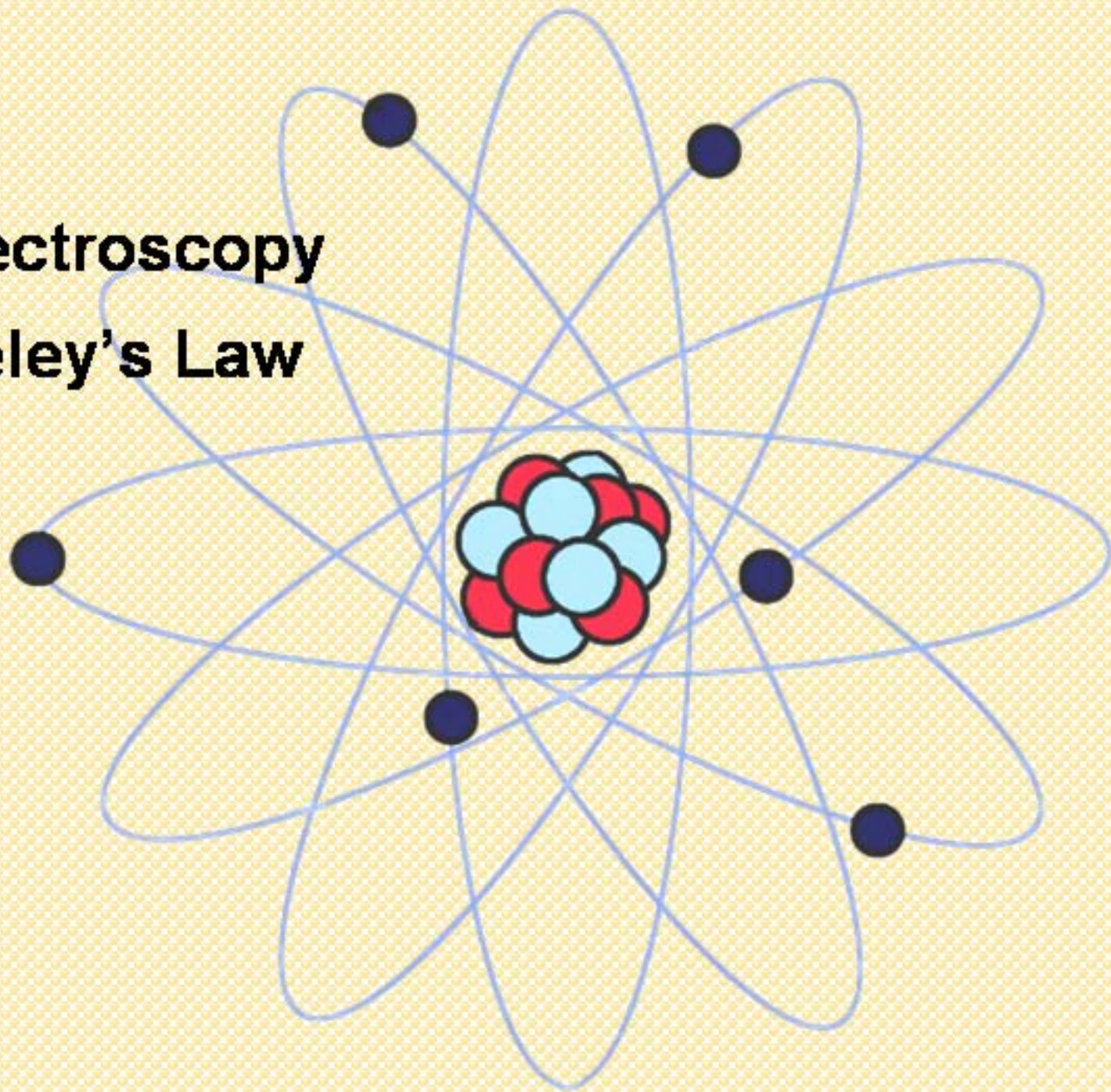


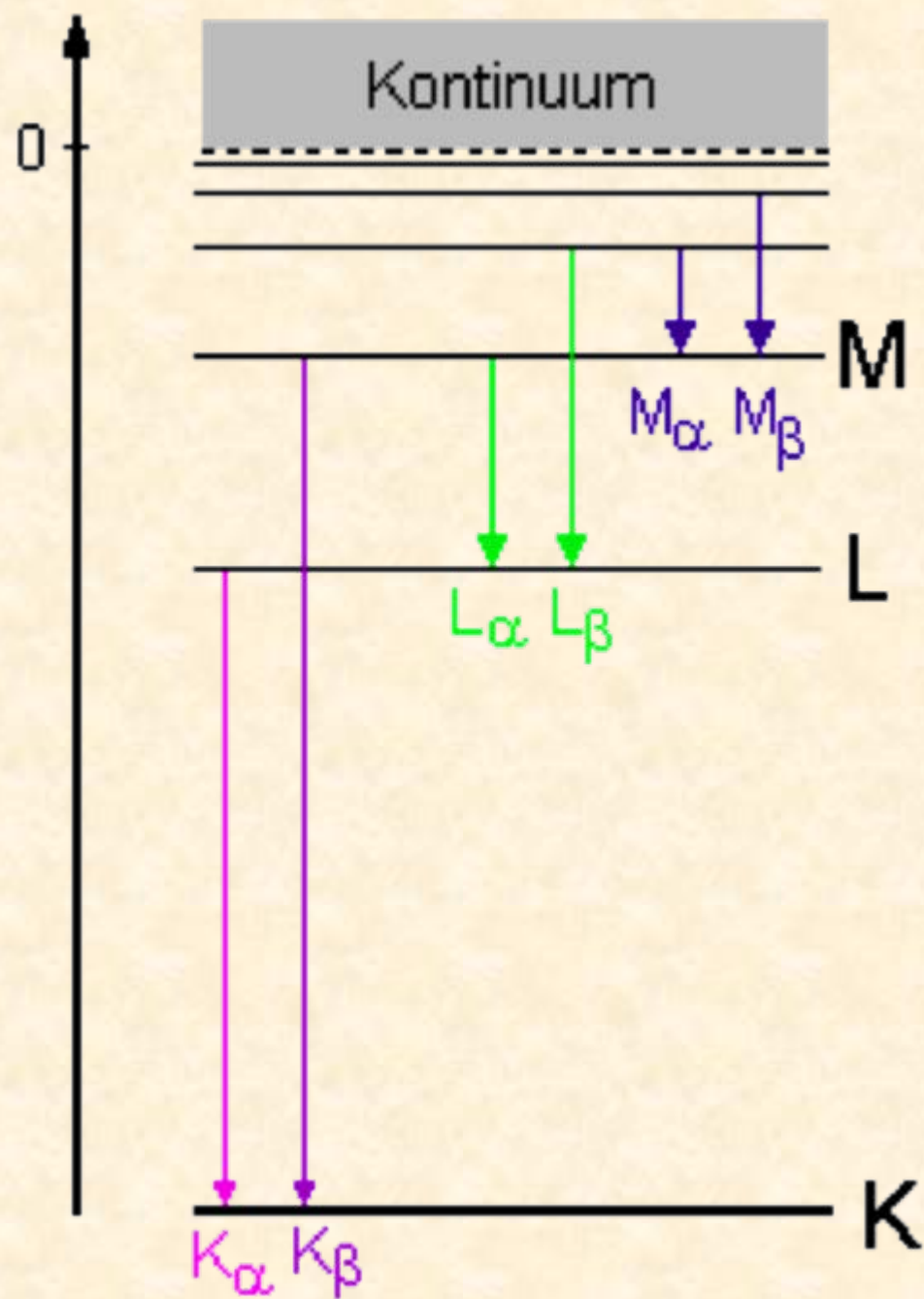
# X-Ray Spectroscopy and Moseley's Law

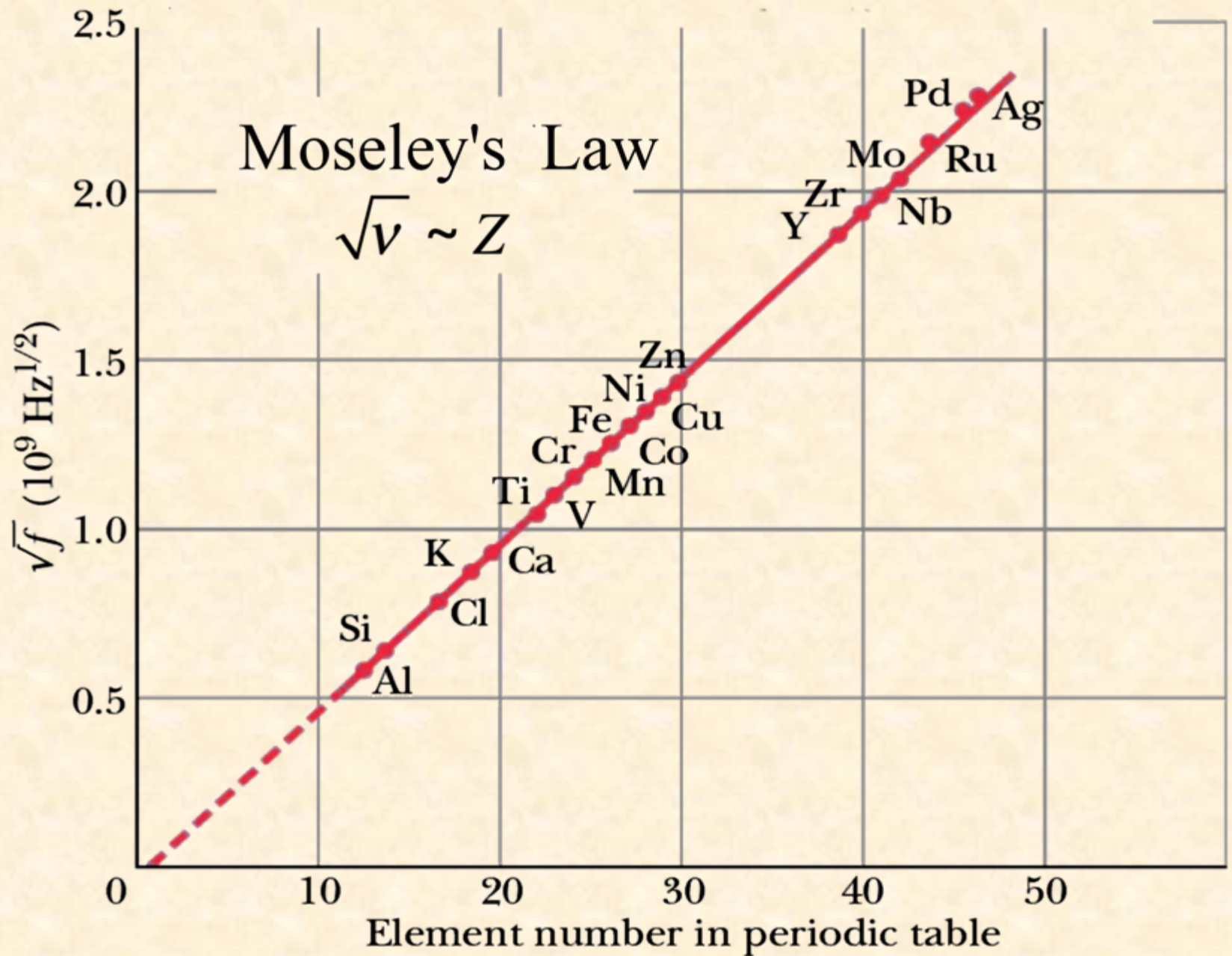




**Henry Gwyn Jeffreys Moseley**  
1887 - 1915





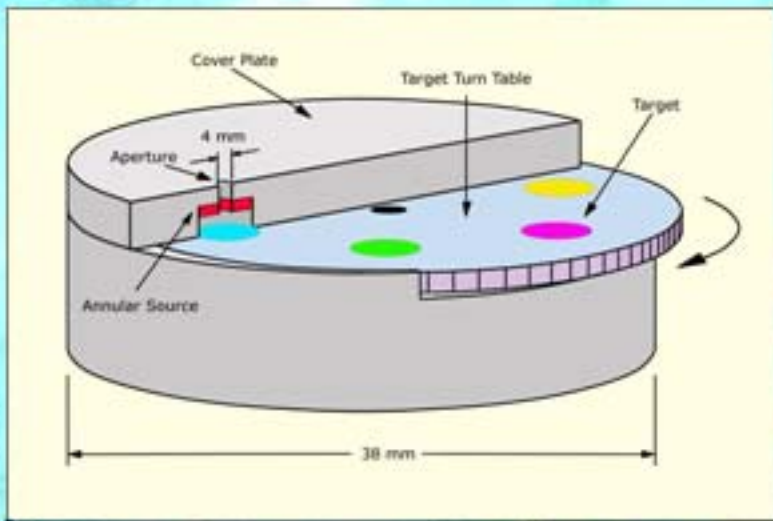






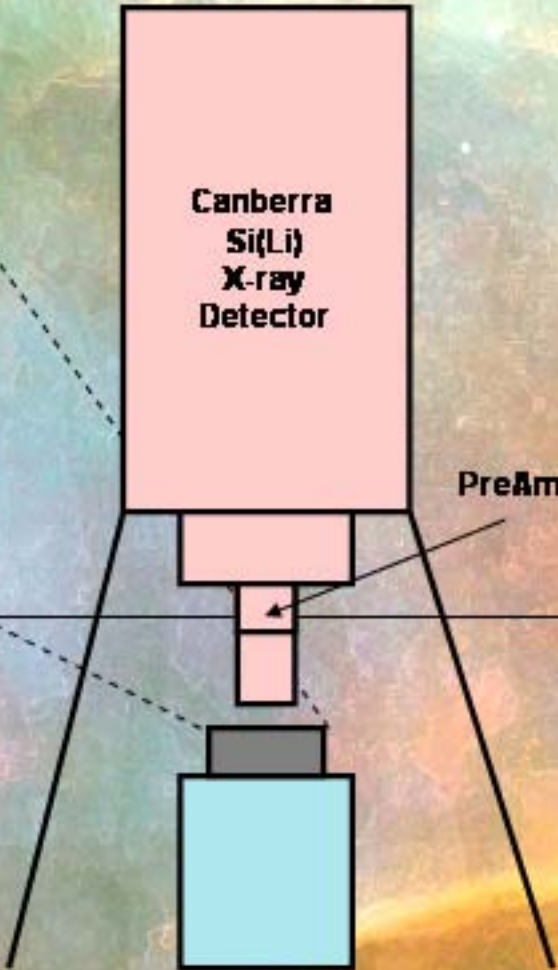
**Material Analysis using X-Ray Fluorescence (XRFA)**





**Source**

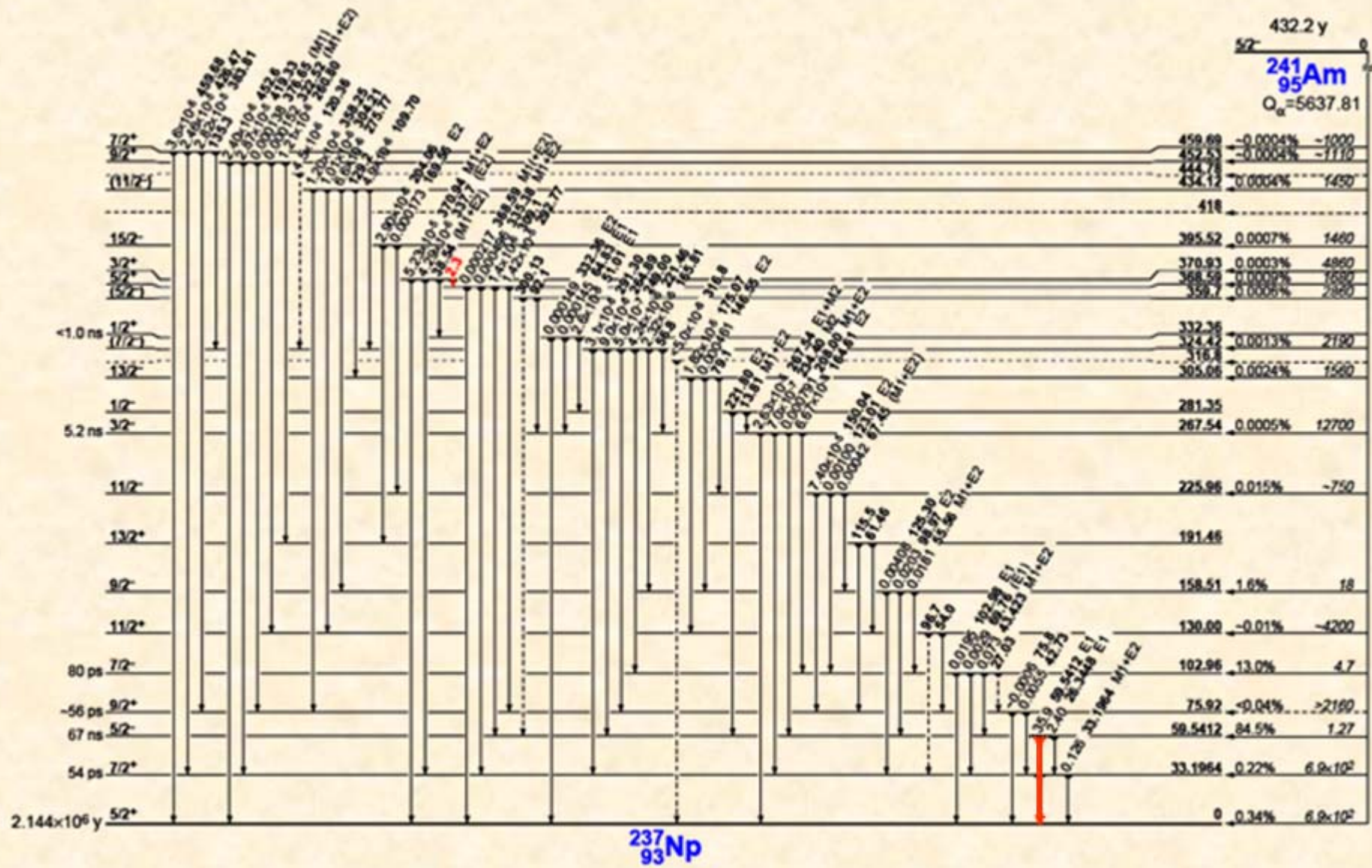
**HV Power Supply  
Canberra  
3122D**



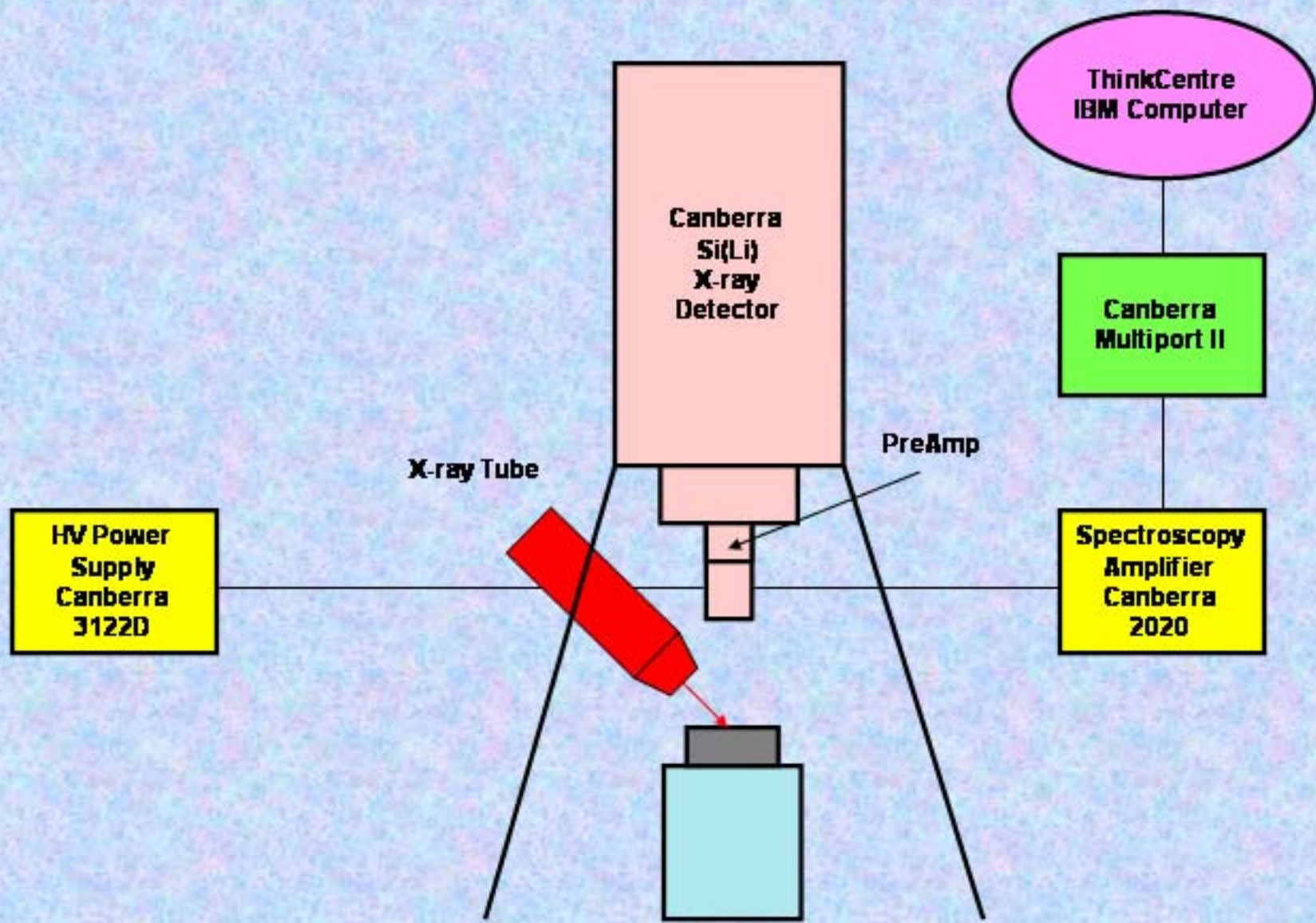
**ThinkCentre  
IBM Computer**

**Canberra  
Multiport II**

**Spectroscopy  
Amplifier  
Canberra  
2020**









**ECLIPSE IV**  
**X-ray tube**  
**AmpTek and Oxford Instr.**

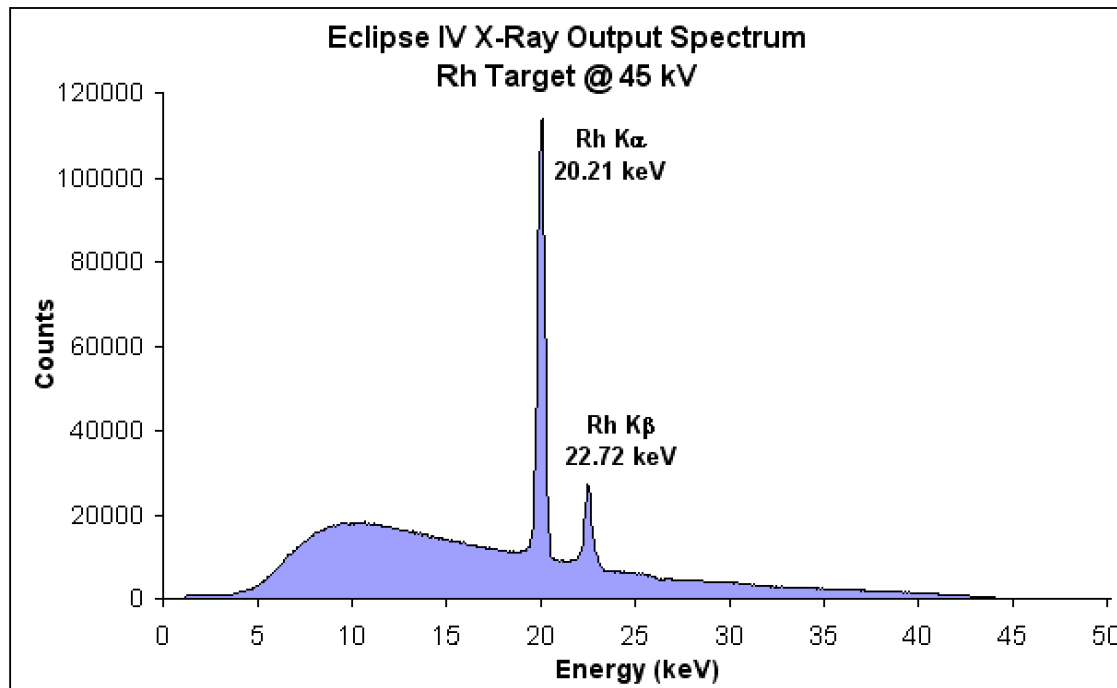
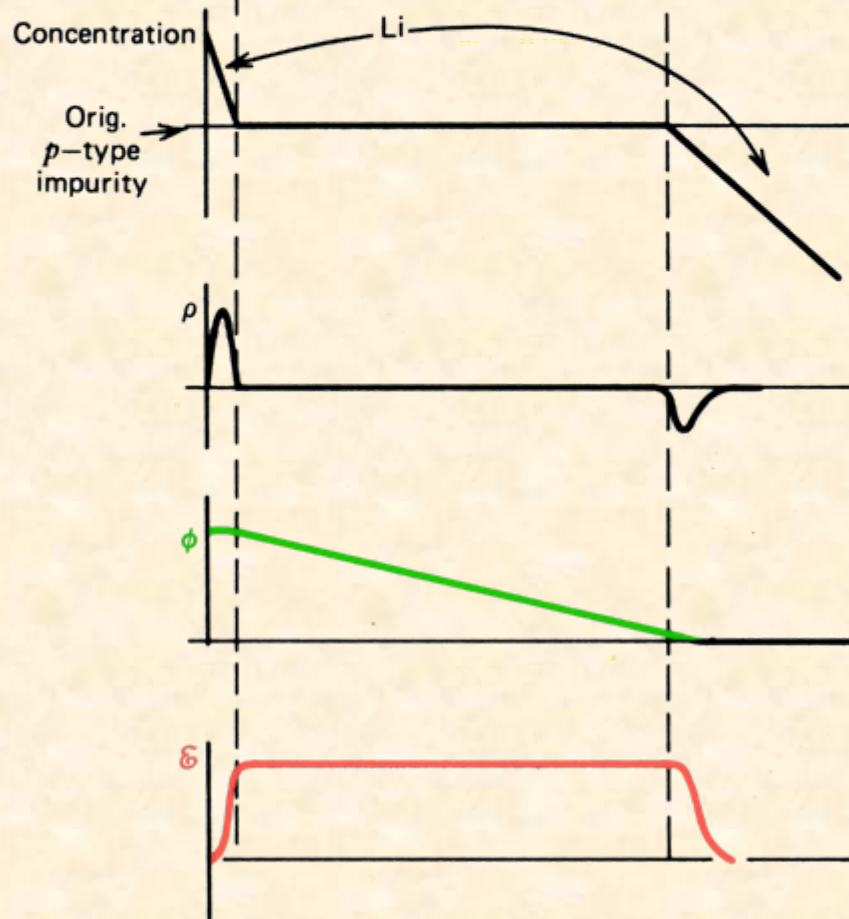
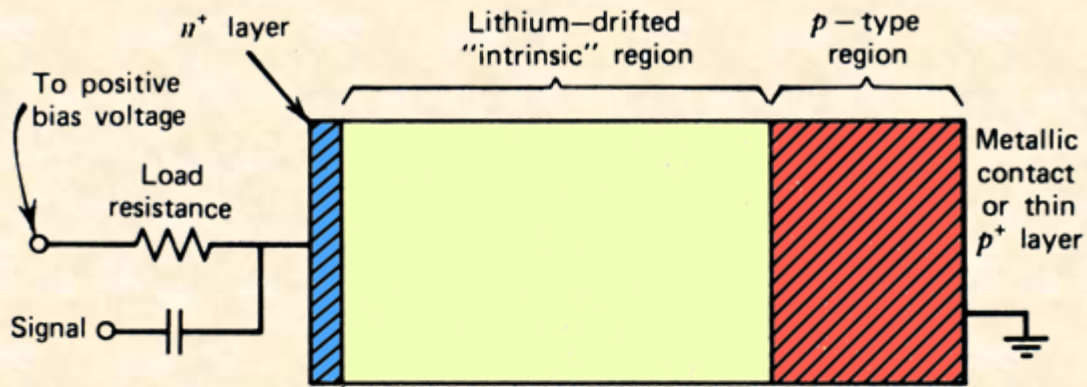
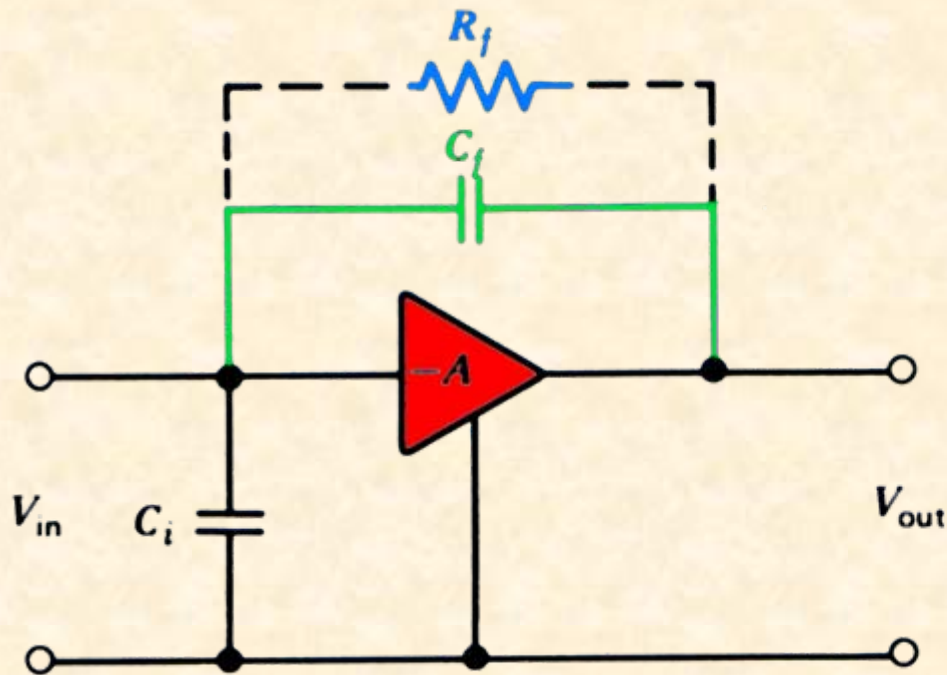


Figure 1. Eclipse-IV output spectrum at 45 kV.





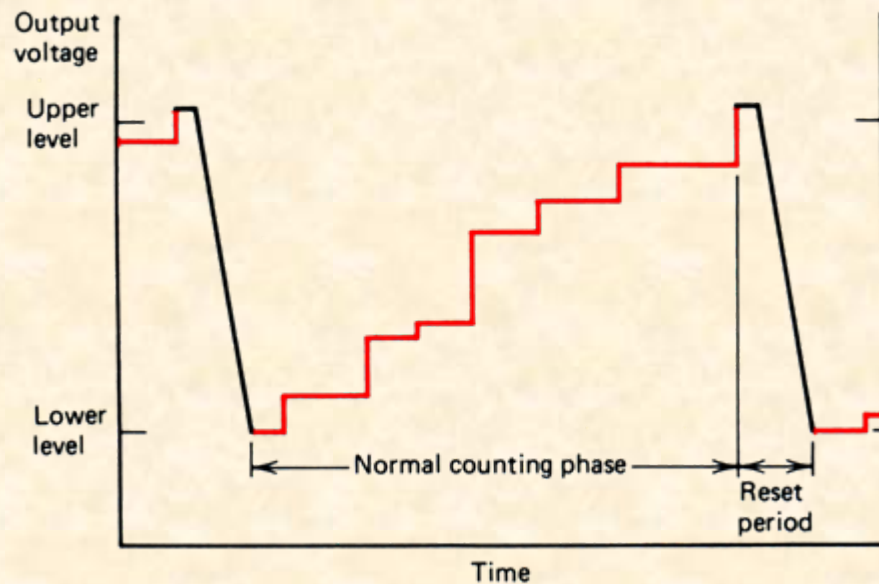
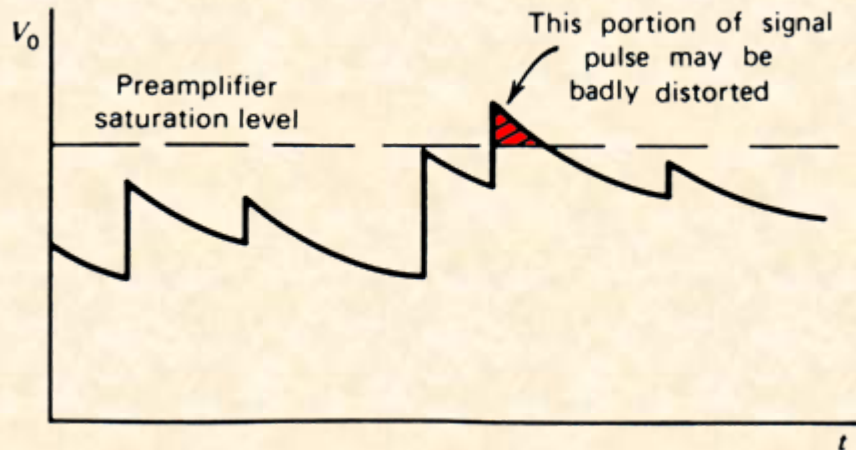


Assume  $A \gg (C_i + C_f)/C_f$

$$V_{out} = -A V_{in}$$

$$V_{out} = -A \frac{Q}{C_i + (A + 1)C_f}$$

$$V_{out} \cong -\frac{Q}{C_f}$$

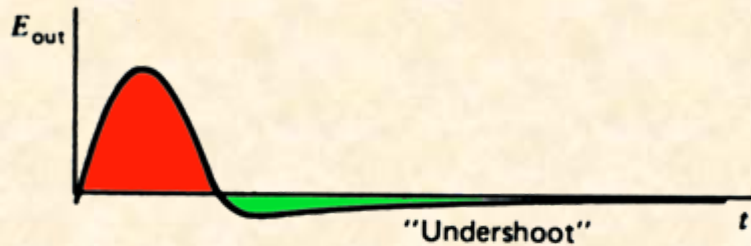


- **A detector is a kind of transducer, a physical entity is coded as an electronic signal**
- **Linear relation between X-ray energy and output pulse amplitude**
- **We are measuring voltages, dynamic range 0 – 8V in ~ 1 microsec; resolution several millivolts**
- **We have to understand the information chain, we need clear and clean conditions**



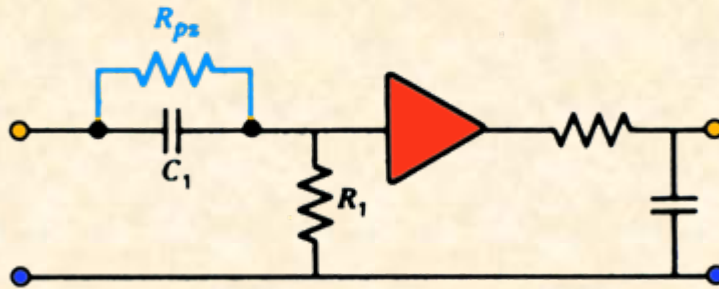


(a)

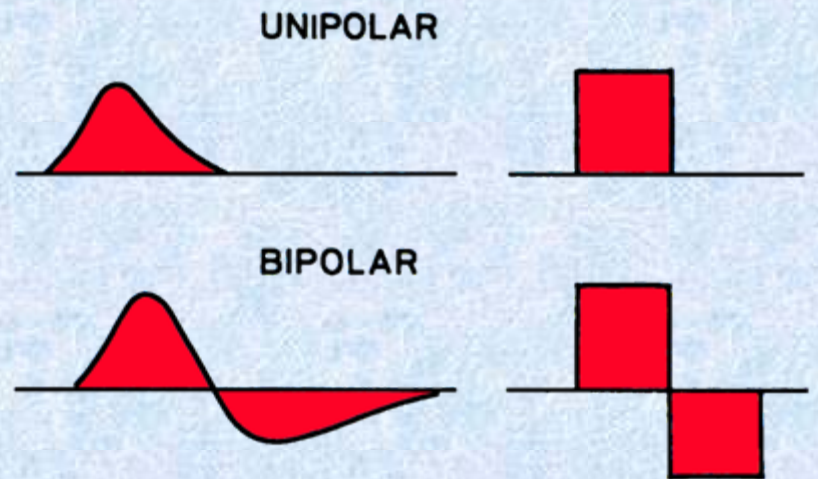
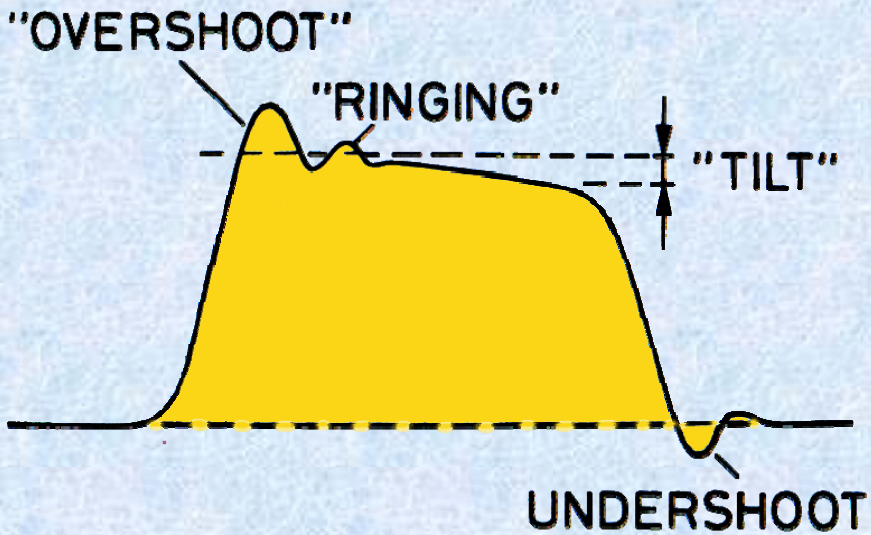
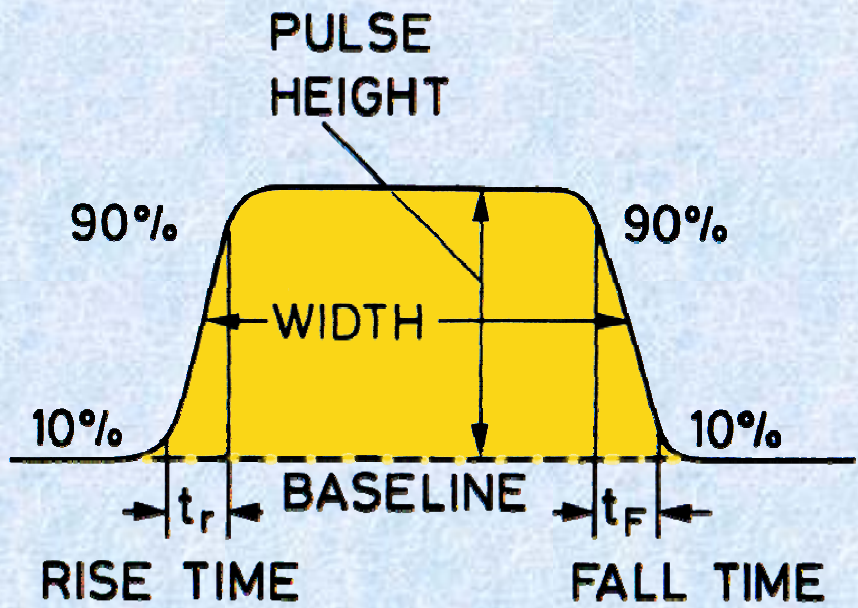


(b)

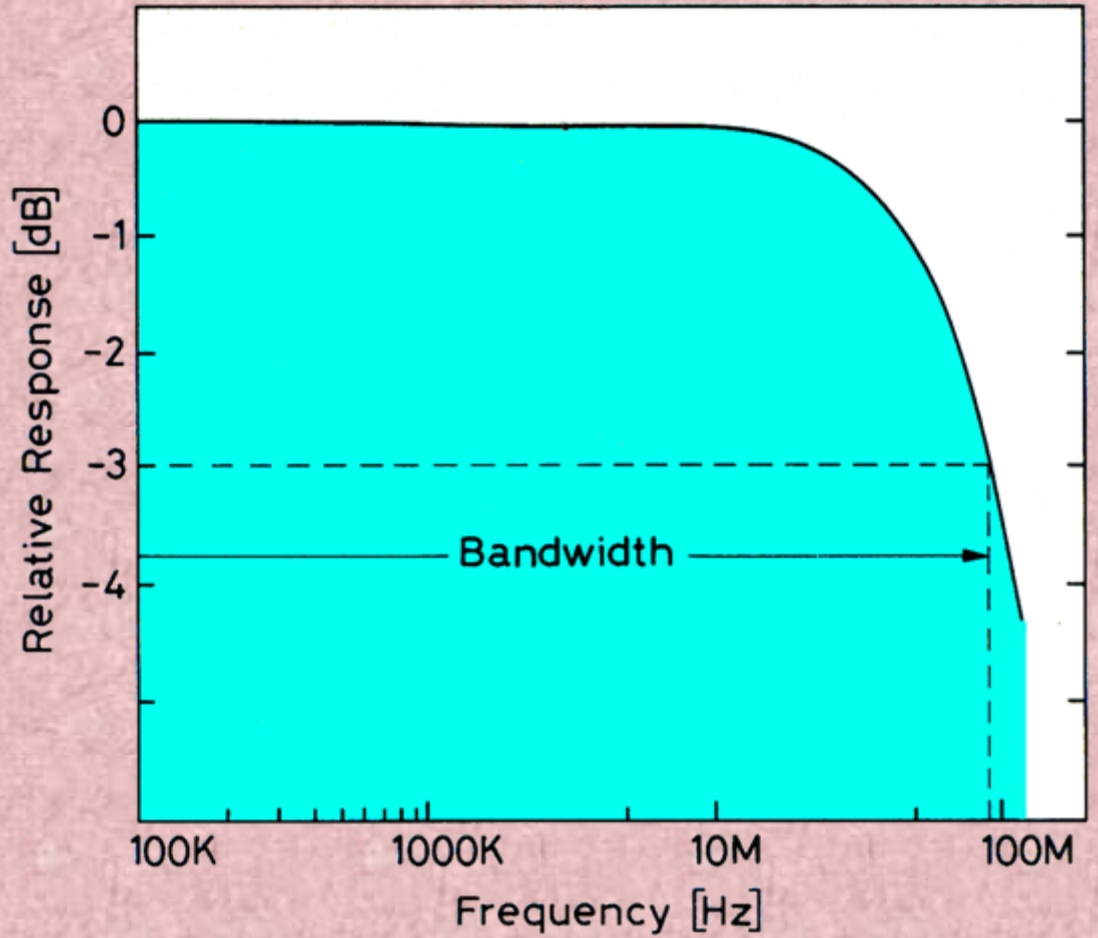
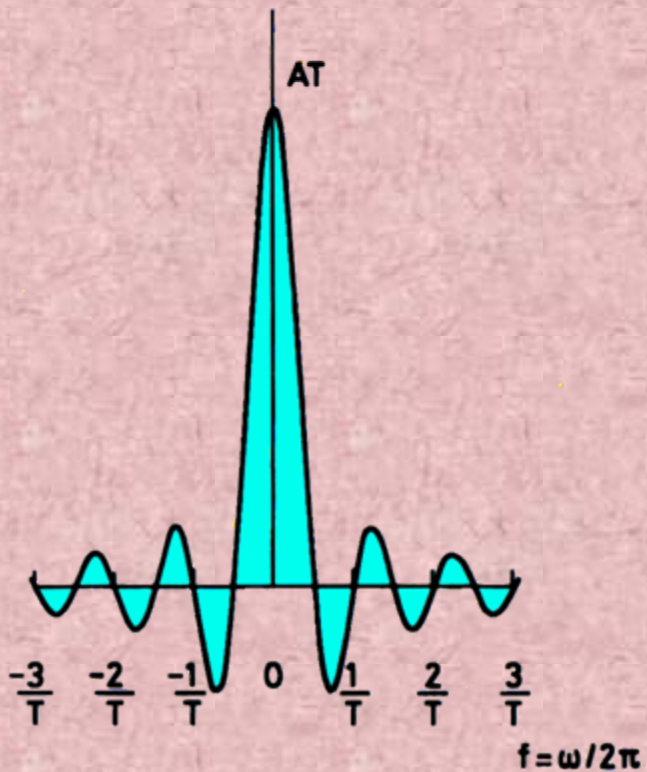
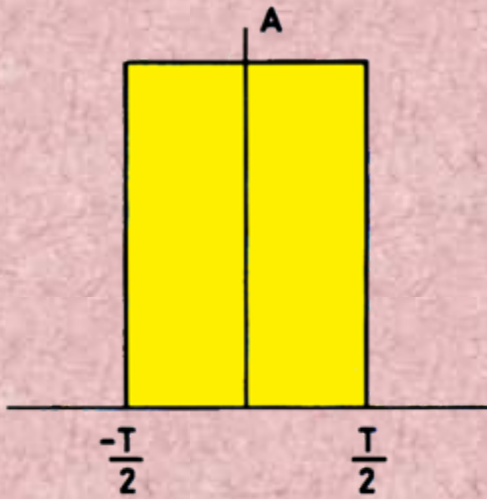
## Pole Zero Cancellation

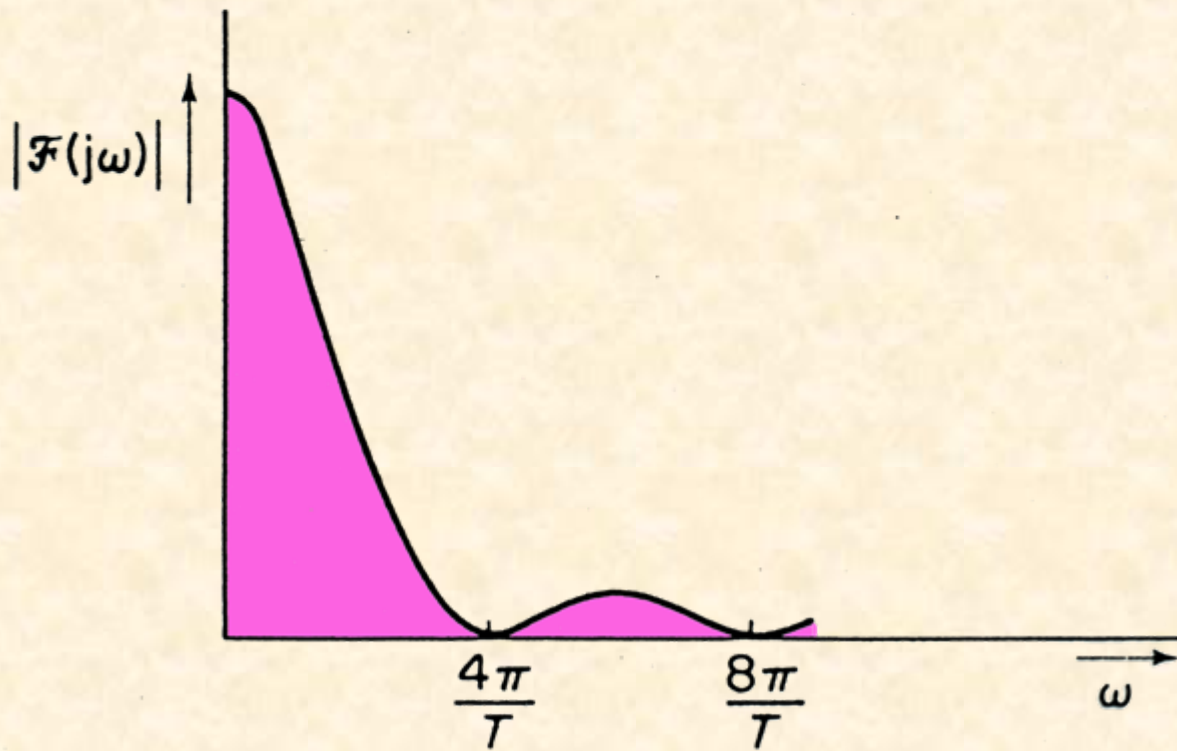
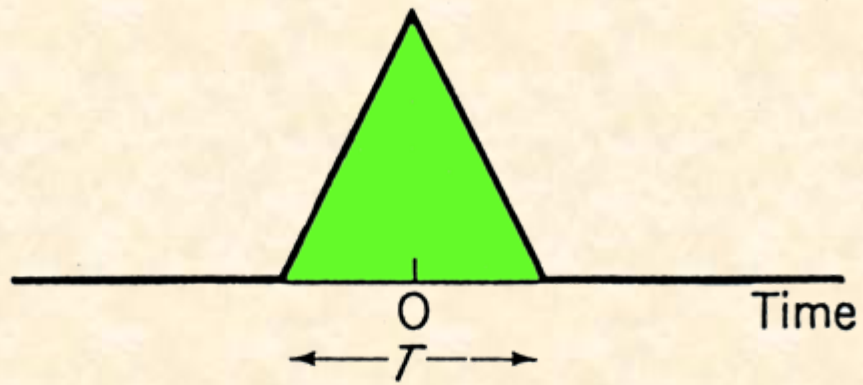


(c)



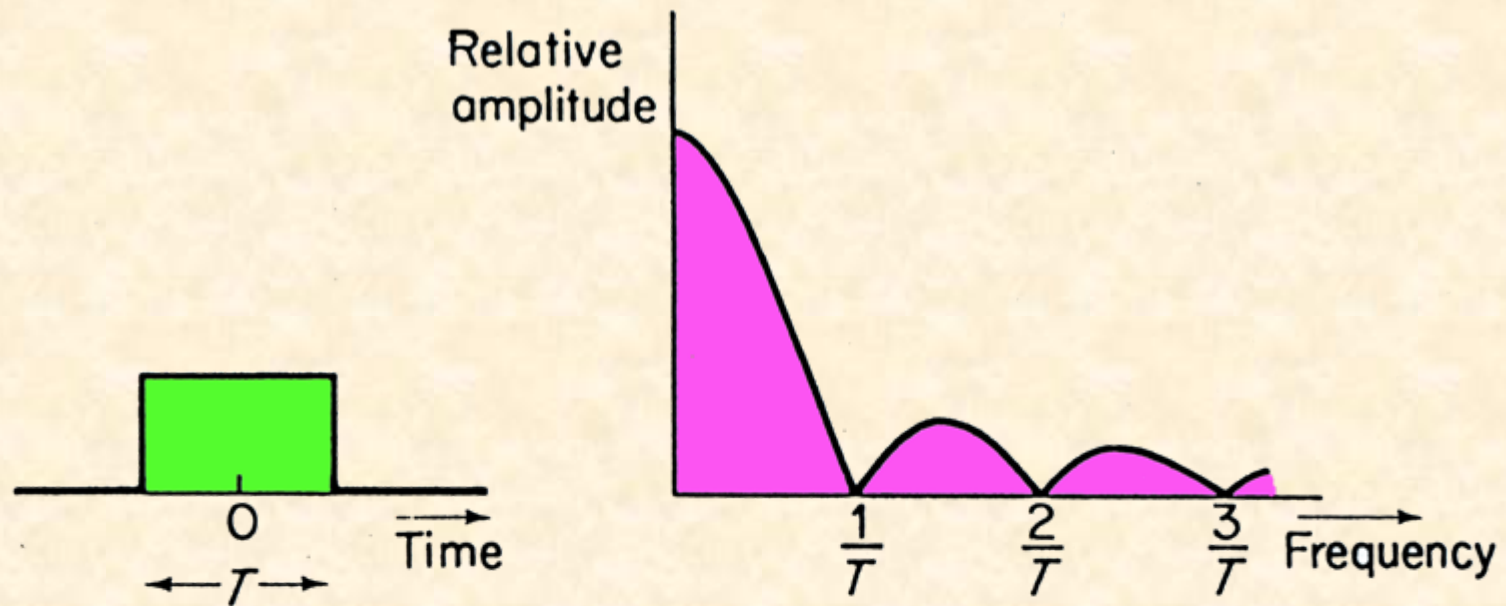




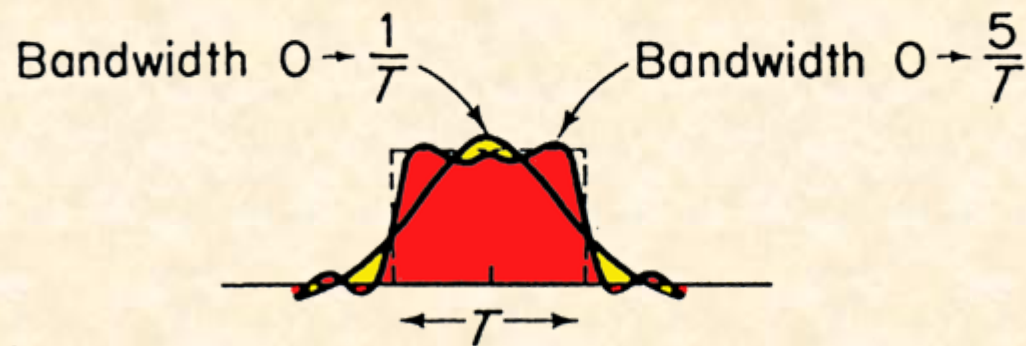


Triangular pulse and its frequency (Fourier) spectrum



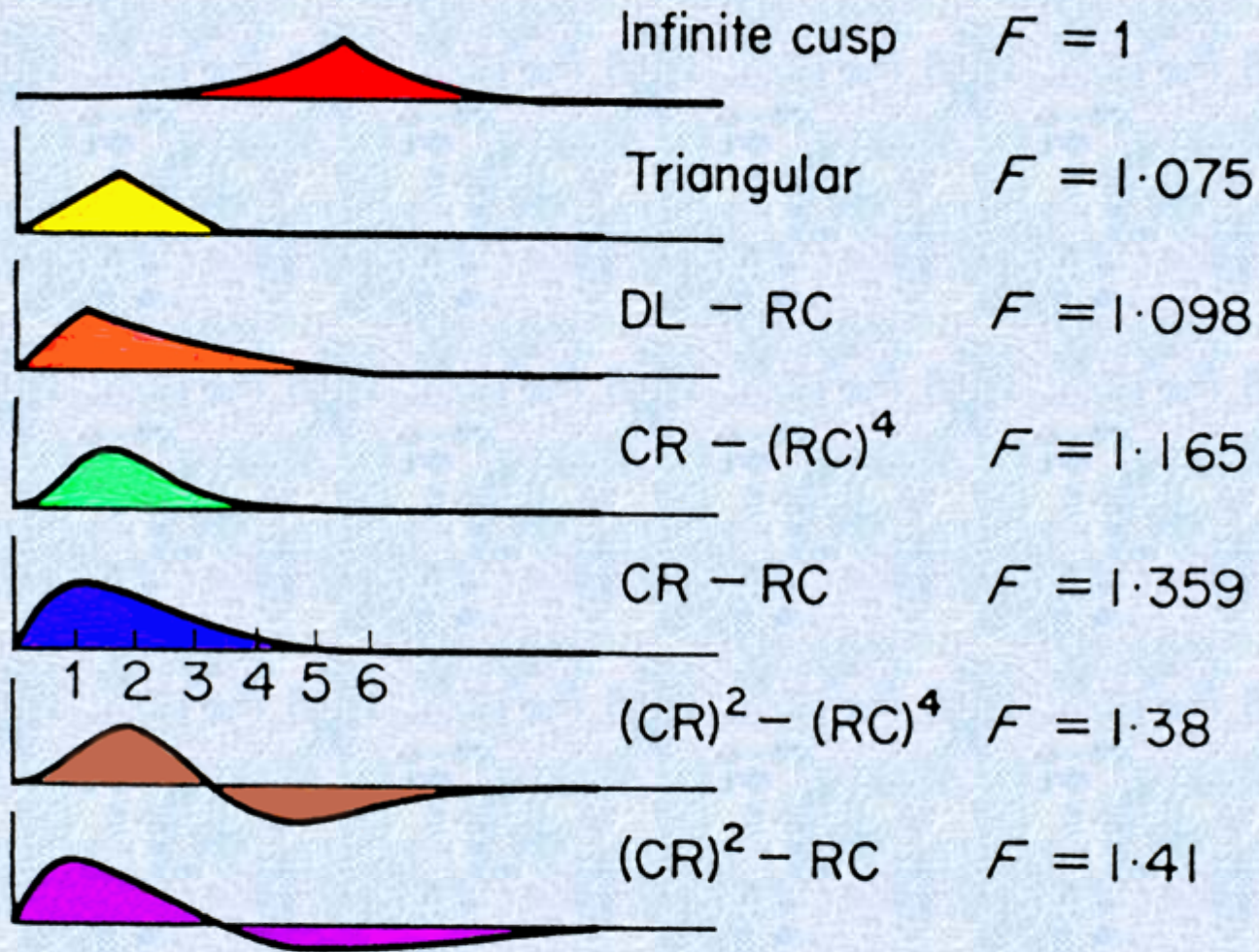


(a)



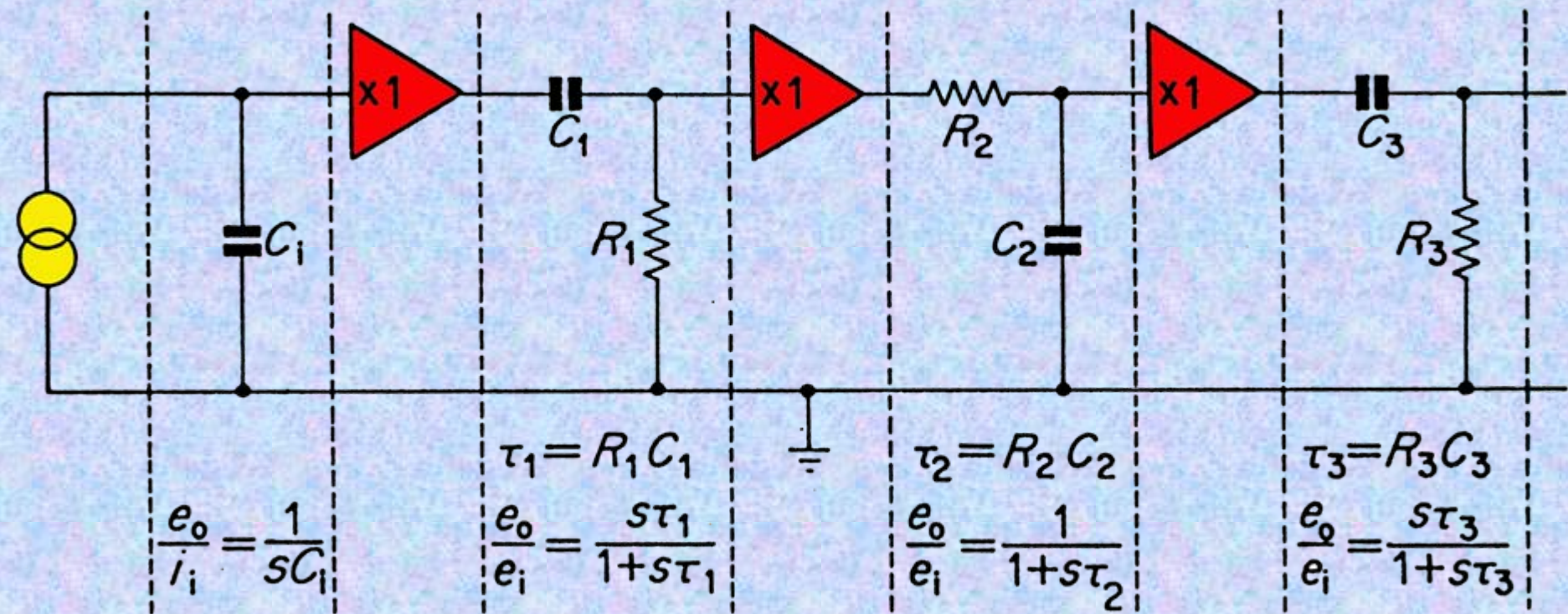
(b)

(a) Frequency (Fourier) spectrum of a rectangular pulse of duration  $T$ . (b) Effect on the pulse shape of the removal of all frequencies above  $1/T$  and above  $5/T$

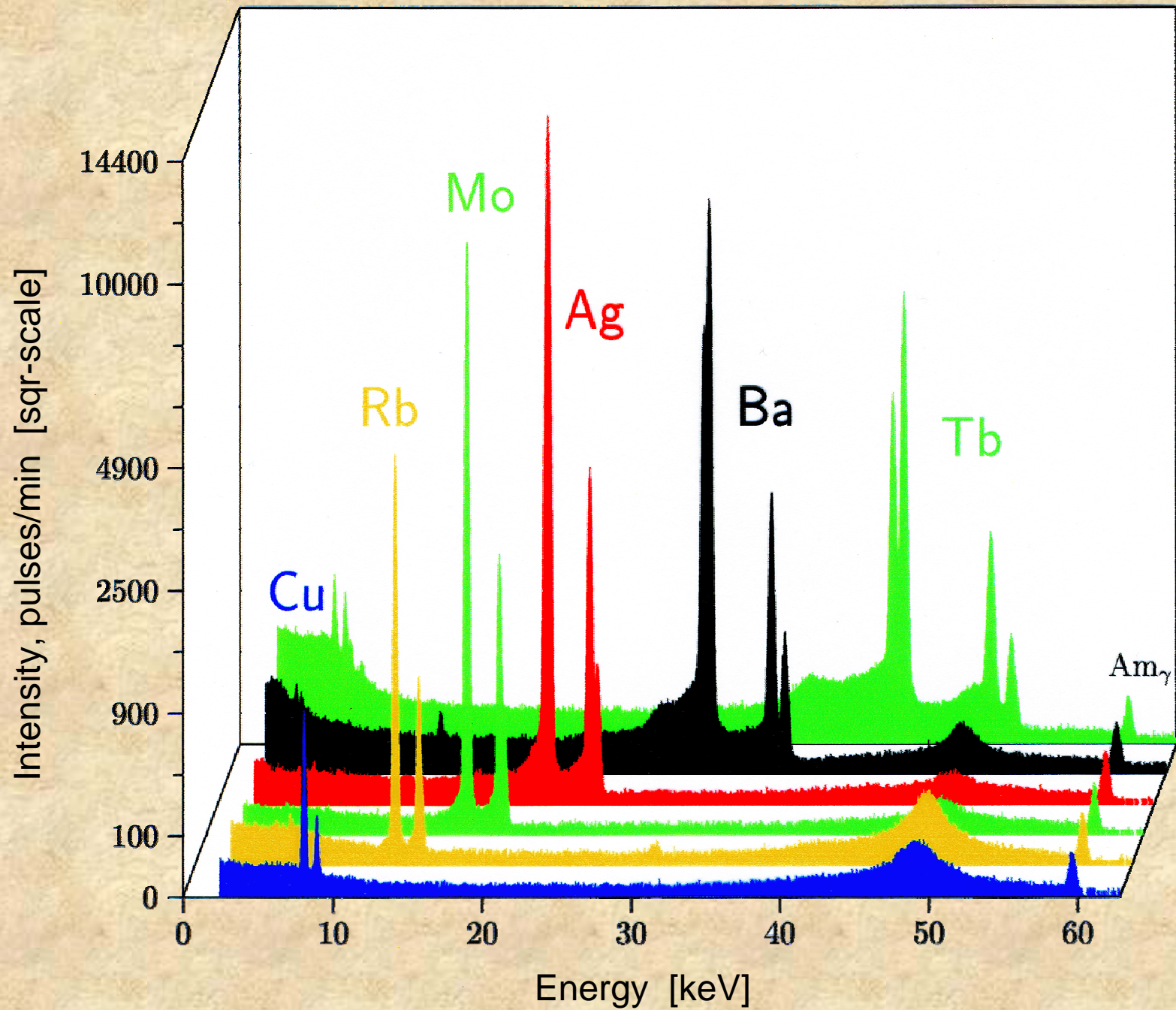


Pulse shapes, normalized to constant height, for various systems when the time scale of each (i.e. the value of  $\tau$ ) is optimized for the case of a noise corner time  $\tau_c$  equal to 1. The curves shown are adapted from those given by Konrad<sup>20</sup>

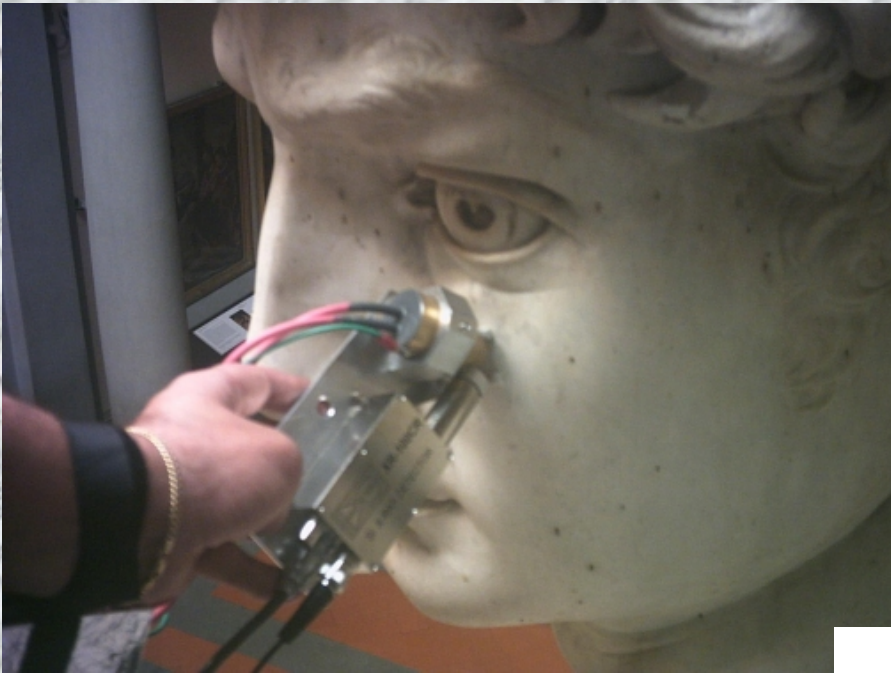




Equivalent circuit for  $(CR)^2$ -RC shaping







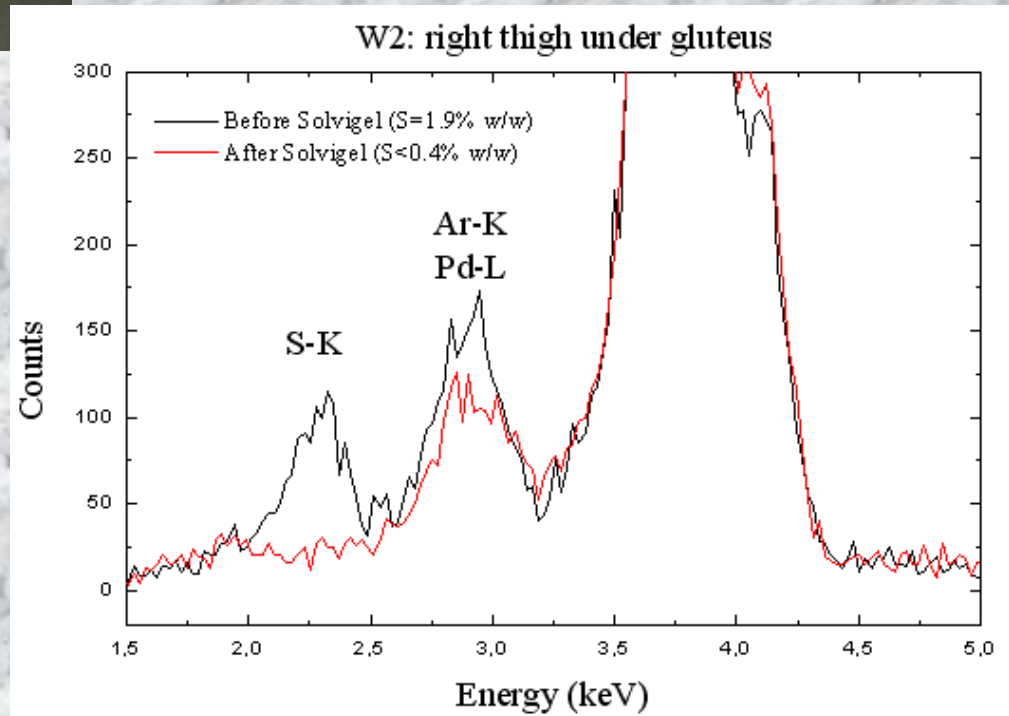
## The Restoration of Michelangelo's David

Typical spectrum before and after cleaning treatment

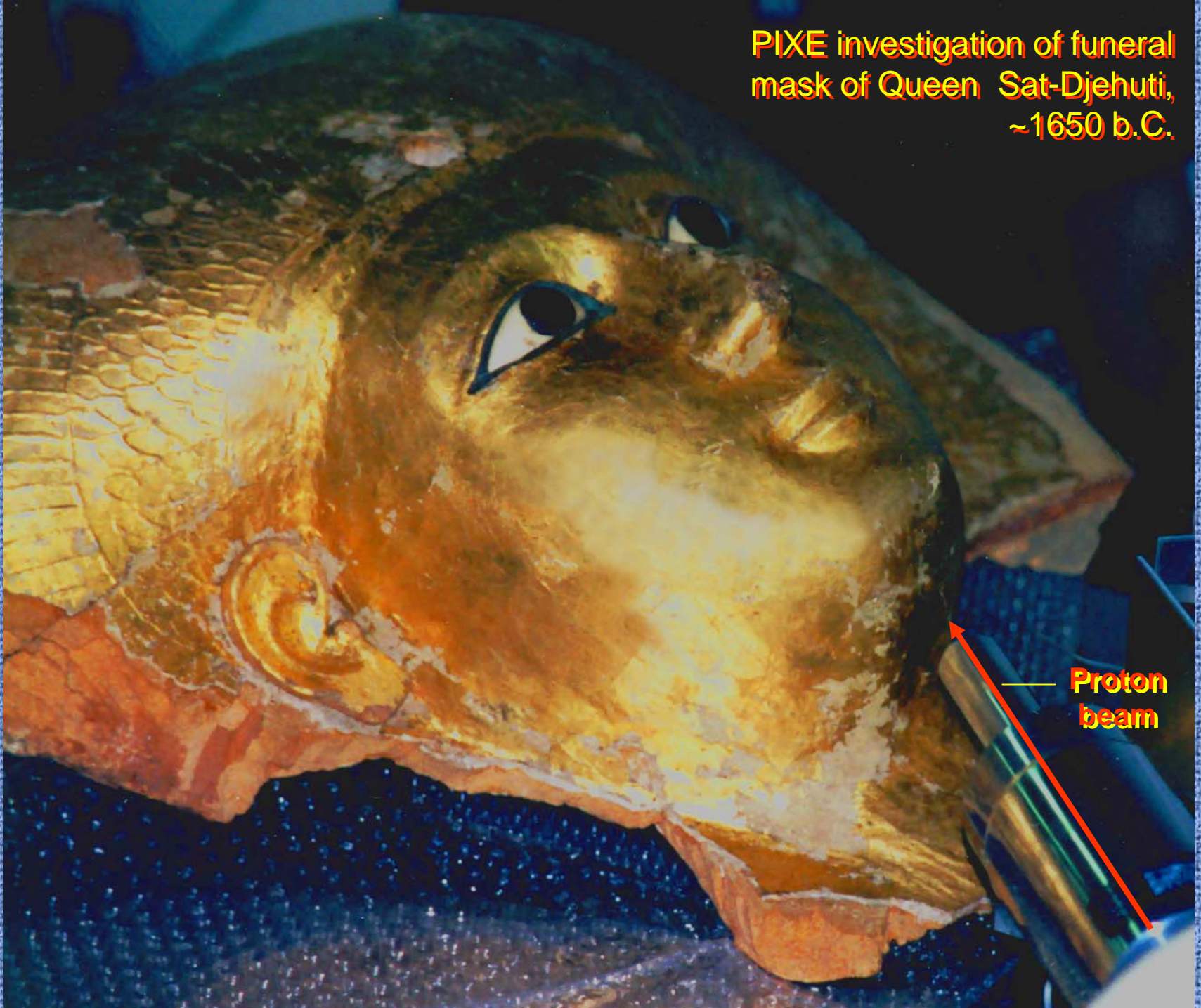
Taking an x-ray fluorescence spectrum of David

*Mapping sulphates on Michelangelo's David using portable EDXRF (from "Exploring David: diagnostic tests and state of conservation," edited by S. Bracci et al., GIUNTI, Florence, 2004)*

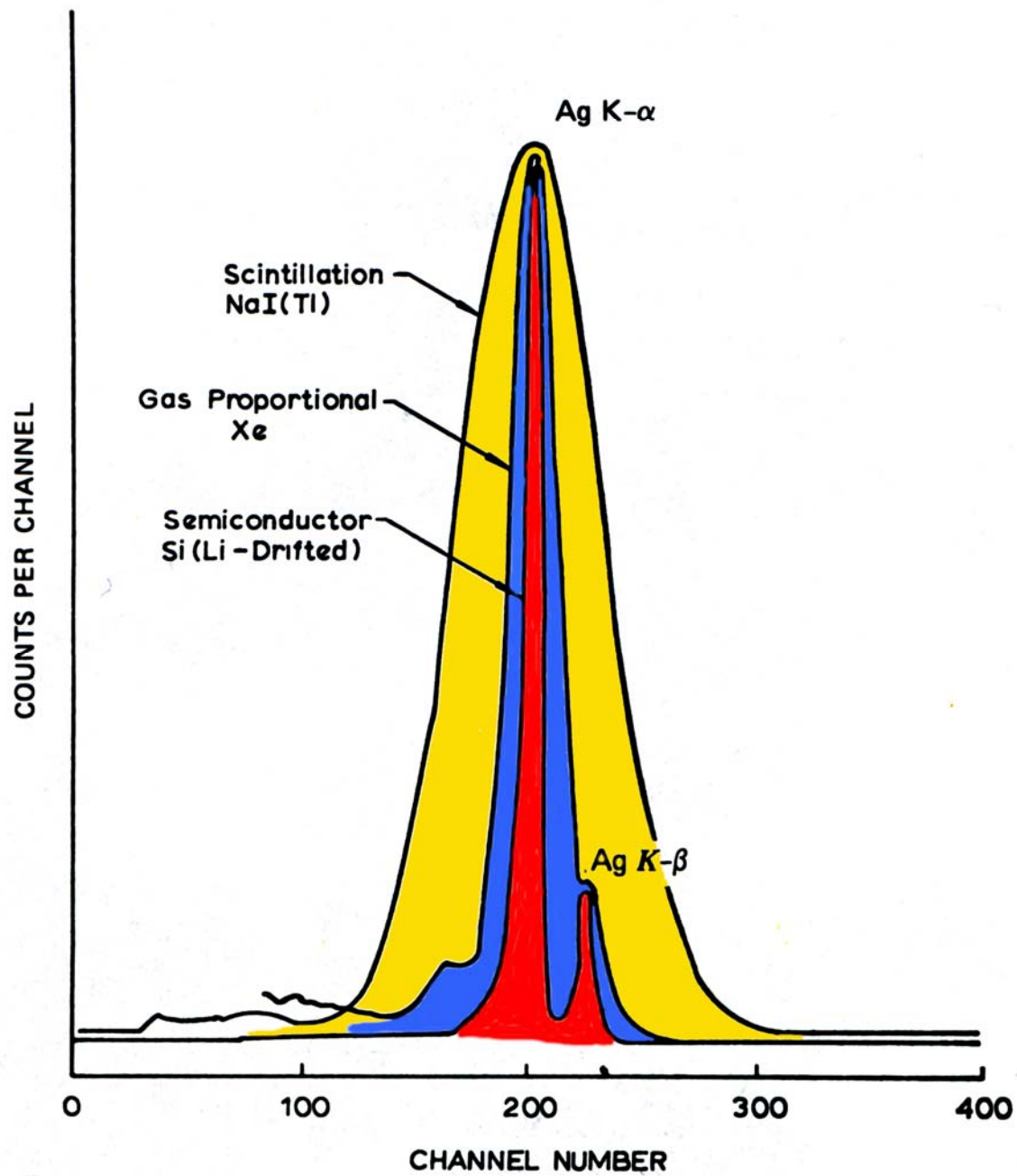
Giovanni Buccolieri, Alfredo Castellano, Marina Donativi, Stefano Quarta  
Universita di Lecce, Dipartimento di Scienza dei Materiali



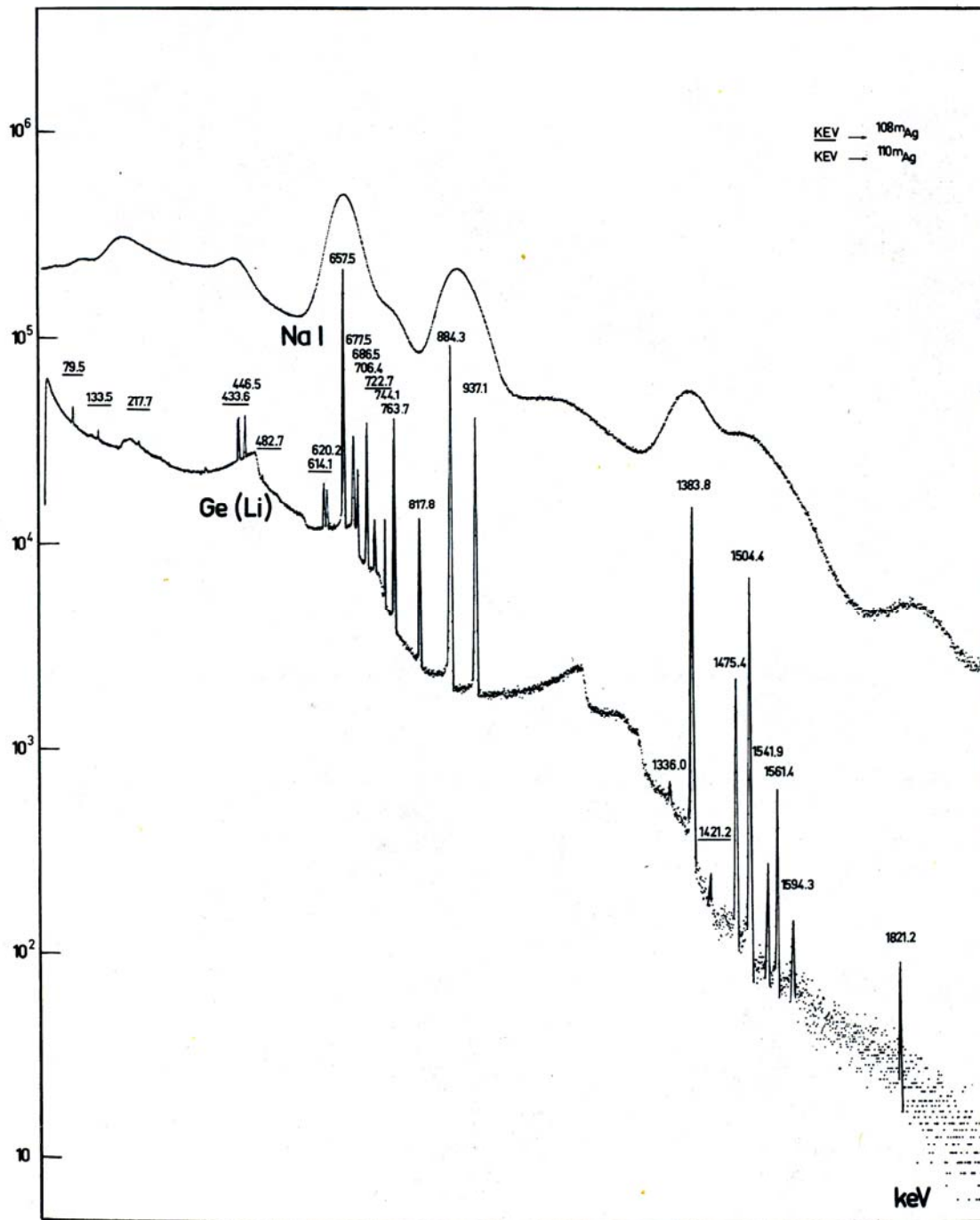
PIXE investigation of funeral  
mask of Queen Sat-Djehuti,  
~1650 b.C.

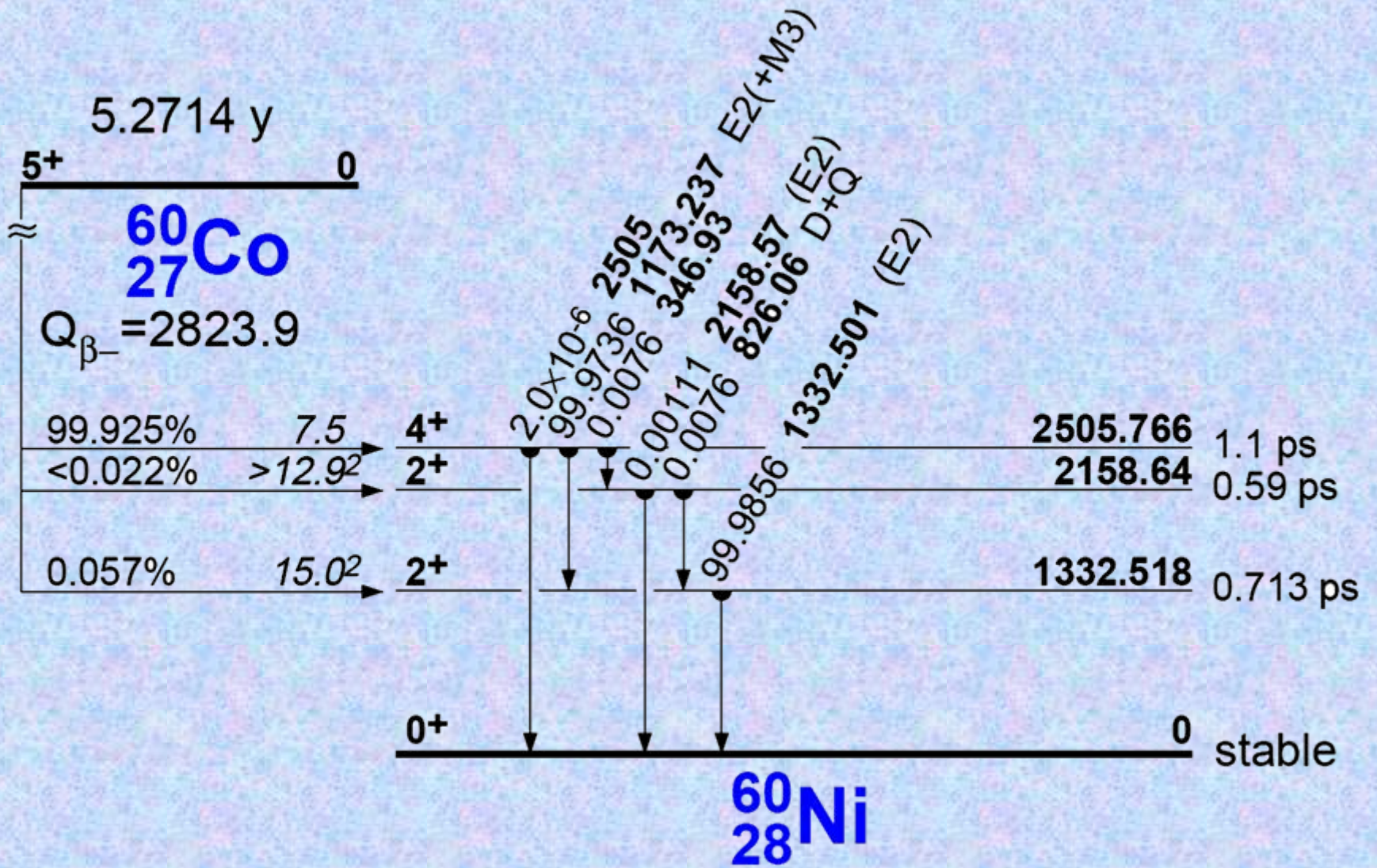




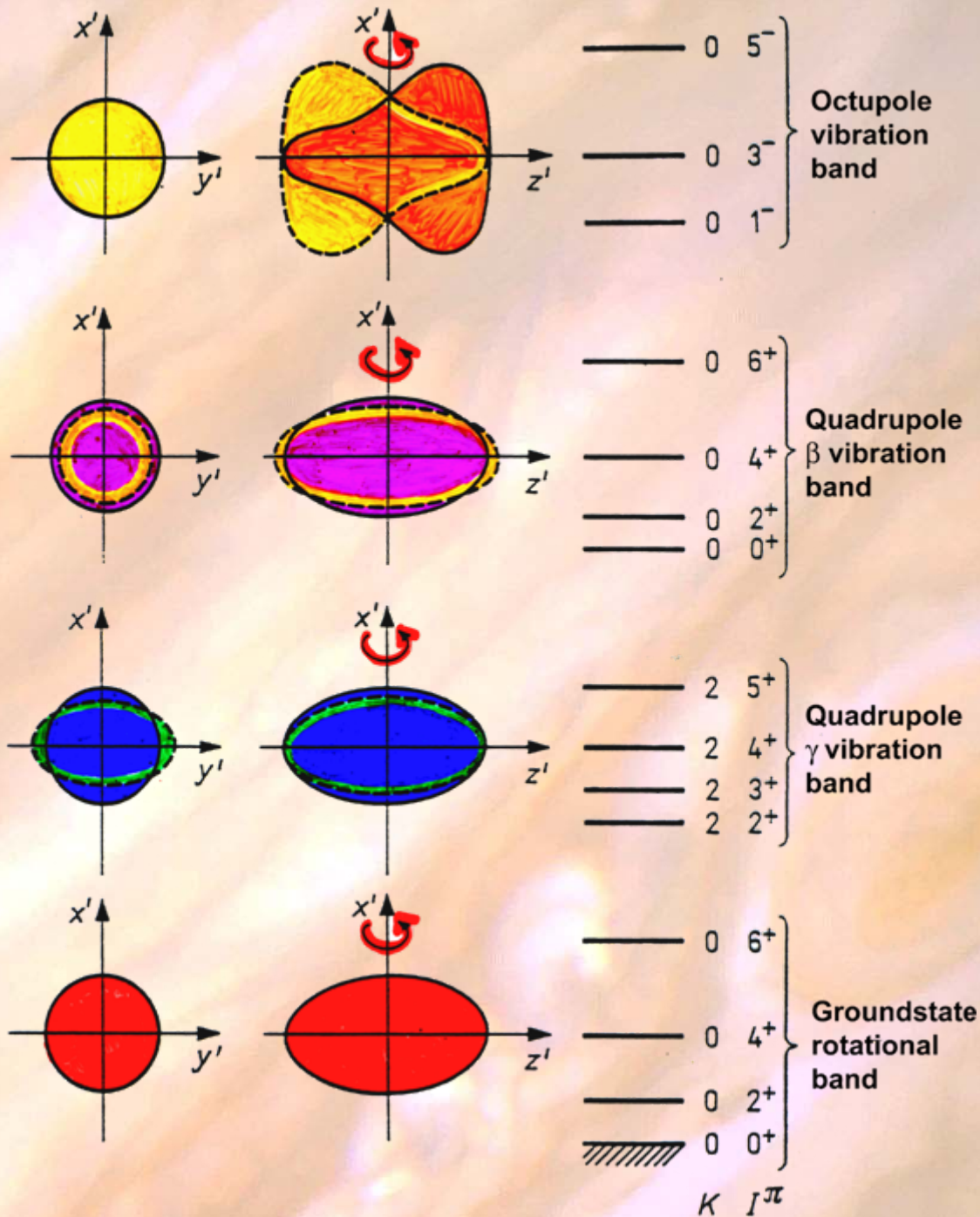






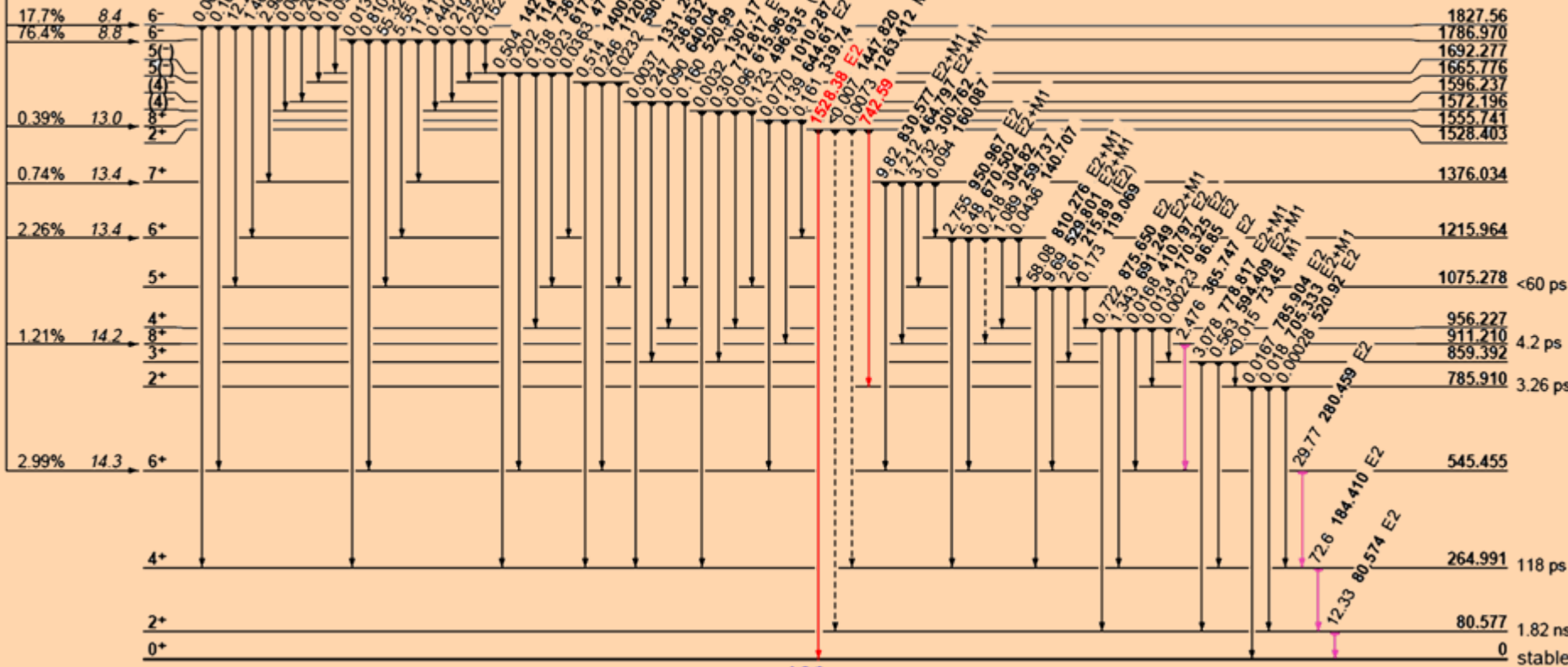


# Collective model of the nucleus (liquid drop)

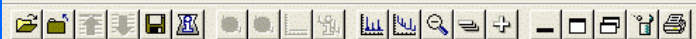




$1.20 \times 10^3$  y  
 $(T) = \frac{5.985}{0}$   
 $^{166}_{67}\text{Ho}$   
 $Q_{\beta^-} = 1854.9$



$^{166}_{68}\text{Er}$



Idle Channel: 1939 : 0.3880 MeV Counts: 2748 Preset: 100000/7035.31

Acquire

Start Stop

Expand Off

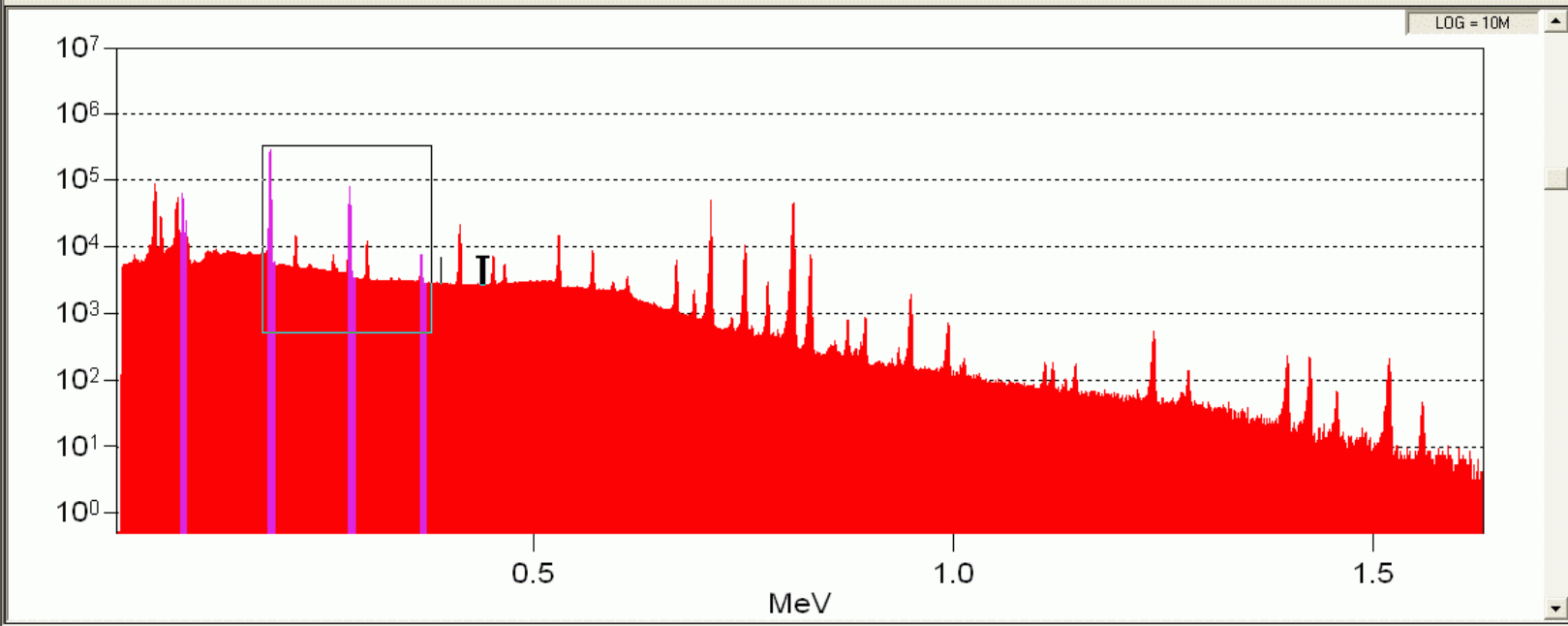
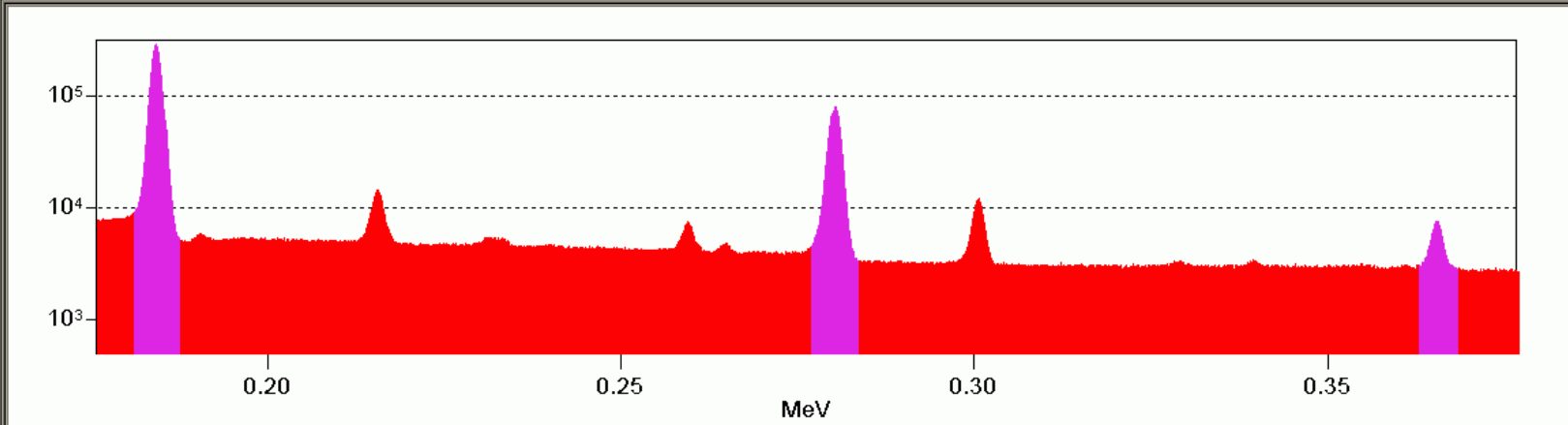
Clear

ROI Index:

- +

Datasource

Prev Next

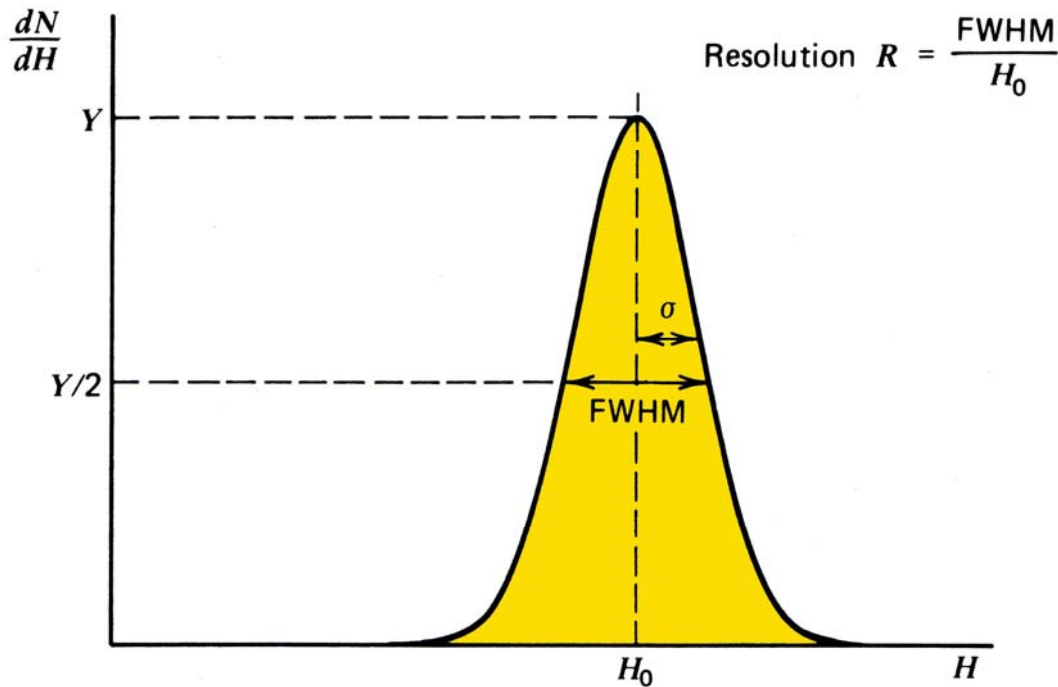


MARKER INFO

Next

Prev

<b>Left Marker:</b> 2186 : 0.4372 MeV	<b>FWHM, FWTM:</b> 0.0001, 0.0001 MeV
<b>Right Marker:</b> 2193 : 0.4385 MeV	<b>Gaussian Ratio:</b> 0.987
<b>Centroid:</b> 2189 : 0.4378 MeV	<b>ROI Type:</b>
<b>Area:</b> 30 ± 535.02%	<b>Integral:</b> 20616



$$\text{Resolution } R = \frac{\text{FWHM}}{H_0}$$

$$\text{FWHM (Gaussian shape)} = 2.35 \sigma$$

$$\text{For } \frac{\Delta E}{E} \leq 1\%$$

$$\gamma \quad N \geq 55\,000$$

$$(\text{=} 235 \times 235)$$

E corresponds to number of charge carrier pairs or photons  $\rightarrow$  number N

Poisson process: fluctuations =  $\sqrt{N}$  = standard deviation

$$\text{pulse amplitude } H_0 = K \cdot N \quad \sigma = K \cdot \sqrt{N}$$

$$\text{FWHM} = 2.35 K \cdot \sqrt{N}$$

$$R_{\text{statist}} = \frac{\text{FWHM}}{H_0} = \frac{2.35 K \cdot \sqrt{N}}{K \cdot N} = \frac{2.35}{\sqrt{N}}$$

$$\text{with correlations and FANO factor: } R_{\text{statist}} = \frac{2.35 \cdot \sqrt{F}}{\sqrt{N}}$$

$$(\text{FWHM})_{\text{total}}^2 = (\text{FWHM})_{\text{statist}}^2 + (\text{FWHM})_{\text{noise}}^2 + (\text{FWHM})_{\text{drift}}^2$$

FANO factor = correlation effect

Poisson statistics is not valid if  $\epsilon$

is mostly the same in all charge

carrier production processes

$$F = \frac{\text{observed variance in } N}{\text{Poisson predicted variance}}$$



	energy [MeV]	charge carrier-pairs, resp. photoelectrons	statistical error rms	full width half maximum [%]
<b>semiconductor detector</b> $\epsilon = 3\text{eV} / \text{e}^- \text{-hole-pair}$ Fano-factor 0.1 – 0.16	3.0	$10^6$	400	0.09
	1.0	$3.3 \times 10^5$	230	0.16
	0.3	$10^5$	127	0.30
	0.1	$3.3 \times 10^4$	73	0.52
<b>gas proportional counter</b> $\epsilon = 30\text{eV} / \text{ion-pair}$ Fano-factor ~ 0.33	3.0	$10^5$	183	0.43
	1.0	$3.3 \times 10^4$	106	0.75
	0.3	$10^4$	58	1.36
	0.1	$3.3 \times 10^3$	33	2.35
<b>scintillation detector</b> $\epsilon = 300\text{eV} / \text{photo-electron}$ no Fano-factor !	3.0	$10^4$	100	2.35
	1.0	$3.3 \times 10^3$	60	4.08
	0.3	$10^3$	30	7.40
	0.1	$3.3 \times 10^2$	15	13.0