Nuclear Equation of State
Used in Astrophysics Models
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Comparative Study of Equations of State used in Neutron Star and Supernova Collapse Models

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Outline

1. General properties of EOS for nuclear matter
2. Classification according to models of N-N interaction
3. Examples of EOS – sensitivity to the choice of N-N interaction
4. Consequences for supernova simulations
5. Constraints on EOS
6. New developments

Equation of State is derived from a known dependence of energy per particle of a system on particle number density:

$$E/A = \mathcal{E}(n) \;\; or \;\; F/A = \mathcal{F}(n)$$

I. E (or Boltzman free energy F = E-TS for system at finite temperature) is constructed in the form of an **effective energy functional** (Hamiltonian, Lagrangian, DFT/EFT functional or an empirical form)

II. An equilibrium state of matter is found at each density n by minimization of $\mathcal{E}(n)$ or $\mathcal{F}(n)$

III. All other related quantities like e.g. pressure *P*, incompressibility *K* or entropy *s* are calculated as derivatives of $\mathcal{E}$ or $\mathcal{F}$ at equilibrium:

$$P(n) = n^2 \frac{\partial \mathcal{E}(n)}{\partial n}$$

$$s(n) = -\frac{\partial \mathcal{F}(n)}{\partial T}\Big|_{n,Y_p}$$

$$K(n) = 9\frac{\partial P(n)}{\partial n} = 18n\frac{\partial \mathcal{E}(n)}{\partial n} + 9n^2 \frac{\partial^2 \mathcal{E}(n)}{\partial n^2}$$

IV. Use as input for model simulations

(Very) schematic sequence of equilibrium phases of nuclear matter as a function of density:

$<\sim 2x10^{-4} fm^{-3}$

Nuclei in electron gas

$\sim 2x10^{-4} fm^{-3}$

Nuclei in Neutron + Electron gas

$\sim 0.06 fm^{-3}$

'Pasta phase'

$\sim 0.1 fm^{-3}$

n,p,e,μ (β–equilibrium)

$0.3\text{-}0.5 fm^{-3}$

Nucleons + heavy baryons + leptons

$>0.5 fm^{-3}$

Quarks ???

To construct the energy functional we need nuclear and particle physics models:

Expectation value of the total energy:

$$E = < \phi, (T + V)\phi >$$

T kinetic energy, V total potential energy of the system
Φ is a Slater determinant of single particle states ϕ_i

Theories

Relativistic ↔ **Non-relativistic**

Potentials

Realistic ↔ Potentials ↔ **Phenomenological**

Reid 93
Nijmegen II
Argon v18 (A18)
} Local non-rel

Skyrme
Gogny
}

Nijmegen I — Non-local non-rel — SMO

CD Bonn — Local rel — NL +

Equation of State of Akmal et al. PRC 58, 1804 (1998) thought of as the most complete study to date of HDNM

Potentials:

A18 (two-body):
static, long range one-pion exchange
+
PHENOMENOLOGICAL medium and short range part dependent on 18 two-body operators

UIX (three-body)
static, long-range two-pion exchange
+
PHENOMENOLOGICAL medium range repulsive term

+

Relativistic boost correction to the two body N-N interaction

+

Ad hoc density dependent term $\gamma_2\rho^2 e^{-\gamma_3\rho}$

which provides 25% (~4 MeV) correction to the E/A of symmetric nuclear matter to reach the empirical value 16 MeV

For densities <0.1 fm^{-3}, EOS by Lorenz et al., PRL 70,379 (1993) is used.

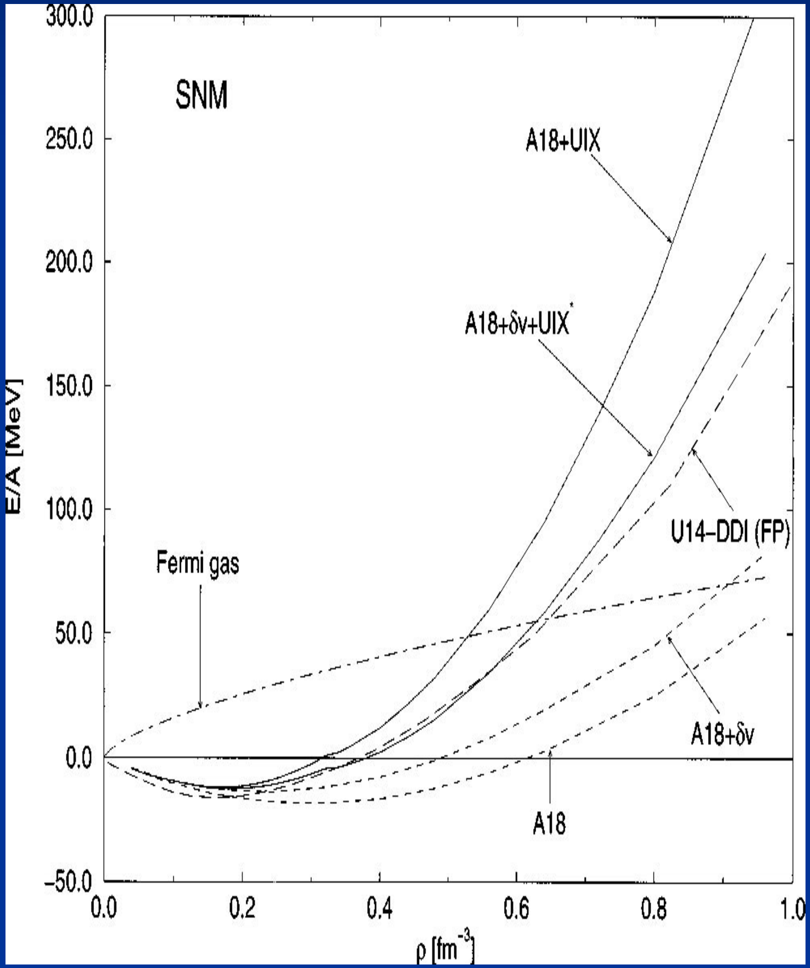
Akmal et al., E/A as a function of baryon number density

TABLE VI. The $E(\rho)$ of SNM in MeV.

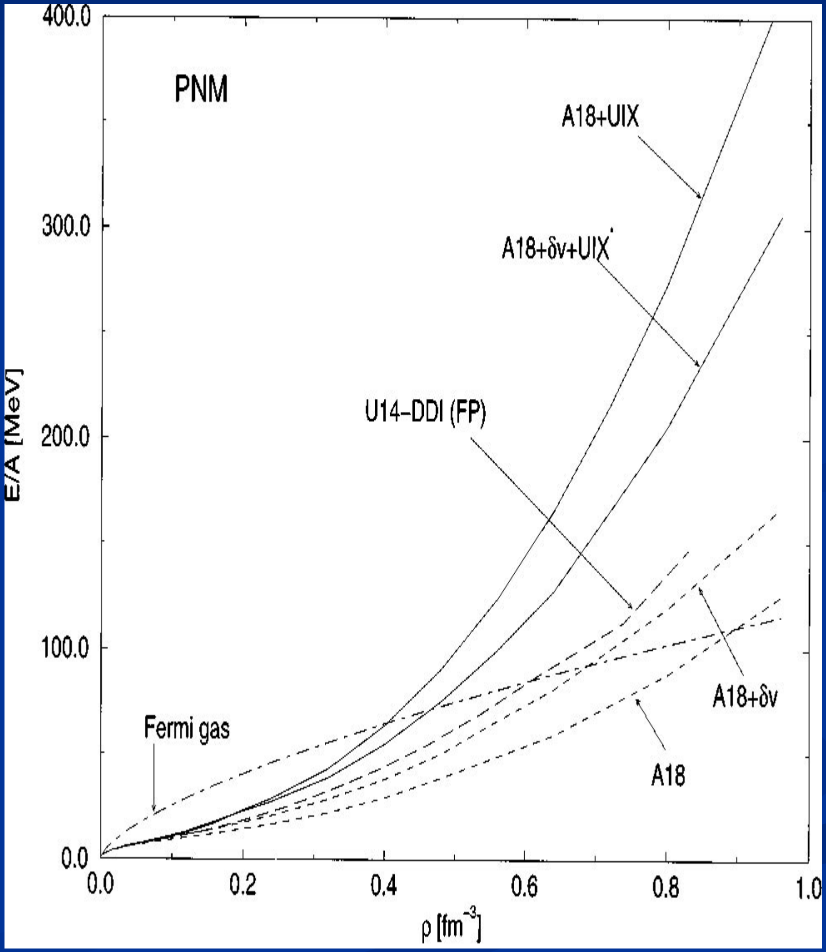
ρ	A18	A18+ δv	A18+UIX	A18+ δv +UIX*	corrected
0.04	−4.28	−4.08	−4.39	−4.31	−6.48
0.08	−8.72	−8.07	−8.06	−7.97	−12.13
0.12			−10.52	−10.54	−15.04
0.16	−14.59	−12.54	−11.85	−12.16	−16.00
0.20			−11.28	−12.21	−15.09
0.24	−17.61	−13.69	−8.99	−10.89	−12.88
0.32	−18.13	−11.87	0.84	−4.21	−5.03
0.40	−16.37	−7.70	12.23	2.42	2.13
0.48	−12.21	−1.01	32.18	15.56	15.46
0.56	−5.79	8.16	59.99	34.42	34.39
0.64	2.76	19.54	95.05	58.36	58.35
0.80	25.01	45.24	188.51	121.25	121.25
0.96	56.51	82.63	313.46	204.02	204.02

At saturation density 0.16 fm^{-3} the expected value of E/A=16 MeV

Akmal et al., Sensitivity of density dependence of E/A to the form of a realistic potential

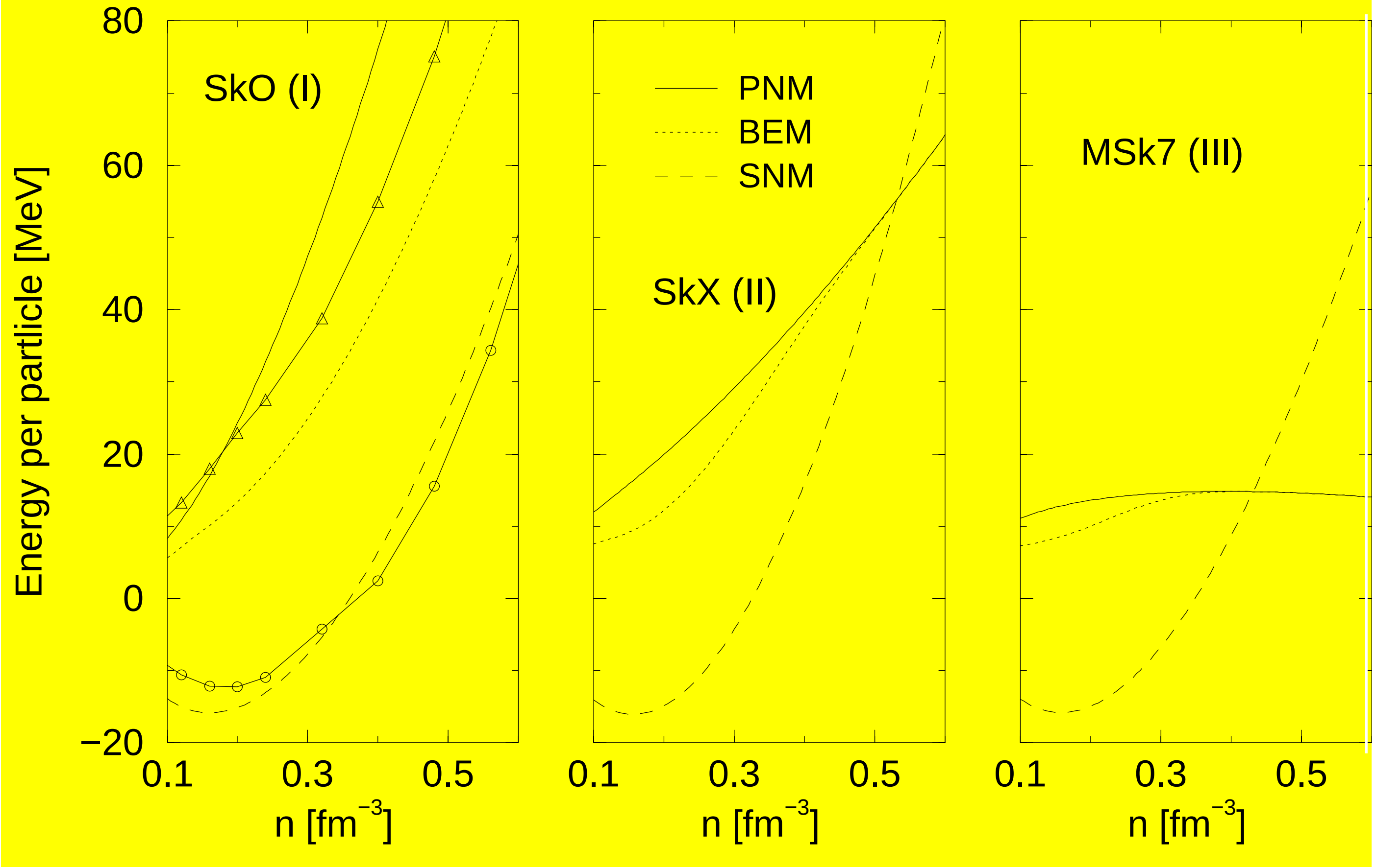


Symmetric nuclear matter



Pure neutron matter

Non-relativistic Mean Field Theory
Various Parameterizations of Skyrme interactions (87 tested)



27

33

27

J.R.Stone et al., PRC 68, 034324 (2003)

EOS at Finite Temperature

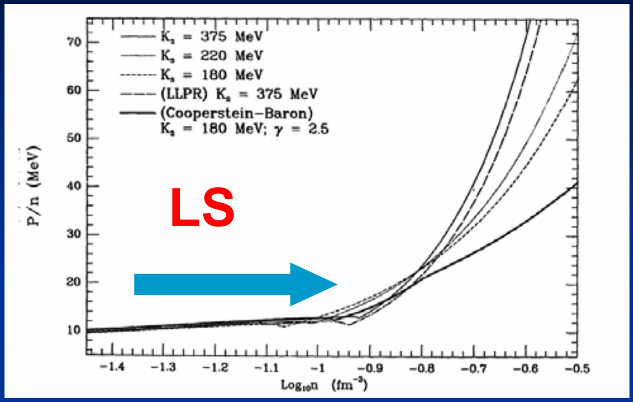
1. LATTIMER and SWESTY, Nucl.Phys. A535, 331 (1991)
   Current standard in supernova modeling

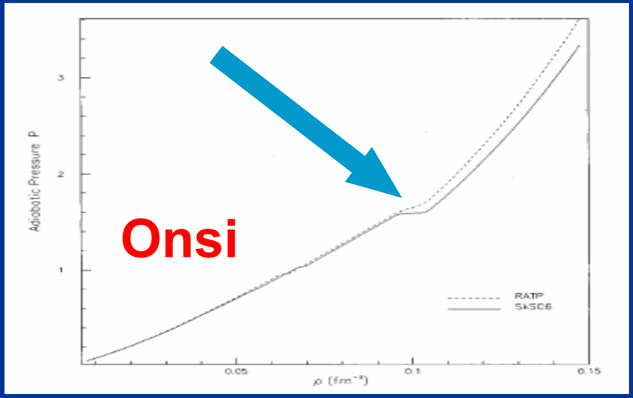
   Liquid drop model + schematic 2- and many-body interactions
   subnuclear + non-equilibrium nuclear matter

2. ONSI et al., Phys.Rev. C55, 3139 (1997)
   Non-Relativistic Mean Field in ETFSI method for
   subnuclear density + nuclear matter

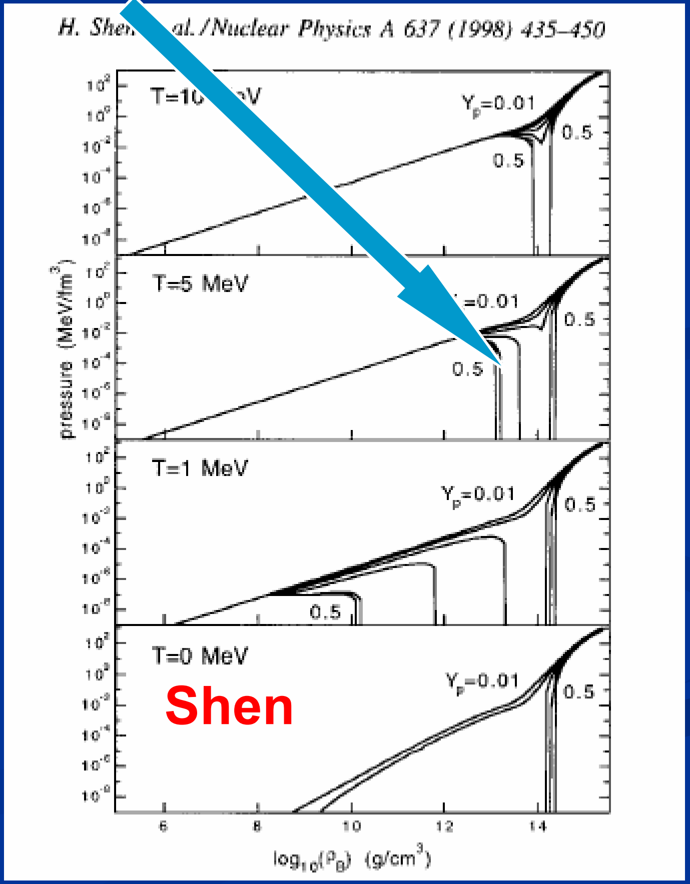
3. SHEN et al., Nucl. Phys. A637, 435 (1998)
   Relativistic Mean Field Theory in TF approximation
   subnuclear density + nuclear matter

Pressure as a function of baryon number density





Discontinuities at phase boundaries



$n[fm^{-3}]$	LS	Onsi
0.1	13	16
0.15	16	23

Adiabatic index:

$$\Gamma = \frac{\partial \ln P}{\partial \ln n}\Big|_{s,Y_p}$$

Very important in hydrodynamic calculations
A measure of stiffness of the matter
At phase boundaries reflects nature and treatment of phase changes

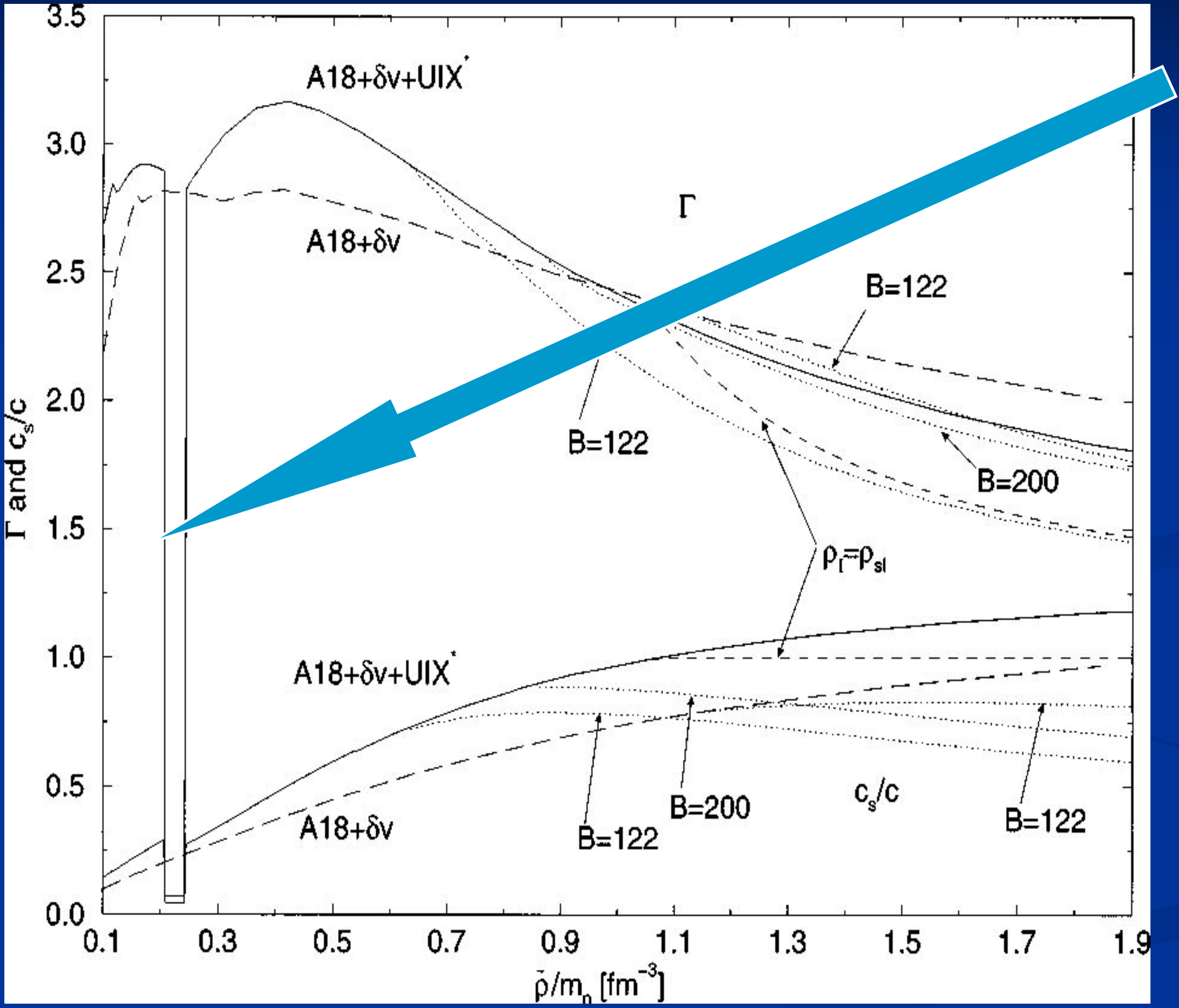
Examples

I. Akmal et al : transition between normal low density matter (LDP) and high density matter with condensed pions (HDP)

II. LS: transition from 'pasta' phase to pure nuclear matter phase
(proton rich nuclei in neutron vapor + α particles,
neutron bubbles + α in dense proton rich matter) } + e

III. Onsi et al: transition between droplet and bubble phase
transition between bubble and homogeneous matter

Adiabatic index as calculated by Akmal et al, PRC 58, 1804 (1998)

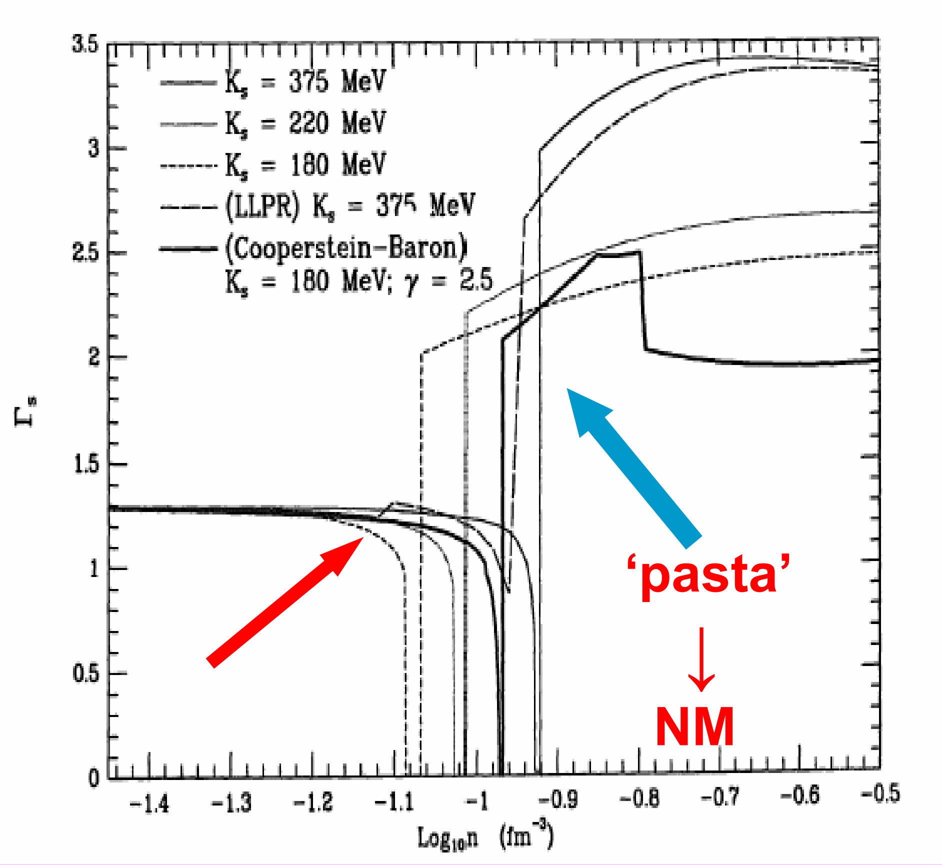


Discontinuity at n~0.2 fm^{-3} represents an LDP -> HDP transition

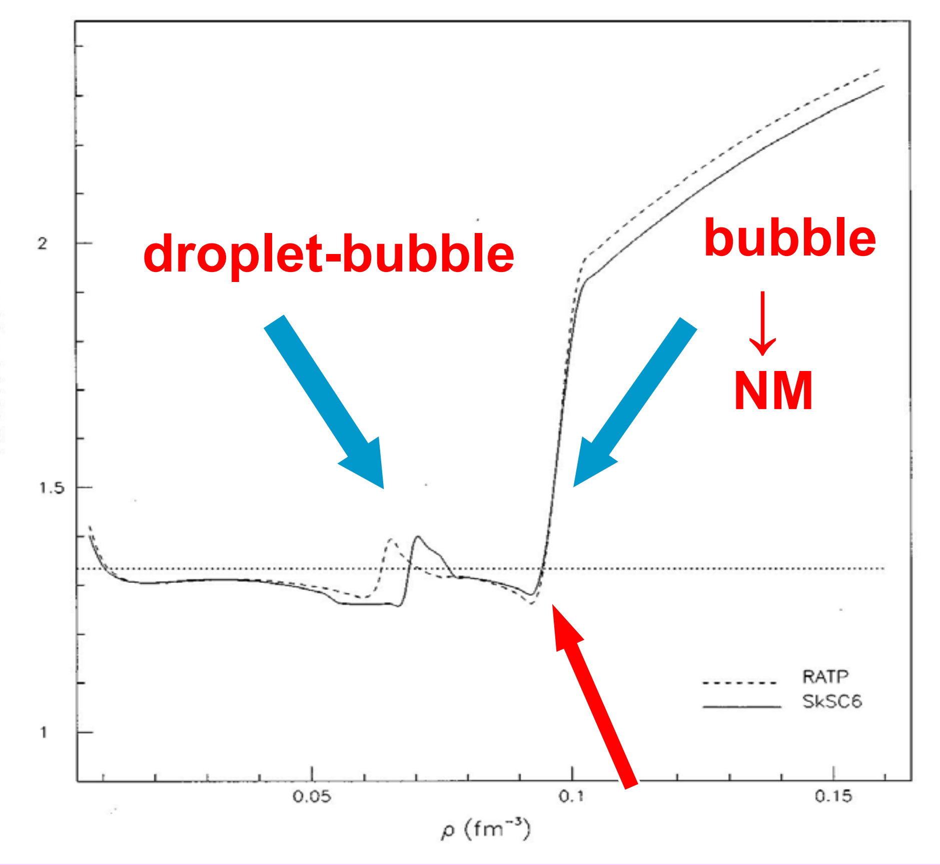
B - bag constant for quark admixtures

Adiabatic index as a function of *ln n* (LS) and *n* (Onsi) for *s=1* and Y_p*=0.3*

LS



Onsi et al.



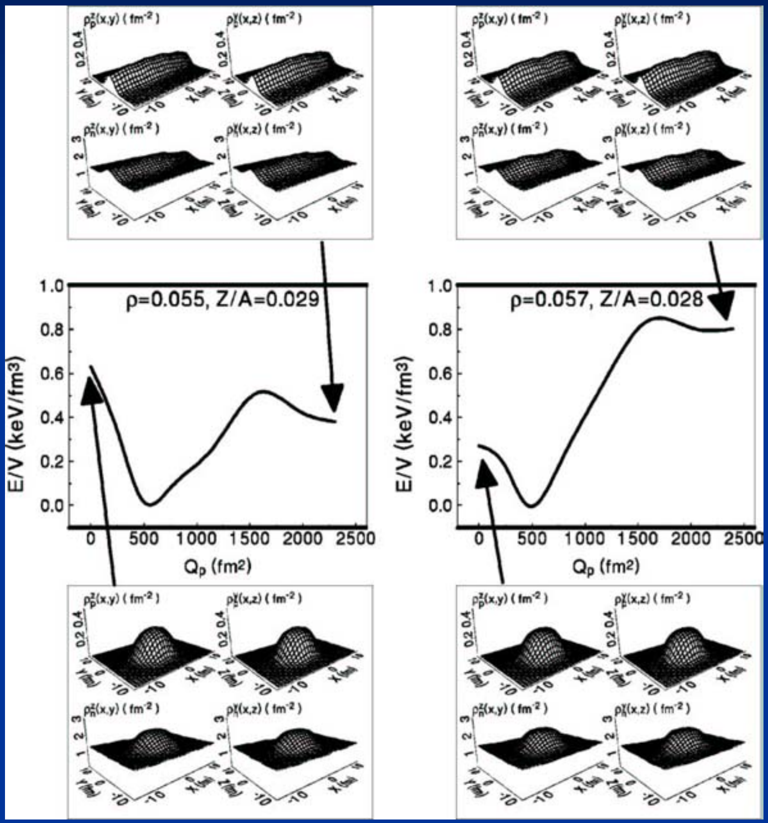
Note different x and y-axis scales

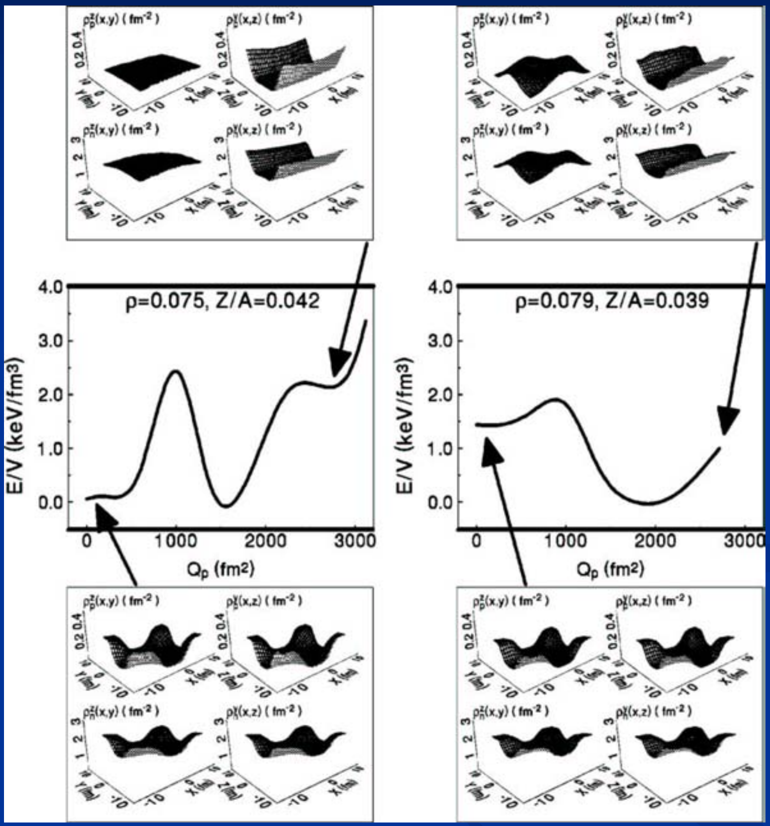
Values of Γ differ from Akmal et al

Bound nuclei in e + n gas beyond Wigner-Seitz model

Magierski and Heenen, PRC 65, 045804 (2002)

3D Mean Field HF Model (T=0)





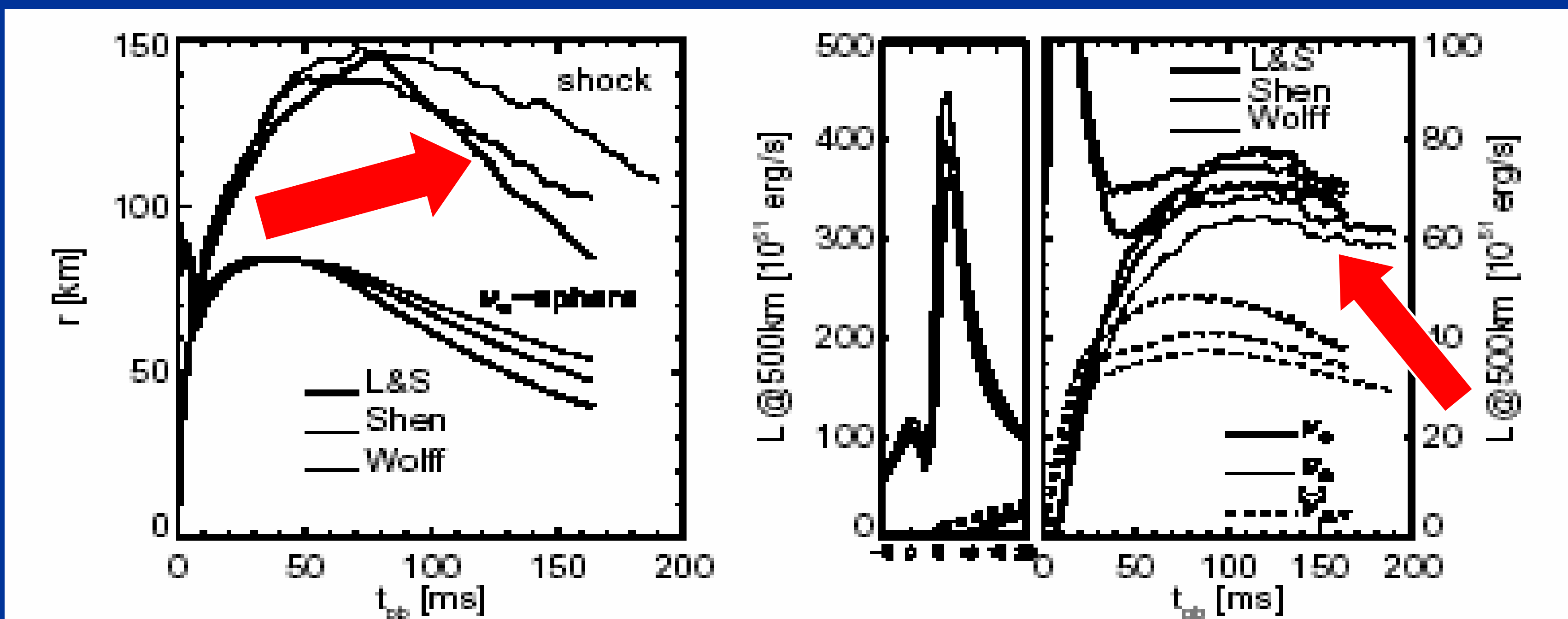
Shell effects in inhomogeneous asymmetric nuclear matter lead to a complicated pattern of density dependent phases and phase transitions not taken into account by previous models

A STATE-OF-ART TREATMENT OF THE 'PASTA' PHASE???

"Core-Collapse Supernovae at the Threshold"
H.-T. Janka, R. Buras, K. Kifonidis, A. Marek and M. Rampp
(Proceedings IAU Coll.~192, Valencia, Spain, April 22--26, 2003,
Eds. J.M.~Marcaide ad K.W.~Weiler, Springer Verlag)

Shen et al, NP A637, 435 (1998) RMF
Hillebrandt and Wolff, 1985 – Skyrme T-dependent 1D Hartree-Fock

W. Hillebrandt, R.G. Wolff: Models of type II supernova explosions. In: *Nucleosynthesis: Challenges and New Developments* ed by D. Arnett, J.W. Truran (Univ. of Chicago Press, Chicago 1985) pp 131–150



Shock radius

Electron neutrino luminosity

Sensitivity of predicted nuclear matter parameters to details of EOS

↓

Search for constraints to narrow down the variety of models.

Example:

Efficient constraint for a non-relativistic Skyrme potential in Hartree-Fock approximation

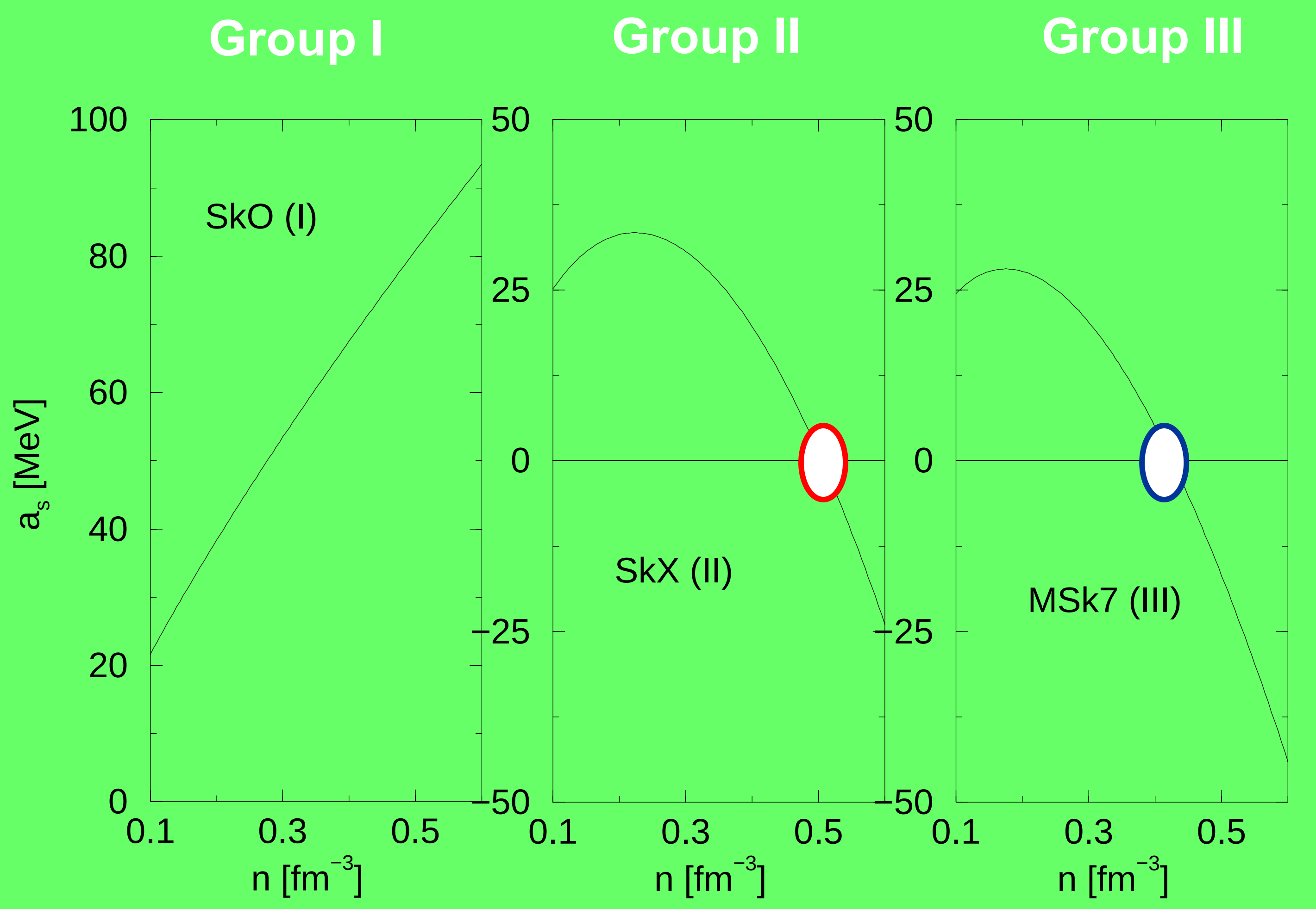
Density dependence of the symmetry energy

$$\mathcal{S}(n) = \mathcal{E}(n, Y_p = 0) - \mathcal{E}(n, Y_p = \frac{1}{2}) \ \ where \ \ Y_p = \frac{n_p}{n}$$

or

$$\mathcal{E}(n, Y_p) = \mathcal{E}(n, Y_p = \frac{1}{2}) + \frac{1}{2}(\frac{d^2\mathcal{E}}{dY_p^2})(n)(Y_p - \frac{1}{2})^{1/2}$$

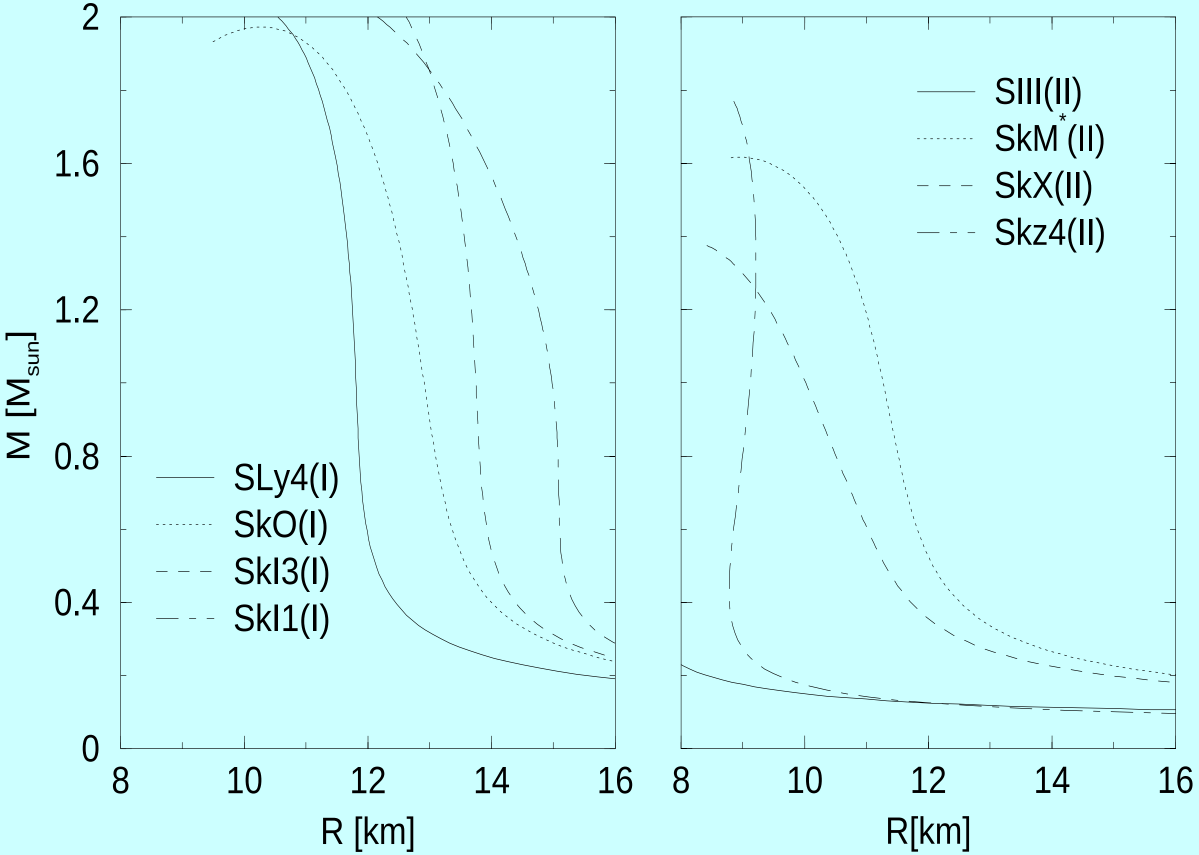
Density dependence of the symmetry energy is the main criterion for distinction between Skyrme parameterizations



SkO, SkX, MSk7 examples of Skyrme potentials

J.R.Stone at al., PRC68, 034324 (2003)

Gravitational mass versus radius for T=0 non-rotating neutron stars calculated using Skyrme EOS based on parameterizations I, II and III



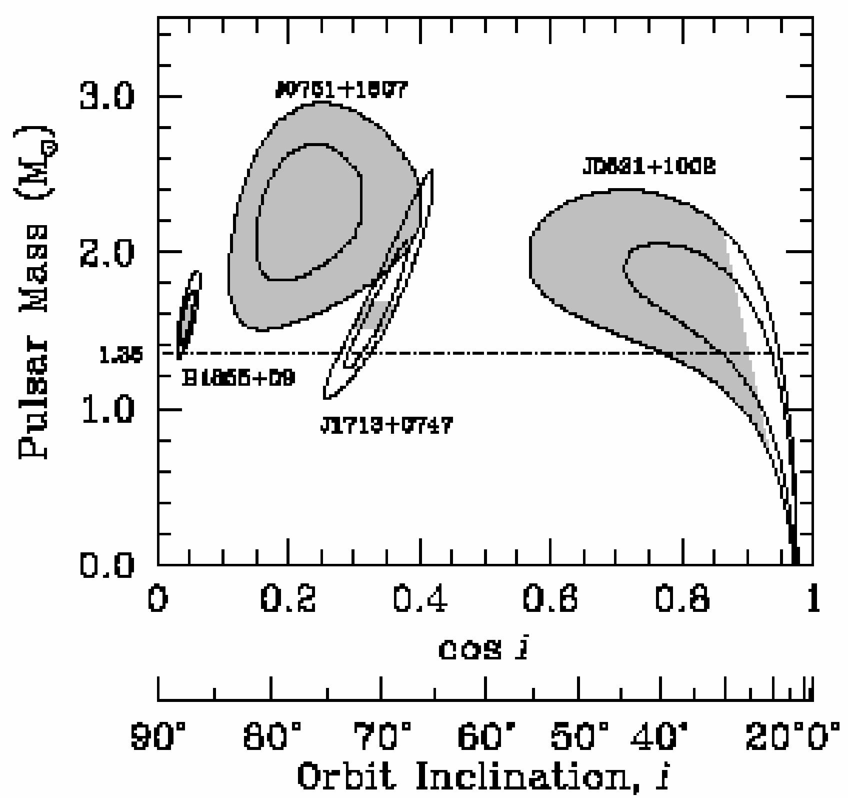
No objects with mass and radius in agreement with observational data on neutron stars

Constraint from maximum mass of a neutron star

Nice et al.,

Young Neutron Stars and Their Environments
IAU Symposium, Vol. 218, 2004
F. Camilo and B. M. Gaensler, eds.

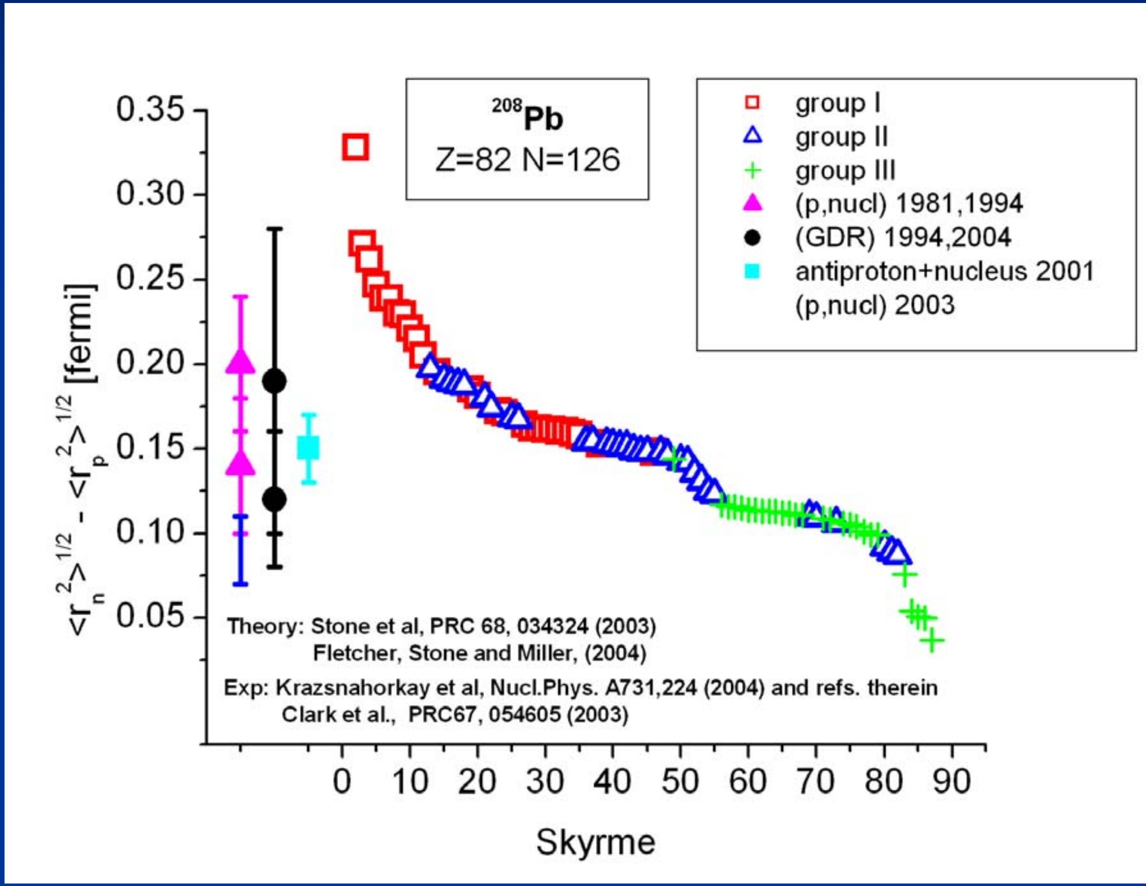
Pulsar+white dwarf binary – Arecibo telescope – timing analysis
orbital period 0.26 days –> 1.6 – 2.8 M_{solar} with 95% confidence



If this observation is confirmed, a large number of EOS would be eliminated

Structure of HDM would be questioned as the presence of hyperons and quarks is known to lower the maximum mass of a neutron star.

Calculated neutron skin in ^{208}Pb for 87 Skyrme models in comparison with experimental data



Experimental data on neutron skins from proton scattering are model dependent!
Clark et al. PRC 67, 054605 (2003)

Most precise data (1.5%) from atomic parity violation measurement in electron scattering at JLAB expected in about 2 years
Horowicz et al. PRC63, 025501 (2001)

No calculation of ground state properties of finite nuclei heavier then C is possible as yet for realistic interactions

Heavy Ion Collisions
Testing of the density dependence of $\mathcal{S}$

Bao-An Li et al., PRL 78, 1644 (1997):88, 192701, (2002)
Danielewicz et al, Science 298, 1592 (2002)

The only terrestrial situation where HD neutron rich matter can be formed – up to several times nuclear saturation density n_o (MSU, Darmstadt, RHIC)

Observables: π- to π+ ratio
neutron-proton collective flow

transverse and elliptical flow of particles from high density regions during collisions

New developments:

W.Newton (+J.R.Stone):

3D self-consistent Hartree-Fock-(Skyrme) approximation
no nuclear shape constraints (previously only spherical)
natural inclusion of shell effects
finite temperature

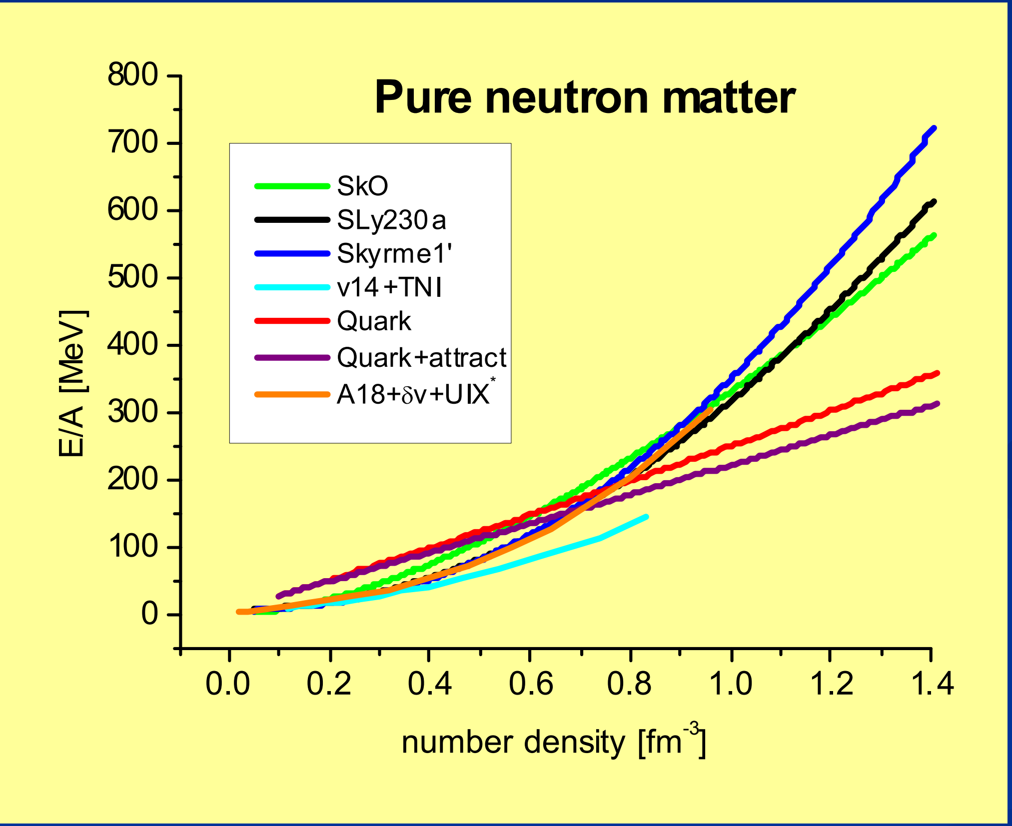
Bonche and Vautherin, NP A372, 496 (1981) (1D, Finite T)
Magierski and Heenen, PRC 65, 045802 (2002) (3D, T=0)

In full range of relevant densities and particle compositions

Comment: Codes are flexible, the Skyrme interaction can be easily replaced by any other phenomenological density dependent effective interaction (separable, Gogny etc) or some other NN potential

N-N potential based on quark and gluon exchange
Only 2 parameters!!!!

T.Barnes et al., PRC 48, 539 (1995)



EOS for pure neutron Matter derived in the Fermi gas approximation

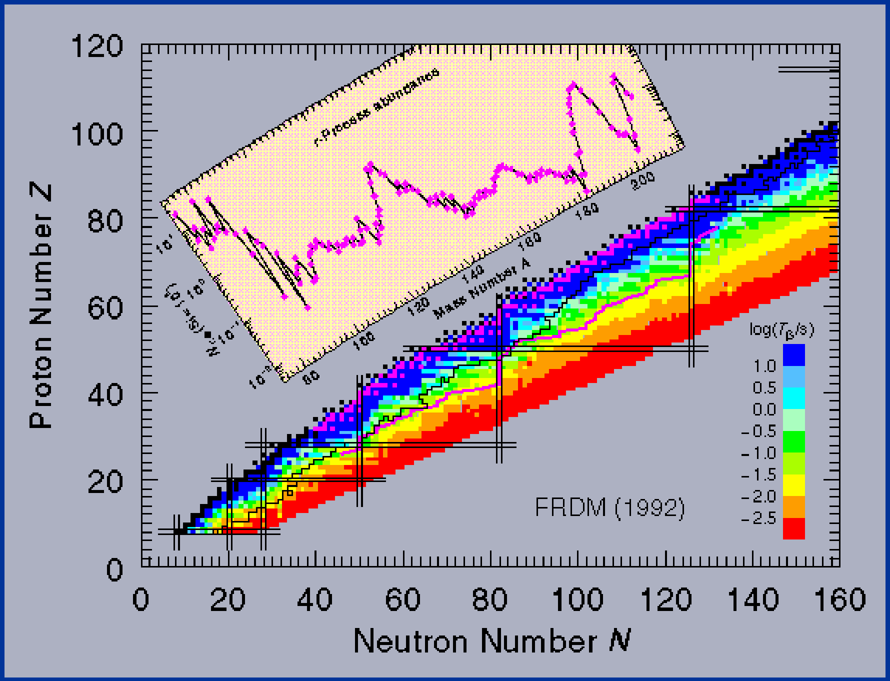
Comments:

1. It has different density dependence
2. It is in the correct energy range
3. The attractive part of the potential is understood in principle

This EOS could produce valid predictions up to unlimited high density

Connection with finite nuclei represents a specific nuclear physics interest:

Recent trend and hope is to find new physics at the boundaries of nuclear stability with the ratio of protons and neutrons very different from unity (e.g. N/Z ~ 2-3)



Neutron stars contain highly asymmetric matter N>>Z

A UNIQUE EXTRAPOLATION POINT FOR POTENTIALS FITTED ALONG THE STABILITY LINE (N ~ Z)

Conclusions

1. Unique selection of EOS may not be possible unless the nucleon-nucleon interaction in nuclear medium is fully understood

2. However constraints on the existing models should be sought and implemented

3. Systematic tests of EOS in supernova simulations may provide additional valuable constraints

4. More effects have to be investigated with ‘successful’ EOS (superfluidity, magnetic fields, presence of boson condensates, mixed phases at high densities, etc)

5. New developments should be pursued