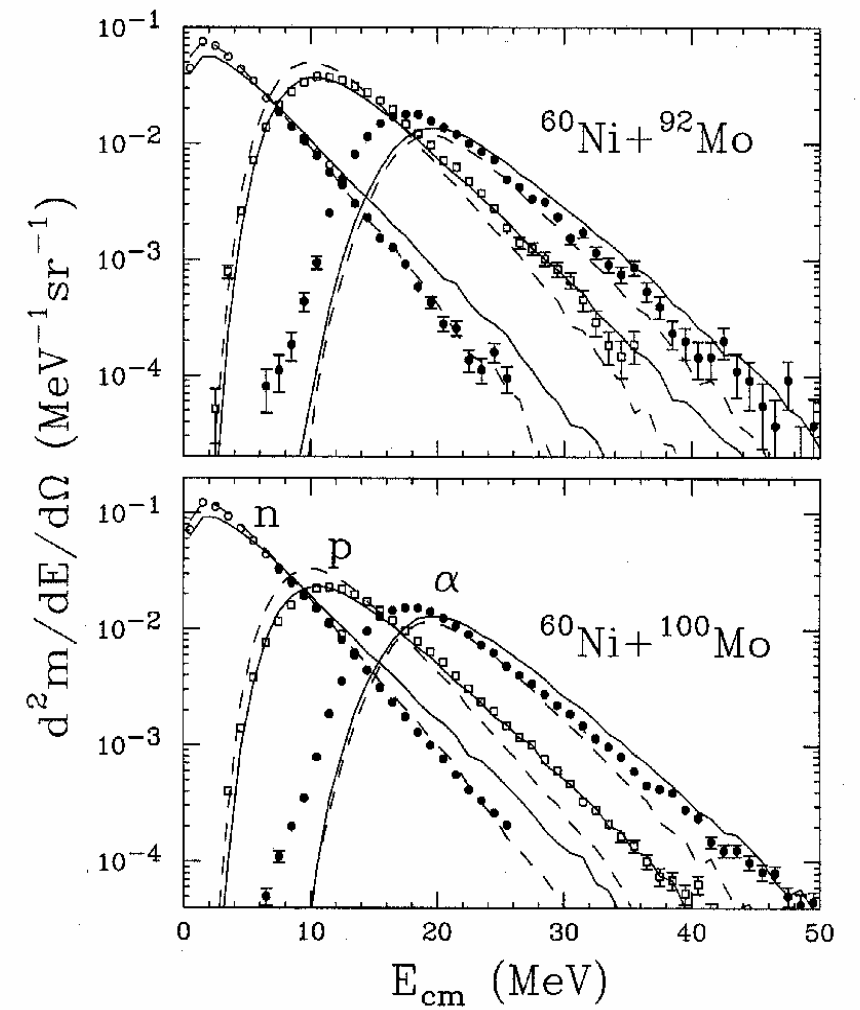
LOGIC

DATA



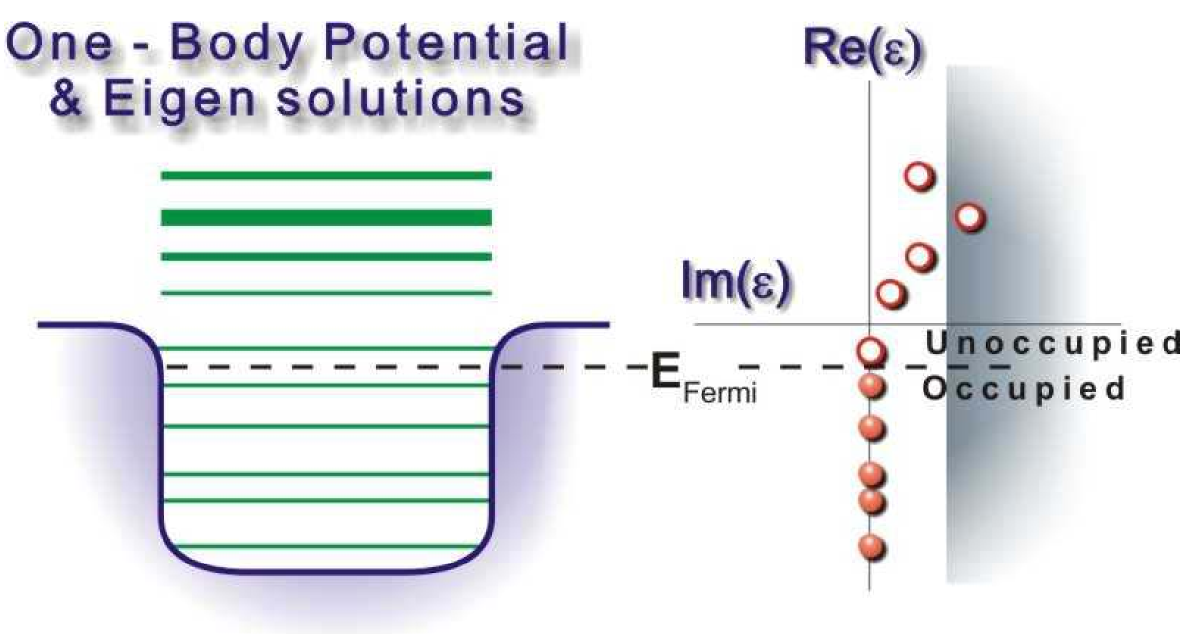
Spectrum of the Evaporated particles

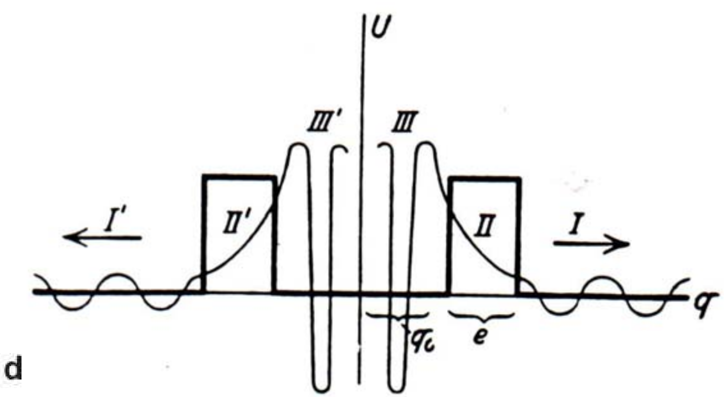




Influence of the continuum – first pass

1) The (multi-particle) level density ω depends exponentially on the single-particle level density g.
2) The first problem is then to solve the SP QM's problem to get the allowed states.
3) After this one can ask, "how many ways are there to load A particles into these levels with the constraint of fixed energy U?" Answer = ω.
4) Imagine a simple single-particle (SP) potential and BOTH its bound states and a decomposition of the continuum into resonance states. The resonances have complex eigen-values the imaginary part of which is proportional to the width and inversely proportional to the lifetime of the state.
5) How should one weight the continuum? Hans Weidenmüller, in a forgotten paper published 30 years ago, said (basically) - consider the experiment and cut in Im(ε) as appropriate.

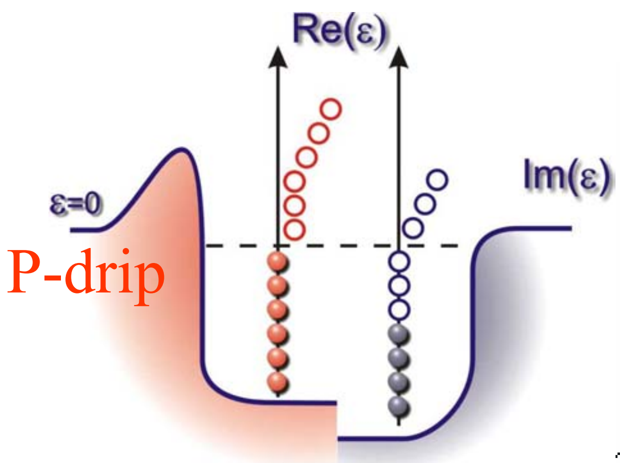




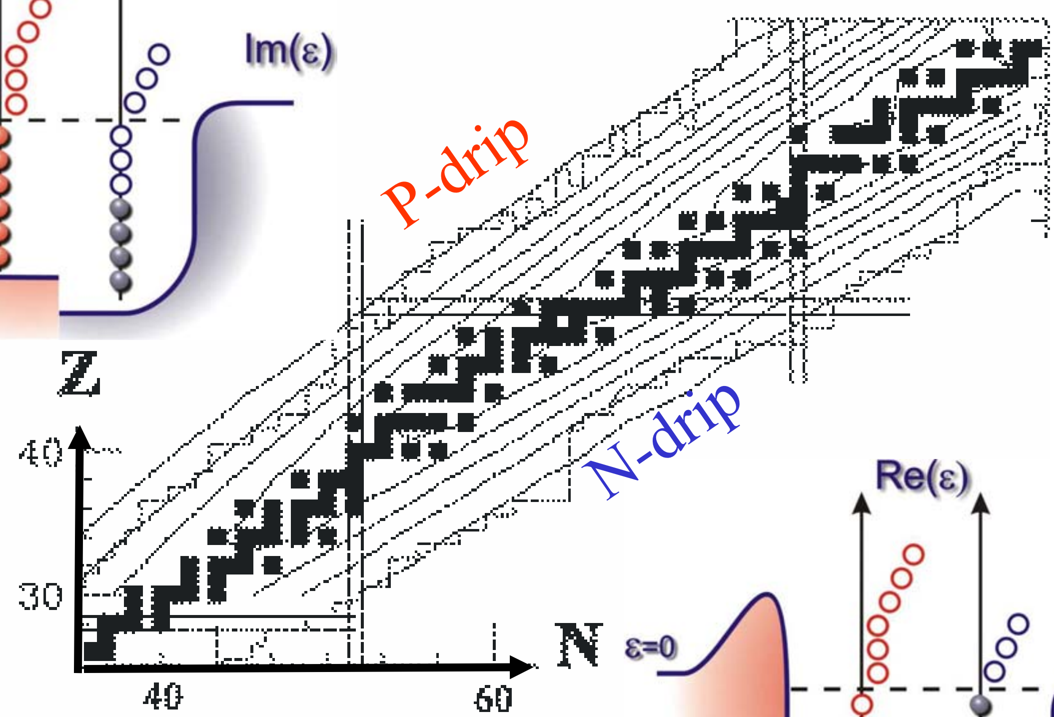
G. Gamow 1928
Unbound solutions are unnormalizable

Nuclei are <u>two-component quantum fluids</u> so we must be concerned with: $\omega(E^*,\delta,Q)$.

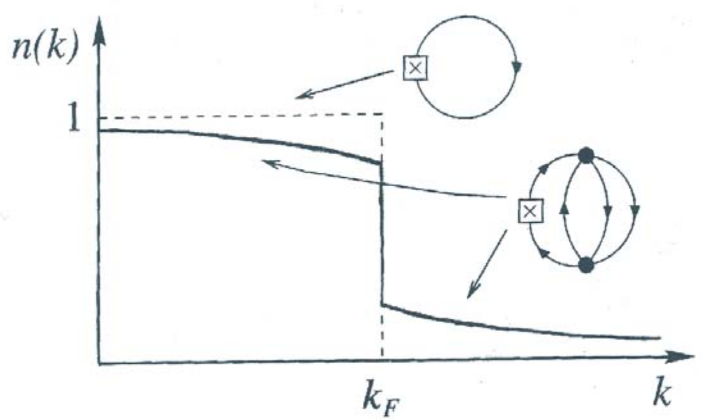
1) SP model =>
Two sets of "single particle" levels



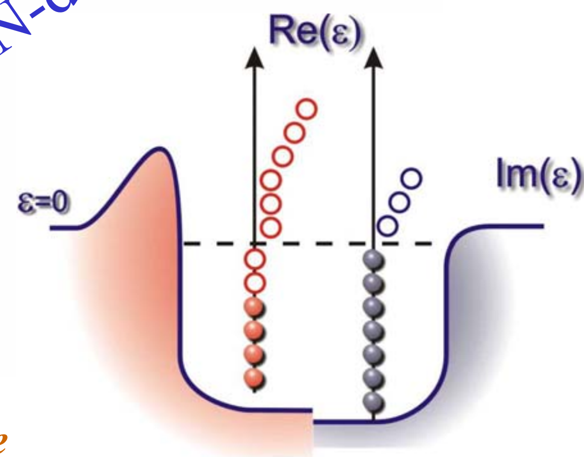
LD contours
Grimes (C)

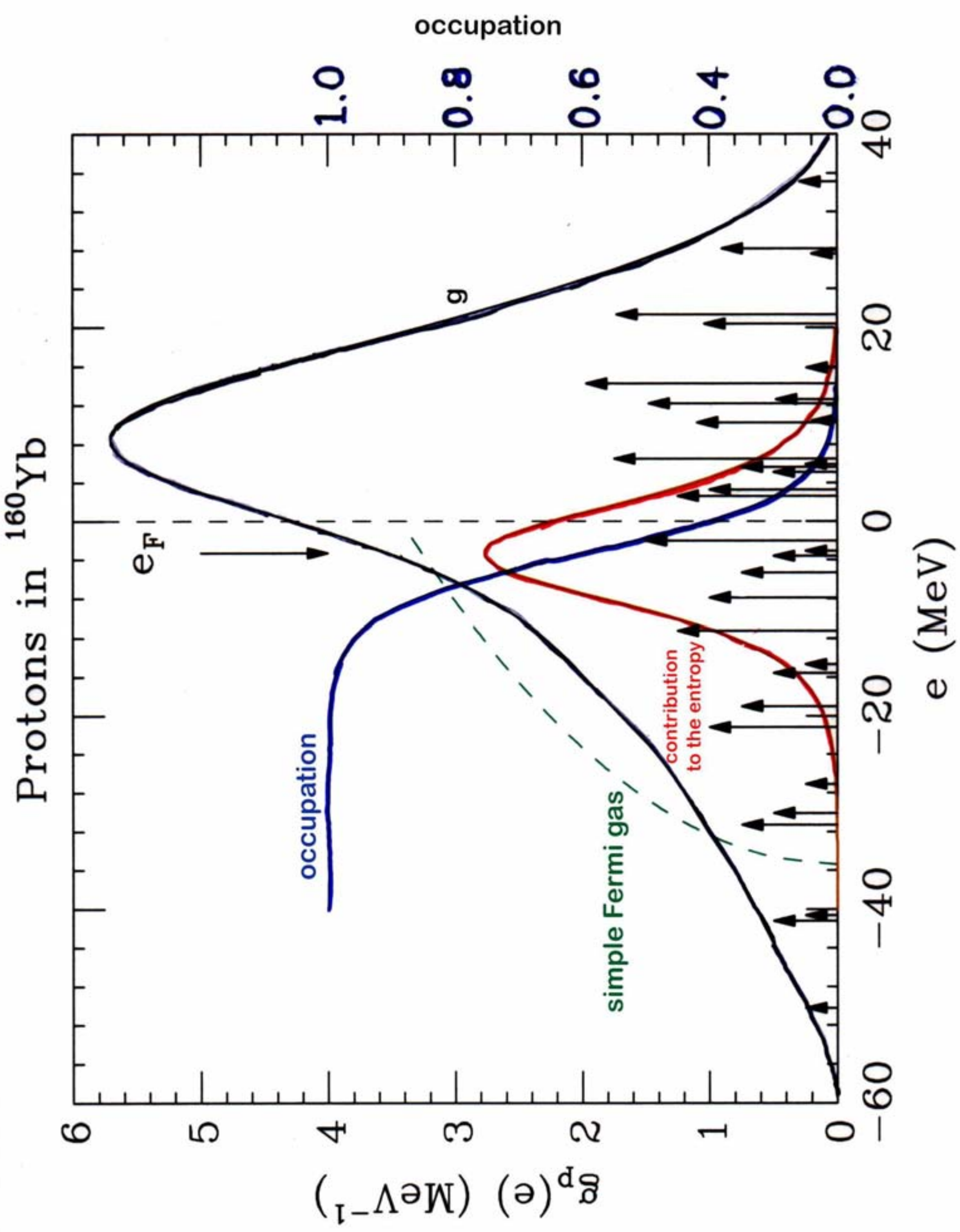


2) But lets not be naïve, we must correct for many-body or in-medium effects. The levels are not fully occupied even at low momentum.



3) These many-body effects *are* the collective enhancements.





Overview from Gamov analysis

A) Calculate SE solutions (⟵)
to single-particle potential problem.
WS +Spin orbit +(deformation + BCS)

B) Smooth to get single-particle g.
C) Set in T => occupation = f(ε)

D) Get S = -∫g(f lnf +(1-f)ln(1-f) dε

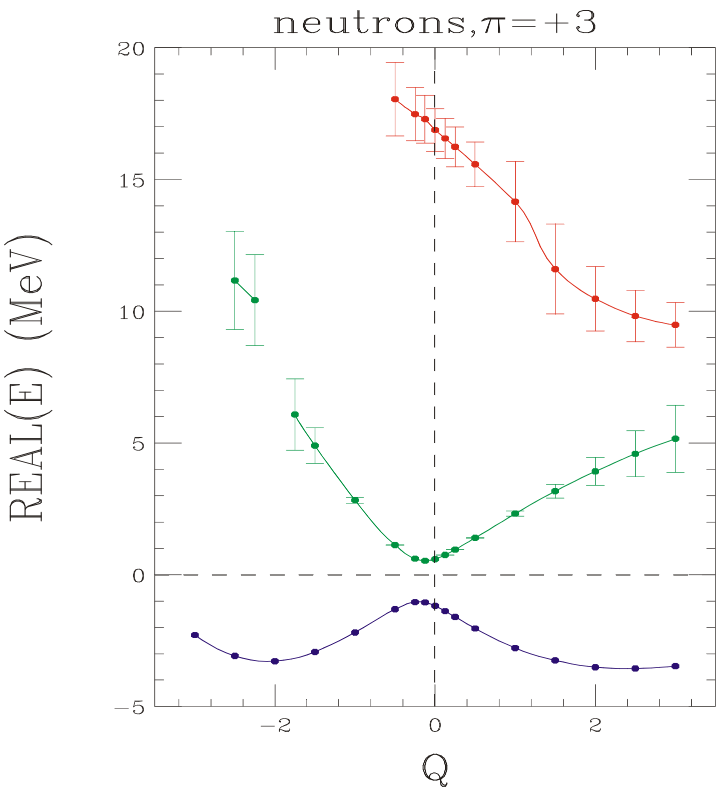
•**Bound states**:
1) Ψ's decay in forbidden region
2) max. occupancy = full spin degeneracy

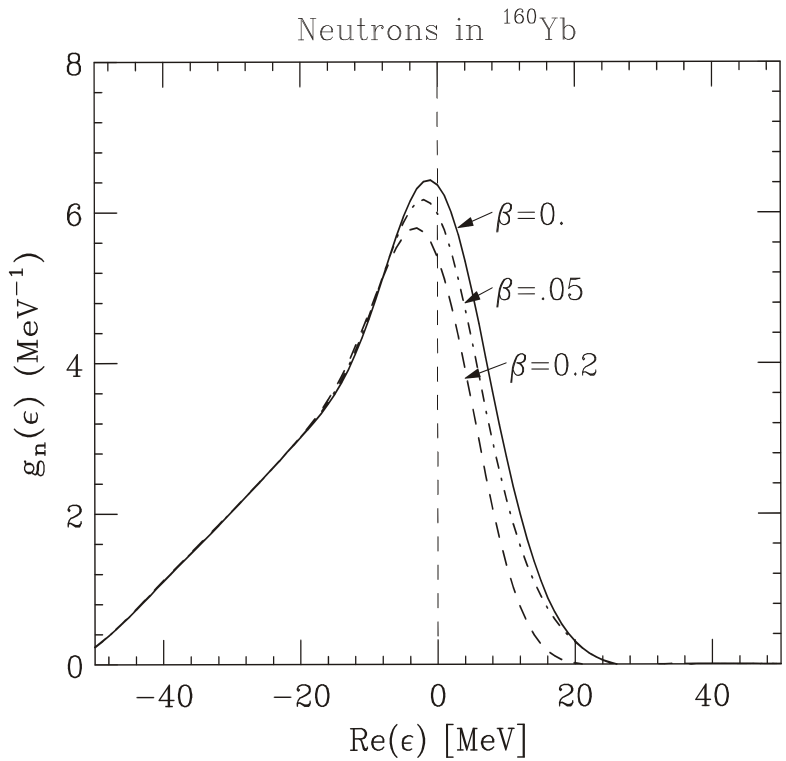
• **Unbound states**:
1) Ψ's oscillate outside
2) max. occupancy reduced by exp(-✍/1MeV)

•The smoothed single particle level density g peaks in the unbound region but decays at high ⚐ due to increasing widths!

Now Deform potential (use Coupled Channels scheme)

Deformation changes ε (both R and I) => g => L.D.





Door knob sphere football

On average the imaginary component ("error" bars) increases with deformation !

=>

Thus the single particle level density g DECREASES with deformation for positive energies.

C. Two methods for treating the continuum

Submitted to PRC 2004 (Charity and Sobotka)

1. The Gamov method: Weidenmüller

Sum over all poles +ve and –ve

$$g_{CN}^{\Gamma} \quad = \Sigma_i \, \delta(\varepsilon-\varepsilon^R{}_i) \;\; [\, e^{-\Gamma i/\,\Gamma o} \,]$$

2. The Subtraction (equilibrium) method: Fowler, Engelbrecht and Woosley

$$g_{CN}^{sub} = g_{tot}(\varepsilon) - g_{gas}(\varepsilon) \quad ; \quad g_{gas}(\varepsilon) = g_{ev}(\varepsilon)$$

$g_{CN}^{sub} = \Sigma_{\,lj}\, g_{lj}$ where $\int_{-\infty}^{\infty} g_{lj}\, d\,\varepsilon = 0$ by Levinson's theorem

$= \Sigma_{\,lj}$ (bound) + [continuum]

$= \Sigma_{\,lj} (2j+1)\{ \, (\Sigma_i \delta(\varepsilon-\varepsilon_i^{lj})] + [(d\delta^{lj} / d\varepsilon)/\pi] \, \}$,

where near a resonance $(d\delta_{lj} / d\varepsilon)$ = Lorentzian.

The +ve energy [continuum] contribution can be written in terms of the S matrix,

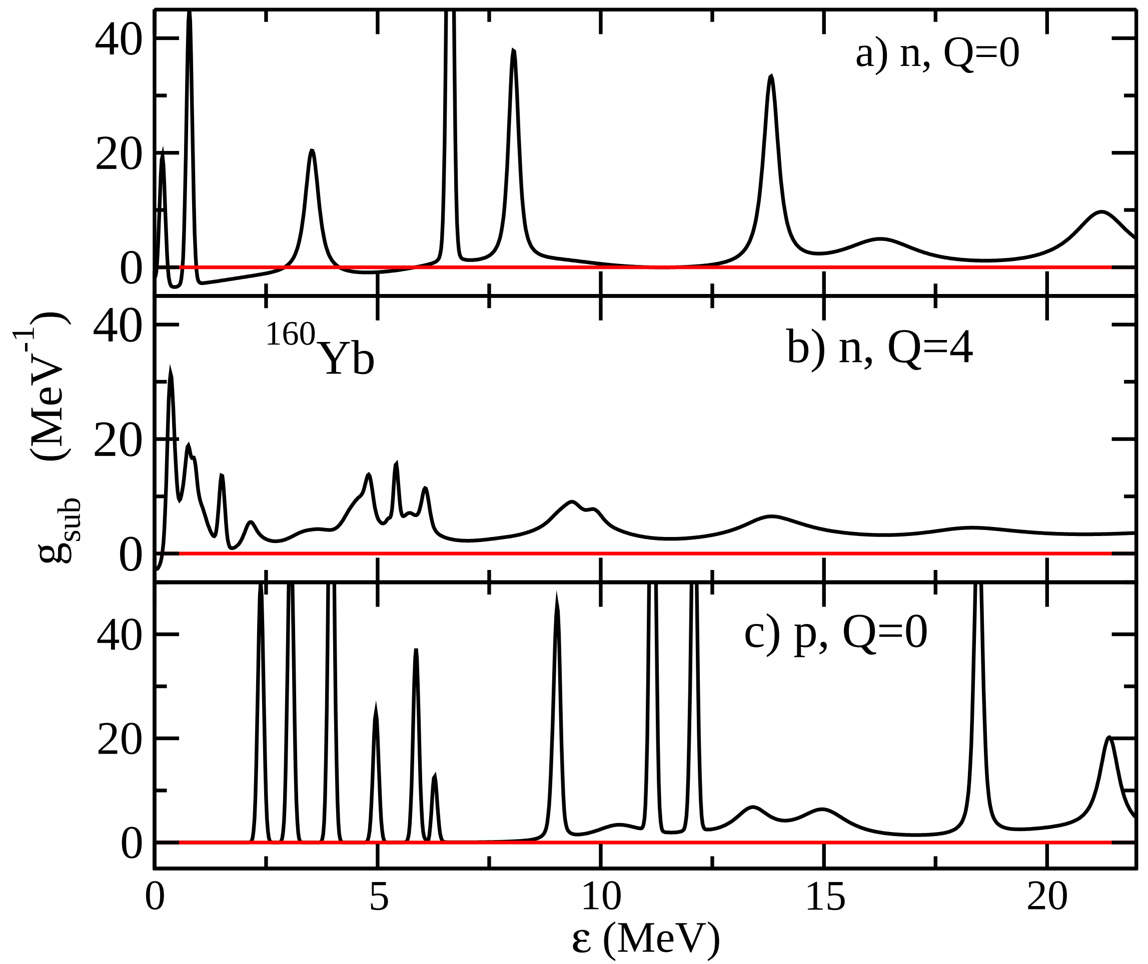
$= [(1/2\, \pi i) Tr \langle S^{-1}(\varepsilon)\, (d / d\varepsilon) S\, (\varepsilon) \rangle]$

Following the lead of nuclear structure community:

Deformed WS pot
+ l-wave < 20 hbar
+Spin orbit
+ BCS
(CC technique)

Discrete states sitting on –ve continuum from low l-wave contribution needed to balance contribution from bound states.

(➔ Levinson's Theorem: Node conservation for each l,j.)



Why Consider Gamov method?

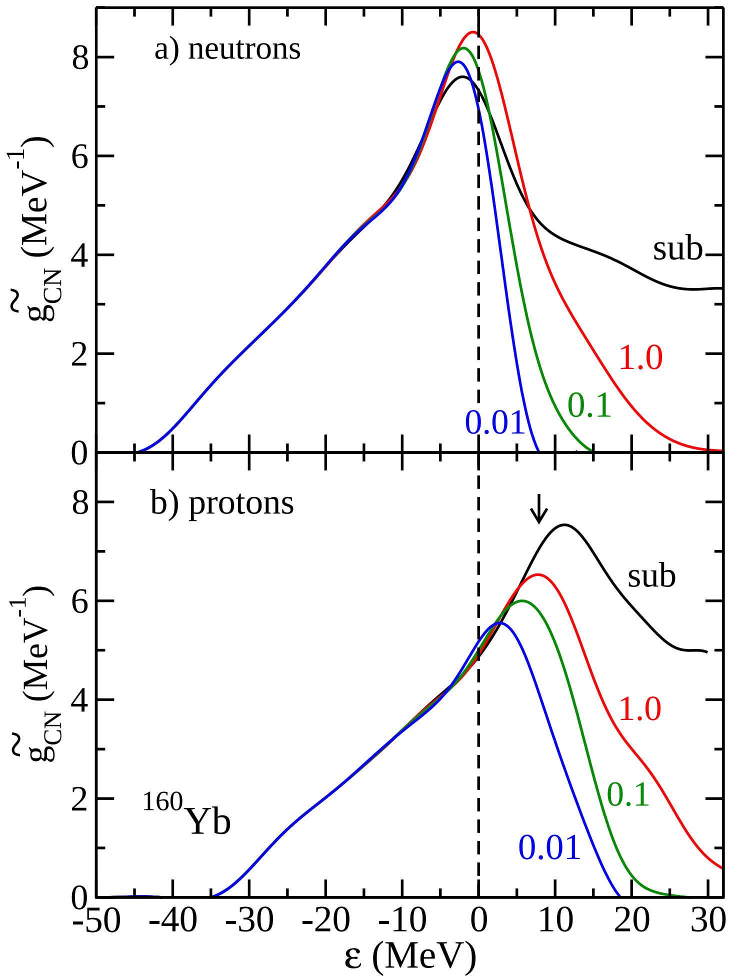
1. The accepted “subtraction” method is an equilibrium (CN-gas) model. If there were indeed a gas phase => No problem as: $\mathbf{g_{tot}^{sub} > 0}$ **for all energies.**

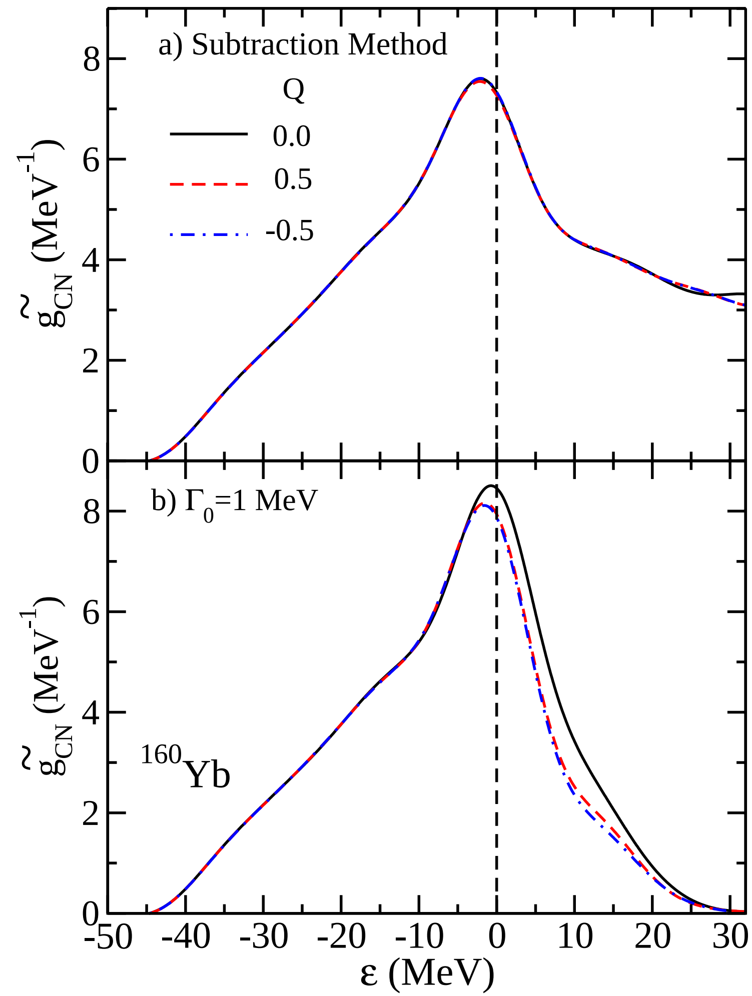
   In the standard reaction scenario **there is no gas**. What does the –ve g_{CN}^{sub} (at certain +ve energies) mean?

2. Neglecting time-scale arguments is an equilibrium fantasy. Hans Weidenmuller almost 40 years ago argued that one MUST cut in the imaginary plane in a fashion what depends on the observable.

   This approach is common in atomic/molecular physics.

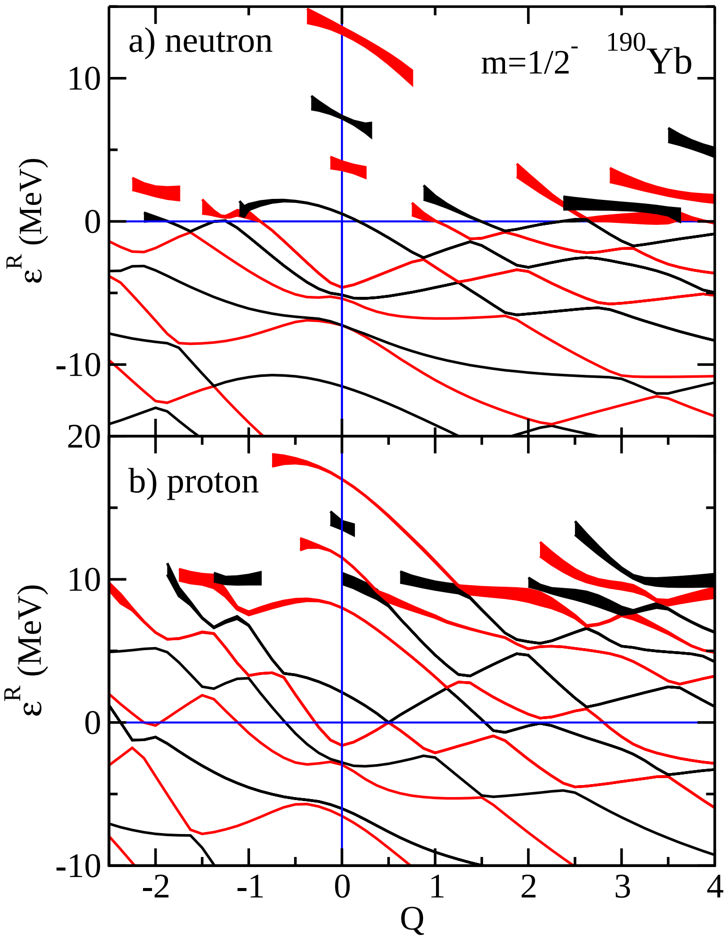
3. There is also a difference in the treatment of virtual states.





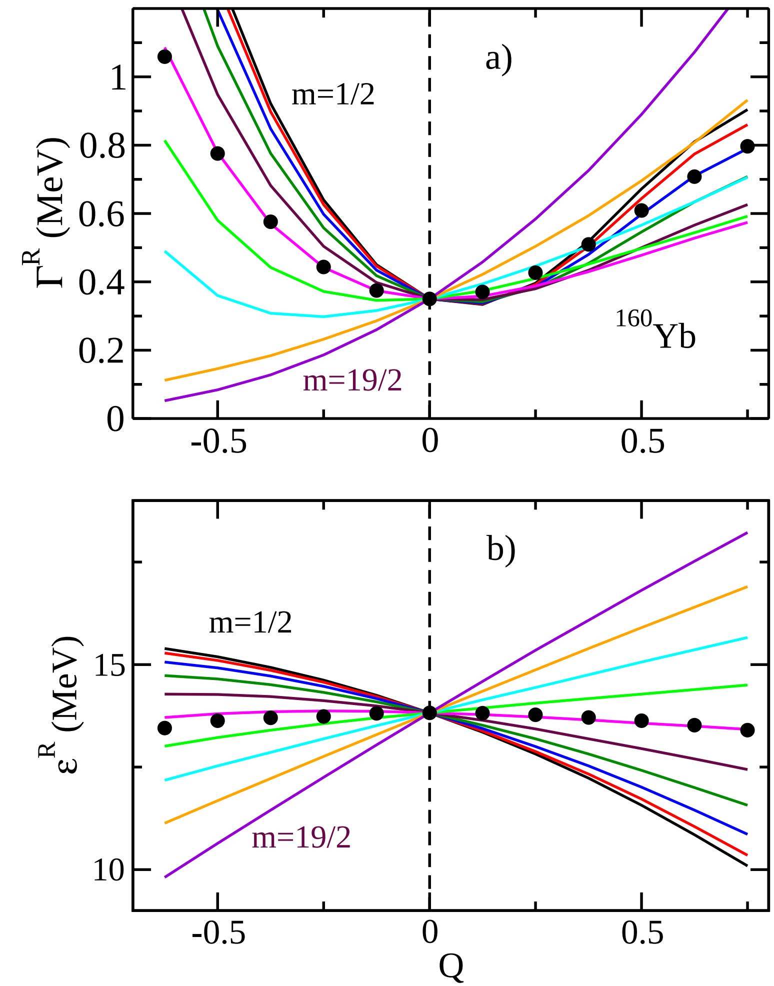
1. g increases to about $\varepsilon = 0$ (higher ε for p than for n) and then decreases.
2. Decrease stronger the smaller Γ and **either** sense deformation.

Follow levels as a fxn of deformation (Q)

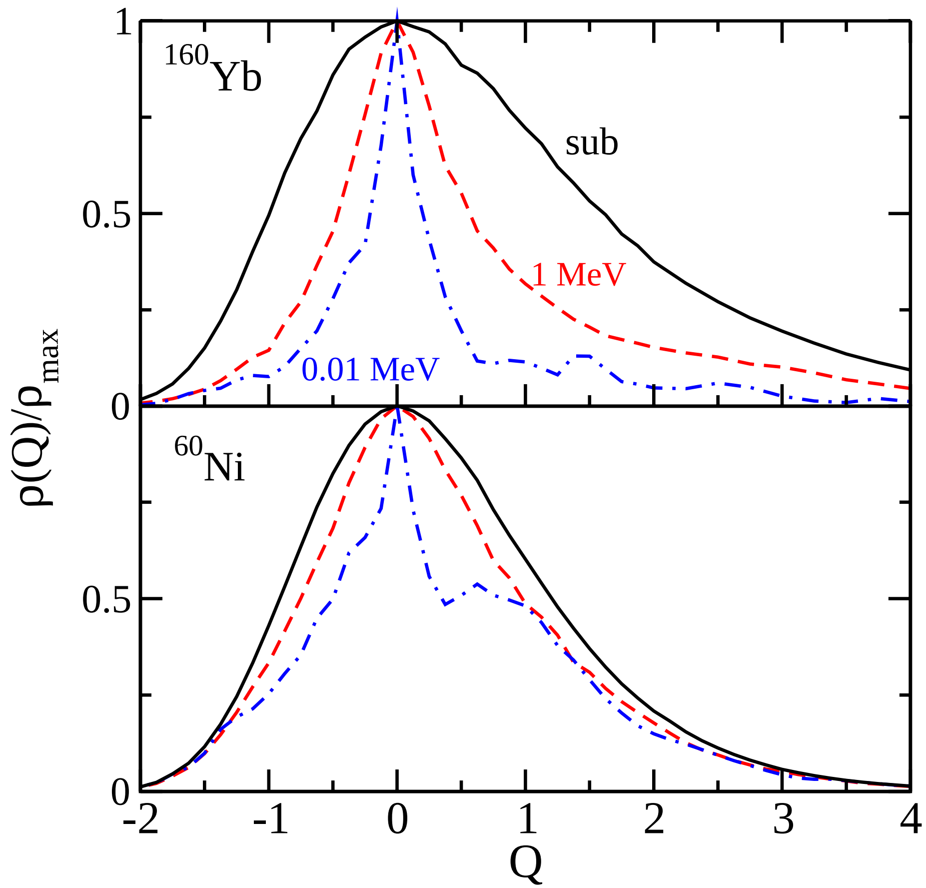


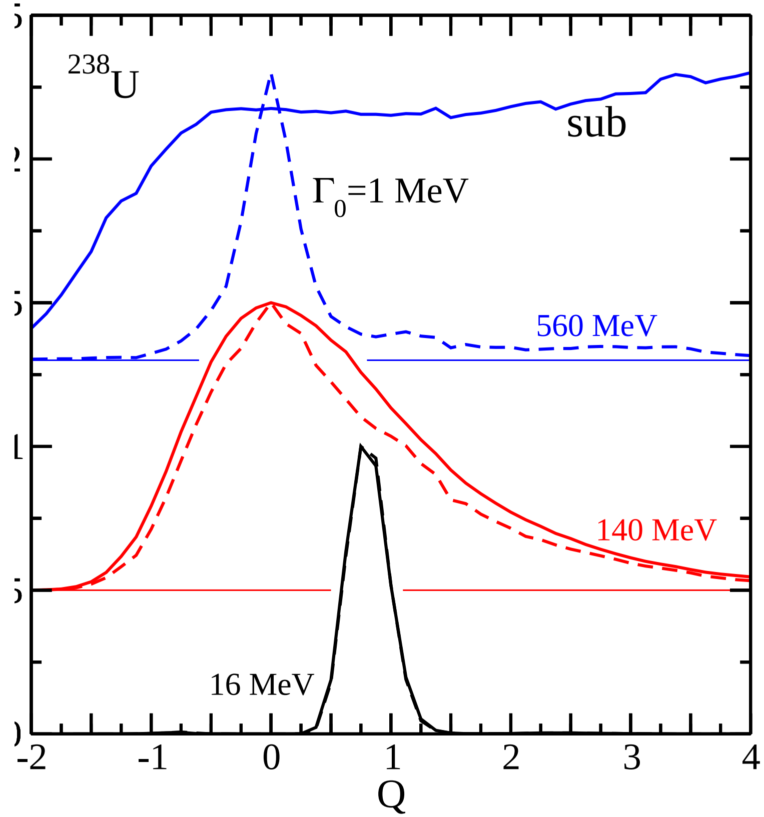
Doorknob football

J=(19/2)$^-$, ● = average



Average width increases with | Q|!!!





Doorknob football

Gamov would reduce fission rate at high E*

BOTH methods confirm basic experimental finding: Very weak δ dependence

also

Grimes et al. propose (and provide fit parameters for)

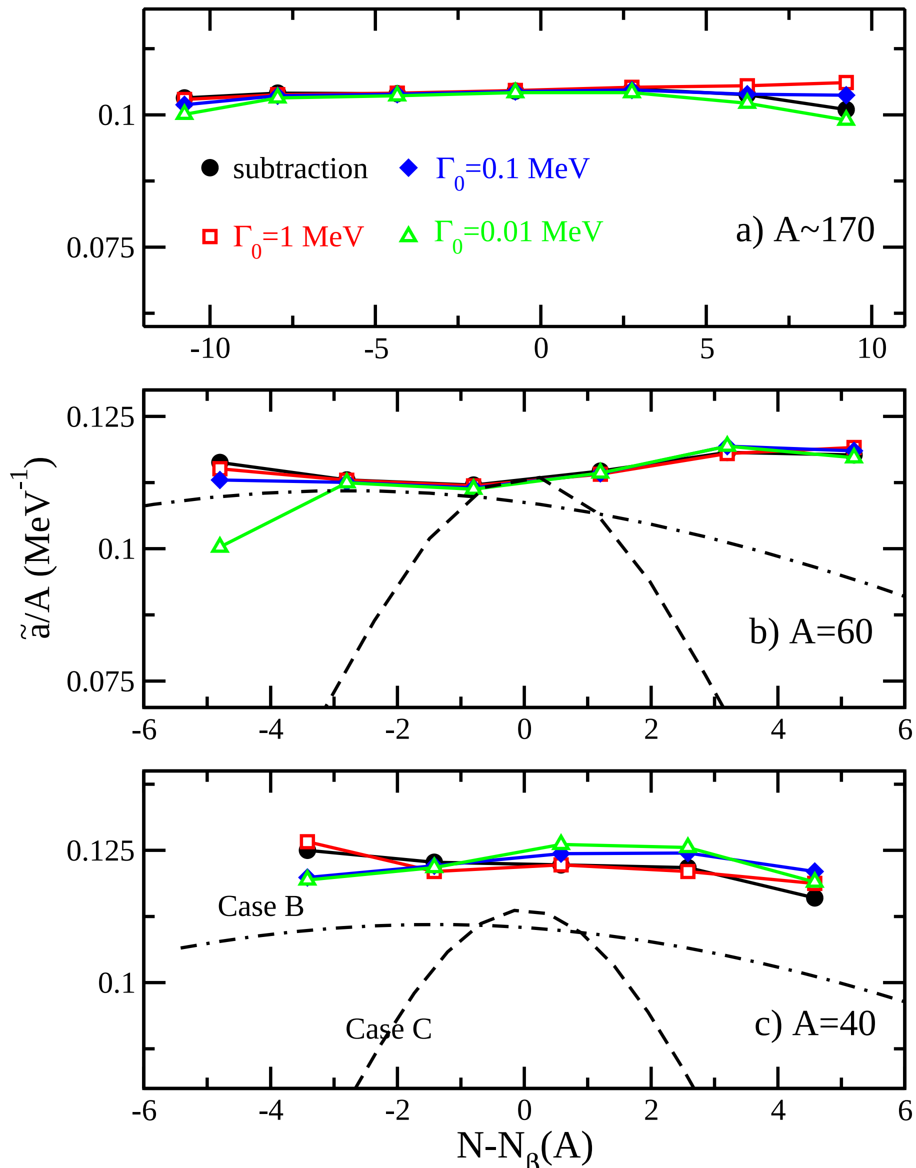
B. ~ continuum logic

$$a_B = a_3 A \exp\{a_4(N-Z)^2\}$$

C. ~ isospin logic

$$a_C = a_1 A \exp\{a_2(Z-Z_\beta)^2\}$$

Neither form is supported by either experiment or theory.



Conclusions

1. **We have some techniques to study $\omega(E^*,\delta)$.**
   (Talked about one - we have another.)

2. **Experiment:** no strong δ dependence close to stability

3. **Calculation:** one should not expect strong δ
   dependence close to stability.

4. **Might be some δ dependence closer to n-drip (not p-drip.)**
   p-drip.)

5. **Gamov method predicts large Q "confinement".**
   Subtraction method does not Q confine.