The Stellar $^{12}C+\alpha$ Fusion Rate: Present Uncertainties and Prospects for their Reduction

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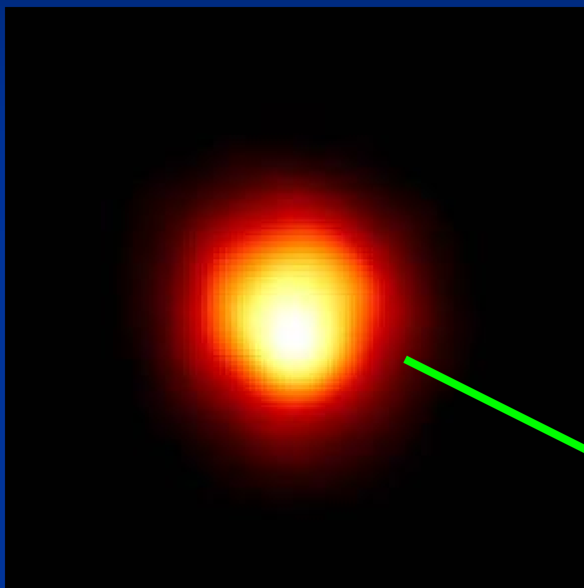
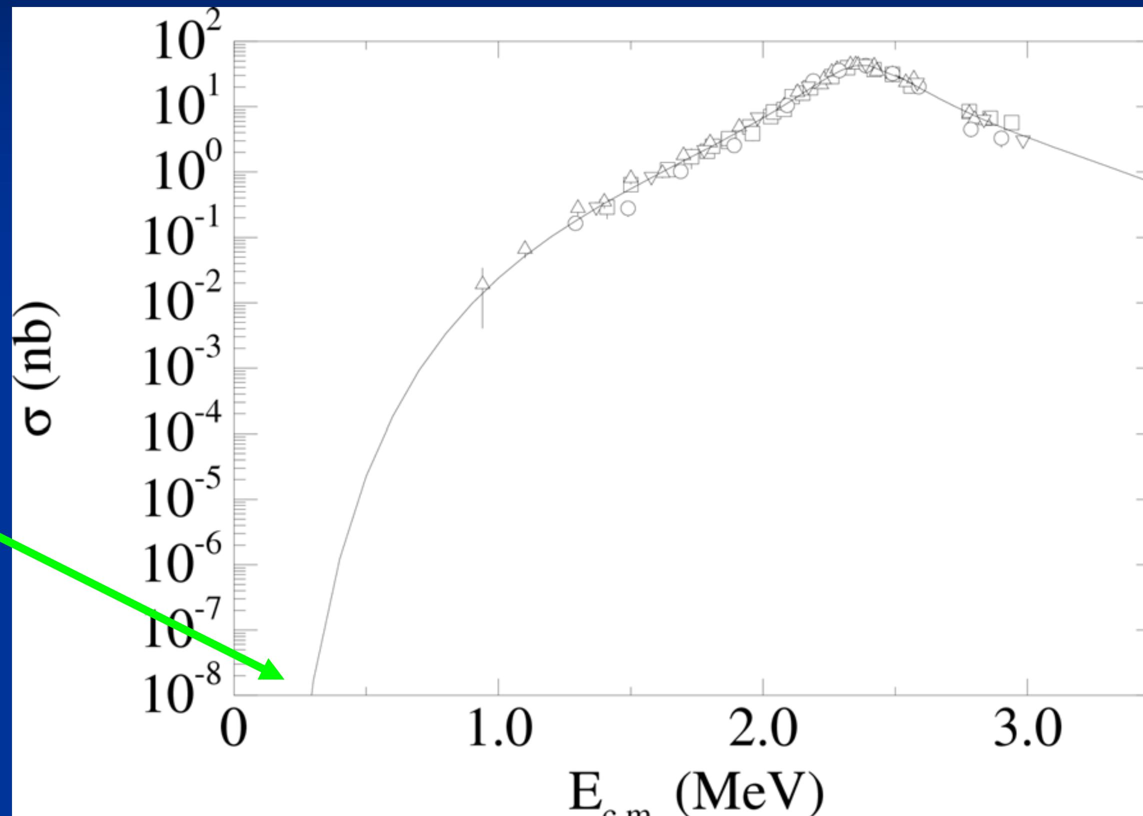
The Problem:

Red Giant



T=(1-3)x10^8 K



- cross section is abnormally small (E1 is isospin-forbidden)
- subthreshold resonances

Outline

- Status of present data
- R-Matrix Analysis
- New Experimental Approaches
- Present Status and Prospects

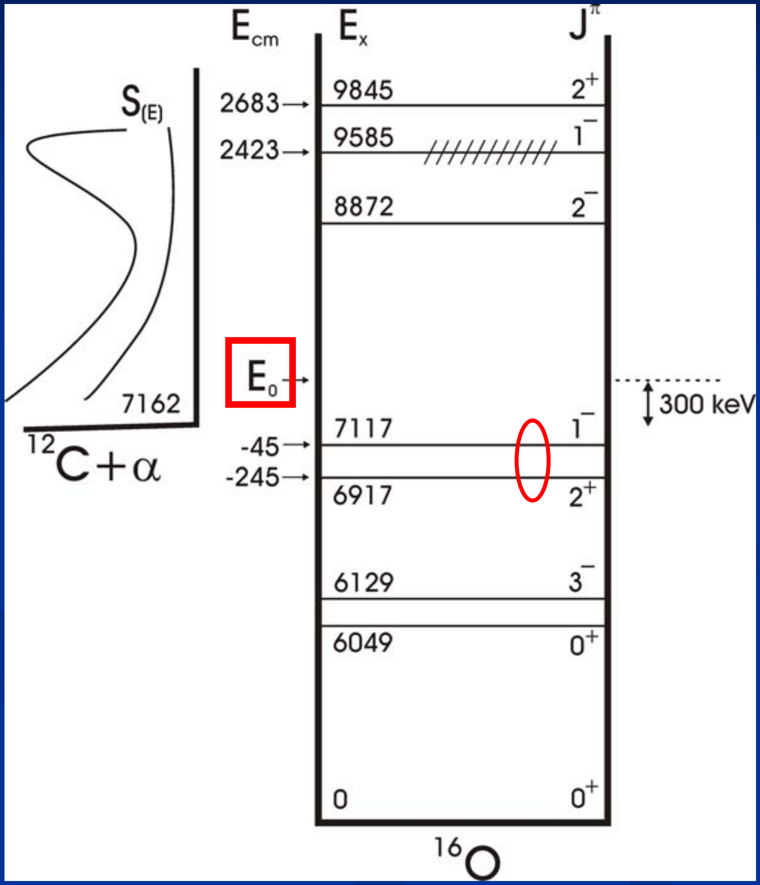
$^{12}C(\alpha,\gamma)^{16}O$ Cross Section

$^{12}C(\alpha,\gamma)$ - extrapolation to helium burning energies $E_0 \approx 300$ keV

$^{12}C(\alpha,\gamma)$ cross section

E1, E2 g.s. transitions thought to be largest

cascade transitions
Up to 30% contribution



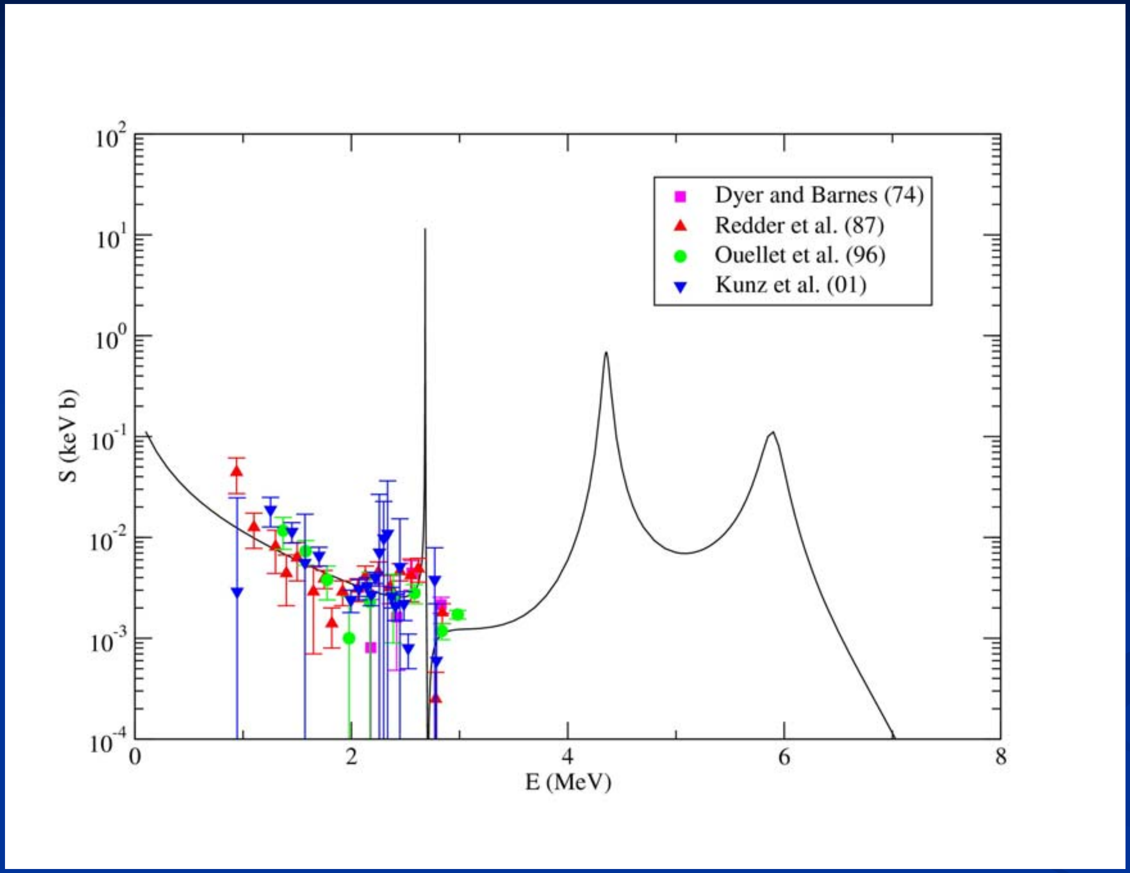
Data Relevant to $^{12}C(\alpha,\gamma)^{16}O$

- $^{12}C(\alpha,\gamma)^{16}O$ cross section data (required!)
- $^{12}C(\alpha,\alpha)$ elastic scattering data
- ^{16}N β-delayed α spectrum
- Bound-state spectroscopy (E_x, Γ_x,…)
- Transfer reactions

This case is ideally suited for R-matrix analysis:

- There are relatively few levels to be considered
- ^{12}C and α are spin-0 nuclei

E2 Ground-State Cross Section



Measurements at higher energies would be helpful-
TRIUMF (Dragon), Bochum (ERNA)

R-Matrix Method

- Exact implimentaton of quantum-mechanical symmetries and conservation laws (Unitarity)
- Treats long-ranged Coulomb potential explicitly
- Wavefunctions are expanded in terms of unknown basis functions
- Energy eigenvalues and the matrix elements of basis functions are adjustable parameters
- A wide range of physical observables can be fitted (e.g. cross sections, E_x, Γ_x,…)
- The fit can then be used to determine unmeasured observables
- Major Approximation: TRUNCATION (levels / channels)

Two Extensions

- The external contribution to capture reactions, which depends of the reduced width of the final state, can be included. Very important for E2 captures.
  - essentially "direct capture"
  - F.C. Barker and T. Kajino, Aus. J. Phys. 44, 369 (1991)
  - R.J. Holt et al., Phys. Rev. C 18, 1962 (1978)
- A mathematically-equivalent formulation is also available which eliminates B_c and the level shift.
  - C.R. Brune, Phys. Rev. C 66, 044611 (2002)
  - C. Angulo and P. Descouvemont, Phys. Rev. C 61, 064611 (2000)

β-Delayed Particles
$A \rightarrow a+b+e+\nu$

- Can supply information about reactions between nuclei a and b (the relative energy spectrum is especially useful)
- But how does one do the analysis?
- Barker has proposed:

$$N_c(E) = f_\beta P_c \left| \sum_{\lambda\mu} B_\lambda \gamma_{\mu c} A_{\lambda\mu} \right|^2$$

- Are the "feeding factors" B_λ real?

Return to First Principles
(with G.M. Hale)

Start from

$$d\Gamma = (2\pi)^4 \delta^3(\vec{p}_A - \vec{p}_a - \vec{p}_b - \vec{p}_e - \vec{p}_\nu)$$
$$\times \delta(E_A + m_A - E_a - m_a - E_b - m_b - W_e - W_\nu)$$
$$\times |T|^2 \frac{d^3\vec{p}_a}{(2\pi\hbar)^3} \frac{d^3\vec{p}_b}{(2\pi\hbar)^3} \frac{d^3\vec{p}_e}{(2\pi\hbar)^3} \frac{d^3\vec{p}_\nu}{(2\pi\hbar)^3}$$

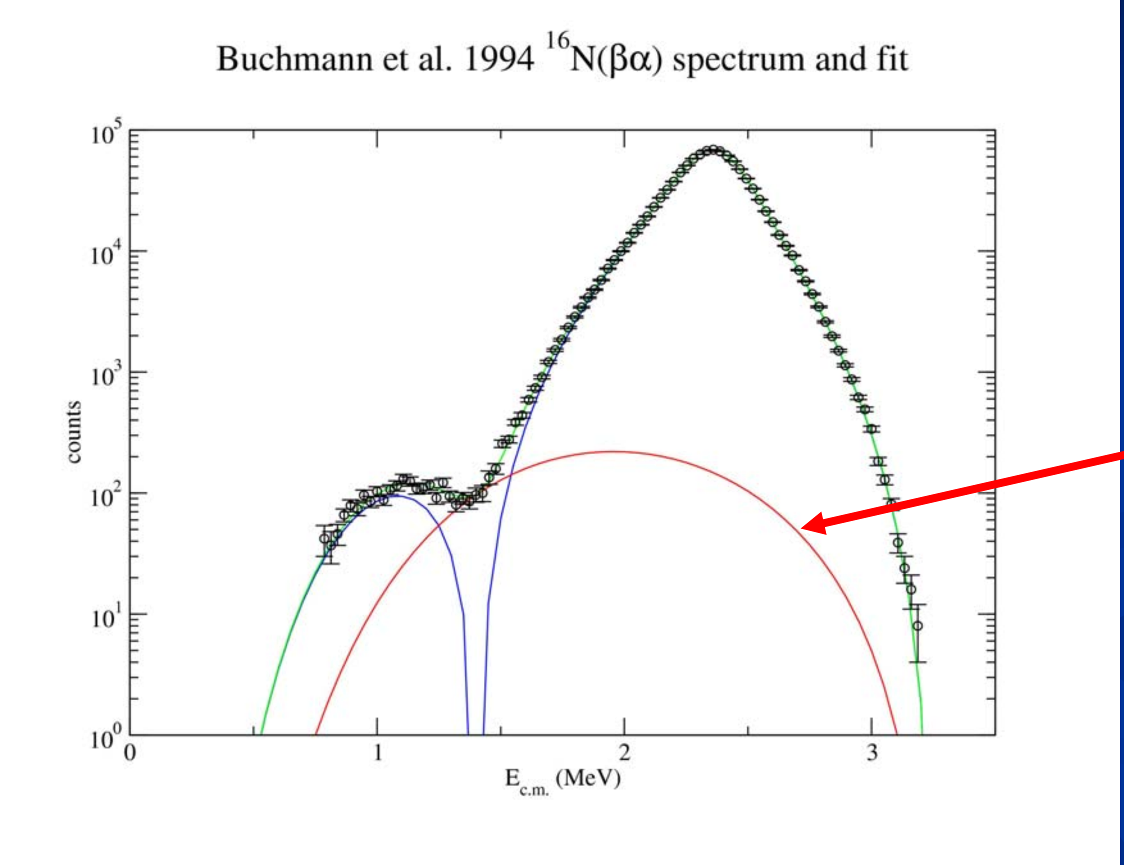
where $T \propto \langle a + b | H_{\mathrm{weak}} | A \rangle$.

An R-matrix expression can be used for the a+b wavefunction!

Summary of Findings

- Barker's formula for the particle energy spectrum is verified (in the "allowed approximation" and ignoring e-ν recoil effects).
- The feeding factors B_λ are related to matrix elements of the R-matrix eigenfunctions.
- The B_λ are real provided that time-reversal invariance holds.
- The framework for calculating higher-order corrections is supplied (e.g. recoil, forbidden transitions).

$^{16}N(\beta\alpha)$ Spectrum



3⁻ strength?

What fills in the interference minimum?

How Reliable are R-Matrix Methods?

- Are the channel radii used in phenomenological analyses (5-7 fm) reasonable?
- What about effects of higher-energy levels (truncation)?
- Phase-equivalent potentials with different bound-state properties have recently been studied: J.-M. Sparenberg, Phys. Rev. C 69, 034601 (2004).

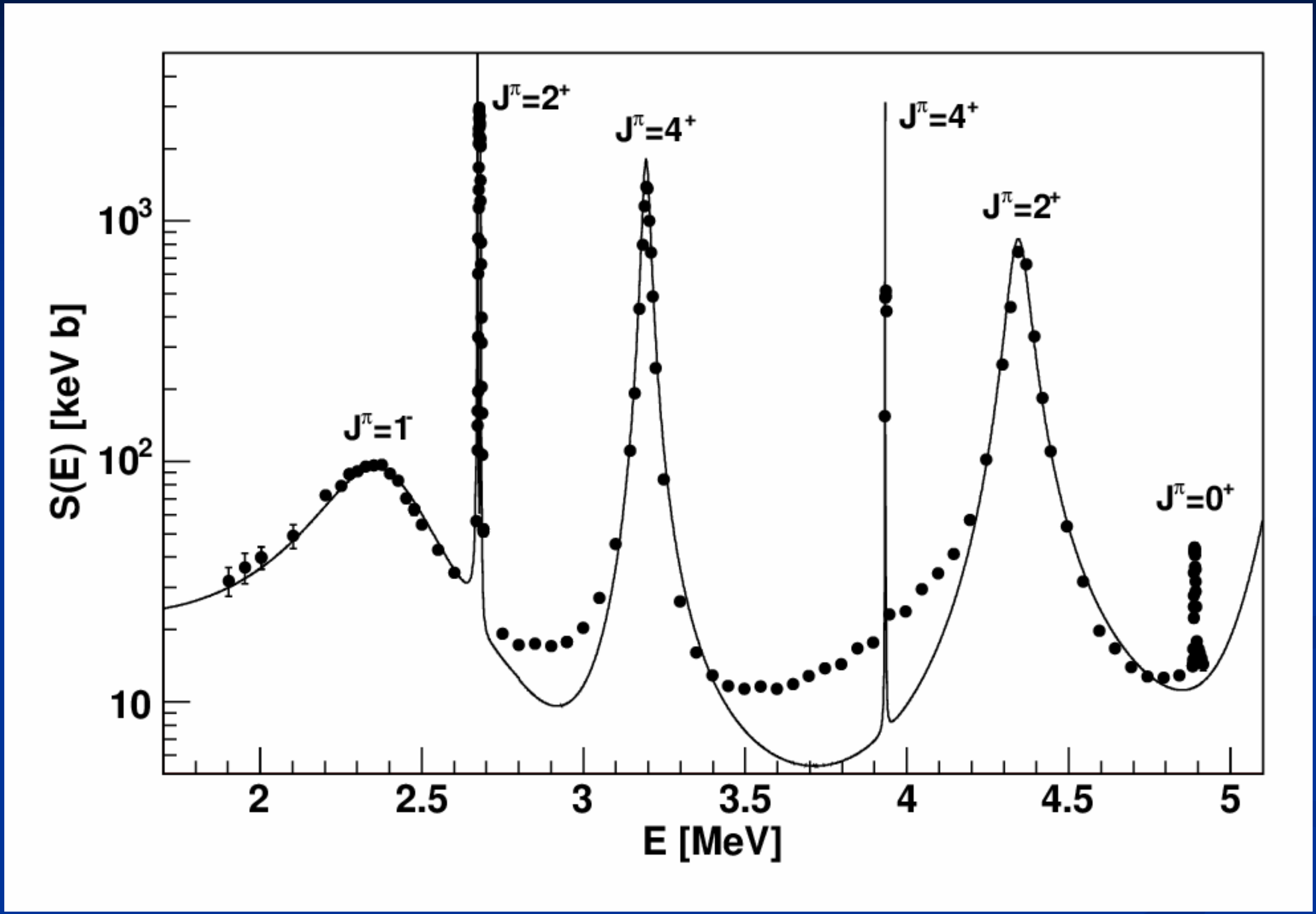
It may be possible to address these questions by applying a phenomenological to cross sections etc… generated by a model.

Summary of Recent Determinations

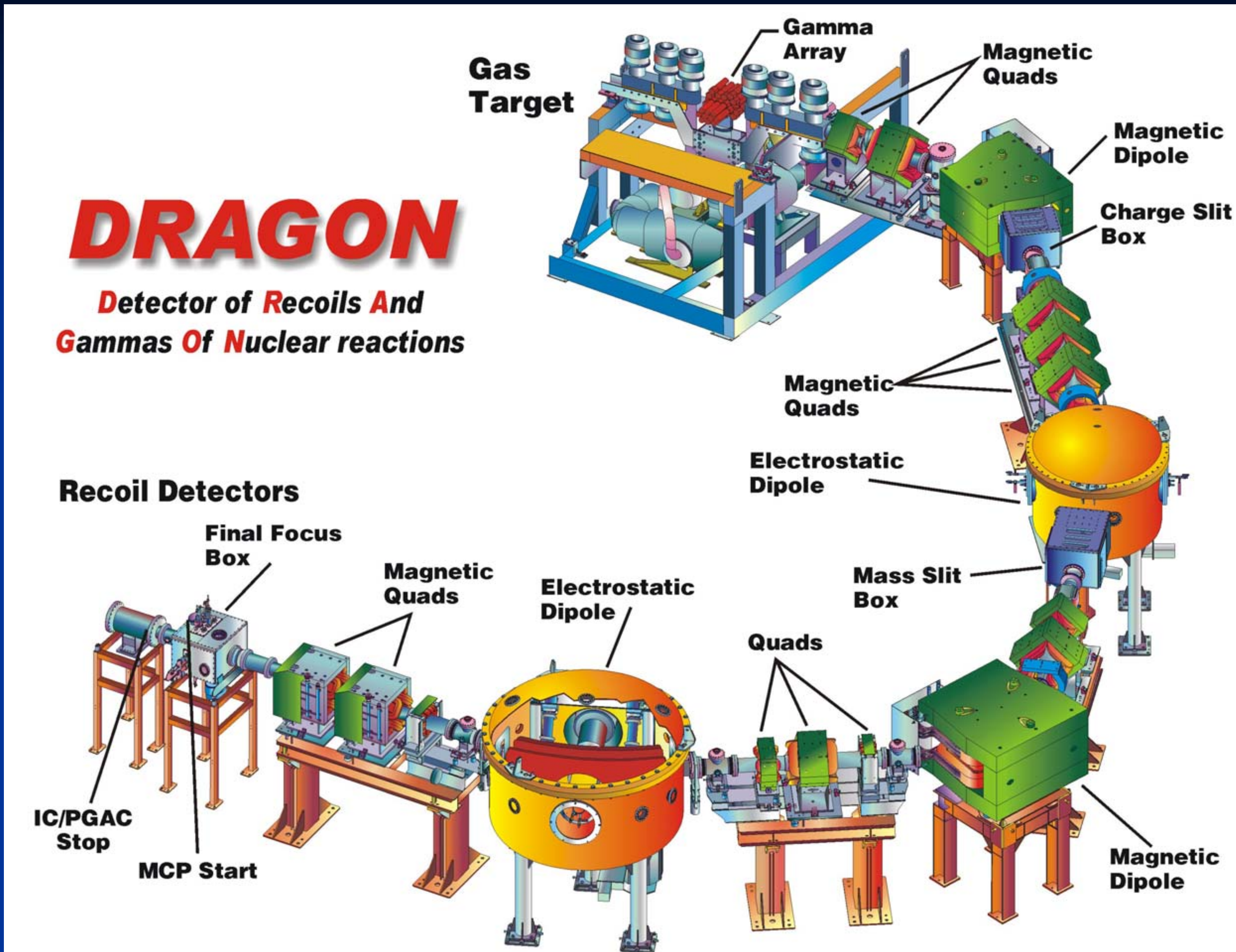
Result @ E=300 keV	source
S_{E1}=79(21) keV-b	$^{16}N(\beta\alpha)$, Buchmann et al. (1994)
S_{E1}=99(44) keV-b	direct measurement, Roters et al. (1999)
S_{E1}=101(17) keV-b	sub-Coulomb α transfer, Brune et al. (1999)
S_{E2}=120(60) keV-b	compilation, NACRE (1999)
$S_{E2}=42^{+16}_{-23}$ keV-b	sub-Coulomb α transfer, Brune et al. (1999)
S_{E2}=85(30) keV-b	direct measurement, Kunz et al. (2001)
$S_{E2}=53^{+13}_{-18}$ keV-b	$^{12}C(\alpha,\alpha)$, Tischhauser et al. (2002)
S_C=16 keV-b	theoretical, Barker and Kajino (91)
S_C=4(4) keV-b	direct measurement, Kunz et al. (2001)

$S_{tot} \approx 160$ keV-b

New Total Cross Section Measurement



ERNA/Bochum/Napoli (D. Schürmann et al.), using a Recoil separator and inverse kinematics – all final states


DRAGON
Detector of Recoils And
Gammas Of Nuclear reactions
Gas
Target
Gamma
Array
Magnetic
Quads
Magnetic
Dipole
Charge Slit
Box
Magnetic
Quads
Electrostatic
Dipole
Mass Slit
Box
Recoil Detectors
Final Focus
Box
Magnetic
Quads
Electrostatic
Dipole
Quads
IC/PGAC
Stop
MCP Start
Magnetic
Dipole

Other Ongoing or Unpublished Work

- Measurement of β-delayed α spectrum of ^{16}N at Argonne National Lab (X.D. Tang et al.)
- Branching-ratio measurements for bound states at Ohio University (C.M. Matei et al.)
- $^{12}C(\alpha,\gamma)$ measurements: Karlsruhe, Stuttgart (?)

Conclusions and Outlook

My take on S(300 keV):

- S(E1-g.s.) = 80(20) keV-b
- S(E2-g.s.) = 45(25) keV-b
- S(Cascade) = 35(20) keV-b
- S(total) = 160 (40) keV-b

- Improvements in low-energy capture measurements are difficult
- The time is right for a new global analysis