Hartree-Fock-Bogoliubov mass formulas and the equation of state of neutron-star matter

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F. Tondeur

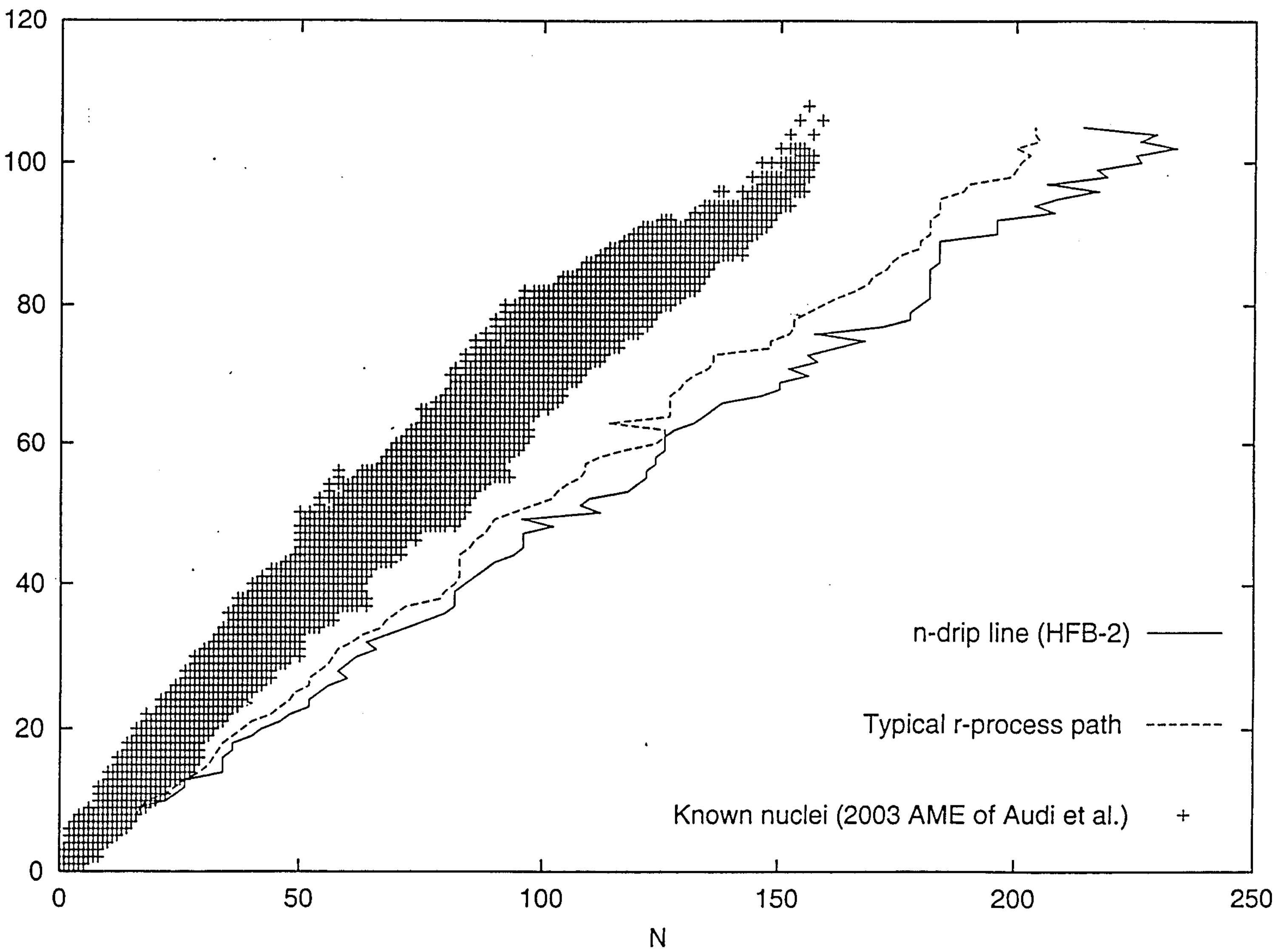
We will describe how nuclear mass formulas that we have developed for studying the r-process of nucleosynthesis can be extended to the calculation of the EOS of neutron-star matter.

Another way of saying this:

How nuclear masses constrain the EOS.

Why does one need a mass formula for the r-process?

Evolution of r-process depends on masses (among other quantities) of intermediate nuclei that are so neutron-rich that there is no hope of measuring them in the foreseeable future.



120
100
80
60
40
20
0
0
50
100
150
200
250
N
n-drip line (HFB-2)
Typical r-process path
Known nuclei (2003 AME of Audi et al.)

- Need reliable mass models to extrapolate from data out to the neutron drip line.

- For reliability mass model must not only fit data but be as *theoretically sound as possible.*

- We take this to mean: *as microscopic as possible.*

- Best that can be done so far is Hartree-Fock-Bogoliubov (HFB).

Forces of HFB Model

- Conventional Skyrme

$$\begin{aligned}
v_{ij} = & \; t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{ij}) \\
& + t_1(1+x_1P_\sigma)\frac{1}{2\hbar^2}\{p_{ij}^2\delta(\mathbf{r}_{ij}) + h.c.\} \\
& + t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\mathbf{p}_{ij}.\delta(\mathbf{r}_{ij})\mathbf{p}_{ij} \\
& + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\gamma\delta(\mathbf{r}_{ij}) \\
& + \frac{i}{\hbar^2}W_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j).\mathbf{p}_{ij}\times\delta(\mathbf{r}_{ij})\mathbf{p}_{ij}
\end{aligned}$$

- Pairing force

$$v_{pair}(\boldsymbol{r}_{ij}) = V_{\pi q}\left[1-\eta\left(\frac{\rho}{\rho_0}\right)^\alpha\right]\delta(\boldsymbol{r}_{ij})$$

cutoff: $$E_F - \varepsilon_\Lambda \leq \varepsilon_i \leq E_F + \varepsilon_\Lambda$$

- Generalized Wigner term for nuclei with $N \simeq Z$

$$E_W = V_W \exp\left\{-\lambda\left(\frac{N-Z}{A}\right)^2\right\} + V_W'|N-Z|\exp\left\{-\left(\frac{A}{A_0}\right)^2\right\}$$

Best fit so far is parameter set BSk8 (mass table HFB-8):

for 2149 data of Audi *et al.* (2003)

$$\sigma = 0.635 \text{ MeV}.$$

But how reliable are extrapolations to neutron-rich nuclei?

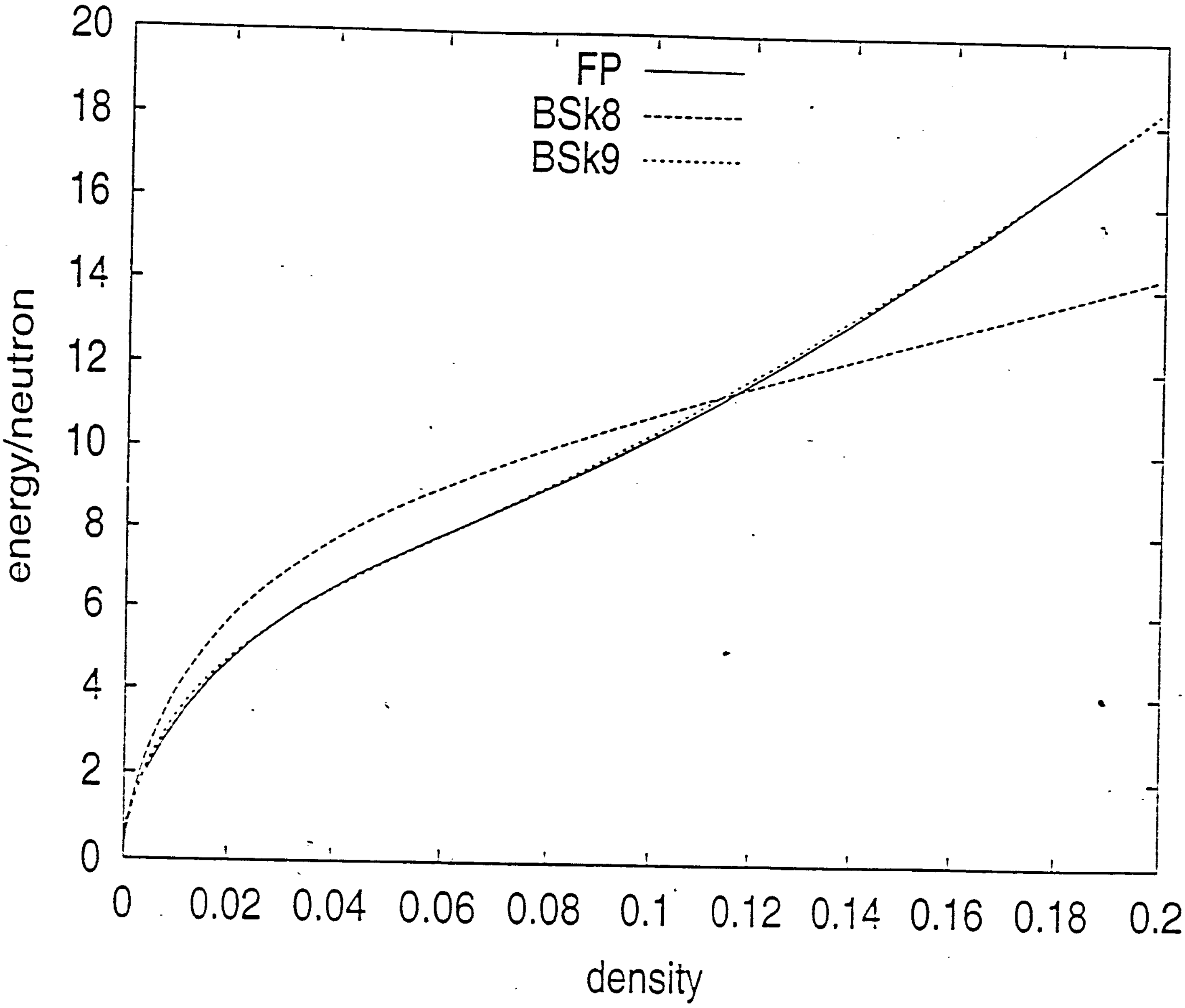
An obvious requirement: fit pure neutron matter

- this can be reliably calculated from realistic two- and three-nucleon forces, e.g., Friedman and Pandharipande (1981) - FP.

A force that fits the masses and satisfies this condition will be highly suitable not only for extrapolation to the neutron-rich nuclei involved in the r-process but also for extrapolating still further, beyond the r-process path, out to neutron-star matter.

Role of mass fit is to tie down the surface properties

- important for inhomogeneous crust.



FP
BSk8
BSk9
energy/neutron
density
20
18
16
14
12
10
8
6
4
2
0
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

• Situation is actually worse than we have shown here for BSk8:

neutron matter collapses at sub-nuclear densities if mass fit is completely optimized.

• Crucial factor here is *symmetry coefficient* a_{sym}:

energy/nucleon of nuclear matter

$$e = a_v + a_{sym}\left(\frac{N-Z}{A}\right)^2 + \cdots$$

In a completely free fit of masses $a_{sym} \simeq 27.5$ MeV.

We stopped the collapse in BSk8 by imposing $a_{sym} = 28.0$ MeV.

• This shows how to get a good fit to FP neutron-matter curve:

increase a_{sym} still further

Fit BSk9 achieved by imposing

$$a_{sym} = \mathbf{30.0\ MeV}$$

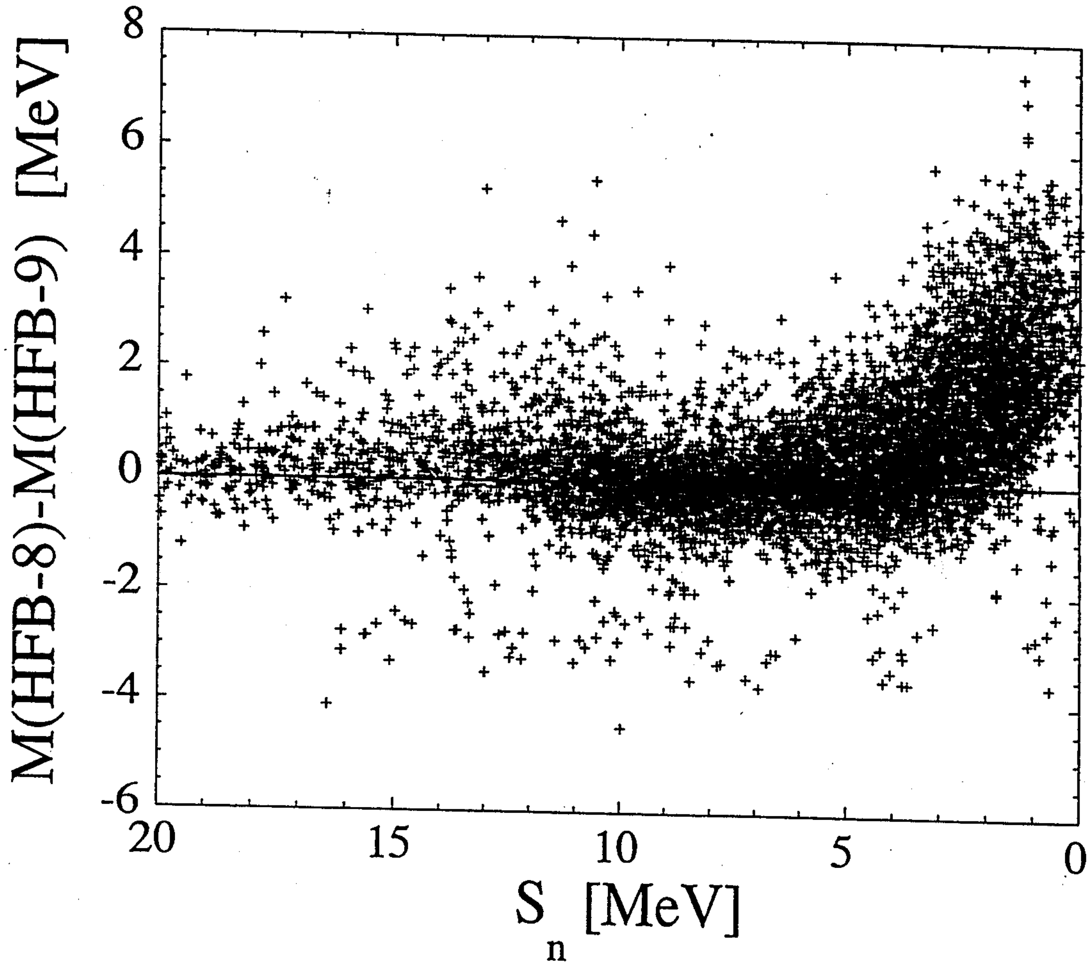
(optimal fit BSk8 has $a_{sym} = 28.0$ MeV).

Rms (σ) and mean (ϵ) deviations between data and predictions for parameter sets BSk8 and BSk9. The first pair of lines refers to absolute masses M (2149 measured values), the second pair to neutron separation energies S_n (1988 measured values), the third pair to beta-decay energies Q_β (1868 measured values), and the last pair to charge radii (523 measured values).

	BSk8	BSk9
$\sigma(M)$ [MeV]	0.635	0.733
$\epsilon(M)$ [MeV]	0.009	0.025
$\sigma(S_n)$ [MeV]	0.564	0.589
$\epsilon(S_n)$ [MeV]	0.013	0.007
$\sigma(Q_\beta)$ [MeV]	0.704	0.721
$\epsilon(Q_\beta)$ [MeV]	-0.027	-0.009
$\sigma(r_c)$ [fm]	0.0250	0.0242
$\epsilon(r_c)$ [fm]	0.0047	0.0028

So imposing neutron-matter constraint has led to a slight deterioration in the quality of the mass fit. However, for r-process what counts is not the absolute masses but rather the neutron-separation energy S_n and the beta-decay energy Q_β. In this crucial respect virtually no deterioration.

Do the two mass models give similar extrapolations out to the neutron drip line?



Again, what counts for the r-process is S_n and Q_β.

Rms and mean differences (in MeV) between the HFB-8 and HFB-9 models for the S_n and Q_β of 1724 neutron-rich nuclei. (Mean differences correspond to HFB-8 - HFB-9.)

	S_n	Q_β
σ	0.553	0.769
ϵ	-0.087	0.263

- Differences between two models for extrapolation comparable to errors with which each model fits the data.
- Thus imposing neutron-matter condition has no practical effect on r-process predictions: BSk8 and BSk9 are equivalent.

But BSk9 more appropriate for neutron-star EOS.

$$(\rho \leq \rho_0)$$

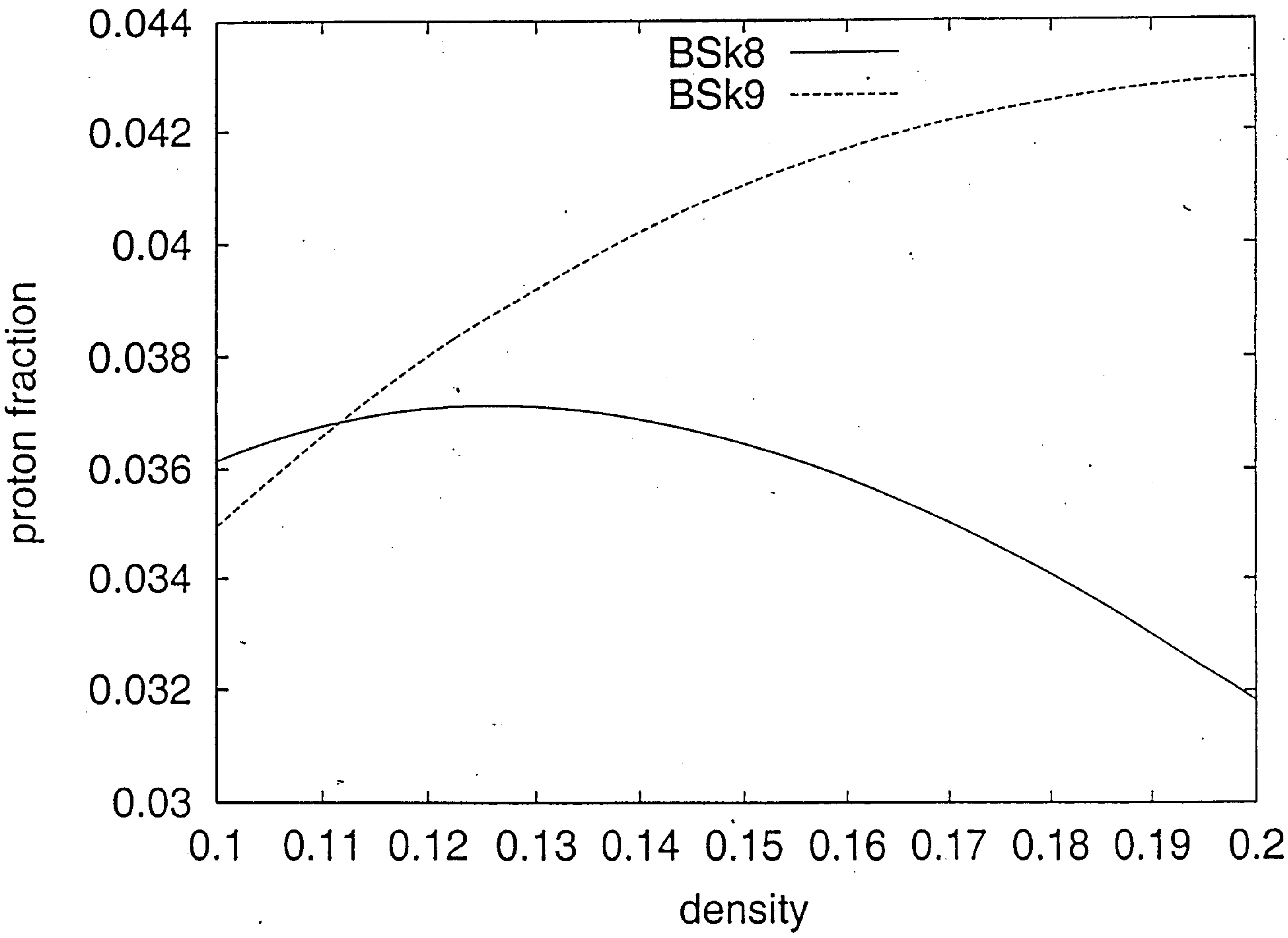
$$M_s = 0.80M$$

Implications for neutron-star EOS

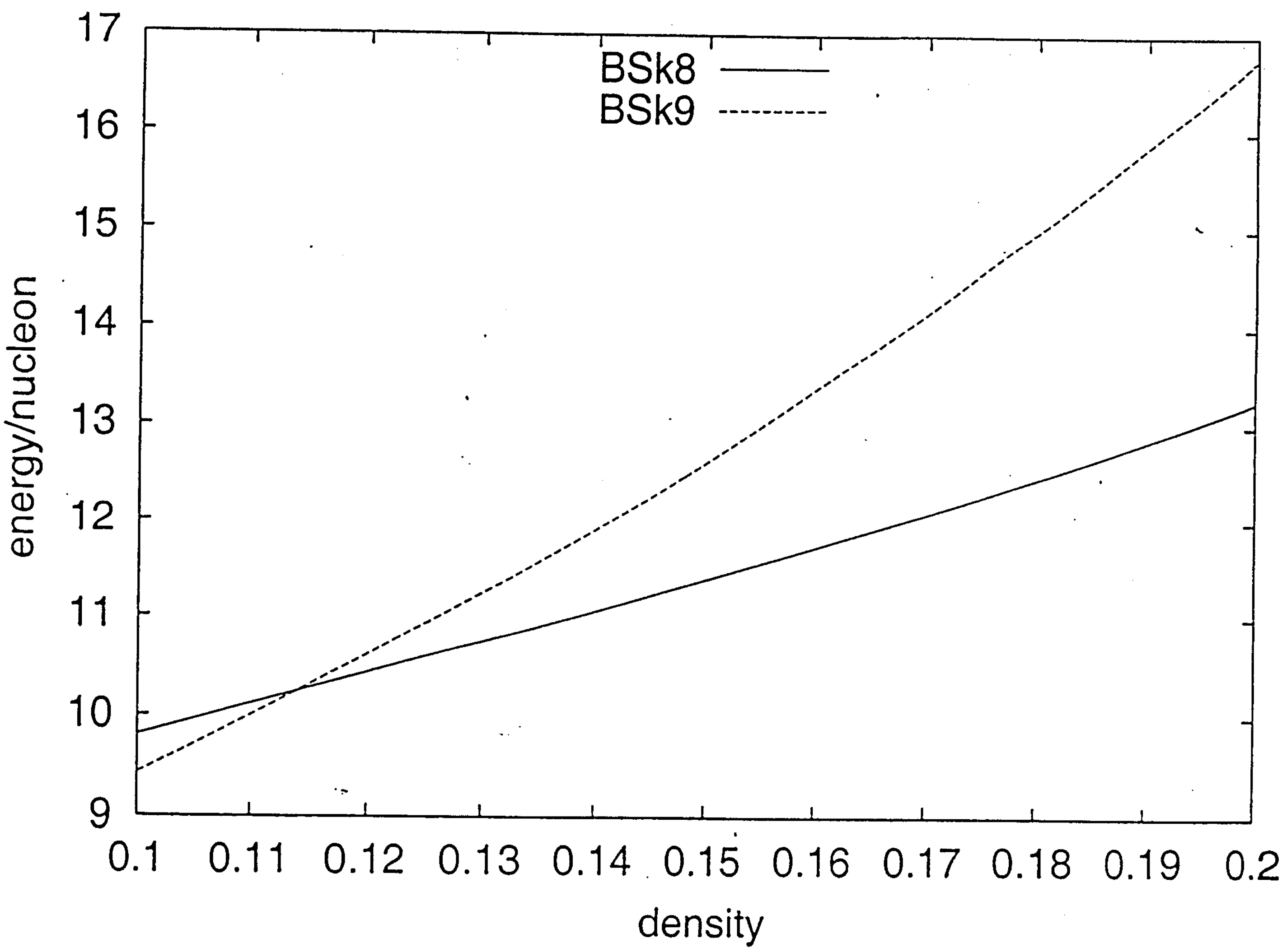
• While choice between BSk8 and BSk9 is of no consequence for the r-process, imposing neutron-matter constraint will obviously have implications for homogeneous core of neutron star ($\rho \geq 0.7\rho_0 \simeq 0.1$ fm^{-3}).

However, note that core is not pure neutron matter, but rather is β-equilibrated.

• Difference between the two forces is actually greater than for pure homogeneous neutron matter. In particular, increase of a_{sym} between BSk8 and BSk9 leads to a considerable increase in number of proton-electron pairs.



BSk8
BSk9
0.044
0.042
0.04
0.038
0.036
0.034
0.032
0.03
proton fraction
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
density



BSk8
BSk9
17
16
15
14
13
12
11
10
9
energy/nucleon
0.1
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
0.19
0.2
density

• Outer Crust

sub-drip

distribution depends only on differences between neighbouring nuclei

∴ no essential difference between BSk8 and BSk9

• Inner Crust

n-p clusters in neutron gas
n bubbles in n-p liquid

Calculations:

Wigner-Seitz model; 4th-order Extended Thomas-Fermi

Onsi et al. Phys. Rev. C **55** 3139 (1997)

Optimal composition of Wigner-Seitz cell: $\bar{\rho} = 0.02\ \mathrm{fm}^{-3}, T = 0$

	BSk8 (28 MeV)	BSk9 (30MeV)
Z	51	38
N	1147	1192

CONCLUSIONS

- EOS for neutron-star matter ($\rho \leq \rho_0$) highly sensitive to theoretical neutron-matter curve.

- Skyrme force BSk9 well adapted to EOS for neutron-star matter (insofar as FP calculation is valid).

- Fit of this force to masses is sub-optimal, but this is of no consequence for r-process.

- Masses are nevertheless well fitted, so surface properties well described - crucial for the inhomogeneous crust.

HOWEVER

• It is disconcerting that optimal fit of force to masses is not consistent with FP neutron-matter curve. However, latter is just theory, and much more serious is the fact that mass fit is not consistent with the observation that neutron stars are stable.

• A possible further contradiction with experiment: neutron-skin thickness

$$\theta_n = R_n - R_p$$

Some suggestion that a_{sym} might be even higher than 30 MeV, but mass fit would be very bad, and also neutron-matter curve would be stiffer than given by FP, for conventional Skyrme force.

Results of forthcoming Jefferson Lab. measurement based on parity-violating electron-nucleus scattering will be crucial.

FUTURE

These problems might reflect inadequacy of conventional Skyrme form.

A POSSIBLE SOLUTION:

add a t_4 term, $\rho^\gamma p^2 \delta(\mathbf{r})$