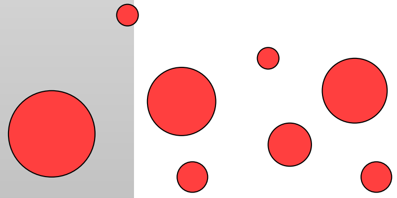
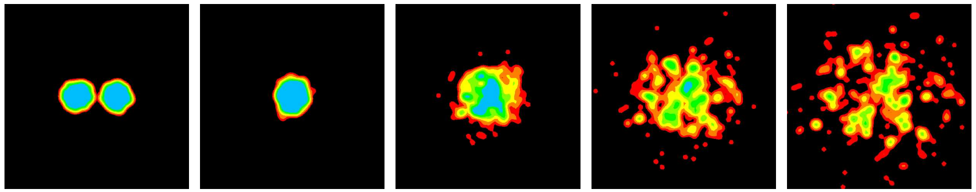
Symmetry energy for excited nuclei produced by multifragmentation

Akira Ono (Tohoku University, Sendai, Japan)



Astrophysical environments



Equilibrium ⇐ Molecular Dynamics (AMD) ⇒ **Dynamics**

- Equation of State
- Liquid-gas phase transition

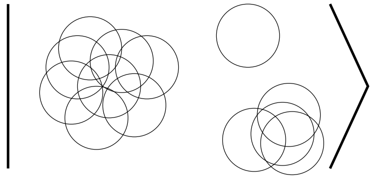
- Flow etc
- Multifragmentation

- Quantum statistics in AMD. ($E^* = aT^2$)
- Any equilibrium in collisions?
  ⇒ Nuclear symmetry free energy

The first version of AMD (before 1995)

A Slater determinant of Gaussian wave packets

$$|\Phi(\mathbf{Z})\rangle = \det_{ij}\Big[\exp\Big\{-\nu(\mathbf{r}_i - \mathbf{Z}_j/\sqrt{\nu})^2\Big\}\chi_{\alpha_j}(i)\Big]$$



$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width Parameter = 0.16 fm^{-2}

χ_{α_i} : Spin-Isospin States = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Time Dependent Variational Principle for $\{\mathbf{Z}_1(t), \ldots, \mathbf{Z}_A(t)\}$

$$\delta\int dt\,\frac{\langle\Phi(\mathbf{Z})|(i\hbar\frac{d}{dt} - H)|\Phi(\mathbf{Z})\rangle}{\langle\Phi(\mathbf{Z})|\Phi(\mathbf{Z})\rangle} = 0 \qquad \Rightarrow \qquad i\hbar\sum_{j\tau} C_{i\sigma,j\tau}\frac{dZ_{j\tau}}{dt} = \frac{\partial\mathcal{H}}{\partial Z^*_{i\sigma}}$$

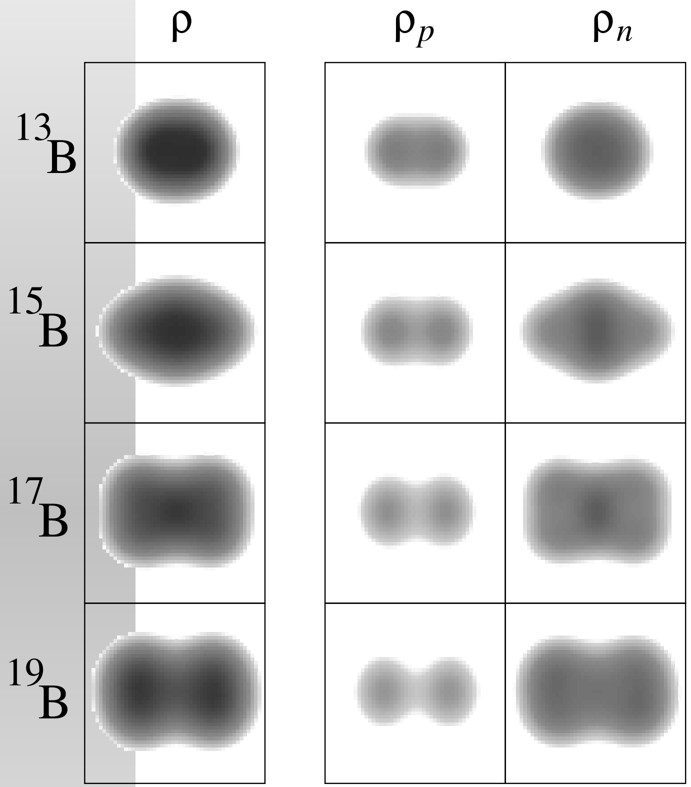
Wave packet motion in the mean field

H: Effective Hamiltonian

+ Two-nucleon collisions (as in other simulation methods)

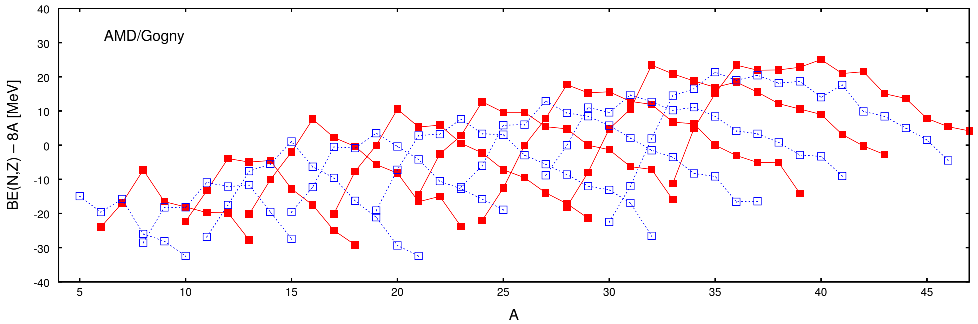
Nuclear structure

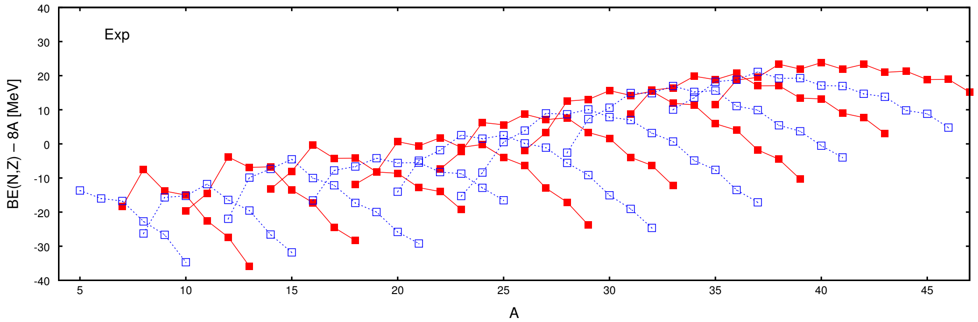
Density of B-isotpes



Kanada-En'yo et al.

$BE(N,Z) - 8A$ MeV





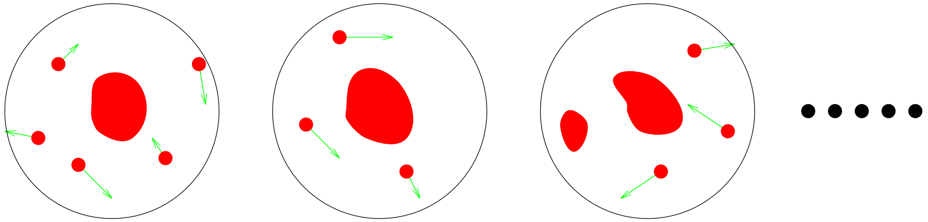
AMD wave function

$$|\Phi(\mathbf{Z})\rangle = \det_{ij}\left[\exp\left\{-\nu(\mathbf{r}_i - \mathbf{Z}_j/\sqrt{\nu})^2\right\}\chi_{\alpha_j}(i)\right]$$

Statistics in AMD

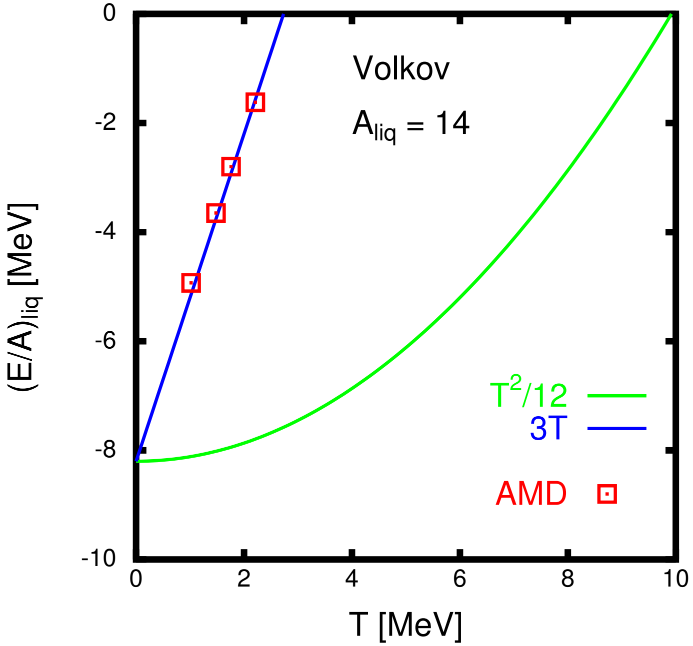
Solve the time evolution of many nucleon system contained in a box.

Statistical ensemble (E, V)



Energy balance between liquid and gas

- Internal energy of nuclei: $(E/A)_{\text{liquid}}$
- Energy of gas nucleons: $E_{\text{gas}} = \frac{3}{2}T$

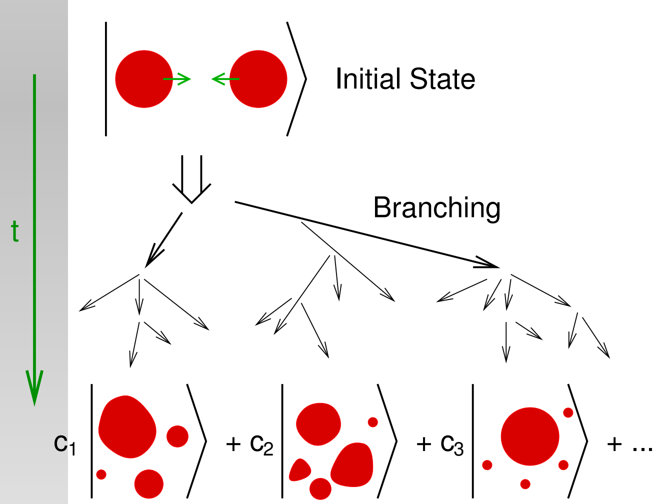


$$E^*_{\text{liq}} = 3AT \quad (\neq aT^2)$$

Ono and Horiuchi, PRC53 (1996) 2341.

How to get quantum statistics

- Fluctuation or stochasticity. (Ohnishi and Randrup)
- Dynamics of wave packet widths. (Schnack and Feldmeier)
- Quantum branching $\Leftarrow$ dynamics of wave packet shape. (Ono and Horiuchi)



AMD wave function for each branch

$$\langle \mathbf{r}_1 \ldots \mathbf{r}_A | \Phi(Z) \rangle = \det_{ij}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

Stochastic equation of motion

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\mathrm{PB}} + (\mathrm{NN\ coll}) + \Delta\mathbf{Z}_i(t)$$

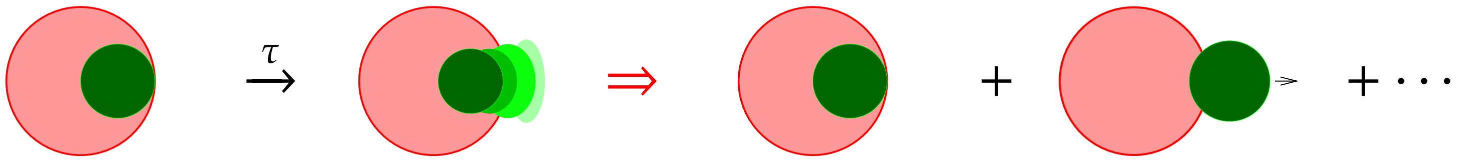
Mean field + Quantum branching

At each time step t_0, for each wave packet k,. . .

Mean field propagation for $t_0 \to t_0 + \tau$ + **Branching at $t_0 + \tau$** τ: Coherence time

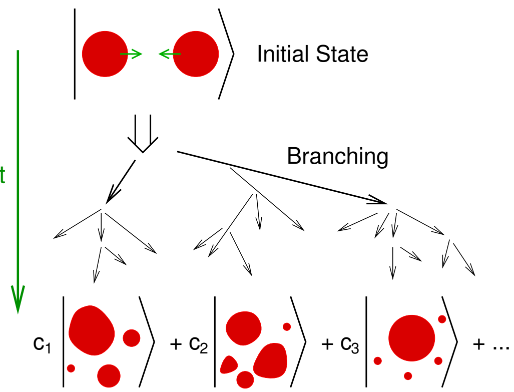
$t = t_0$ $\qquad\qquad$ $t = t_0 + \tau$

$$|\mathbf{Z}_k\rangle\langle\mathbf{Z}_k| \xrightarrow[\text{Mean field}]{} |\psi_k\rangle\langle\psi_k| \xrightarrow[\text{Branching}]{} \int |\mathbf{z}\rangle\langle\mathbf{z}|\, w_k(\mathbf{z}) d\mathbf{z} \qquad \text{for } k = 1, \ldots, A$$



$$i\hbar \frac{d}{dt}|\psi_k(t)\rangle = h^{\mathrm{HF}}|\psi_k(t)\rangle$$

$$\frac{\partial f_k}{\partial t} = -\frac{\partial h^{\mathrm{HF}}}{\partial \mathbf{p}} \cdot \frac{\partial f_k}{\partial \mathbf{r}} + \frac{\partial h^{\mathrm{HF}}}{\partial \mathbf{r}} \cdot \frac{\partial f_k}{\partial \mathbf{p}}$$

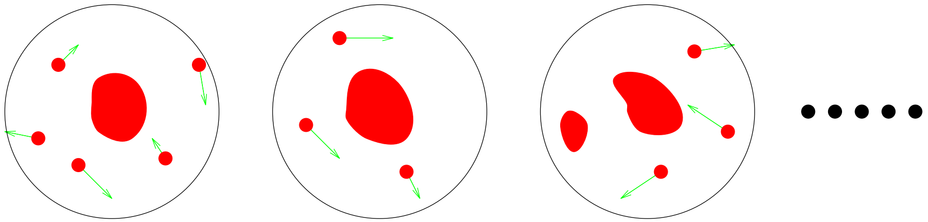


$$|\Phi(Z)\rangle\langle\Phi(Z)| \qquad |\Psi\rangle\langle\Psi| \xrightarrow[\text{Branching}]{} \int |\Phi(z)\rangle\langle\Phi(z)|\, w(z) dz$$

Statistics in AMD

Solve the time evolution of many nucleon system contained in a box.

Statistical ensemble (E, V)



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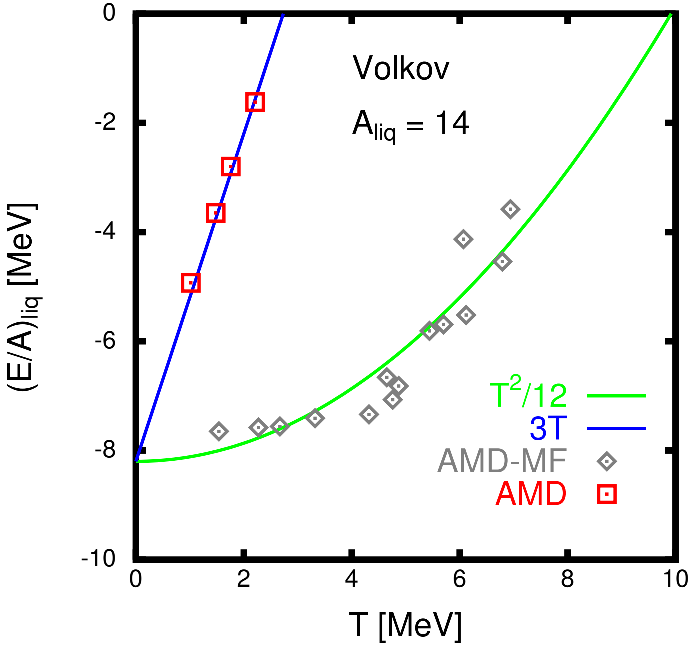
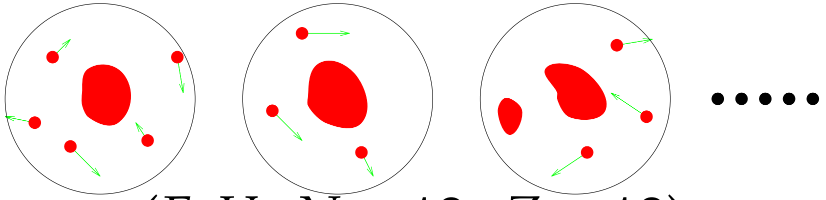


$$E^*_{\text{liq}} \approx aT^2$$

Ono and Horiuchi, PRC53 (1996) 2341.

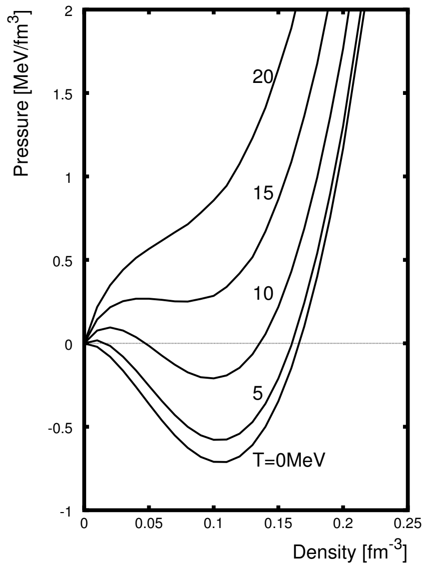
Study of liquid-gas phase transition

Microcanonical ensemble ⇐ Long-time solution of AMD



$(E, V,\ N = 18,\ Z = 18)$

Matter/Gogny

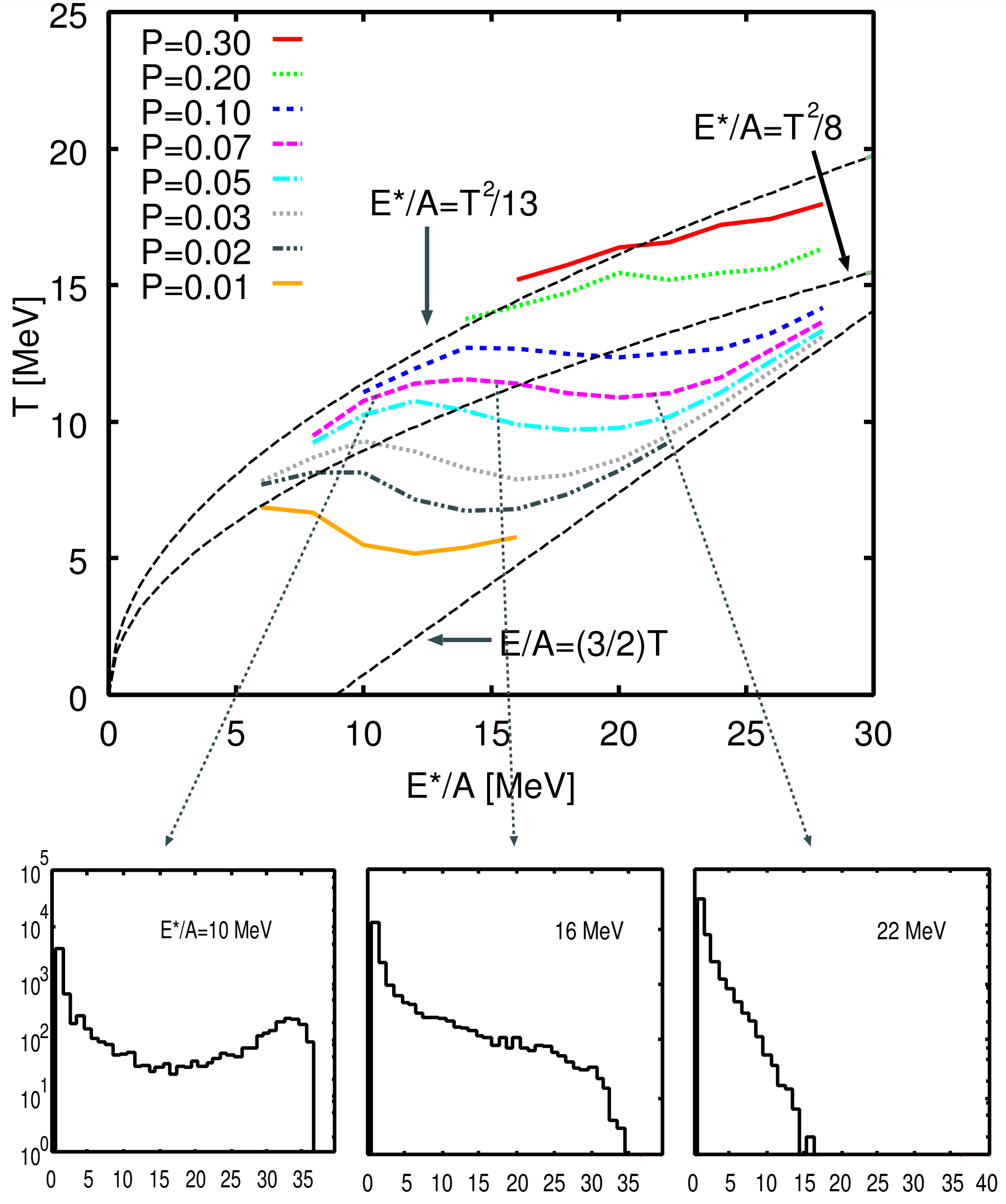


- Gogny force (consistent with saturation property)
- Microcanonical temperature T (⇐ isolated nucleons)

$$\frac{1}{T} = \frac{\partial S(E)}{\partial E} = \left\langle \frac{\partial S_{\rm iso}(E_{\rm iso})}{\partial E_{\rm iso}} \right\rangle_E = \left\langle \frac{\frac{3}{2}N_{\rm iso} - 1}{E_{\rm iso}} \right\rangle_E \approx \frac{3}{2} \left\langle \frac{E_{\rm iso}}{N_{\rm iso}} \right\rangle_E^{-1}$$

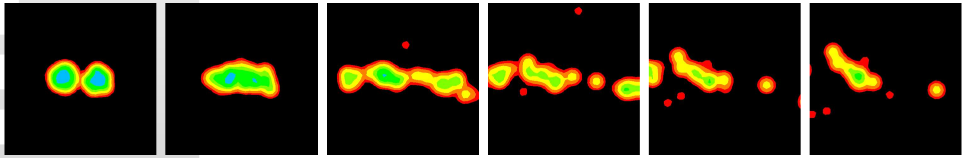
- Pressure P (⇐ reflection at the wall)

Caloric curve by AMD

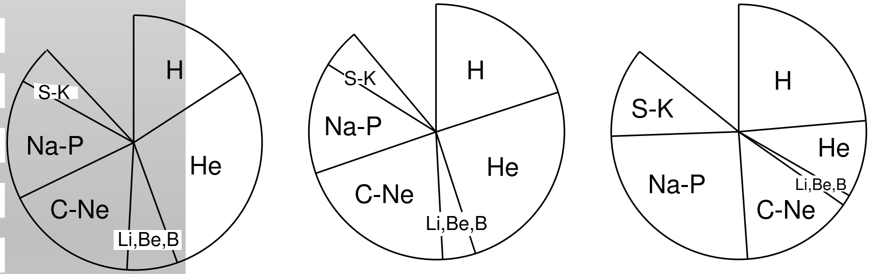


AMD results for fragmentation

^{40}Ca + ^{40}Ca at 35 MeV/u, $b = 0$



Experiment AMD AMD



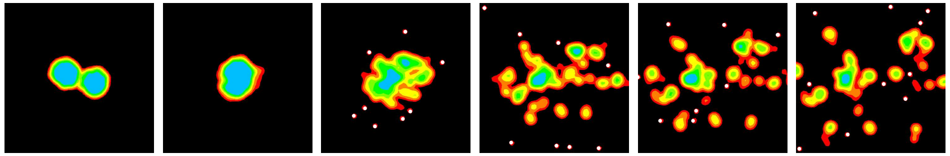
(Gogny force) (SKG2 force)

Soft EOS, Stiff EOS,

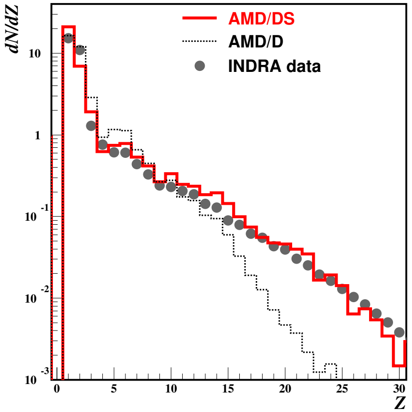
p-dep U p-indep U

AMD with $\tau \to 0$.

Xe + Sn at 50 MeV/u, $0 \le b \le 4$ fm

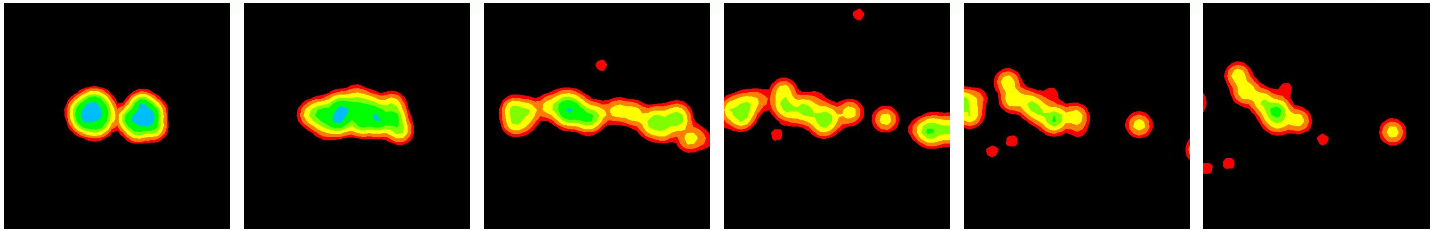


Charge distribution



- AMD/D ($\tau = 0$)
- AMD/DS (finite τ)

Isoscaling in dynamical collisions



Can we find any “equilibrium” in dynamical collisions?

AMD, $t = 300$ fm/c

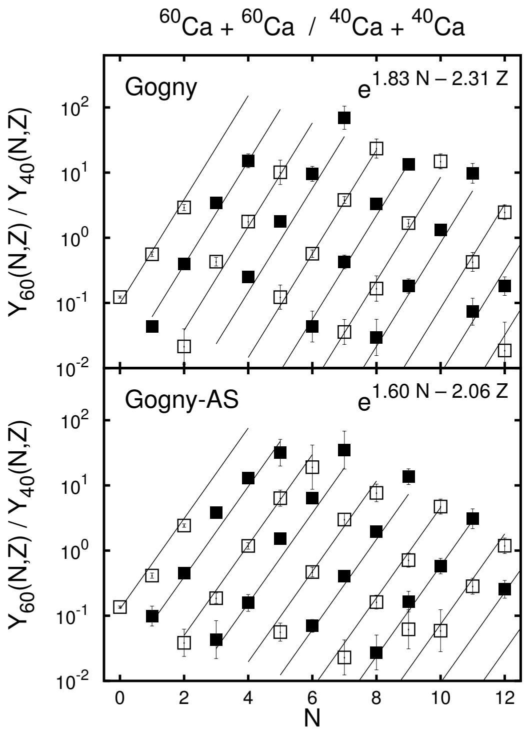
Isoscaling

— Fragment yeilds from two systems

1: ^{40}Ca + ^{40}Ca at 35 MeV/u

2: ^{60}Ca + ^{60}Ca at 35 MeV/u

$$\frac{Y_2(N,Z)}{Y_1(N,Z)} \propto e^{\alpha N + \beta Z}$$



The fragment isospin composition is largely governed by a statistical law.

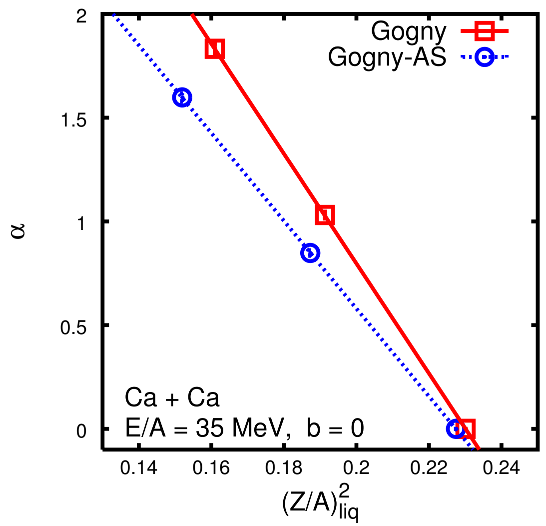
A statistical relation in the simulation results

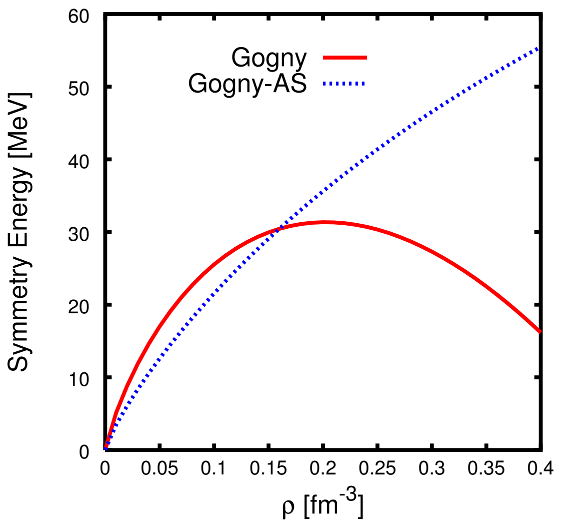
A statistical relation between

- the isoscaling parameter α
- the fragment isospin asymmetry $(Z/A)^2_{\text{liq}}$

$$\frac{\alpha_{21}}{(Z/A)^2_{\text{liq},1} - (Z/A)^2_{\text{liq},2}} = \frac{4C}{T}$$

C: Symmetry energy coefficient





- $4C(\text{Gogny})/T = 26.5$
- $4C(\text{Gogny-AS})/T = 21.2$

$\Rightarrow$

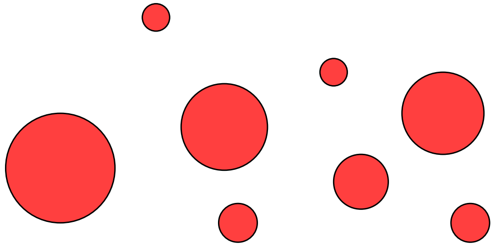
- $\rho \sim 0.08$ fm^{-3}
- $T \sim 3.4$ MeV

Ono, Danielewicz, Friedman, Lynch, Tsang, PRC **68**, 051601(R) (2003)

Fragment yields and nuclear free energies

- Isoscaling $Y^{(1)}_{NZ}/Y^{(2)}_{NZ} \propto e^{\alpha N+\beta Z}$
- Relation $\alpha/\Delta(Z/A)^2 = 4C/T$

$$\Uparrow$$



Fragment yields $\quad Y_{NZ} \propto \exp\left[-\frac{G_{NZ}(T,P)}{T} + \frac{\mu_n}{T}N + \frac{\mu_p}{T}Z\right]$

$Y_{NZ} \Rightarrow G_{NZ}$ (symmetry energy)

Liquid drop form $\quad G_{NZ}(T,P) = f(A)A + c(A)\frac{(N-Z)^2}{A}$ + Coulomb

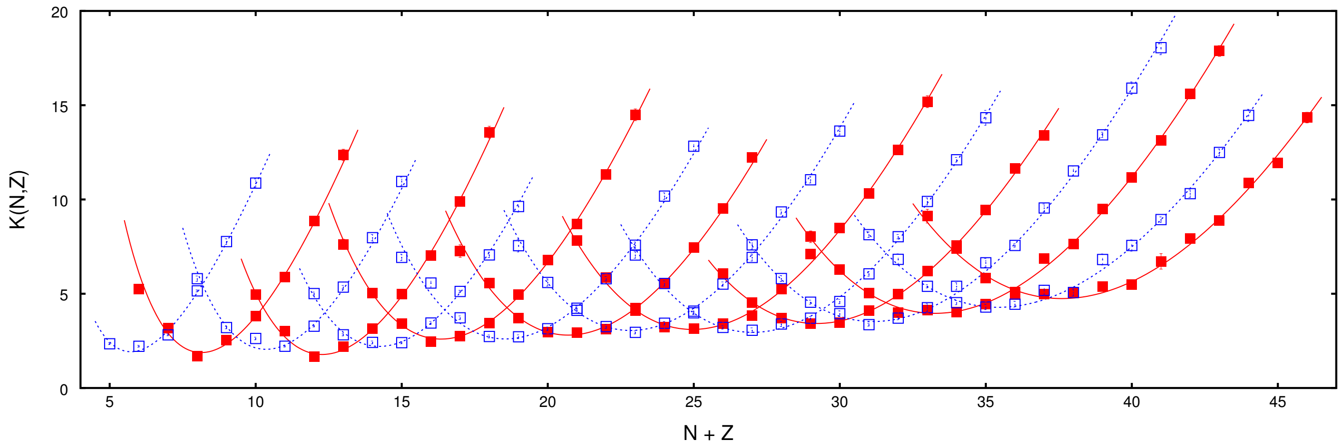
Fragment yields ⇒ Symmetry energy

$Y_i(N,Z)$ from many systems

- $i = 1$: $^{40}Ca + ^{40}Ca$
- $i = 2$: $^{48}Ca + ^{48}Ca$
- $i = 3$: $^{60}Ca + ^{60}Ca$
- $i = 4$: $^{46}Fe + ^{46}Fe$

By employing isoscaling,

$$\left.\begin{array}{ll} & Y_1(N,Z) \\ \approx & Y_2(N,Z)\, e^{-\alpha_2 N - \beta_2 Z} \\ \approx & Y_3(N,Z)\, e^{-\alpha_3 N - \beta_3 Z} \\ \approx & Y_4(N,Z)\, e^{-\alpha_4 N - \beta_4 Z} \end{array}\right\} \equiv e^{-K(N,Z)}$$

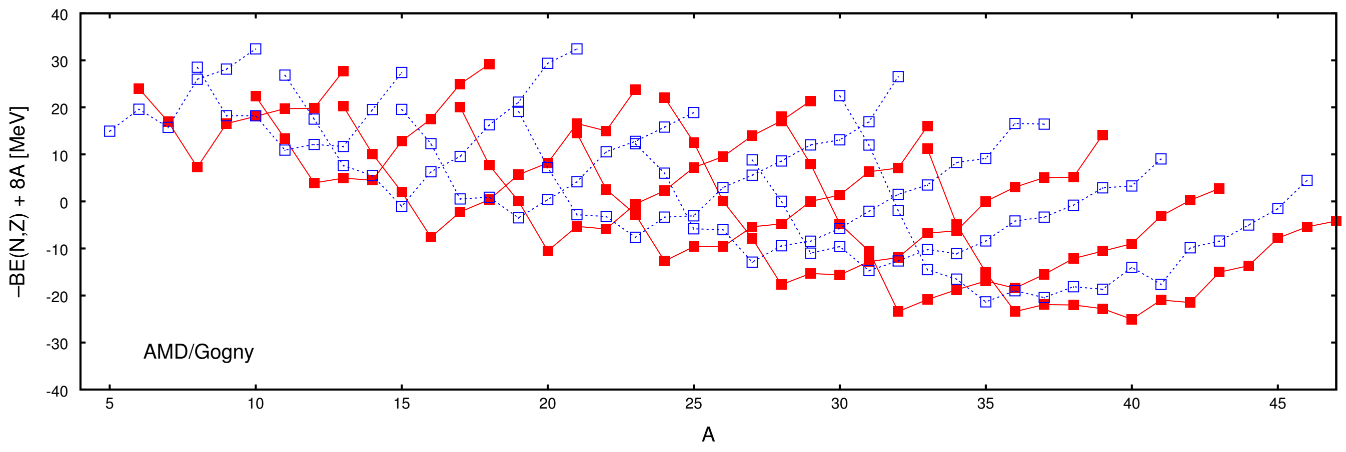


$$K(N,Z) = \xi(Z)N + \eta(Z) + \zeta(Z)\frac{(N-Z)^2}{N+Z} = \frac{G_{\text{nuc}}(N,Z)}{T} - \frac{\mu_n}{T}N - \frac{\mu_p}{T}Z$$

Symmetry energy coefficient in G_{nuc} is $C(Z) = T\zeta(Z)$

Binding energies ($T = 0$)

$-\mathrm{BE}(N, Z) + 8A$ MeV $\Leftarrow$ AMD/Gogny



Shell, pairing and clustering effects are stronger than $K(N, Z)$.

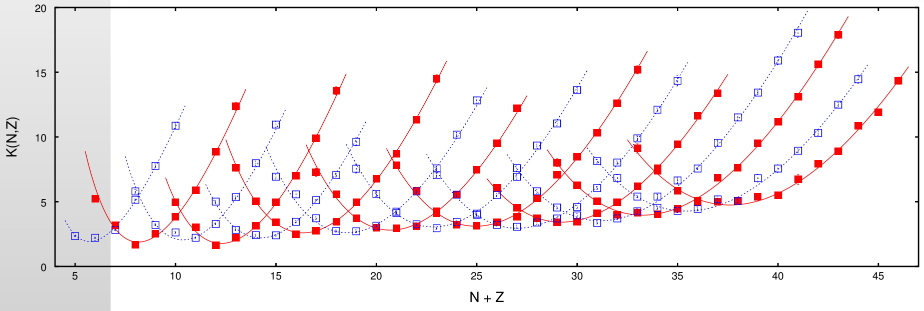
$\Rightarrow$ Fitting is not easy for the binding energies.

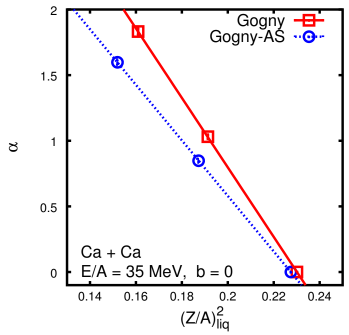
The AMD ground state masses are fitted by a liquid-drop formula:

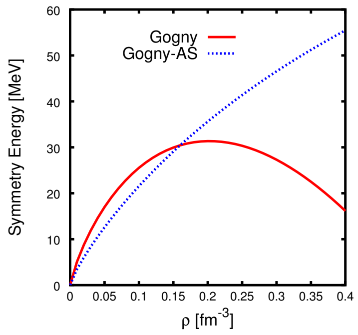
$$-\mathrm{BE}(N, Z) = a_{\mathrm{v}} A + a_{\mathrm{s}} A^{2/3} + a_{\mathrm{c}} Z^2 A^{-1/3} + [c_{\mathrm{v}} + c_{\mathrm{s}} A^{-1/3}] \frac{(N-Z)^2}{A} + \{\pm, 0\}\delta / A^{-1/2}$$

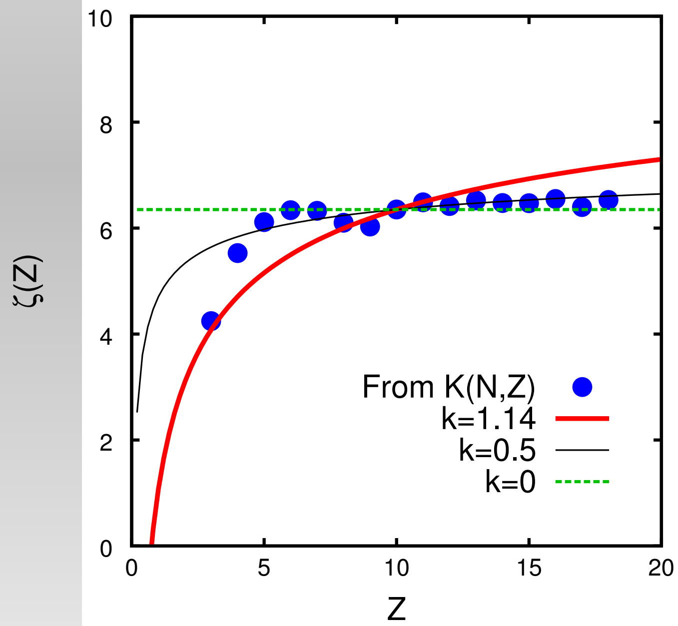
$a_{\mathrm{v}} = -14.6, \quad a_{\mathrm{s}} = 14.9, \quad c_{\mathrm{v}} = 30.9, \quad c_{\mathrm{s}} = -35.2, \quad a_{\mathrm{c}} = 0.65, \quad \delta = 10.1$ in MeV

Size dependence of symmetry energy





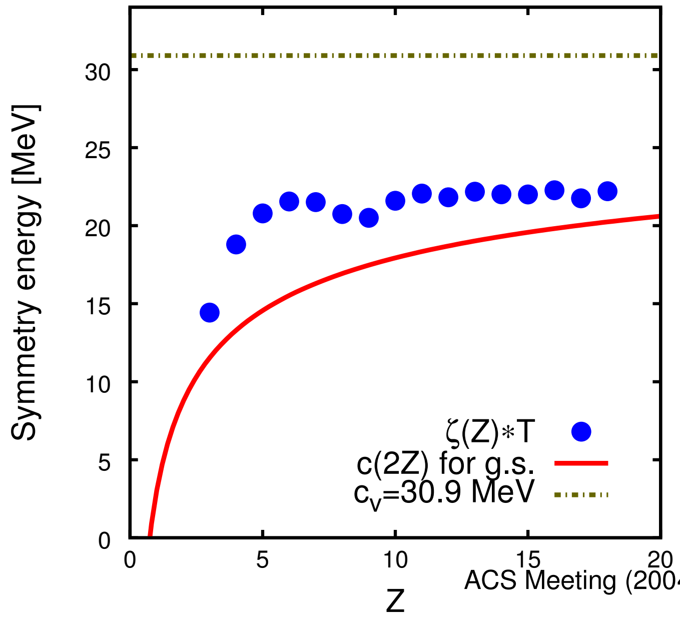




$$\zeta(Z) \propto 1 - k(2Z)^{-1/3}$$

Previous work $\Rightarrow T = 3.4$ MeV

$$\boxed{C(Z) = T\zeta(Z)}$$



Surface effect is very weak.

Direct information of the bulk term

Why is the surface effect missing?

- Because of finite T?

$$G_{NZ}(T,P) = f(A,T,P)A + c(A,T,P)\frac{(N-Z)^2}{A} + \text{Coulomb}$$

$$f(A,T,P) = a_\text{v}(T,P) + a_\text{s}(T,P)A^{-1/3}, \qquad c(A,T,P) = c_\text{v}(T,P) + c_\text{s}(T,P)A^{-1/3}$$

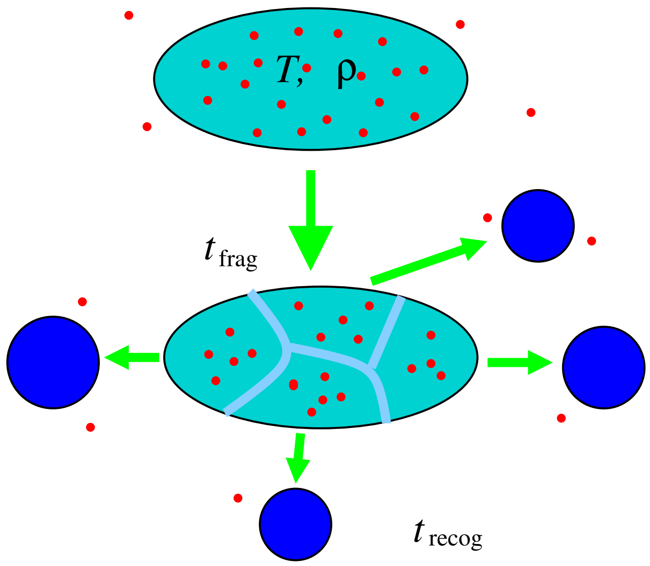
$$c_\text{s}(T) = \left[1 - \left(\frac{T}{T_\text{c}}\right)^2\right]^2 c_\text{s}(T=0) \sim 0.9 \times c_\text{s}(T=0) \qquad \text{for } T = 3.4 \text{ MeV}$$

Lattimer et al., NPA535, 331(1991)

- Because of the fragmentation dynamics?

$\rho \sim 0.08\ \text{fm}^{-3}$

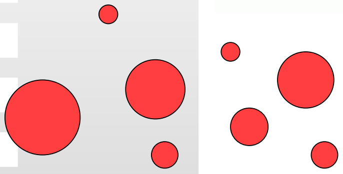
$T \sim 3.4$ MeV



Summary

- AMD with quantum branching
  - Quantum statistics
    $E^* \approx aT^2$, Liquid-gas phase transition
  - Fragmentation in dynamical nuclear collisions
- Nuclear collisions $\Rightarrow$ Isoscaling and symmetry energy
  - Symmetry energy at $\rho \sim 0.08\ \mathrm{fm}^{-3}$ for $T \sim 3.4$ MeV
  - Surface effect is weak for the symmetry energy.
    $\Rightarrow Y(N, Z)$ is directly related to bulk symmetry energy.

A relation under equilibrium



under given T, P, $(N_i^{\rm tot}, Z_i^{\rm tot})$

Yield of nucleus (N, Z)

$$Y_i(N,Z) = Y_{0i} \exp\Big[-\Big(G_{\rm nuc}(N,Z) - \mu_{ni}N - \mu_{pi}Z\Big)/T\Big]$$

$G_{\rm nuc}(N, Z; T, P)$: Free energy of a nucleus

Isoscaling

$$Y_2(N,Z)/Y_1(N,Z) \propto e^{\alpha N + \beta Z}$$

$$\alpha = (\mu_{n2} - \mu_{n1})/T$$

$$\beta = (\mu_{p2} - \mu_{p1})/T$$

For each Z, the most probable value of N: $\bar{N}_i(Z)$

$$\frac{\partial}{\partial N}[G_{\rm nuc}(N,Z) - \mu_{ni}N - \mu_{pi}Z]\Big|_{N=\bar{N}_i(Z)} = 0$$

$$\Downarrow \quad (i=1)-(i=2)$$

$$\frac{\alpha}{[Z/\bar{A}_1(Z)]^2 - [Z/\bar{A}_2(Z)]^2} = 4C(Z)/T \qquad C(Z) \approx c_{\rm v} + c_{\rm s}\bar{A}(Z)^{-1/3}$$

$$G_{\rm nuc}(N,Z) = a_{\rm v}A + a_{\rm s}A^{2/3} + a_{\rm c}Z^2A^{-1/3} + [c_{\rm v}A + c_{\rm s}A^{2/3}]\left(\frac{N-Z}{A}\right)^2 + \cdots$$

Langevin-like equation of motion

Equation of motion for the wave packet centroids

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\} + \Delta\mathbf{Z}_i(t) + \mu\Big(\mathbf{Z}_i,\ \mathcal{H} + \sum_m \beta_m Q_m\Big) + \text{NN-Collision}$$

If $\mathbf{Z}_i$ were canonical for simplicity,

$$\{\mathbf{Z}_i, \mathcal{H}\} = \frac{1}{i\hbar}\frac{\partial\mathcal{H}}{\partial\mathbf{Z}_i^*}$$

$$\overline{\Delta Z_{ia}(t)} = 0, \qquad \overline{\Delta Z_{ia}(t)\Delta Z_{jb}(t)} = D_{iab}(t)\delta_{ij}\delta(t-t')$$

$$(\mathbf{Z}_i, \mathcal{H}') = \frac{1}{\hbar}\frac{\partial\mathcal{H}'}{\partial\mathbf{Z}_i^*}$$

- Legendre parameters β_m are determined so that Q_m are not changed by the $(\mathbf{Z}_i, \mathcal{H}')$ term.
- μ is determined by the total energy conservation.