# Self-Consistent Equation of State for Hot Dense Matter: A Work in Progress

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# Outline

- Aim
- Self-consistent EOS calculation
- Context
- Preliminary Results
- Summary

# Aim

- Equation of state of matter in the density range $n_b \approx 0.0001 - 0.5\ \text{fm}^{-3}$ and the temperature range $T \approx 0 - 100\text{MeV}$, for use in the study of supernovae and neutron stars
- Calculate in such a way that the micro-physical structure and bulk properties of the matter emerge self-consistently.

# Aim: Features of Hot Dense Matter

- Low densities: nuclei + electrons
- $0.001 < n_b < 0.1\ fm^{-3}$: massive neutron rich nuclei + neutron gas
- Shape transitions in the range $0.01 < n_b < 0.1 fm^{-3}$
- $> 0.1\ fm^{-3}$ uniform nuclear matter
- Shell structure
- (+ fermions etc.)

# Aim: Self-Consistency

- Work with a non-relativistic theory
- Select a single Hamiltonian

  $H = T_{nucleons} + T_{fermions} + V_{nucleons} + V_{Coulomb}$
- Select a nuclear potential containing all relevant microscopic interactions
- Employ one method to solve Schrödinger equation, apply consistently at all $n_b$, T.

# Self-Consistent Calculation

- Choose phenomenological potential, e.g. Skyrme
- Choose the nuclear ground state to be Slater determinants (Hartree-Fock approx.)
- Fermion species treated as ideal Fermi gases
- Minimize the Hamiltonian with respect to all free parameters
  - nuclear s.p. wavefunctions
  - baryon number
  - proton fraction $y_p$

# Self-Consistent Calculation

- HF equations and Poisson equation solved within a 3-D rectangular unit cell with periodic boundary conditions
- Initial wavefunctions: harmonic oscillator (lower densities), plane waves (higher densities)
- Iterative method used: imaginary time step[1]

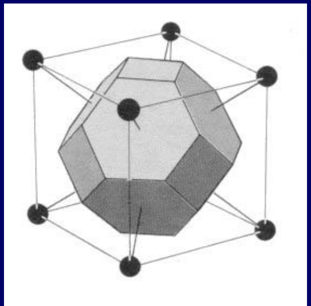
$$|\phi_{i,q}^{n+1}\rangle = e^{-\Delta t \, h_{HF}/\hbar} |\phi_{i,q}^{n}\rangle$$

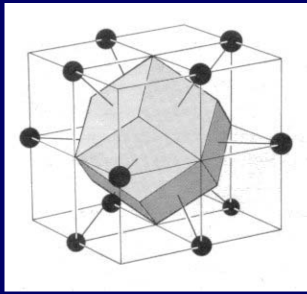
[1] Davies et al Nucl Phys A342, 111 (1980)

# Self-Consistent Calculation

- Free of the limitations of the Wigner-Seitz approximation…

BCC…





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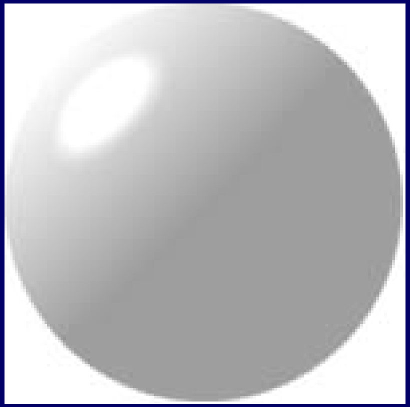


...any lattice type can be represented

# Self-Consistent Calculation

- Impose parity conservation in the three dimensions: tri-axial shapes allowed, but not asymmetric ones.
- Solution only in one octant of cell
- Still computationally intensive: to calculate one configuration takes of order 24hrs
- Testing: code reproduces binding energies and single particle structure of laboratory nuclei

# EOS construction

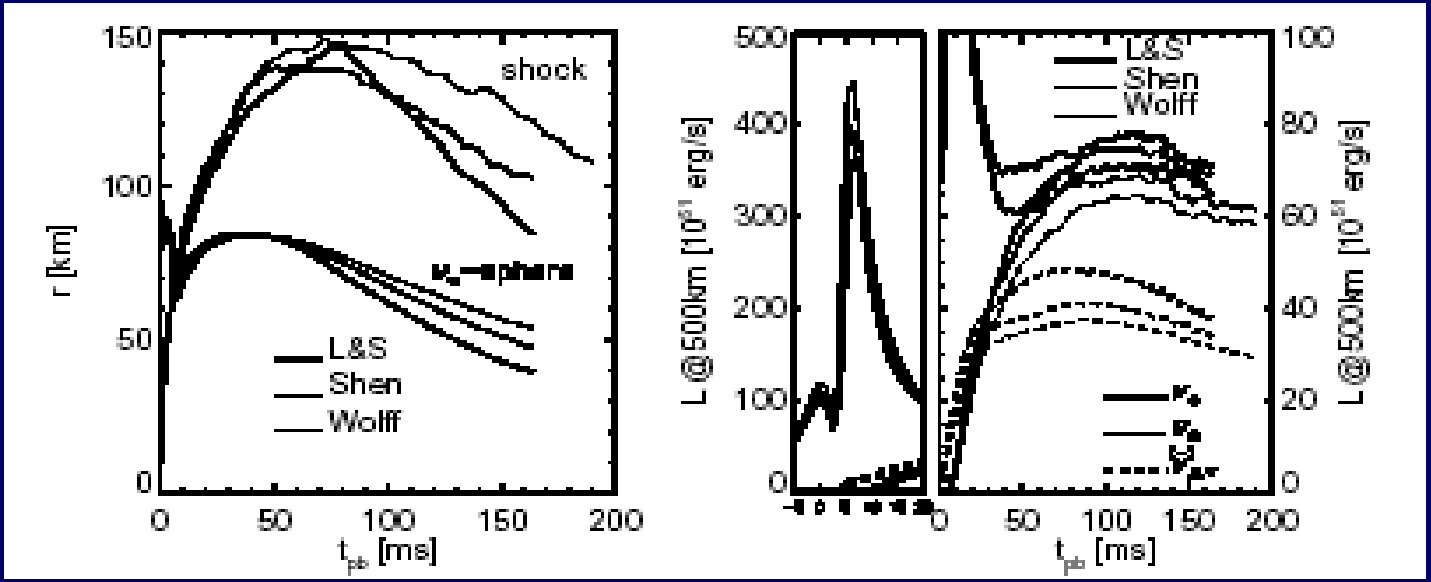
- For a given $n_b$,T, calculate the energy density of the matter for a range A (cell size), $y_p$: select the minimum value
- To explore shape effects: minimize with respect to deformation (constrained calculation)

# Context…

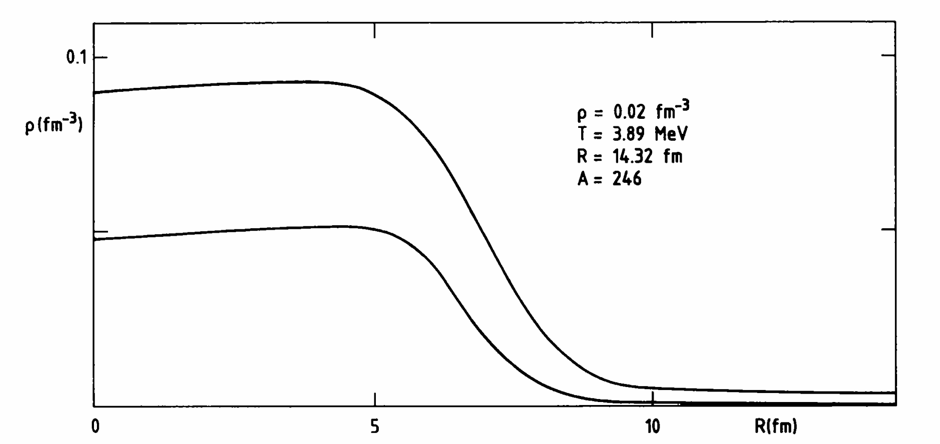
- Our calculation is continuing on from where others left off…
- Bonche and Vautherin, Nucl Phys A372 (1981), A&A 115 (1982): 1-D HF calculations in the Wigner-Seitz approximation, finite T
- Hillebrandt and Wolff (1985) conduct supernova simulations based on the resultant EOS

# Context…

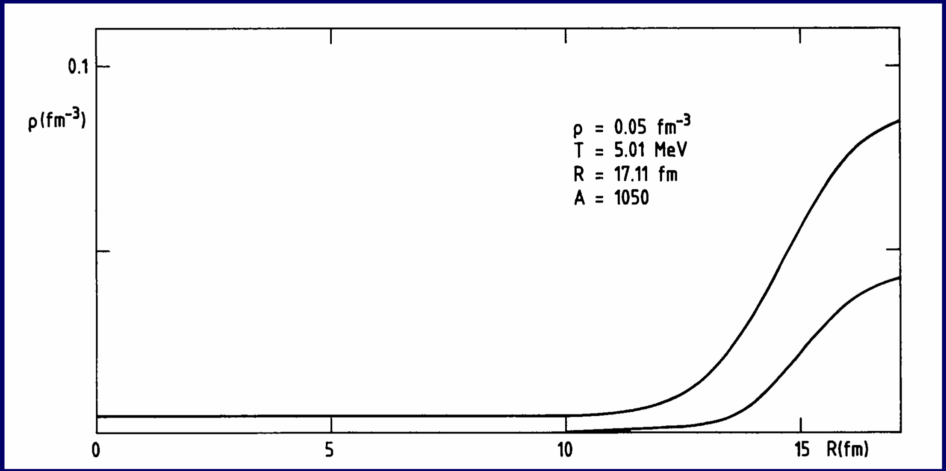
1-D Skyrme-HF EOS in supernova simulations



# Bonche and Vautherin




Neutron gas and nuclei coexist self-consistently…

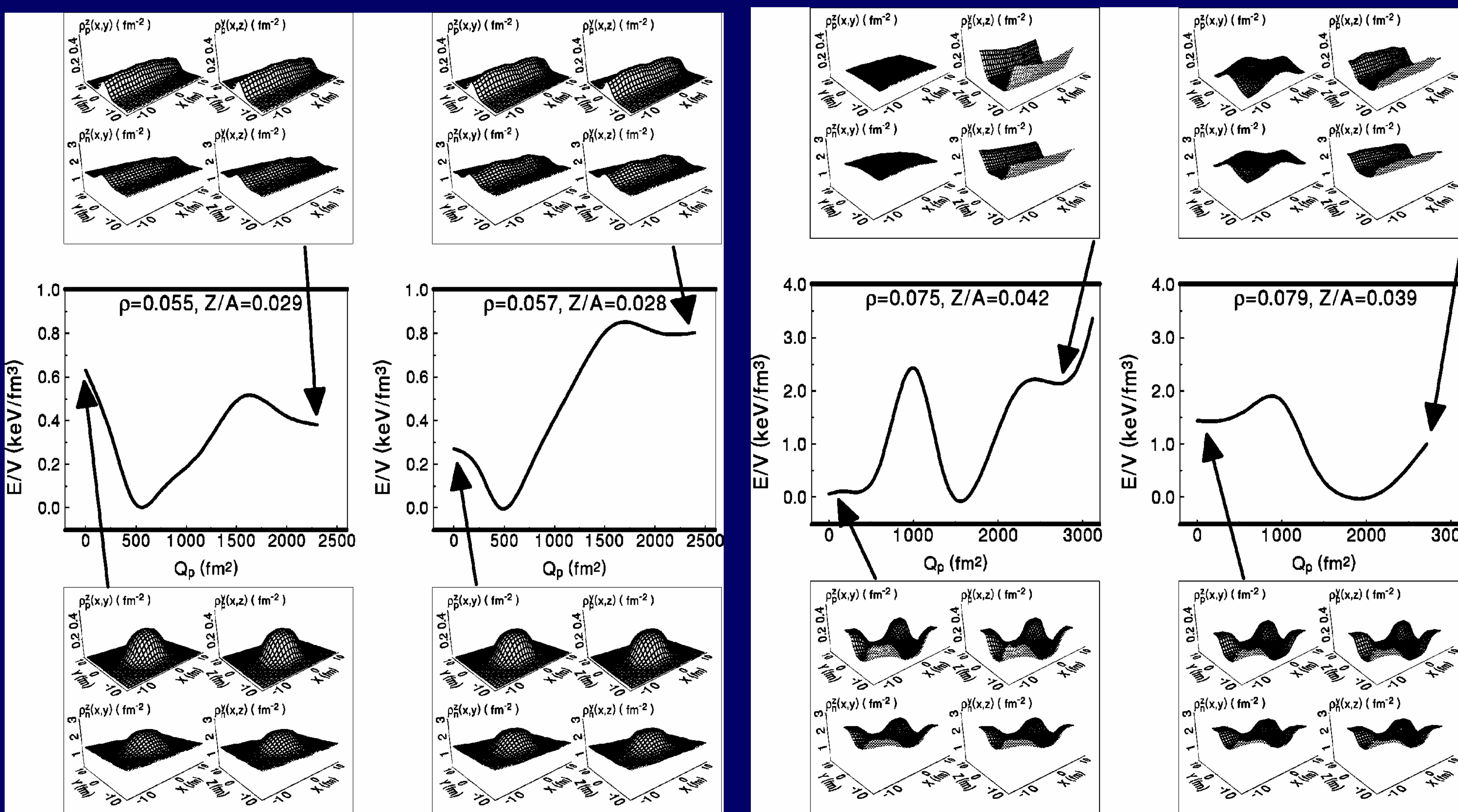



…and the bubble phase too

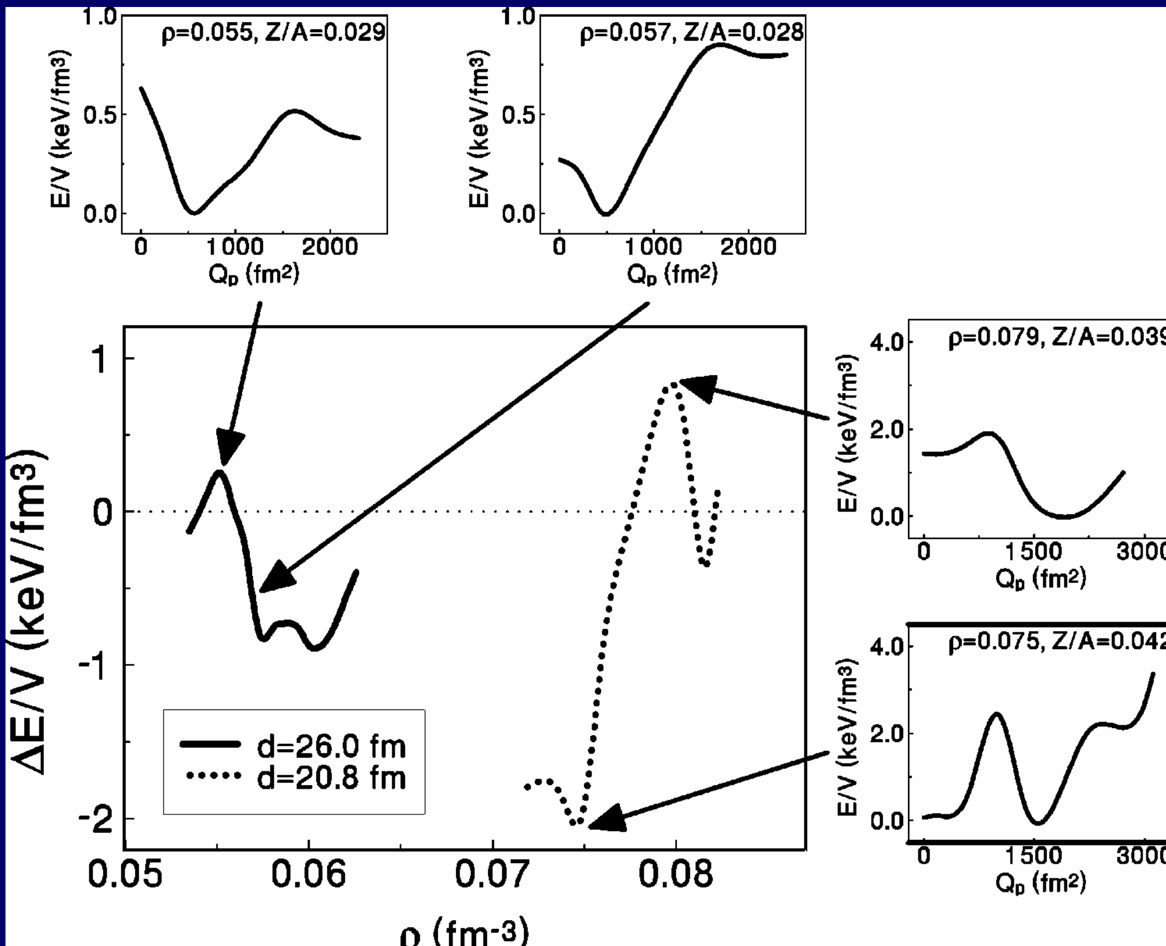
# Context…

- Magierski and Heenen PRC65 045804 (2001): 3-D HF calculation of nuclear shapes at bottom of neutron star crust at zero T
- When treated in 3 dimensions, series of shape transitions become complex

# Context: Magierski and Heenen



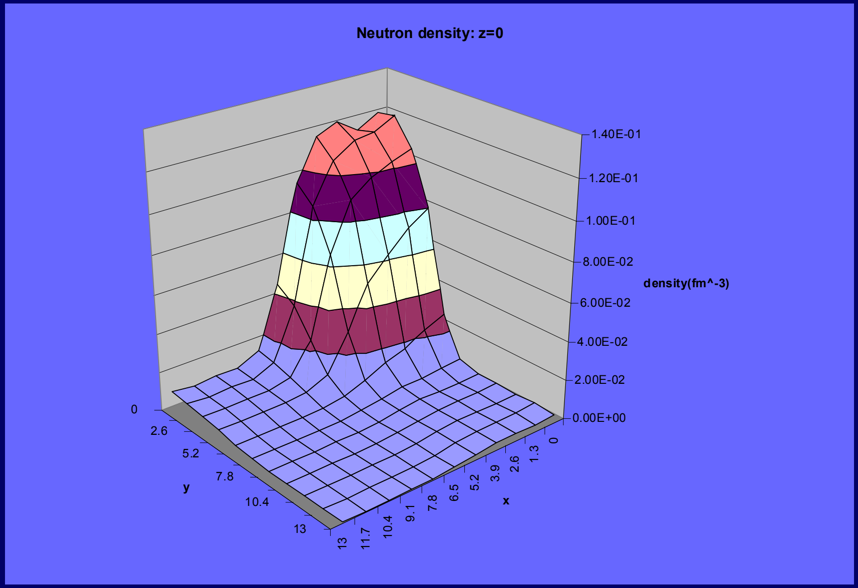
# Context… Magierski and Heenen

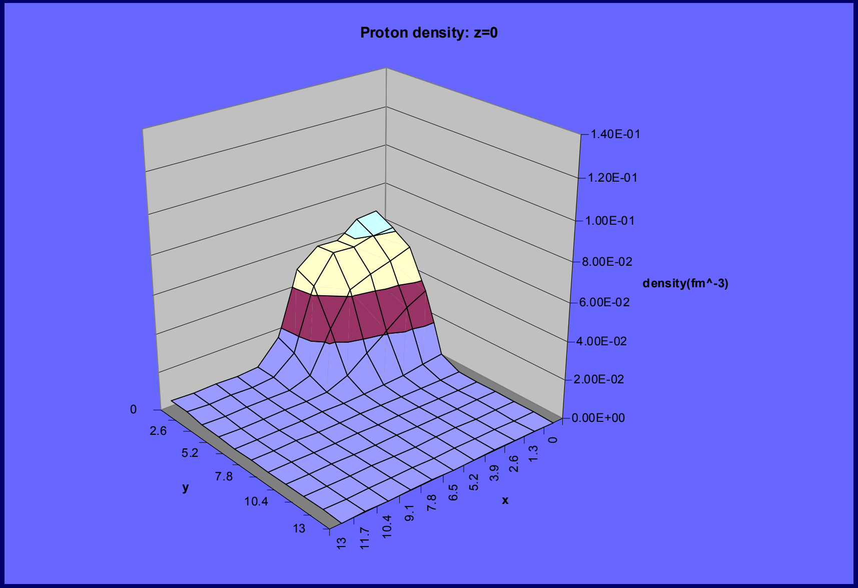



- Rapid fluctuation of energy difference between phases with density
- Fluctuation is from shell energy of the unbound neutrons: a Casimir-like Effect
- Confirmed by comparing with shell energy of neutrons in semi-classical approx.
- Such effects may even reverse order of phase transitions

# Preliminary Results

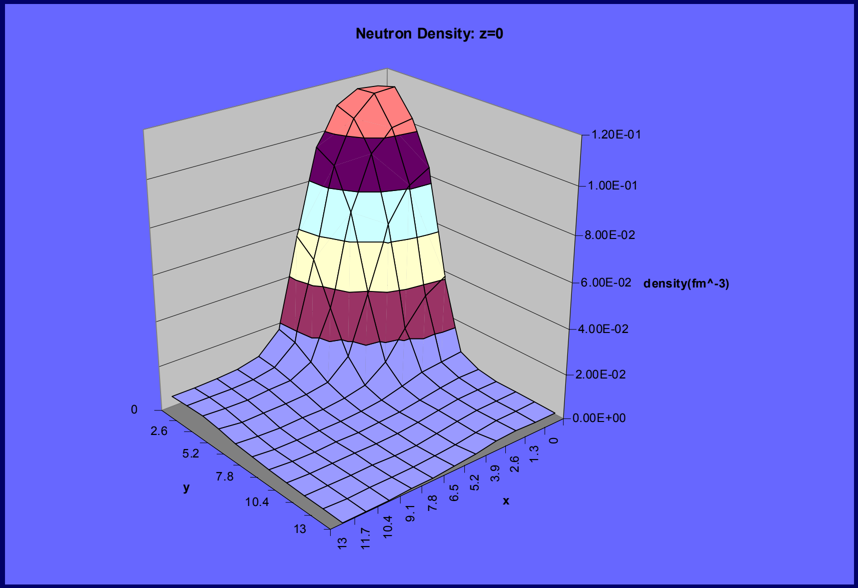
A=120 Z=0.25, T=0MeV, $n_b$ = 0.007$fm^{-3}$

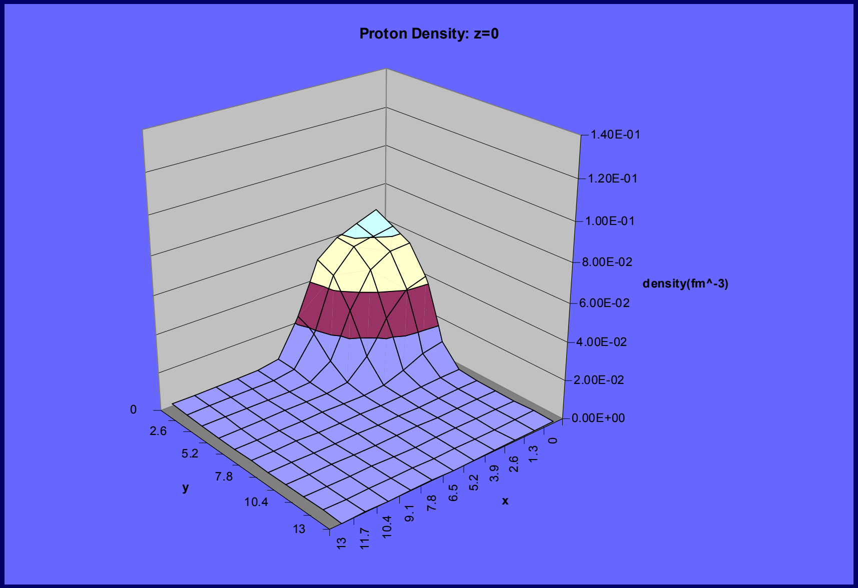


# Preliminary Results

A=120 Z=0.25, T=2.5 MeV, $n_b$ = 0.007$fm^{-3}$



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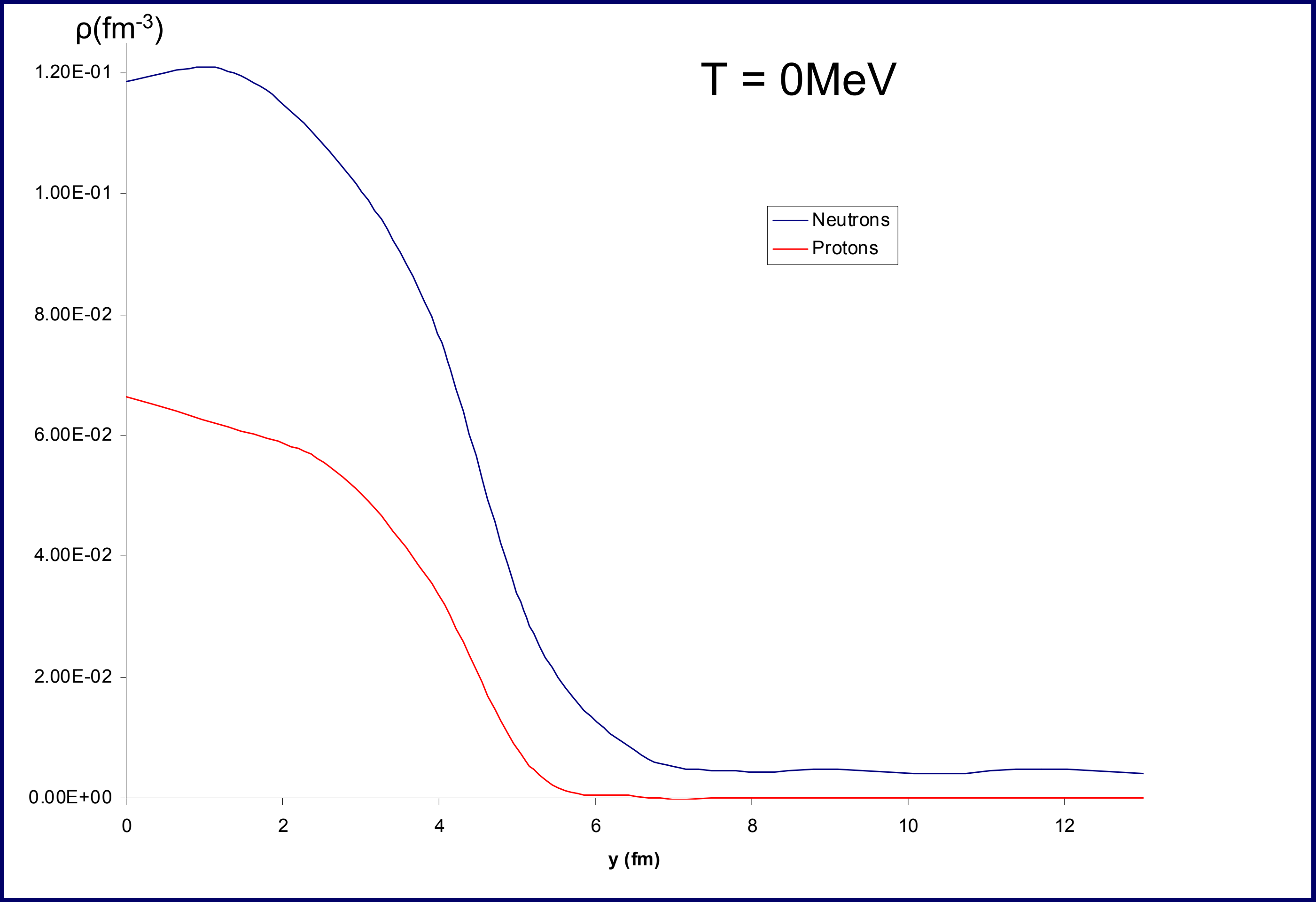

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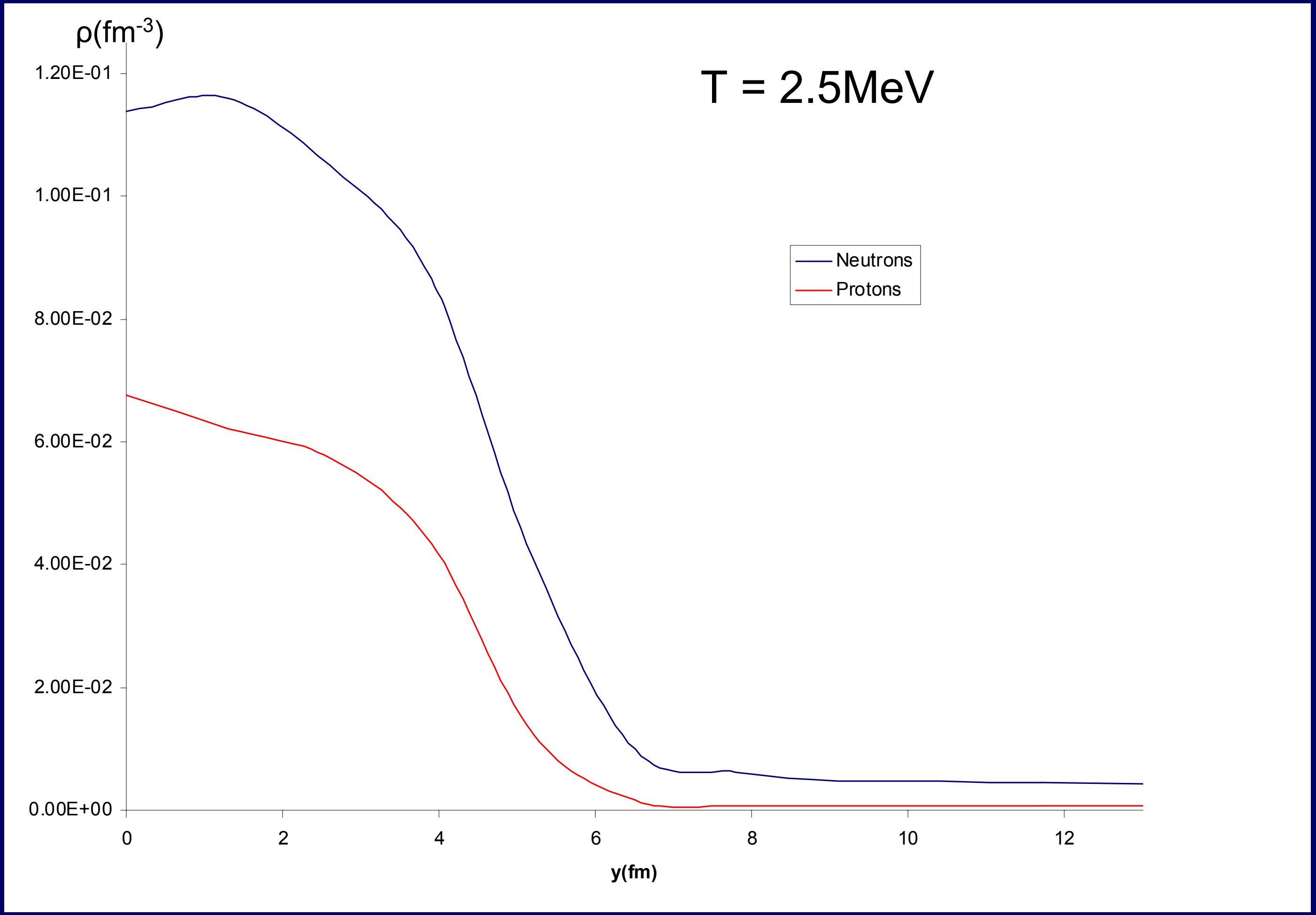
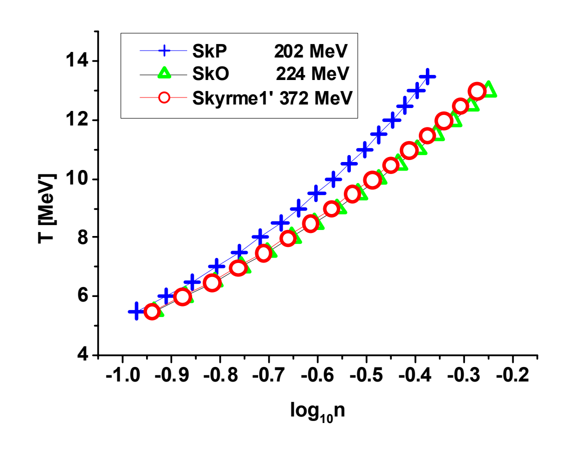
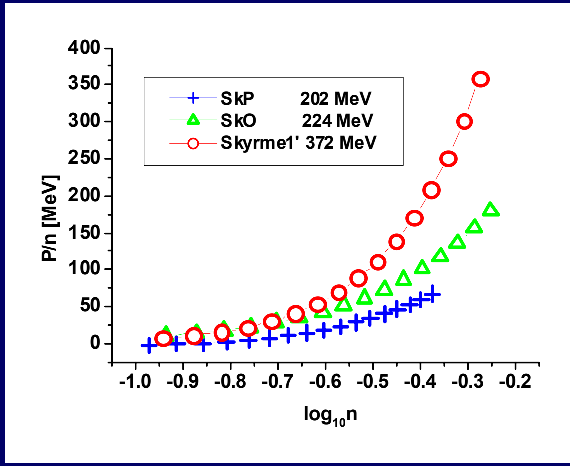


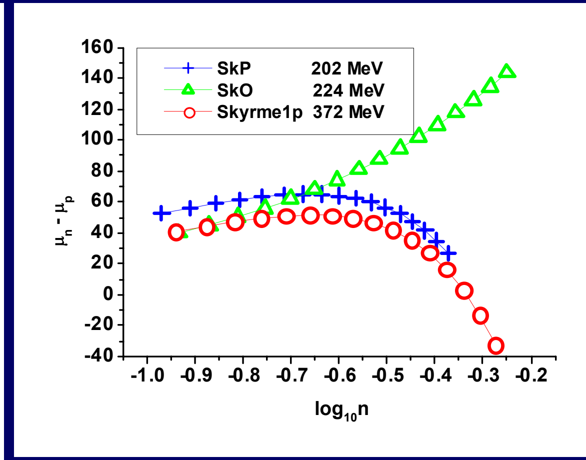
# EOS of Uniform nuclear matter

S=1, $Y_p$ = 0.3

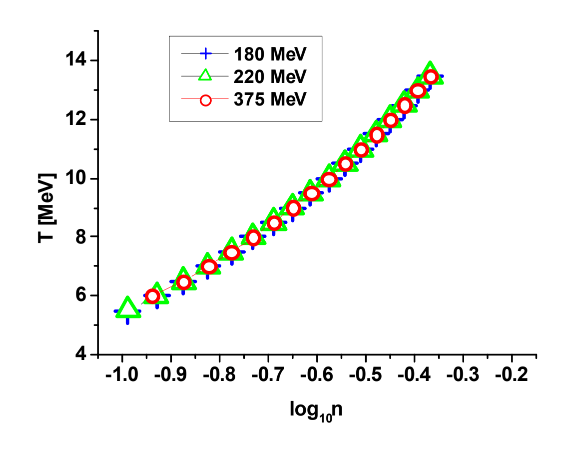
NS

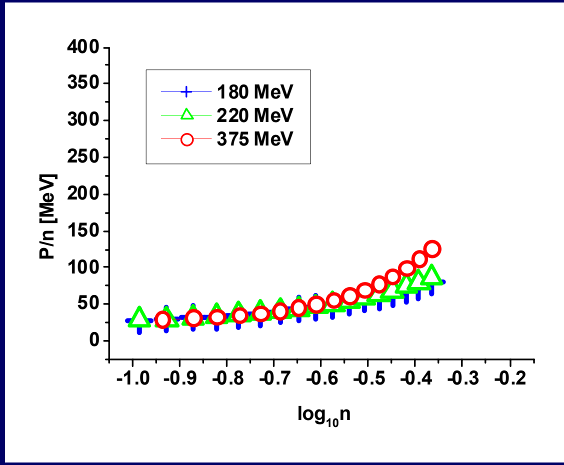


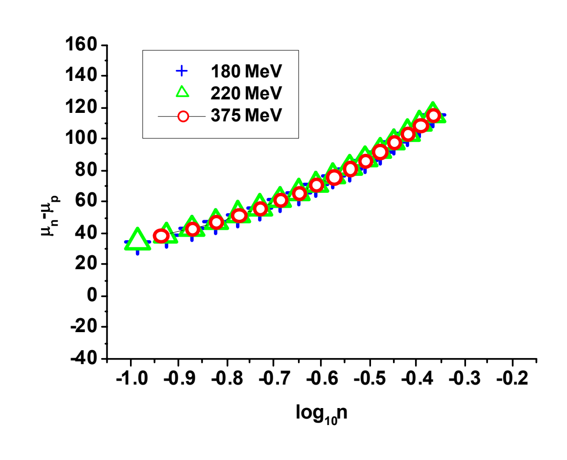




LS



# Summary

- We are constructing a self-consistent EOS using Skyrme-HF in 3D
- Will take into account the temperature and density effects on 3D nuclear shapes
- Basic testing of the code is complete
- First set of calculations (minimization w.r.t mass number) under way