Nuclear Symmetry Energy

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Volume & Surface Symmetry Energy

Bethe-Weizsäcker formula: $\propto A$

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + a_A\, \frac{(N-Z)^2}{A} + \delta$$

No surface symmetry energy...

Surface energy: $E_S = a_S\, A^{2/3} = \frac{a_S}{4\pi\, r_0^2}\, 4\pi\, r_0^2\, A^{2/3} = \frac{a_S}{4\pi\, r_0^2}\, \mathcal{S}$

$$\frac{E_S}{\mathcal{S}} = \sigma = \frac{a_S}{4\pi\, r_0^2} \quad \text{(tension – work per area)}$$

$\rightarrow$ As nucleons at surface less bound, creating surface requires work.

Symmetry energy reduces the binding, so, as n-p asymmetry increases, the work to create surface should drop (you cannot subtract same thing twice from volume!)

$$\sigma = \frac{\partial\, E_S}{\partial\, \mathcal{S}} \quad \searrow$$

(in the general definition of tension)

σ as intensive should depend on an intensive quantity characterizing neutron-proton (n-p) asymmetry $\rightarrow \mu_A$

$$\mu_A = \frac{\partial\, E}{\partial\,(N - Z)}$$

Since tension should drop no matter whether more neutrons or protons $\rightarrow$ quadratic in chemical potential

$$\sigma = \sigma_0 - \gamma\, \mu_A^2$$

Surface energy E_S must then also depend on $\mu_A \ldots$

Thermodynamic consistency then requires:
Surface must contain n-p excess!

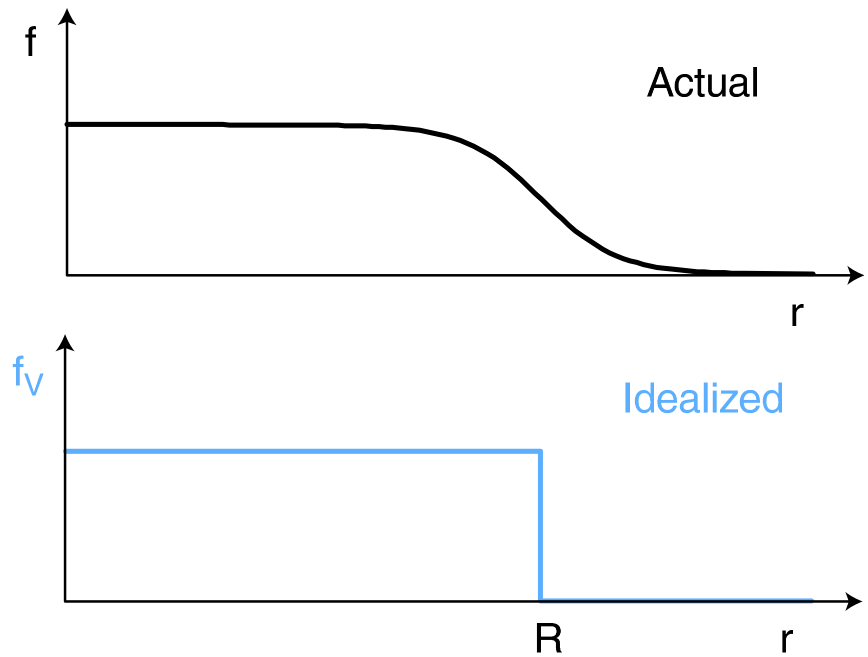
$$(N_S - Z_S) \propto \mu_A$$

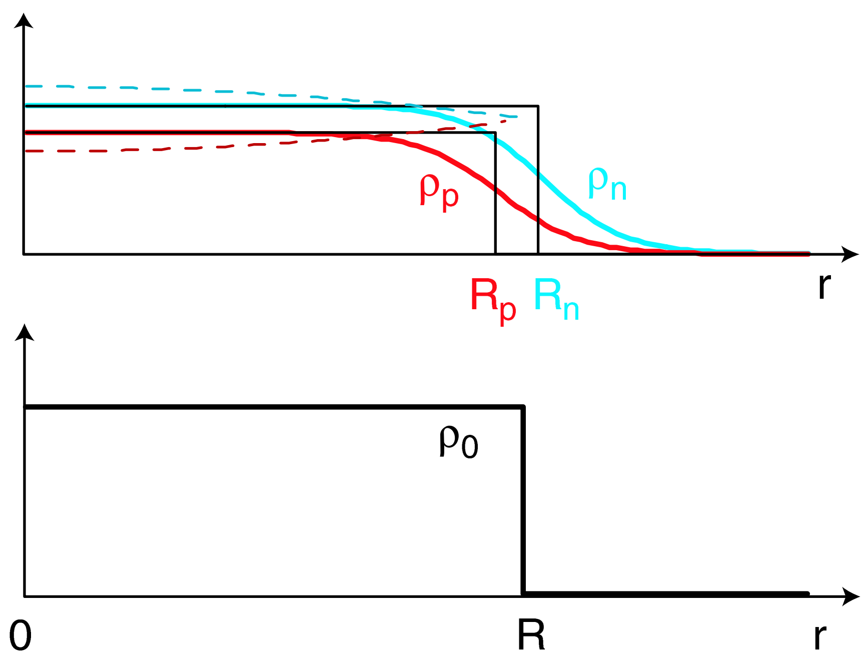
Surface energy must be quadratic in the excess and/or μ_A.
?How can surface hold particles?!

Gibbs definition for surface quantities - difference between actual and idealized where volume contribution only: $F_S = F - F_V$

result depends on surface position R

$\rightarrow A_S = A - A_V = 0$





2-component system: surfaces for neutrons and protons may be displaced.

Net surface position set demanding: $A_S = 0$.

However, $N_S - Z_S \neq 0$!

With thermodynamic consistency resolved, $\sigma = \sigma_0 - \gamma\, \mu_A^2$ yields for surface energy

$$\begin{aligned} E_S &= \sigma_0\, \mathcal{S} + \gamma\, \mu_A^2\, \mathcal{S} = E_S^0 + \frac{1}{4\gamma}\, \frac{(N_S - Z_S)^2}{\mathcal{S}} \\ &= E_S^0 + a_A^S\, \frac{(N_S - Z_S)^2}{A^{2/3}} \qquad \text{(surface capacitor)} \end{aligned}$$

Volume similarly: $E_V = E_V^0 + a_A^V\, \dfrac{(N_V - Z_V)^2}{A}$ (volume capacitor)

Net Energy & Asymmetry: $E = E_S + E_V\,, \quad N - Z = N_S - Z_S + N_V - Z_V$

Minimization of E with respect to the asymmetry partition: analogous to coupled capacitors, $q_X = N_X - Z_X\,,$ $E_X = E_X^0 + q_X^2/2C_X$, with the result

$$E = E^0 + \frac{q^2}{2C} = E^0 + \frac{(N-Z)^2}{\frac{A}{a_A^V} + \frac{A^{2/3}}{a_A^S}}$$

volume capacitance surface capacitance

Modified Binding Formula

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \frac{a_A^V}{1 + A^{-1/3}\, a_A^V / a_A^S}\, \frac{(N-Z)^2}{A}$$

$a_A(A)$

Regular formula for $a_A^V/a_A^S = 0$ - i.e. surface not accepting the asymmetry excess ($a_A^S = \infty$) - or for $A \rightarrow \infty$.
Modified formula: weakening of the symmetry term for low A.

Whether one can replace $a_A(A)$ by a_A^V for large A depends on the ratio a_A^V/a_A^S.

The ratio may be determined from surface asymmetry excess, as surface-to-volume asymmetry ratio:

$$\frac{N_S - Z_S}{N_V - Z_V} = \frac{C_S}{C_V} = \frac{A^{2/3}/a_A^S}{A/a_A^V} = A^{-1/3}\, a_A^V/a_A^S$$

Asymmetry Skins

Measurements of n-p skin sizes difficult: two different probes required.

E.g. electrons + protons, $\pi^+ + \pi^-$, protons + neutrons

Issues:

1. Data expressed in terms of difference of n and p rms radii.

Conversion straightforward, if diffuseness similar for n and p.

2. For heavy nuclei, Coulomb competes with symmetry energy, pushing protons out.

$\Rightarrow$ minimize sum of 3 energies w/respect to asymmetry:

$$E = E_V + E_S + E_C \qquad E_C = \frac{e^2}{4\pi\epsilon_0} \frac{1}{R} \left(\frac{3}{5} Z_V^2 + Z_V Z_S + \frac{1}{2} Z_S^2 \right)$$

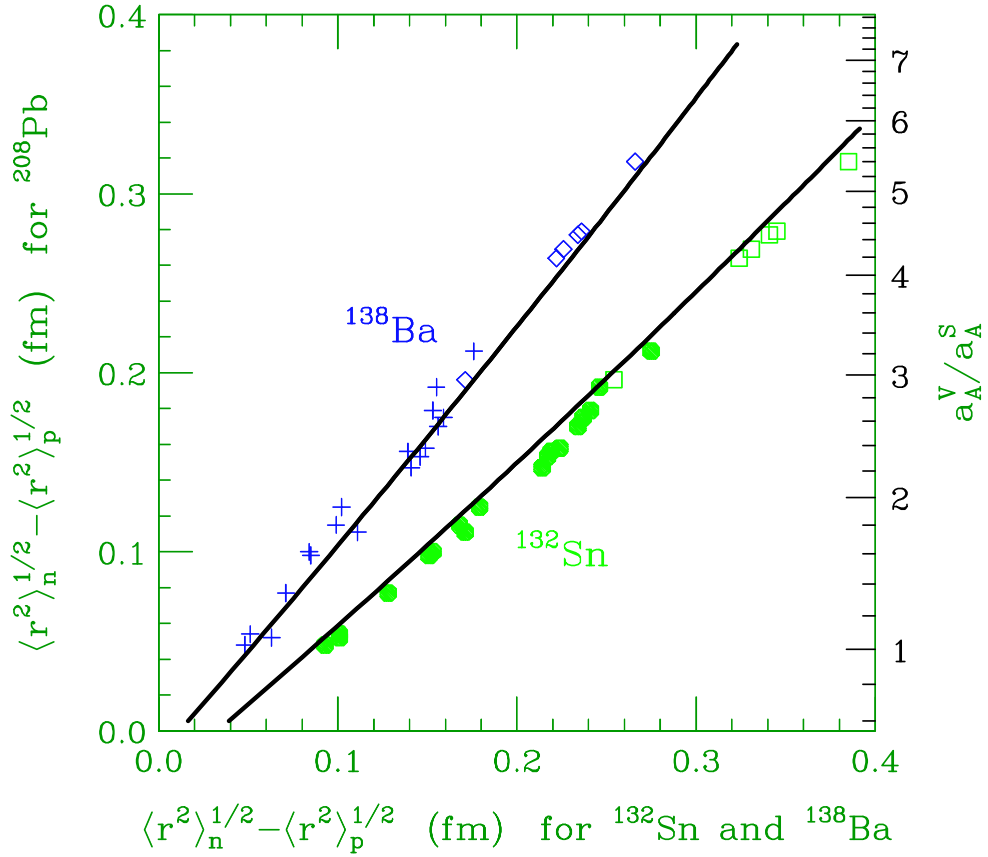
From the modified minimization, analytic difference of rms radii:

$$\frac{\langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}}{\langle r^2 \rangle^{1/2}} = \frac{A}{6NZ} \frac{N-Z}{1 + A^{1/3} a_A^S / a_A^V} - \frac{a_C}{168 a_A^V} \frac{A^{5/3}}{N} \frac{\frac{10}{3} + A^{1/3} a_A^S / a_A^V}{1 + A^{1/3} a_A^S / a_A^V}$$

symmetry energy only — Coulomb correction

Test of the macroscopic formula

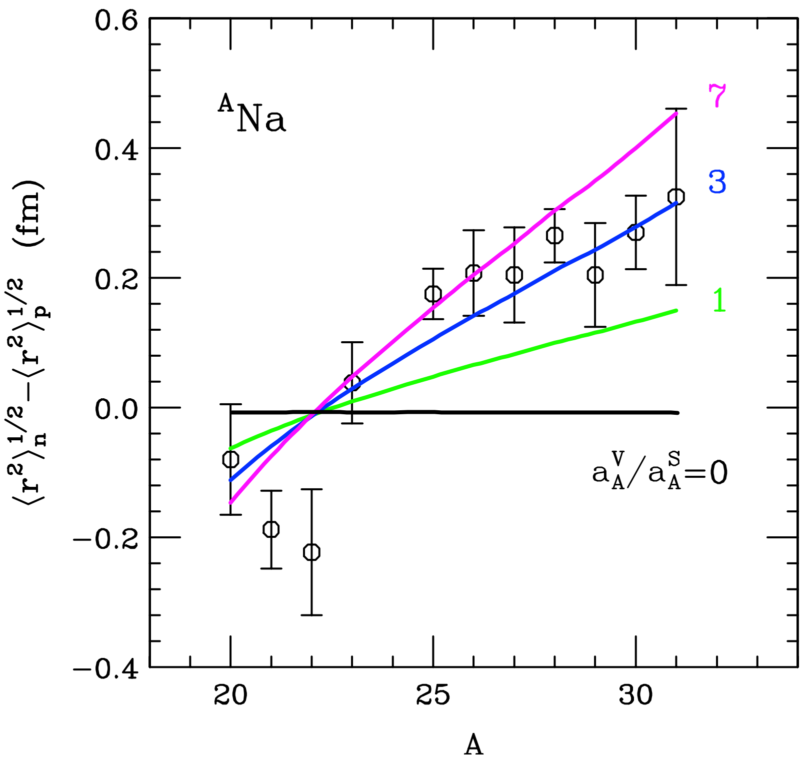
Comparison of the formula (lines) with a multitude of nonrelativistic and relativistic mean-field calculations by Typel and Brown PRC64(01)027302 (symbols)

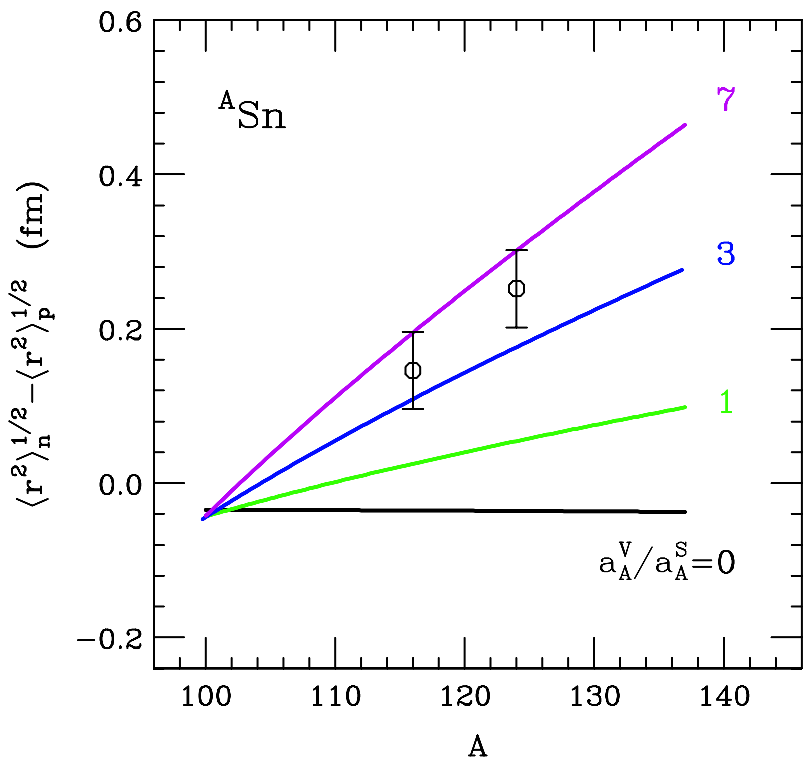


Accuracy, in reproducing microscopic theory, of ∼0.01 fm ?!

⇒ next data

Systematic of n-p skin sizes for different Na isotopes by Suzuki et al., PRL75(95)3241 + other data



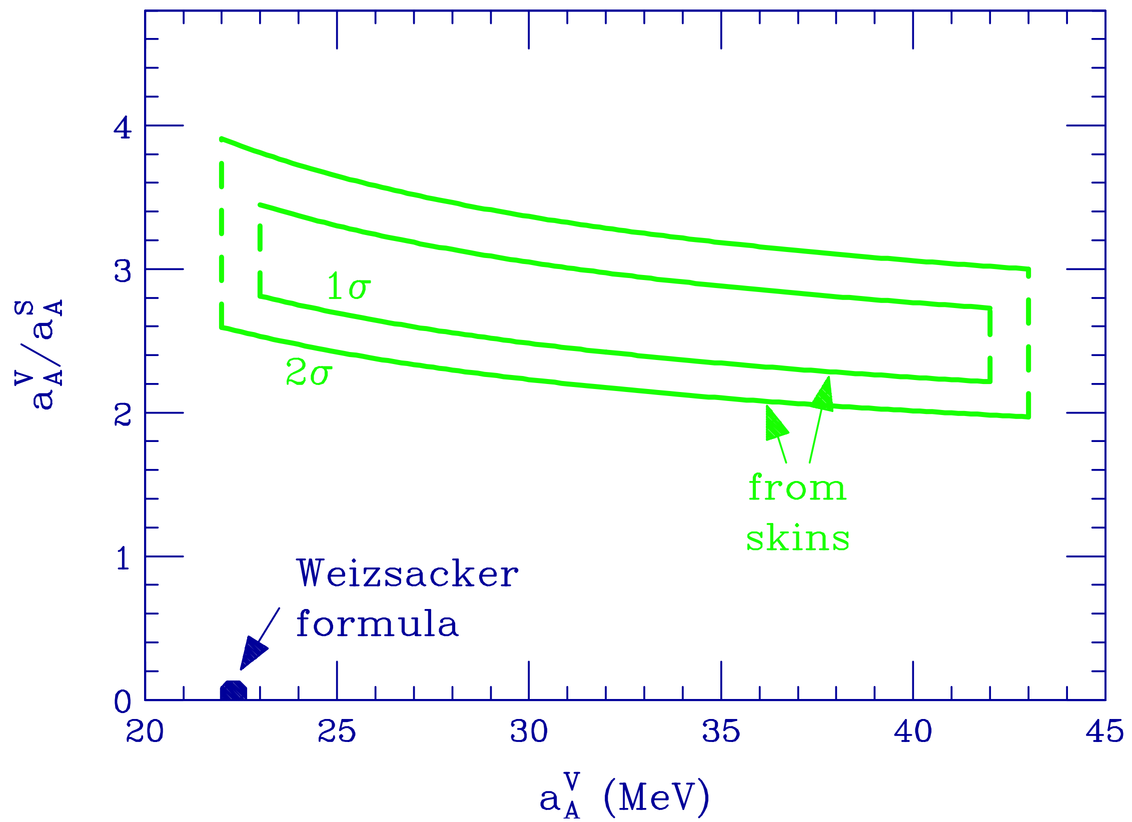


difference between the rms n and p radii vs A

$$a_A^V/a_A^S \sim 3$$

Global fit to skin data

1-σ & 2-σ limits on a_A^V/a_A^S as a function of a_A^V:



dependence on a_A^V due to Coulomb

As $A^{-1/3}\, a_A^V/a_A^S$ never small, symmetry term not expandable;
Bethe-Weizsäcker not acceptable at the macroscopic level.

Charge Invariance

Conclusions on details in the symmetry term, following mass-formula fits, are, unfortunately, interrelated with conclusions on details in other terms: isospin-dependent Coulomb, Wigner & pairing + isospin-independent, due to $(N-Z)/A$ - A correlations along the line of stability (PD NPA727(03)233)!

Best would be to study the symmetry term in isolation from the rest of mass formula! Absurd?!

Charge invariance comes to rescue: nuclear states characterized by different isospin values (T,T_z), $T_z = (Z-N)/2$. Nuclear energy scalar in isospin space:

$$E_A = a_A(A)\,\frac{(N-Z)^2}{A} = 4\,a_A(A)\,\frac{T_z^2}{A}$$

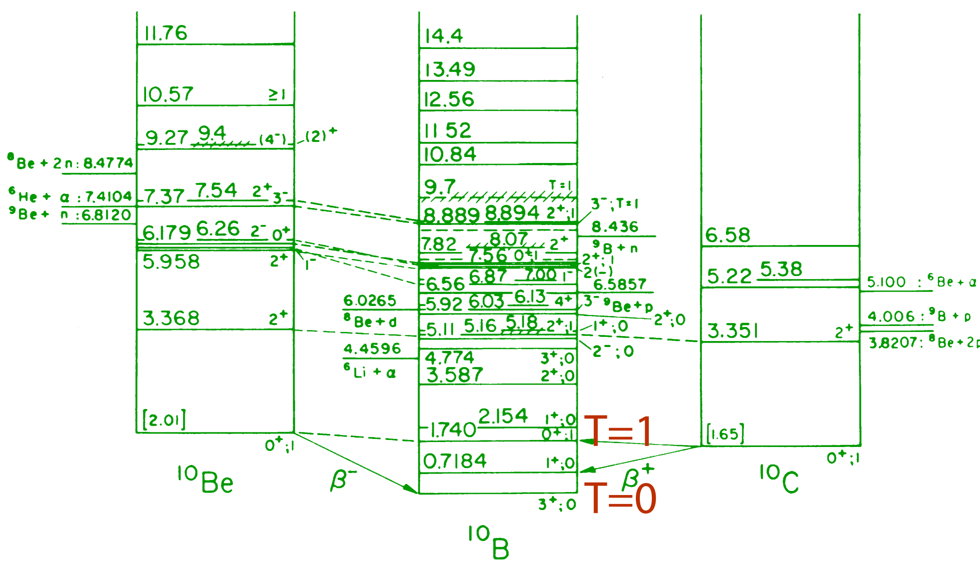
$$\rightarrow\; E_A = 4\,a_A(A)\,\frac{T^2}{A} = 4\,a_A(A)\,\frac{T(T+1)}{A}$$

$$\rightarrow\; E_A = 4\, a_A(A)\, \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T. Pairing term, in the generalization, contributes depending on evenness of T.

?Lowest state of a given T: isobaric analogue state (IAS) of some neighboring nucleus ground-state.



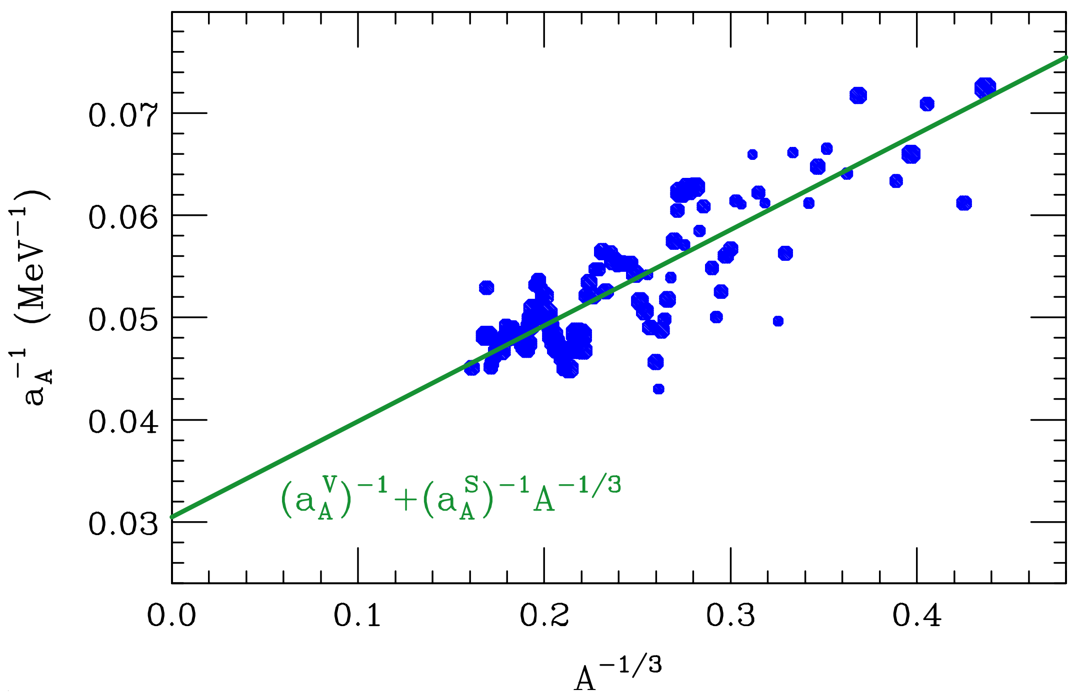
Study of changes in the symmetry term possible within one nucleus

In the same nucleus, when pairing drops out:

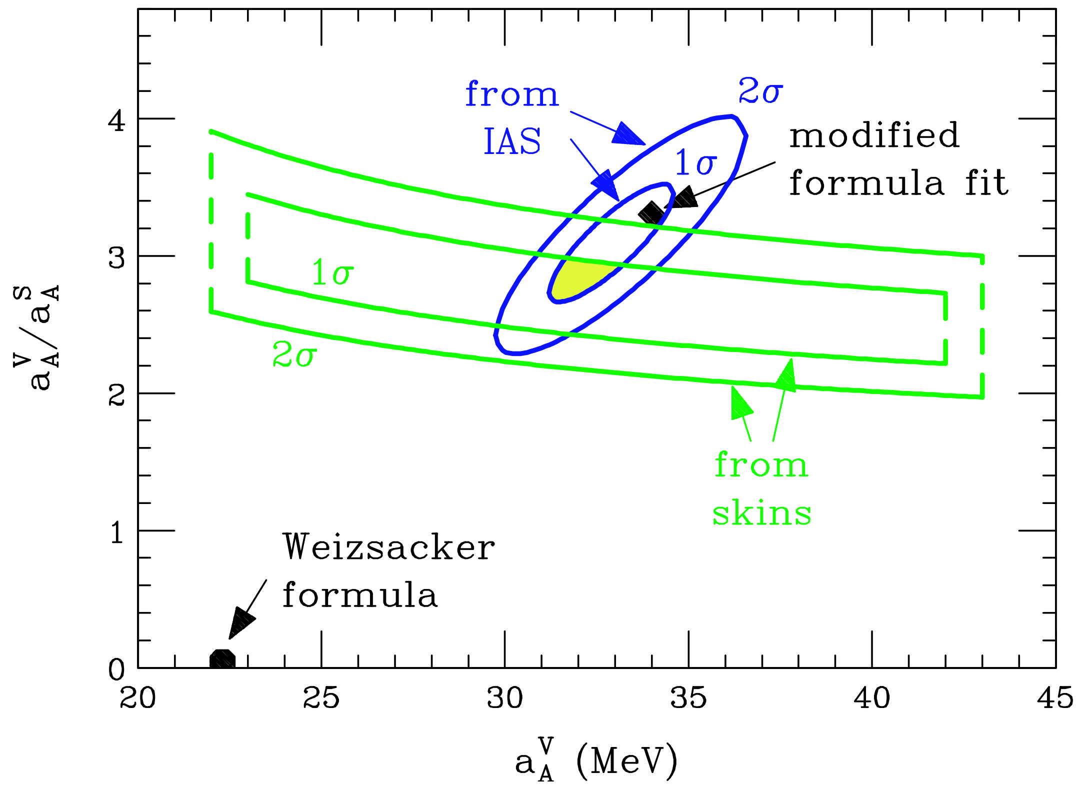
$$E_2(T_2) - E_1(T_1) = \frac{4\,a_A}{A}\{T_2(T_2+1) - T_1(T_1+1)\}$$

$$a_A^{-1}(A) = \frac{4\,\Delta T^2}{A\,\Delta E} \qquad \stackrel{?}{=} (a_A^V)^{-1} + (a_A^S)^{-1}\,A^{-1/3}$$

IAS analysis with largest available energy differences used:



Fit combination



Conclusions: $31\,\mathrm{MeV} \lesssim a_A^V \lesssim 33\,\mathrm{MeV}$, $2.7 \lesssim a_A^V/a_A^S \lesssim 3.0$

next: Symmetry-coeff ratio constraints low-ρ dependence of E_A.

Microscopic Background

In Thomas-Fermi approx with $E = E_0 + \int d^3r\, \rho\, E_A(\rho)\, \left(\frac{\rho_n - \rho_p}{\rho}\right)^2$, where E_A - symmetry energy ($E_A(\rho_0) = a_A^V$), Gibbs prescription for semiinfinite matter yields:

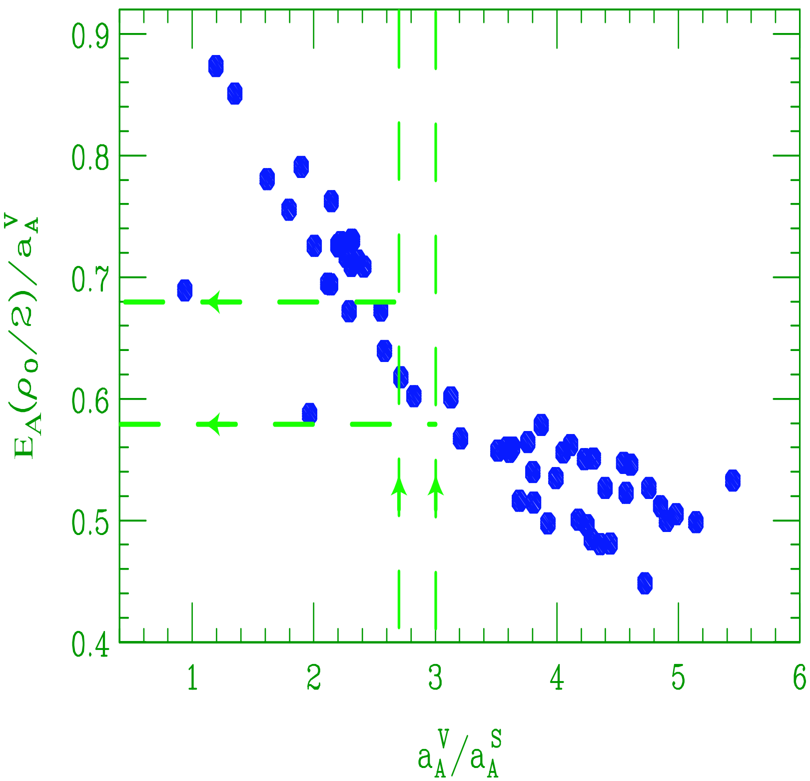
$$\frac{a_A^V}{a_A^S} = \frac{3}{r_0} \int dr\, \frac{\rho(r)}{\rho_0} \left[\frac{E_A(\rho_0)}{E_A(\rho(r))} - 1\right]$$

$\Rightarrow a_A^V/a_A^S$ probes shape of $E_A(\rho)$!

For $E_A(\rho) \equiv a_A^V$, $a_A^V/a_A^S = 0$! Surface capacitance emerges, because E_A drops with ρ.

From $2.7 \lesssim a_A^V/a_A^s \lesssim 3.0$ for mean-field structure calcs (Furnstahl, NPA706(02)85 - symbols), we deduce symmetry energy reduction at half the normal density:

$$0.58 \lesssim E_A(\rho_0/2)/a_A^V \lesssim 0.68$$



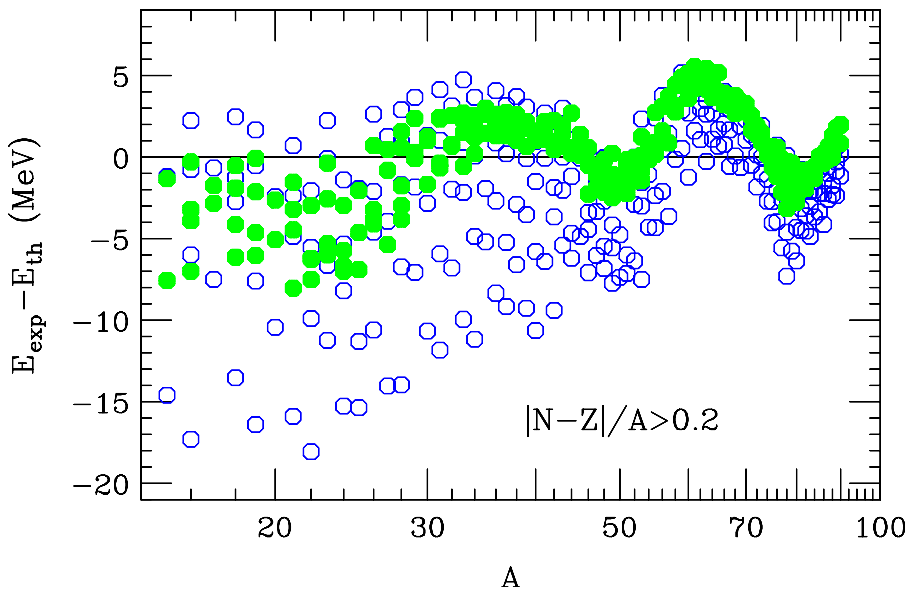
Consequences for neutron stars

Pressure estimate from $E_A(\rho)$ + Lattimer-Prakash scaling, $R\,P^{1/4} \simeq$ const, yields $11.7\,\mathrm{km} \lesssim R \lesssim 13.7\,\mathrm{km}$ for an $1.4\,M_\odot$ star.

Density dependence appears too weak for the direct Urca cooling.

Mass formula performance

Fit residuals for light asymmetric nuclei, when either following the Bethe-Weizsäcker formula (open symbols) or the modified formula with $a_A^V/a_A^S = 2.8$ imposed (closed), i.e. the same parameter No.



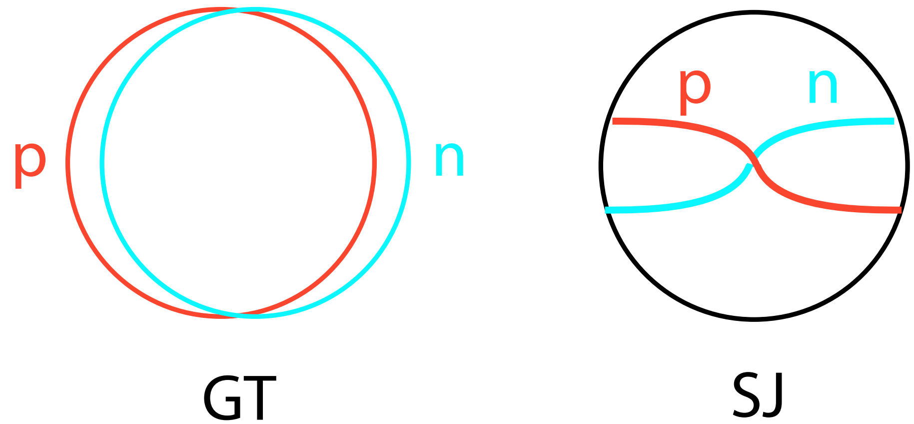
Conclusions

- Bringing macroscopic consistency introduces surface symmetry energy into the binding formula, with the volume and surface symmetry energies combining as energies of coupled capacitors.
- Formula scope gets extended; it predicts surface asymmetry skins and weakening of the symmetry term for light nuclei.
- Skins restrict ratio of symmetry coefficients; charge invariance allows to study symmetry term without leaving a nucleus.
- Skin/IAS fits: $31\,\text{MeV} \lesssim a_A^V \lesssim 33\,\text{MeV}$ and $2.7 \lesssim a_A^V/a_A^S \lesssim 3.0$
- Surface symmetry energy emerges due to a weakening of the symmetry energy with density. a_A^V/a_A^S ratio places E_A within $(0.58 - 0.68)a_A^V$ at $\rho_0/2$. Consequences for neutron stars follow.
- Description of giant dipole resonances improves with an inclusion of the surface symmetry energy. The resonances are more of a GT type for light nuclei and of an SJ type for heavy.

Asymmetry Oscillations

Movement of neutrons vs protons - giant resonances visible in excitation cross sections

Two classical models of the simplest giant dipole resonance (GDR)



Goldhaber-Teller (GT): n & p distributions oscillate against each other as rigid entities:

$$E_{GDR} = \hbar\Omega \propto \sqrt{A^{2/3}/A} = A^{-1/6}$$

Steinwedel-Jensen (SJ): Standing wave of n-p in the interior with vanishing flux at the surface

$$E_{GDR} = \hbar c_a/\lambda \propto A^{-1/3}$$

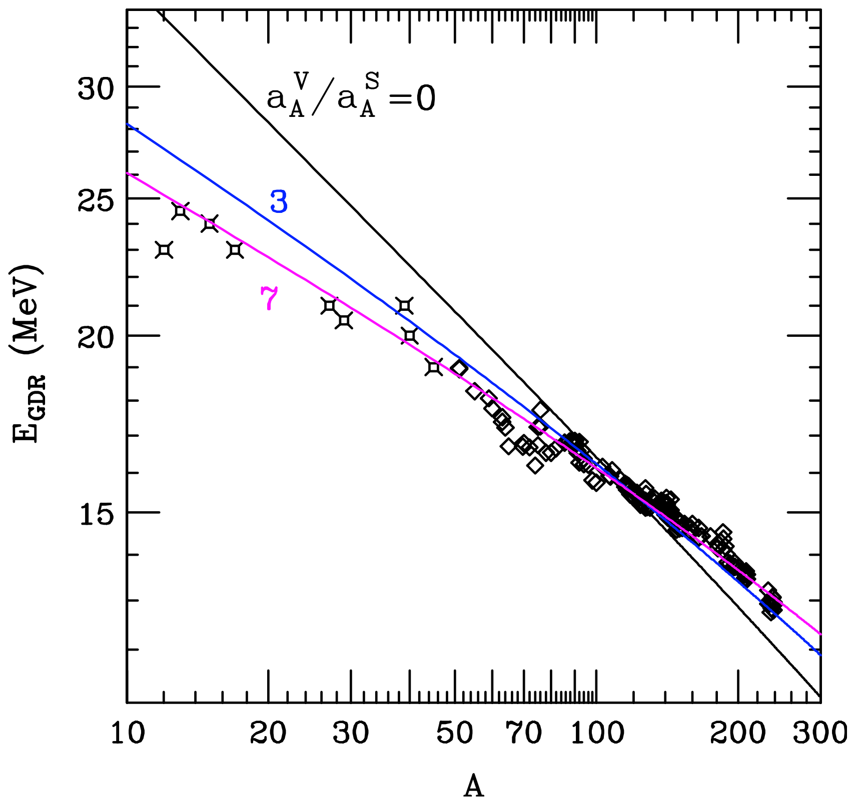
GT model: $a_a^V \to \infty$ SJ model: $a_a^S \to \infty$

Realistic model: SJ but asymmetry flux may flow in and out of the surface… The boundary condition produces:

$$qR\, j_1(qR) = \frac{3\, a_a^S\, A^{1/3}}{a_a^V}\, j_1'(qR)$$

j_1 - spherical Bessel function, typical for waves when spherical symmetry; q - wavenumber, $E_{GDR} = \hbar\, c_a\, q$

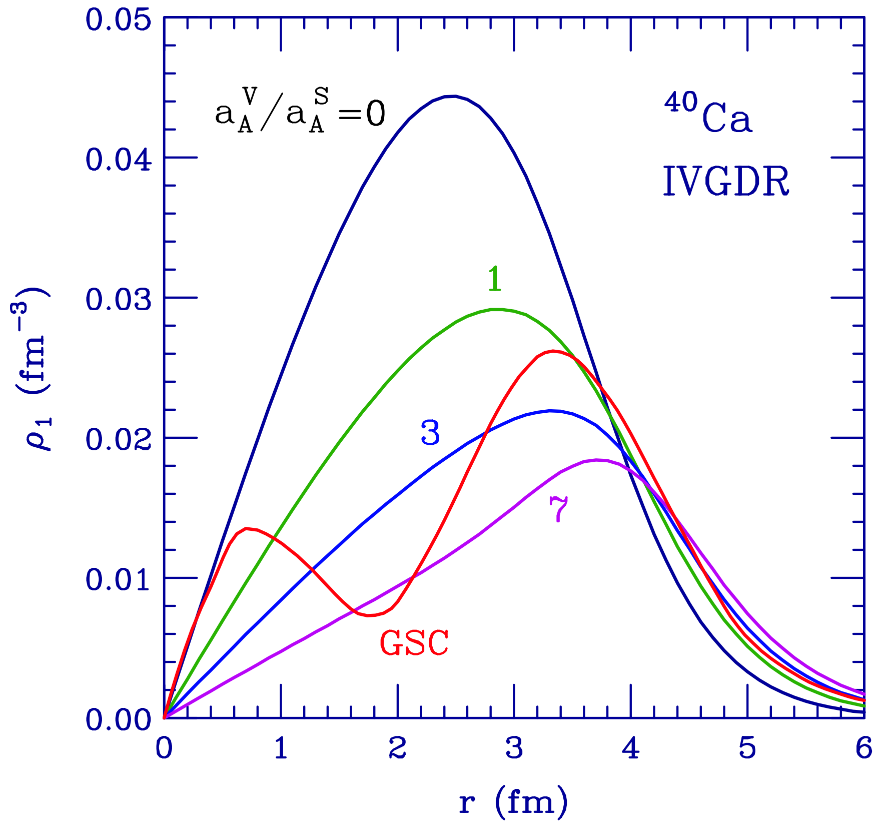
As $a_a^S\, A^{1/3}/a_a^V$ changes, the condition changes between that of open and close pipe and the resonance evolves between GT and SJ



Local Amplitude ≡ Transition Density

$$\rho_1(r) = \frac{D_V}{\rho_0}\, j_1(qr) \left[\rho(r) - \frac{a_a^V}{3\, a_a^S\, A^{1/3}}\, r\, \frac{d\rho}{dr}\right]$$

Compared to microscopic calculations (Khamerdzhiev et al., NPA624(97)328) GSC, including 2p-2h excitations and ground-state correlations:



Different Mass Formulas

Liquid droplet model (Myers & Swiatecki '69)

$$E = \left(-a_1 + J\,\overline{\delta}^2 - \frac{1}{2}\,K\,\overline{\epsilon}^2 + \frac{1}{2}\,M\,\overline{\delta}^4\right) A + \left(a_2 + Q\,\tau^2 + a_3\,A^{-1/3}\right) A^{2/3} + c_1\,\frac{Z^2}{A^{1/3}}\left(1 + \frac{1}{2}\,\tau\,A^{-1/3}\right) - c_2\,Z^2\,A^{1/3} - c_3\,\frac{Z^2}{A} - c_4\,\frac{Z^{4/3}}{A^{1/3}}$$

where

$$\overline{\epsilon} = \frac{1}{K}\left(-2a_2\,A^{-1/3} + L\,\overline{\delta}^2 + c_1\,\frac{Z^2}{A^{4/3}}\right), \qquad \tau = \frac{3}{2}\,\frac{J}{Q}\,(\overline{\delta} + \overline{\delta}_s)$$

$$\overline{\delta} = \frac{I + \frac{3}{8}\,\frac{c_1}{Q}\,\frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4}\,\frac{J}{Q}\,A^{-1/3}}, \qquad \overline{\delta}_s = -\frac{c_1}{12J}\,\frac{Z}{A^{1/3}}, \qquad I = \frac{N-Z}{N+Z}$$

$Q = H/(1 - \frac{2}{3}\,P/J)$. Expansion in asymmetry yields results consistent with current, but approach more complex…

The current formula:

$$E = -a_V\, A + a_S\, A^{2/3} + a_C\, \frac{Z^2}{A^{1/3}} + \alpha\, \frac{(N-Z)^2}{A}\, \frac{1}{1 + \frac{\alpha}{\beta}\, A^{-1/3}}$$

Liquid drop model [LDM] (Myers & Swiatecki '66)

$$\begin{aligned} E \;\; = \;\; & -a_V\, \left(1 - \kappa_V\, I^2\right)\, A + a_S\, \left(1 - \kappa_S\, I^2\right)\, A^{2/3} \\ & + a_C\, \frac{Z^2}{A^{1/3}} - a_4\, \frac{Z^2}{A} \end{aligned}$$

with $I = (N-Z)/A$. LDM corresponds to the expansion in the current formula:

$$\frac{1}{\frac{A}{\alpha} + \frac{A^{2/3}}{\beta}} \simeq \frac{\alpha}{A}\, \left(1 - \frac{\alpha}{\beta}\, A^{-1/3}\right)$$

But that expansion only accurate for $A \gtrsim 1000$, i.e. never!