The Joint Institute for Nuclear Astrophysics

JINA

An Introduction to Ion-Optics

Series of Five Lectures
JINA, University of Notre Dame
Sept. 30 – Dec. 9, 2005

Georg P. Berg

The Lecture Series

1st Lecture: 9/30/05, 2:00 pm: Definitions, Formalism, Examples

2nd Lecture: 10/7/05, 2:00 pm: Ion-optical elements, properties & design

3rd Lecture: 10/14/05, 2:00 pm: Real World Ion-optical Systems

4th Lecture: 12/2/05, 2:00 pm: Separator Systems

5th Lecture: 12/9/05, 2:00 pm: Demonstration of Codes (TRANSPORT, COSY, MagNet)

1st Lecture

1st Lecture: 9/30/05, 2:00 – 3:30 pm: Definitions, Formalism, Examples

- Motivation, references, remarks (4 - 8)
- The driving forces (9)
- Definitions & first order formalism (10 - 16)
- Phase space ellipse, emittance, examples (17 - 25)
- Taylor expansion, higher orders (26 - 27)
- The power of diagnostics (28 - 30)
- Outlook 2nd Lecture: sample magnetic elements (31-34)
- Q & A

Motivation

- Manipulate charged particles ($\beta^{+/-}$, ions, like p,d,α, …)
- Beam lines systems
- Magnetic & electric analysis/ separation (e.g. St. George)
- Acceleration of ions

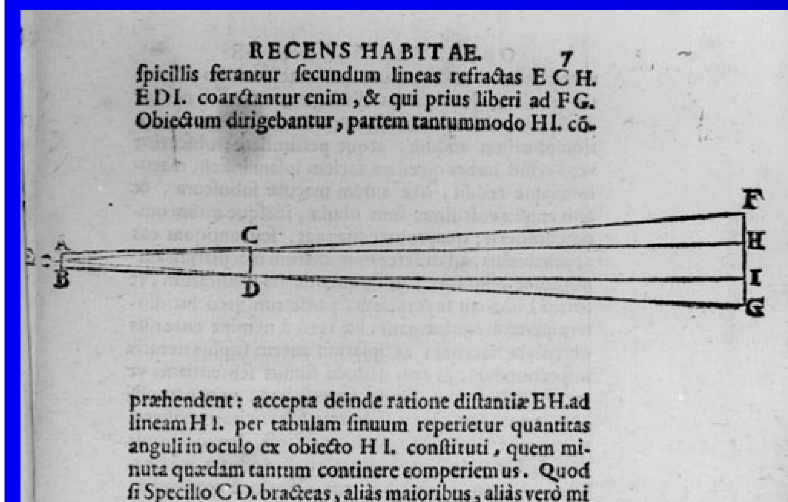
Who needs ion-optics anyway?

- Over **$6*10^9$ people** have - I hope so - happy lives without!
- A group of **accelerator physicists** are using it to build machines that enables physicists to explore the unkown!
- **Many physicists** using accelerators, beam lines and magnet system (or their data) needs some knowledge of ion-optics.
- This lecture series is an **introduction** to the last group and I will do my best to let you in on the basics first and than we will discuss some of the applications of ion-optics and related topics.

Introductory remarks

- Introduction for physicists → Focus on ion-optical definitions, and tools that are useful for **physicist** at the NSL & future users of St. George recoil separator.
- Light optics can hardly be discussed without **lenses** & optical instruments → ion-optics requires knowledge of **ion-optical elements**.
- **Analogy** between Light Optics and Ion-Optics is useful but limited.
- **Ion-optical & magnet design** tools needed to understand electro-magnet systems. Ion-optics is not even 100 years old and (still) less intuitive than optics developed since several hundred years

Historical remarks: Rutherford, 1911, discovery of atomic nucleus



RECENS HABITAE. 7

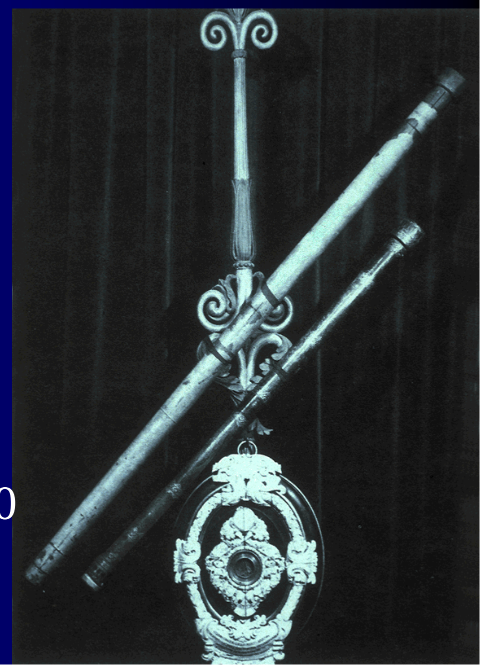
ſpicillis ferantur ſecundum lineas refractas E C H. E D I. coarctantur enim, & qui prius liberi ad F G. Obiectum dirigebantur, partem tantummodo H I. cō-

præhendent: accepta deinde ratione diſtantiæ E H. ad lineam H I. per tabulam ſinuum reperietur quantitas anguli in oculo ex obiecto H I. conſtituti, quem minuta quædam tantum continere comperiemus. Quod ſi Specillo C D. bracteas, aliàs maioribus, aliàs verò mi

Galileo

Telescope 1609 →

← Optics in Siderus Nuncius 1610



Basic tools of the trade

- Geometry, drawing tools, CAD drafting program (e.g. AutoCad)

- Linear Algebra (Matrix calculations), first order ion-optics (e.g. TRANSPORT)

- Higher order ion-optics code to solve equation of motion, (e.g. COSY Infinity, GIOS, RAYTRACE (historic)

- Electro-magnetic field program (solution of Maxwell's Equations), (e.g. finite element (FE) codes, 2d & 3d: POISSON, TOSCA, MagNet)

- Properties of incoming charged particles and design function of electro-magnetic facility, beam, reaction products (e.g. kinematic codes, charge distributions of heavy ions, energy losses in targets)

- Many other specialized programs, e.g for accelerator design (e.g. synchrotrons, cyclotrons) not covered in this lecure series.

Literature

- Optics of Charged Particles, Hermann Wollnik, Academic Press, Orlando, 1987

- The Optics of Charged Particle Beams, David.C Carey, Harwood Academic Publishers, New York 1987
- Accelerator Physics, S.Y. Lee, World Scientific Publishing, Singapore, 1999

- TRANSPORT, A Computer Program for Designing Charged Particle Beam Transport Systems, K.L. Brown, D.C. Carey, Ch. Iselin, F. Rotacker, Report CERN 80-04, Geneva, 1980

- Computer-Aided Design in Magnetics, D.A. Lowther, P. Silvester, Springer 1985

Ions in static or quasi-static electro-magnetic fields

Lorentz Force

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \quad (1)$$

Electric force — Magnetic force

q = electric charge

B = magn. induction

E = electric field

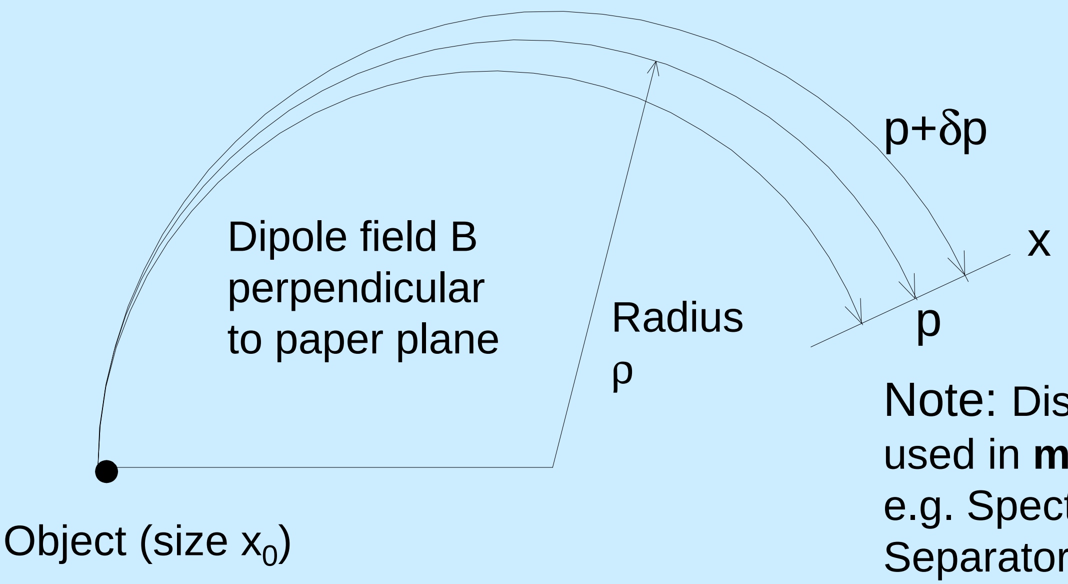
v = velocity

For ion **acceleration electric** forces are used.

For **momentum analysis** the magnetic force is preferred be[illegible] the force is always perpendicular to **B**. Therefore v, p and E are co[illegible]

Force in magnetic dipole B = const: $p = q B \rho$

p = mv [illegible]
ρ = bendi[illegible]
Bρ = magn. [illegible]



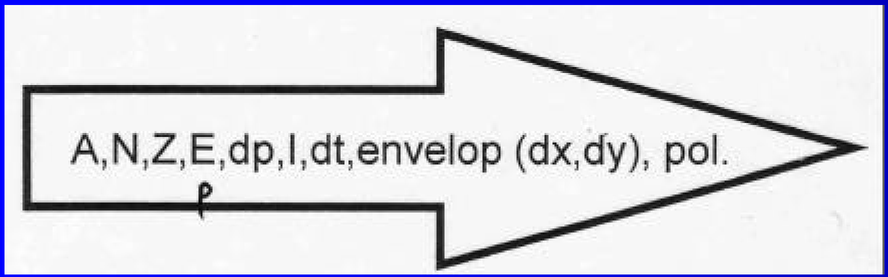
General rule:
Scaling of magnetic[illegible]
in the linear region re[illegible]
in the **same ion-optic**[illegible]

Note: Dispersion δx/δp used in **magnetic analysis**, e.g. Spectrometers, magn. Separators,

Definition of BEAM for mathematical formulation of ion-optics

What is a beam, what shapes it,
how do we know its properties ?

- Beam parameters, the long list
- Beam rays and distributions
- Beam line elements, paraxial lin. approx.
  higher orders in spectrometers
- System of diagnostic instruments



Not to forget: Atomic charge Q
Number of particles n

Defining a RAY

Code TRANSPORT:

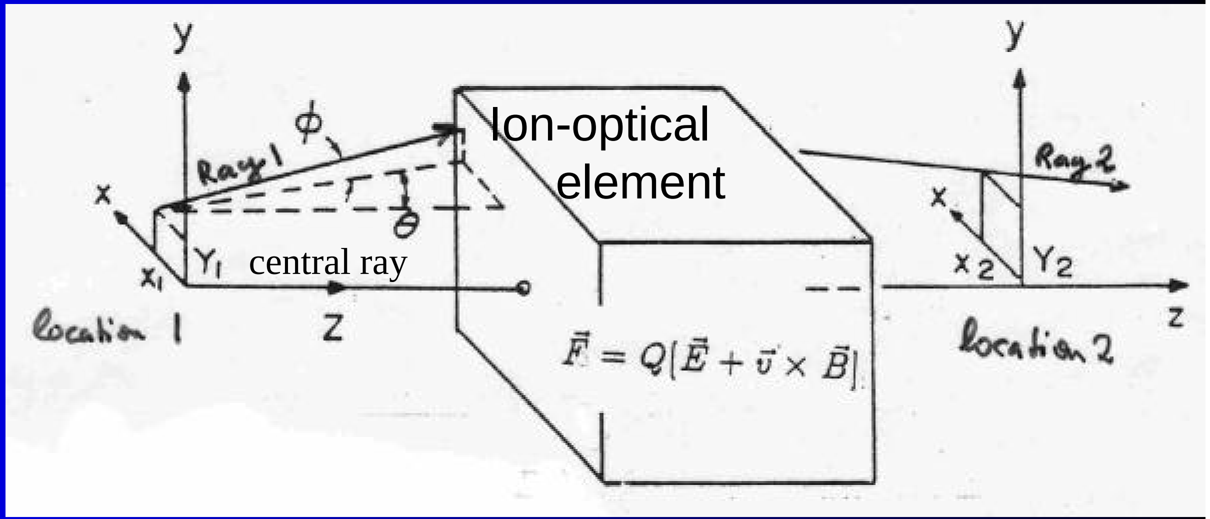
(x, Θ, y, Φ, 1, dp/p)
(1, 2, 3, 4, 5, 6)

Convenient "easy to use" program for beam lines with paraxial beams

Not defined in the figure are:

dp/p = rel. momentum
l = beam pulse length

All parameters are relative to "central ray"



Code: COSY Infinity:

(x, p_x/p_0, y, p_y/p_0, l, dK/K, dm/m, dq)

Needed for complex ion-optical systems including several
- charge states
- different masses
- velocities (e.g. Wien Filter)
- higher order corrections

Not defined in the figure are:

dK/K = rel. energy
dm/m = rel. energy
dq = rel. charge of ion

All parameters are relative to "central ray"

Note: Notations in the Literature is not consistent! Sorry, neither will I be.

TRANSPORT
Coordinate System

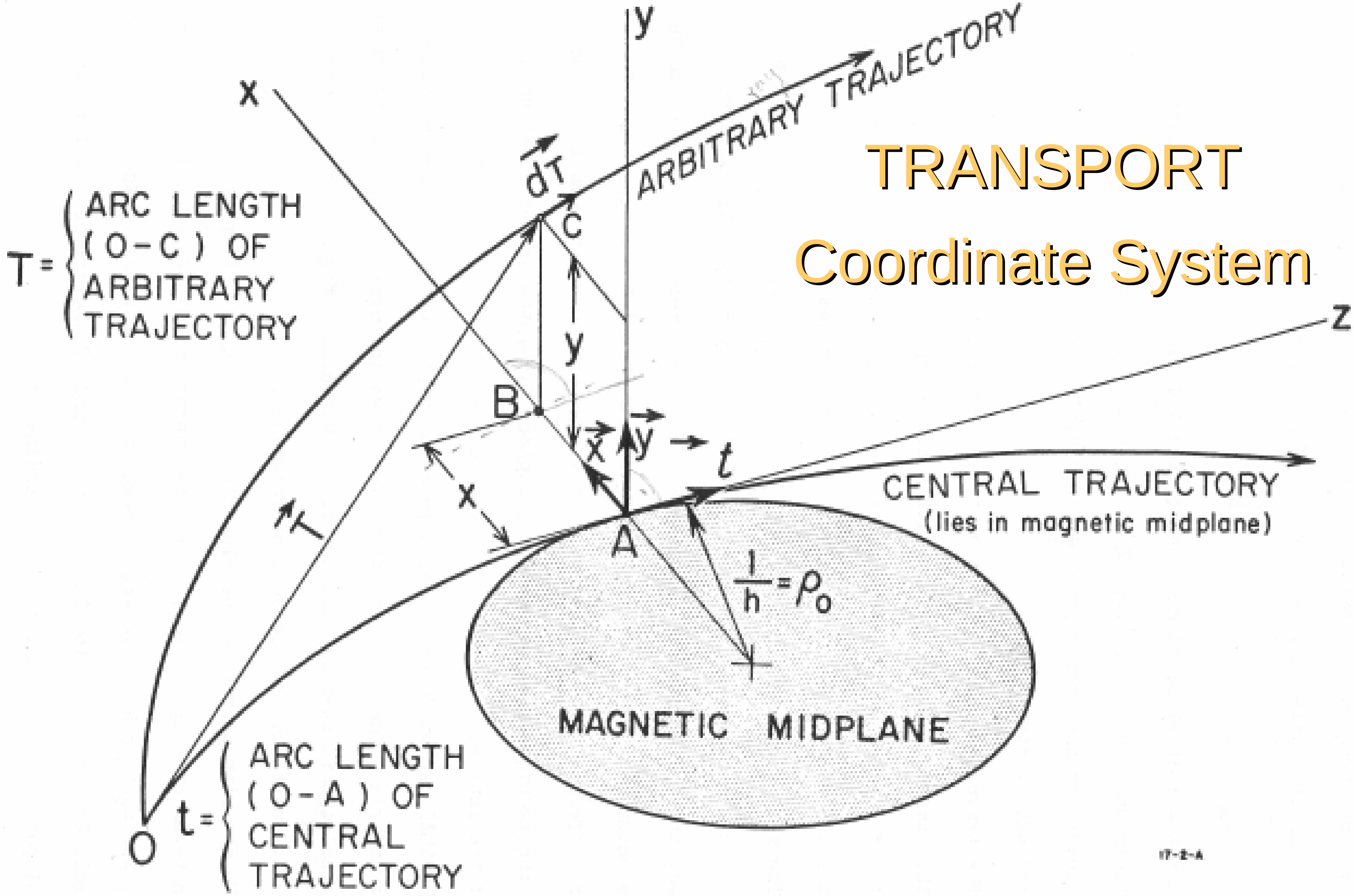
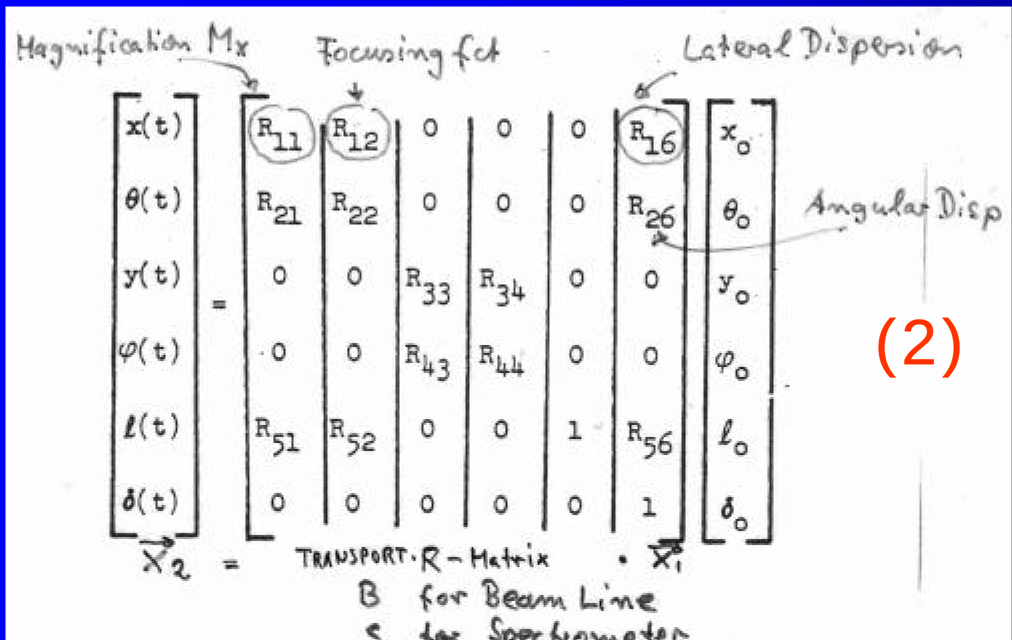


FIG. 1--CURVILINEAR COORDINATE SYSTEM USED IN DERIVATION OF EQUATIONS OF MOTION.

Transport of a ray

6x6 Matrix
representing
optic element
(first order)

↓



(2)

↑ Ray after element at Location t

↑ Ray at initial Location 0

Note: We are not building "random" optical elements. Many matrix elements = 0 because of symmetries, e.g. mid-plane symmetry

TRANSPORT matrices of a Drift and a Quadrupole

DRIFT space matrix $\vec{E} \approx \vec{B} \approx 0$

The first-order R matrix for a drift space is as follows:

$$\begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where

L = the length of the drift space.

First-order quadrupole matrix $\frac{dB}{dx} \neq 0 \quad \frac{dB}{dy} \neq 0$

$$\begin{bmatrix} \cos k_q L & \frac{1}{k_q} \sin k_q L & 0 & 0 & 0 & 0 \\ -k_q \sin k_q L & \cos k_q L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh k_q L & \frac{1}{k_q} \sinh k_q L & 0 & 0 \\ 0 & 0 & k_q \sinh k_q L & \cosh k_q L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

Definitions: L = the effective length of the quadrupole

a = the radius of the aperture

B_0 = the field at radius a

$k_q^2 = (B_0/a)(1/B\rho_0)$, where $(B\rho_0)$ = the magnetic rigidity (momentum) of the central trajectory.

For reference of TRANSPORT code and formalism:

K.L. Brown, F. Rothacker, D.C. Carey, and Ch. Iselin, TRANSPORT: A computer program for designing charged particle beam transport systems, SLAC-91, Rev. 2, UC-28 (I/A), also: CERN 80-04 Super Proton Synchrotron Division, 18 March 1980, Geneva, Manual plus Appendices available on Webpage: ftp://ftp.psi.ch/psi/transport.beam/CERN-80-04/

David. C. Carey, The optics of Charged Particle Beams, 1987, Hardwood Academic Publ. GmbH, Chur Switzerland

Transport of a ray though a system of beam line elements

6x6 Matrix
representing
first optic element
(usually a Drift)

$$x_n = R_n R_{n-1} \ldots R_0 \, x_0 \qquad (3)$$

Ray at final
Location n

Ray at initial
Location 0
(e.g. a target)

Complete system is represented by
one Matrix $R_{system} = R_n R_{n-1} \ldots R_0$ (4)

Geometrical interpretation of some TRANSPORT matrix elements

Achromatic system:
$R_{16} = R_{26} = 0$

Focusing Function

$(x\|a)$	Wollnik
$= dx/d\Theta$	physical meaning
$= (x\|\Theta)$	RAYTRACE
$= R_{12}$	TRANSPORT

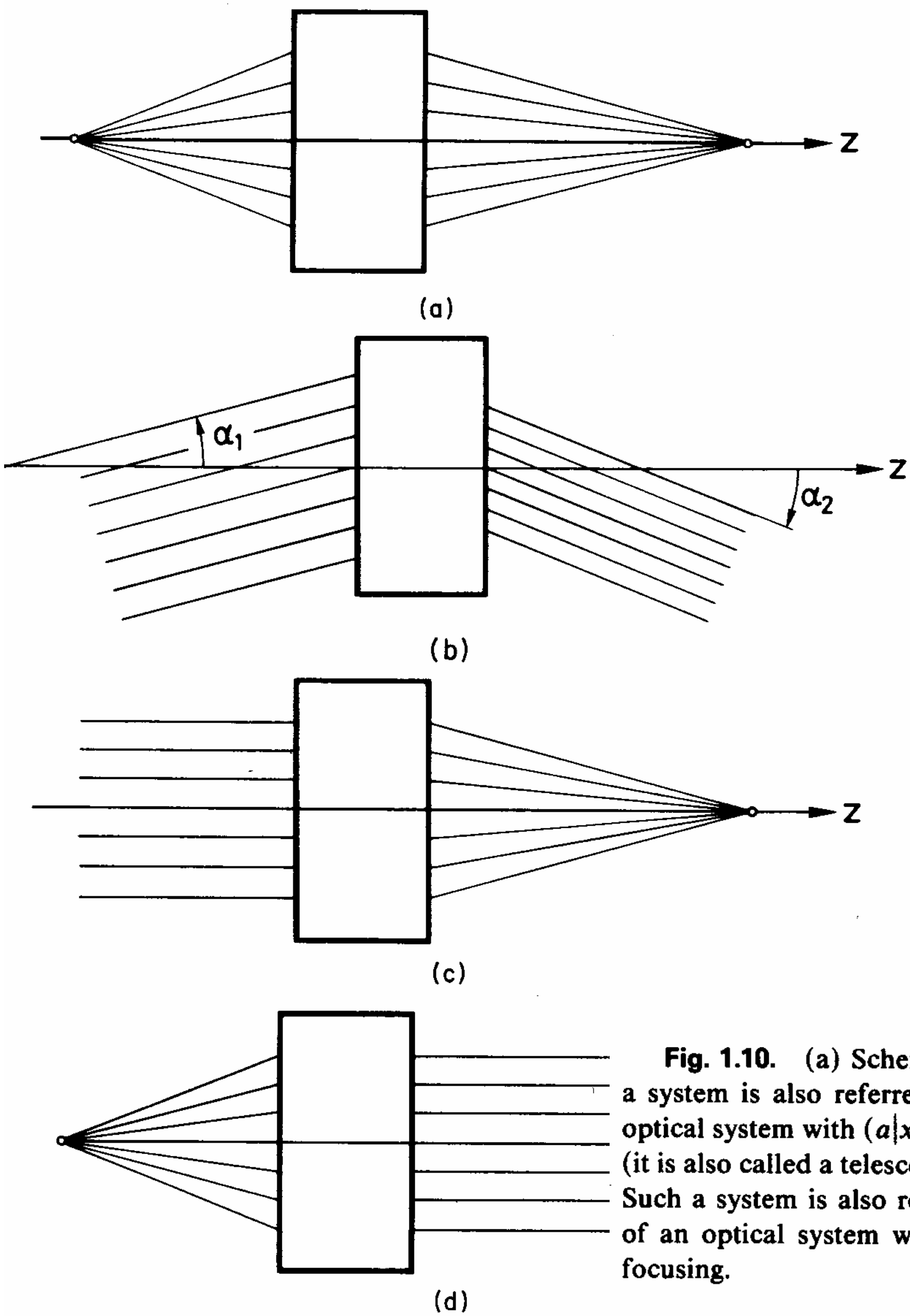
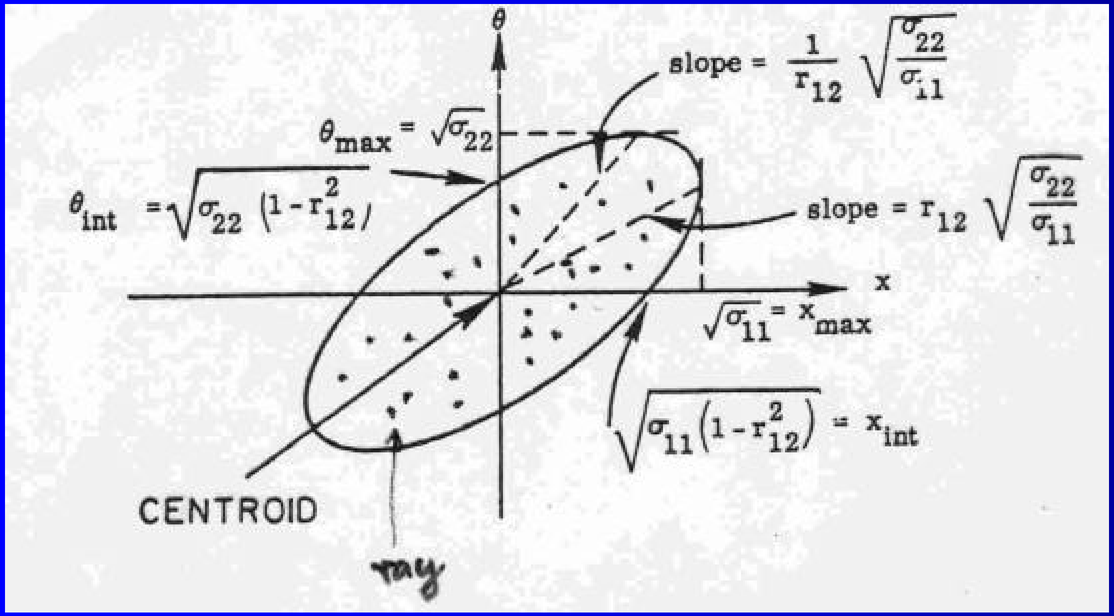


Fig. 1.10. (a) Schematic representation of an optical system with vanishing $(x|a)$. Such a system is also referred to as point-to-point focusing. (b) Schematic representation of an optical system with $(a|x) = 0$. Such a system is also referred to as parallel-to-parallel focusing (it is also called a telescope). (c) Schematic representation of an optical system with $(x|x) = 0$. Such a system is also referred to as parallel-to-point focusing. (d) Schematic representation of an optical system with $(a|a) = 0$. Such a system is also referred to as point-to-parallel focusing.

Short Break?



θ
slope = $\frac{1}{r_{12}}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
$\theta_{max} = \sqrt{\sigma_{22}}$
$\theta_{int} = \sqrt{\sigma_{22}(1-r_{12}^2)}$
slope = $r_{12}\sqrt{\frac{\sigma_{22}}{\sigma_{11}}}$
x
$\sqrt{\sigma_{11}} = x_{max}$
$\sqrt{\sigma_{11}(1-r_{12}^2)} = x_{int}$
CENTROID
ray

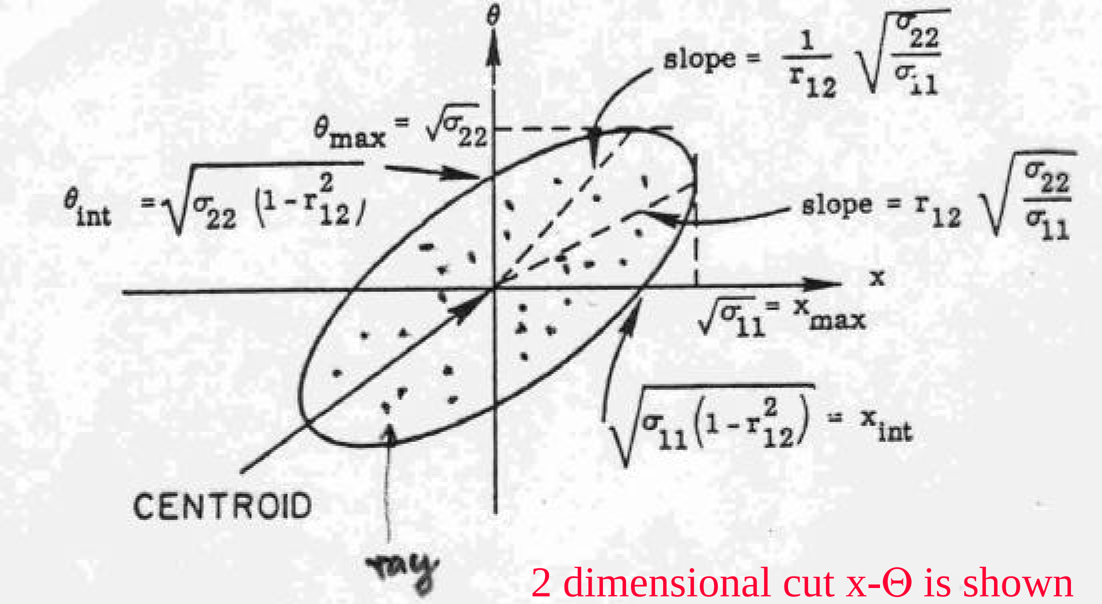
- (2 (5)

Equivalence of Transport of

ONE Ray ⇔ Ellipse

Defining the σ Matrix representing a Beam

The 2-dimensional case (x, Θ)



2 dimensional cut x-Θ is shown

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

Real, pos. definite symmetric **σ Matrix**

$$\sigma^{-1} = 1/\varepsilon^2 \begin{pmatrix} \sigma_{22} & -\sigma_{21} \\ -\sigma_{21} & \sigma_{11} \end{pmatrix}$$

Inverse Matrix

Exercise 1:
Show that: $\sigma\sigma^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$ (Unity Matrix)

Ellipse Area = $\pi(\det \sigma)^{1/2}$

Emittance $\varepsilon = \det \sigma$ is constant for fixed energy & conservative forces (Liouville's Theorem)

Note: ε shrinks (increases) with acceleration (deceleration); Dissipative forces: ε increases in gases; electron, stochastic, laser cooling

2-dim. Coord.vectors (point in phase space)

$$X = \begin{pmatrix} x \\ \Theta \end{pmatrix} \qquad X^T = (x\ \Theta)$$

Ellipse in Matrix notation:

$$X^T \sigma^{-1} X = 1 \qquad (6)$$

Exercise 2: Show that Matrix notation is equivalent to known Ellipse equation:

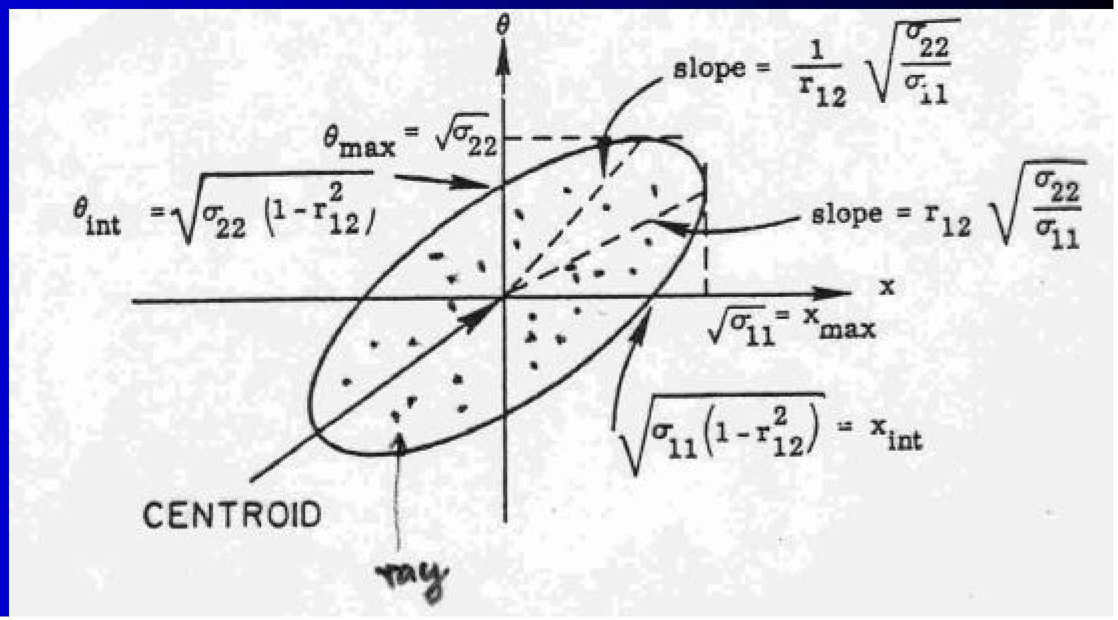
$$\sigma_{22}\, x^2 - 2\sigma_{21}\, x\,\Theta + \sigma_{11}\Theta^2 = \varepsilon^2$$

Courant-Snyder Notation

In their famous "Theory of the Alternating Synchrotron" Courant and Snyder used a Different notation of the σ Matrix Elements, that are used in the Accelerator Literature.

For you r future venture into accelerator physics here is the relationship between the σ matrix and the betatron amplitue functions α, β, γ or Courant Snyder parameters

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$



Transport of 6-dim σ Matrix

Consider the 6-dim. ray vector in TRANSPORT: $X = (x, \Theta, y, \Phi, l, dp/p)$

Ray X_0 from location 0 is transported by a 6 x 6 Matrix R to location 1 by: $X_1 = RX_0$ (7)

Note: R maybe a matrix representing a complex system (3) is : $R = R_n R_{n-1} \ldots R_0$

Ellipsoid in Matrix notation (6), generized to e.g. 6-dim. using σ Matrix: $X_0^T \sigma_0^{-1} X_0 = 1$ (6)

Inserting Unity Matrix $I = RR^{-1}$ in equ. (6) it follows $X_0^T (R^T R^{T-1}) \sigma_0^{-1} (R^{-1} R) X_0 = 1$

from which we derive $(RX_0)^T (R\sigma_0 R^T)^{-1} (RX_0) = 1$ (8)

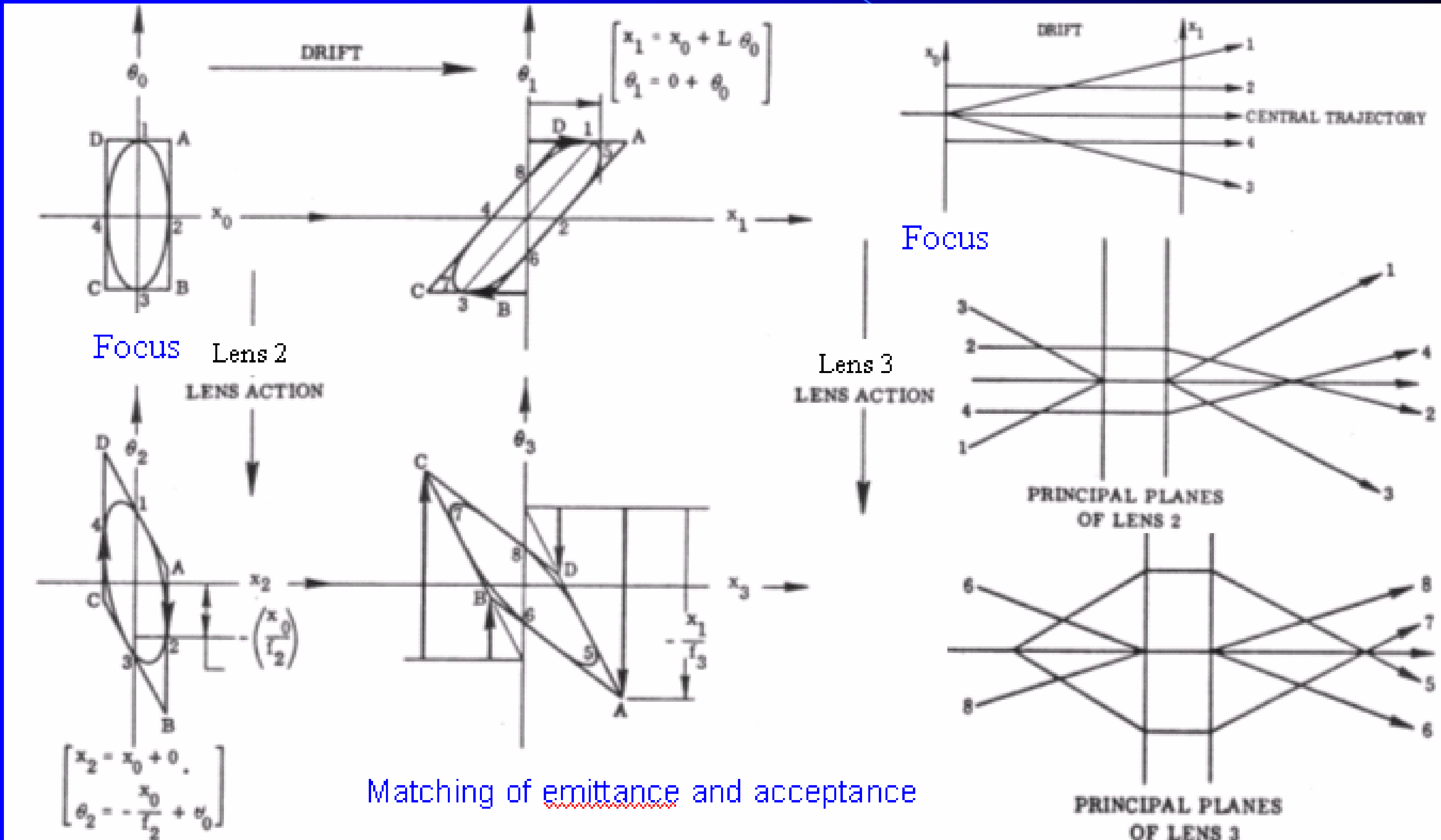
The equation of the **new ellipsoid after transformation** becomes $X_1^T \sigma_1^{-1} X_1 = 1$ (9)

where $\sigma_1 = R\sigma_0 R^T$ (10)

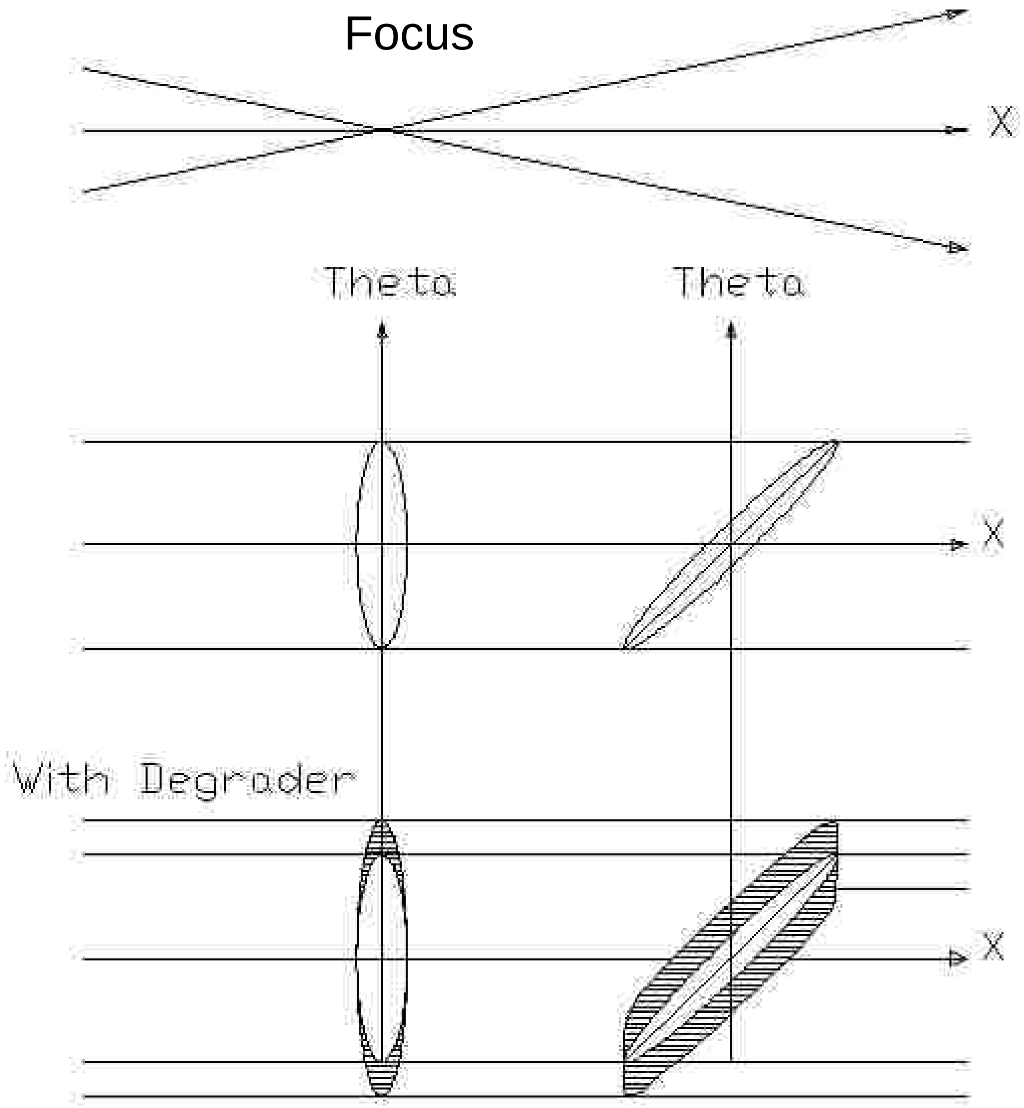
Conclusion: Knowing the TRANSPORT matrix R that transports one ray through an ion-optical system using **(7)** we can now also transport the phase space ellipse describing the initial beam using **(10)**

The transport of rays and phase ellipses

in a Drift and focusing Quadrupole, Lens



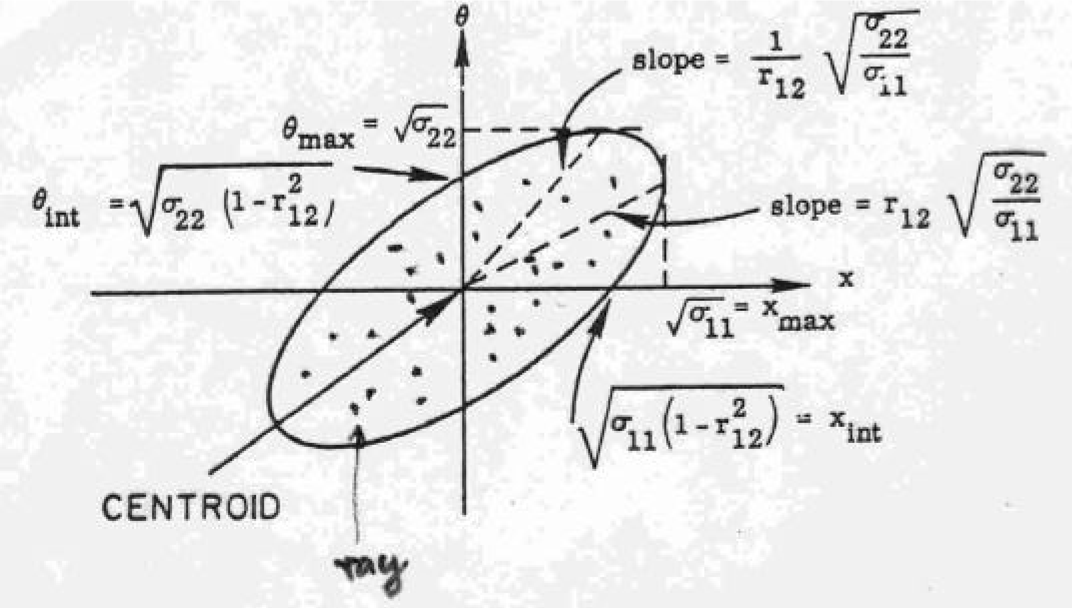
Increase of Emittance ε due to degrader



for back of the envelop discussions!

A degrader / target increases the emittance ε due to multiple scattering.

The emittance growth is minimal when the degrader in positioned in a focus As can be seen from the schematic drawing of the horizontal x-Theta Phase space.



Emittance ε measurement by tuning a quadrupole

The emittance ε is an important parameter of a beam. It can be measured as shown below.

$$x_{max} = \sigma_{11} (1 + \sigma_{12} L/\sigma_{11} - L g) + (\varepsilon L)^2/\sigma_{22}$$

$$g = \frac{\partial Bz/\partial x * l}{B\rho}$$

(Quadr. field strength
l = eff. field length)

L = Distance between quadrupole and beam profile monitor

Take minimum 3 measurements of $x_{max}(g)$ and determine Emittance ε

Exercise 3:

In the accelerator reference book σ_{22} is printed as σ_{11}

Verify which is correct

Emittance ε measurement by moving viewer method

Viewer V1 V2 V3

Beam —|———|———|——→

L_1 L_2

The emittance ε can also be measured in a drift space as shown below.

L = Distances between viewers
(beam profile monitors)

$$(x_{max}(V2))^2 = \sigma_{11} + 2\,L_1\,\sigma_{12} + L_1^{\,2}\,\sigma_{22}$$

$$(x_{max}(V3))^2 = \sigma_{11} + 2\,(L_1 + L_2)\sigma_{12} + (L_1 + L_2)^2\,\sigma_{22}$$

where $\sigma_{11} = (x_{max}(V1))^2$

Emittance: $\varepsilon = \sqrt{\sigma_{11}\sigma_{22} - (\sigma_{12})^{2}}$

Discuss practical aspects
No ellipse no ε? Phase space!

Taylor expansion

Taylor expansion in $x_1, \theta_1, y_1, \phi_1$, and δ

R_{11} R_{12} R_{16}

$$x_2 = (x/x)x_1 + (x/\theta)\theta_1 + (x/\delta)\delta + (x/x^2)x_1^2$$
$$+ (x/x\theta)x_1\theta_1 + (x/\theta^2)\theta_1^2 + (x/x\delta)x_1\delta$$
$$+ (x/\theta\delta)\theta_1\delta + (x/\delta^2)\delta^2 + (x/y^2)y_1^2 + (x/y\phi)y_1\phi_1$$
$$+ (x/\phi^2)\phi_1^2 + \text{higher order terms}$$

e.g. Transfer coeff. $R_{11} = (x/x) = \frac{\partial x_2}{\partial x_0}$, Magnification

$R_{16} = (x/\delta) = \frac{\partial x_2}{\partial \delta}$, Lateral Dispersion

Higher orders: e.g. $(x/\theta^2) = T_{122} = \frac{\partial x_2}{\partial \theta \partial \theta}$

Note: Several notations are in use for 6 dim. ray vector & matrix elements.

R_{nm}	=	$(n\|m)$
TRANSPORT		RAYTRACE
	Notation	

Linear (1st order) TRANSPORT Matrix R_{nm}

Magnification M_x Focusing fct Lateral Dispersion Angular Disp

$$\begin{bmatrix} x(t) \\ \theta(t) \\ y(t) \\ \varphi(t) \\ \ell(t) \\ \delta(t) \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ \theta_0 \\ y_0 \\ \varphi_0 \\ \ell_0 \\ \delta_0 \end{bmatrix}$$

$\vec{X}_2$ = TRANSPORT-R-Matrix $\cdot \vec{X}_1$

B for Beam Line

S for Spectrometer

Remarks:

- Midplane symmetry of magnets reason for many matrix element = 0
- Linear approx. for “well” designed magnets and paraxial beams
- TRANSPORT code calculates 2nd order by including Tmno elements explicitly
- TRANSPORT formalism is not suitable to calculate higher order (>2).

Solving the equations of Motion

$$d(m\dot{x})/dt = Q(E_x + v_y B_z - v_z B_y)$$
$$d(m\dot{y})/dt = Q(E_y + v_z B_x - v_x B_z)$$
$$d(m\dot{z})/dt = Q(E_z + v_x B_y - v_y B_x)$$

Methods of solving the equation of motion:

1) Determine the TRANSPORT matrix, possibly including 2nd order.

2) Code RAYTRACE slices the system in small sections along the z-axis and integrates numerically the particle ray through the system.

3) Code COSY Infinity uses Differential Algebraic techniques to arbitrary orders using matrix representation for fast calculations

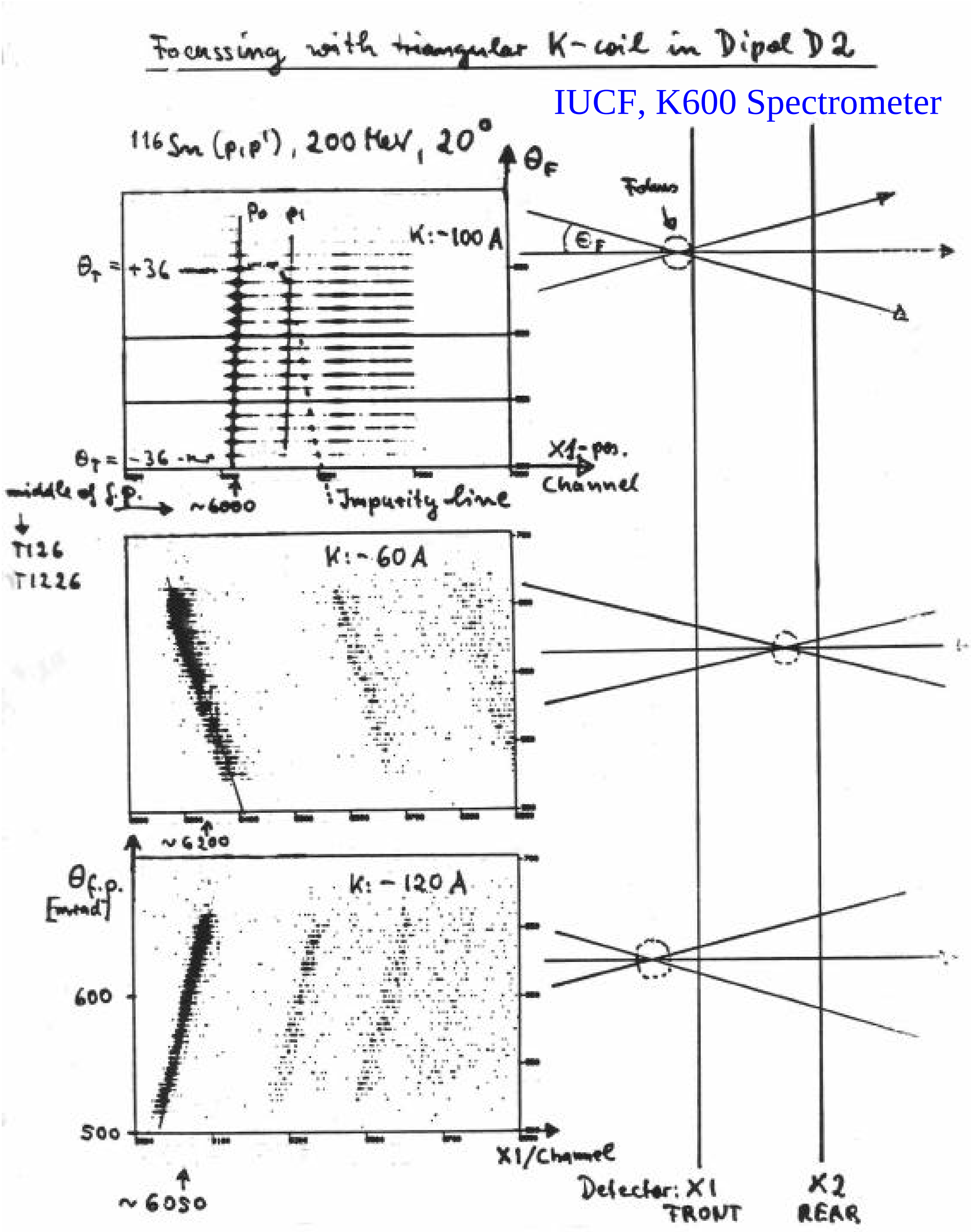
Discussion of Diagnostic Elements

Some problems:

- Range < 1 to $> 10^{12}$ particles/s
- Interference with beam, notably at low energies
- Cost can be very high
- Signal may not represent beam properties (blind viewer spot)

Some solutions:

- Viewers, scintillators with CCD readout
- Slits (movable) Faraday cups (current readout)
- Harps, electronic readout, semi- transparent
- Film (permanent record, dosimetry)
- Wire chambers (Spectrometer)
- Faint beam $10^{12} \longrightarrow 10^{3}$



Diagnostics in focal plane of spectrometer

Typical in focal plane of Modern Spectrometers:

Two position sensitive Detectors:
Horizontal: X1, X2
Vertical: Y1, Y2

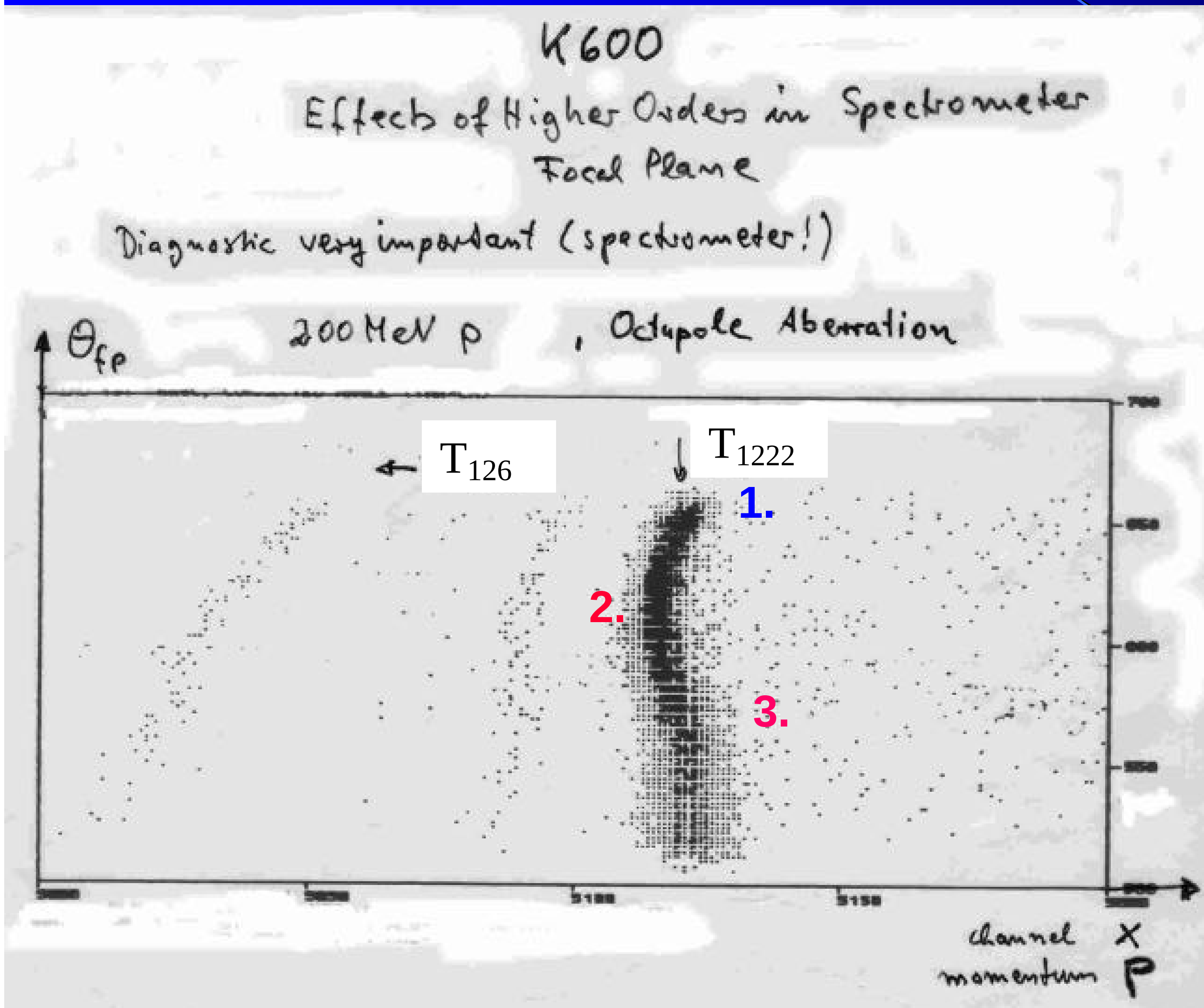
Fast plastic scintillators:
Particle identification
Time-of-Flight

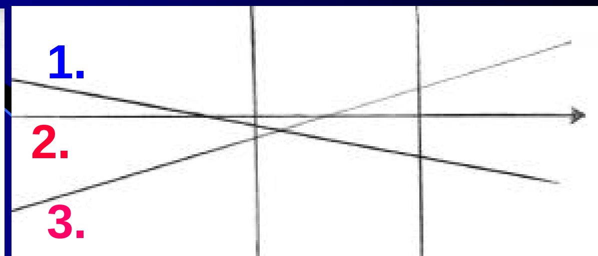
Measurement with IUCF K600 Spectrometer illustrates from top to bottom: focus near, down-stream and upstream of X1 detector, respectively

Higher order beam aberrations

Example Octupole
(S-shape in x-Θ plane

3 rays in focal plane



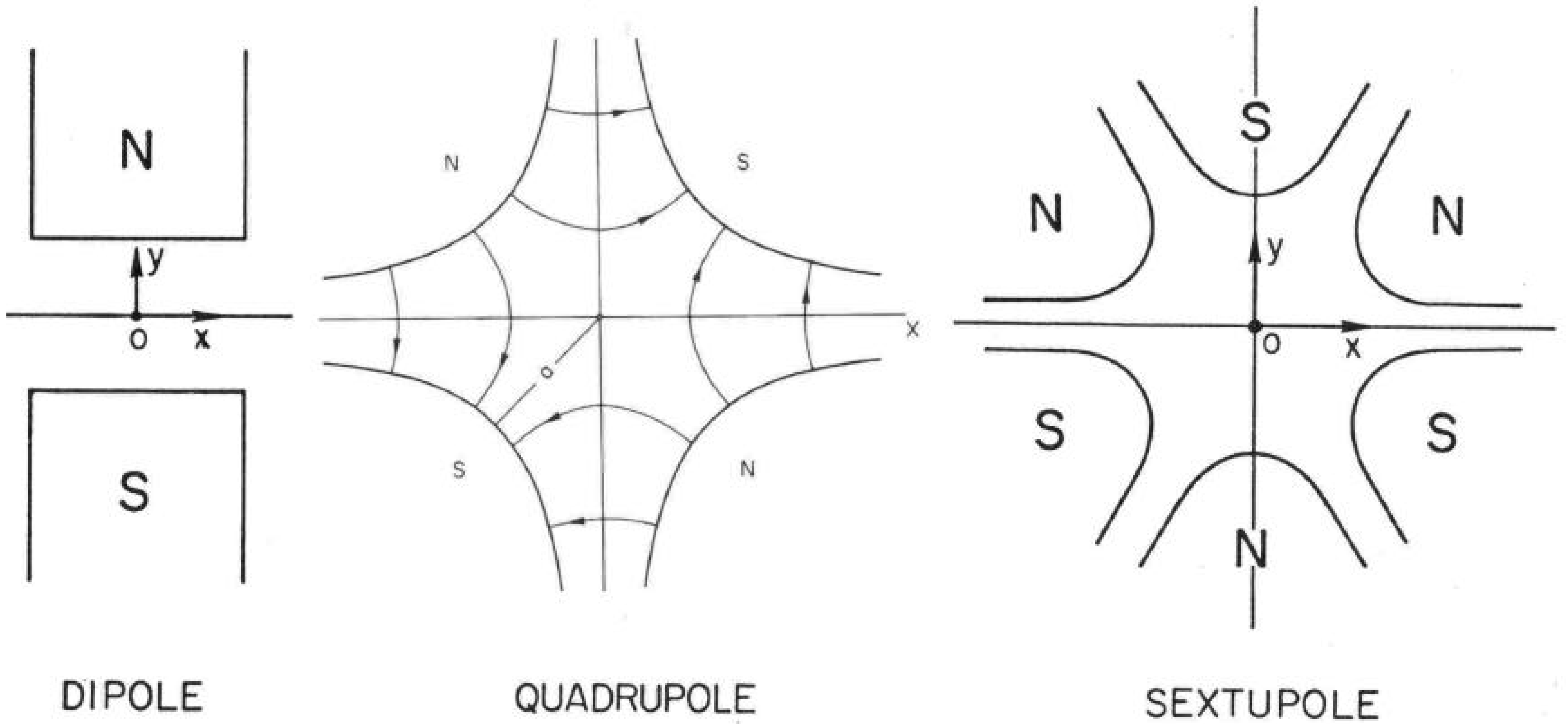


Detector X1 X2

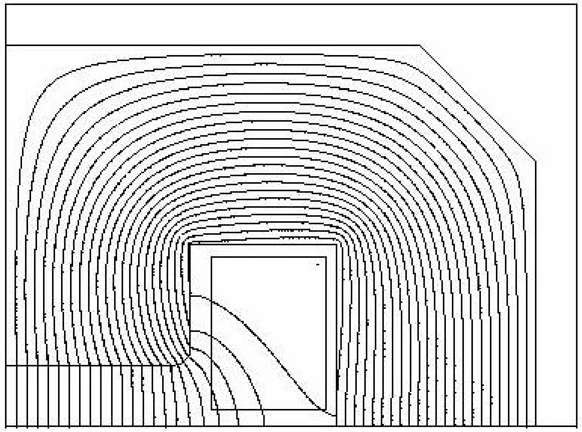
Other Example:

Sextupole T_{122}
C-shape in x-Θ plot

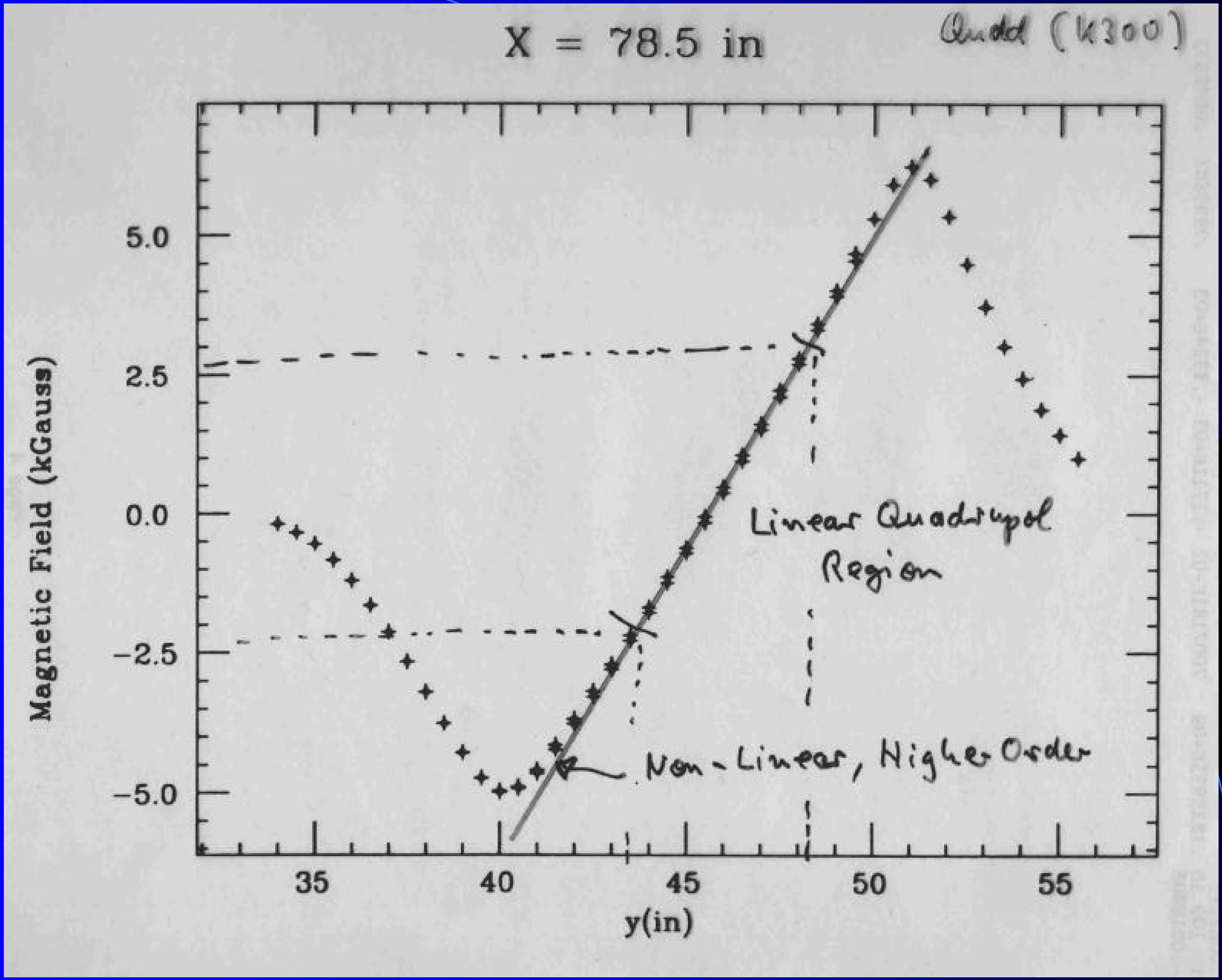
Schematic view of magnetic dipole, quadrupole, sextupole



Field lines of H-frame dipole



Map of a magnetic quadrupole in midplane with sextupole



Example: Collins Quad

Note: Magnet is Iron/Current configuration with field as needed in ion-optical design. 2d/3d finite elements codes solving POISSON equation are well established

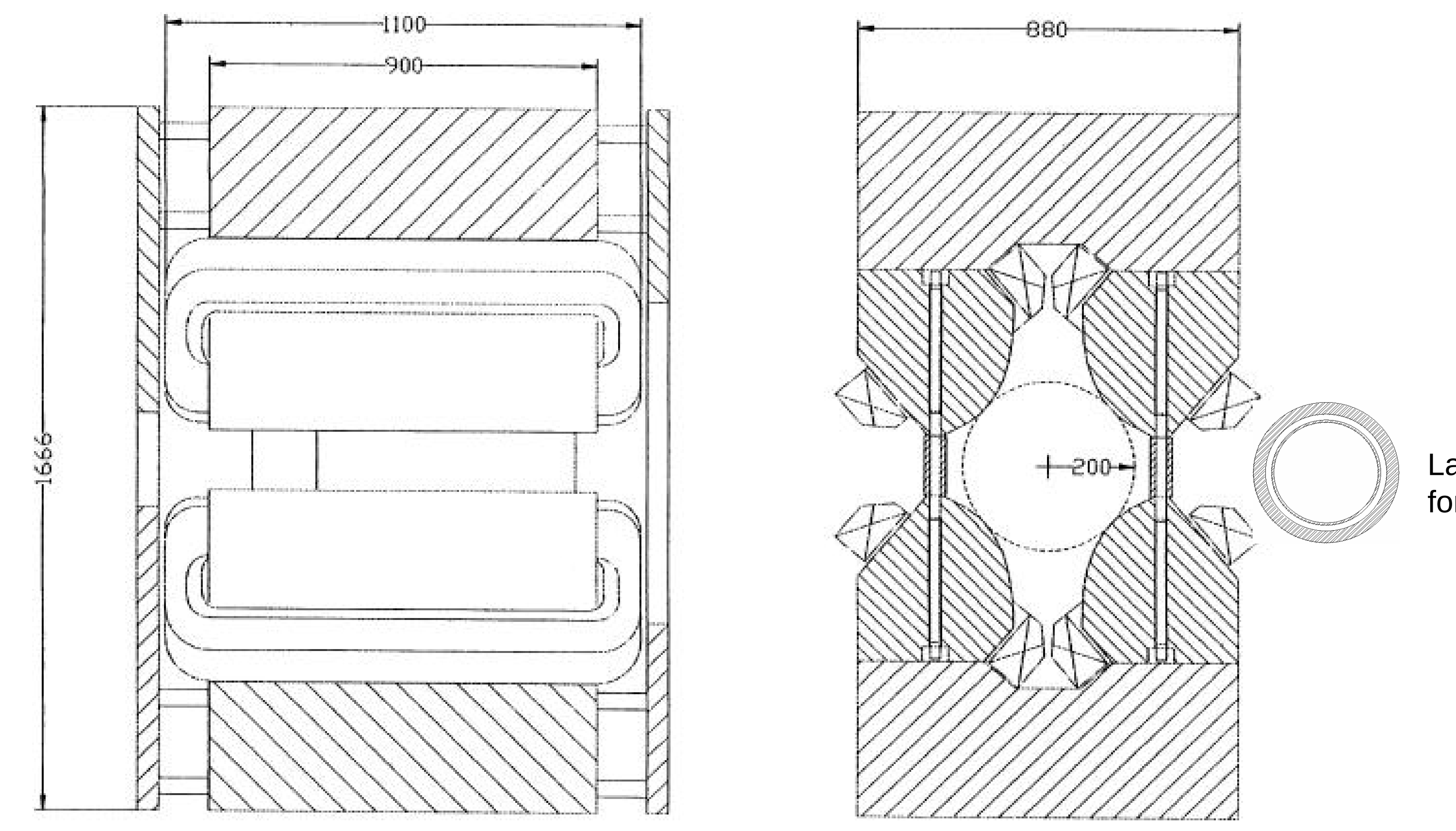


Fig. 4. Section through (right) and perpendicular to (left) the mid-plane of the quadrupole of spectrometer A. Lengths are in mm. The 50 mm thick mirror plates are mounted in a distance of 115 mm to the poles.

Ref. K.I. Blomqvist el al. NIM A403(1998)263

Focusing with a quadrupole doublet

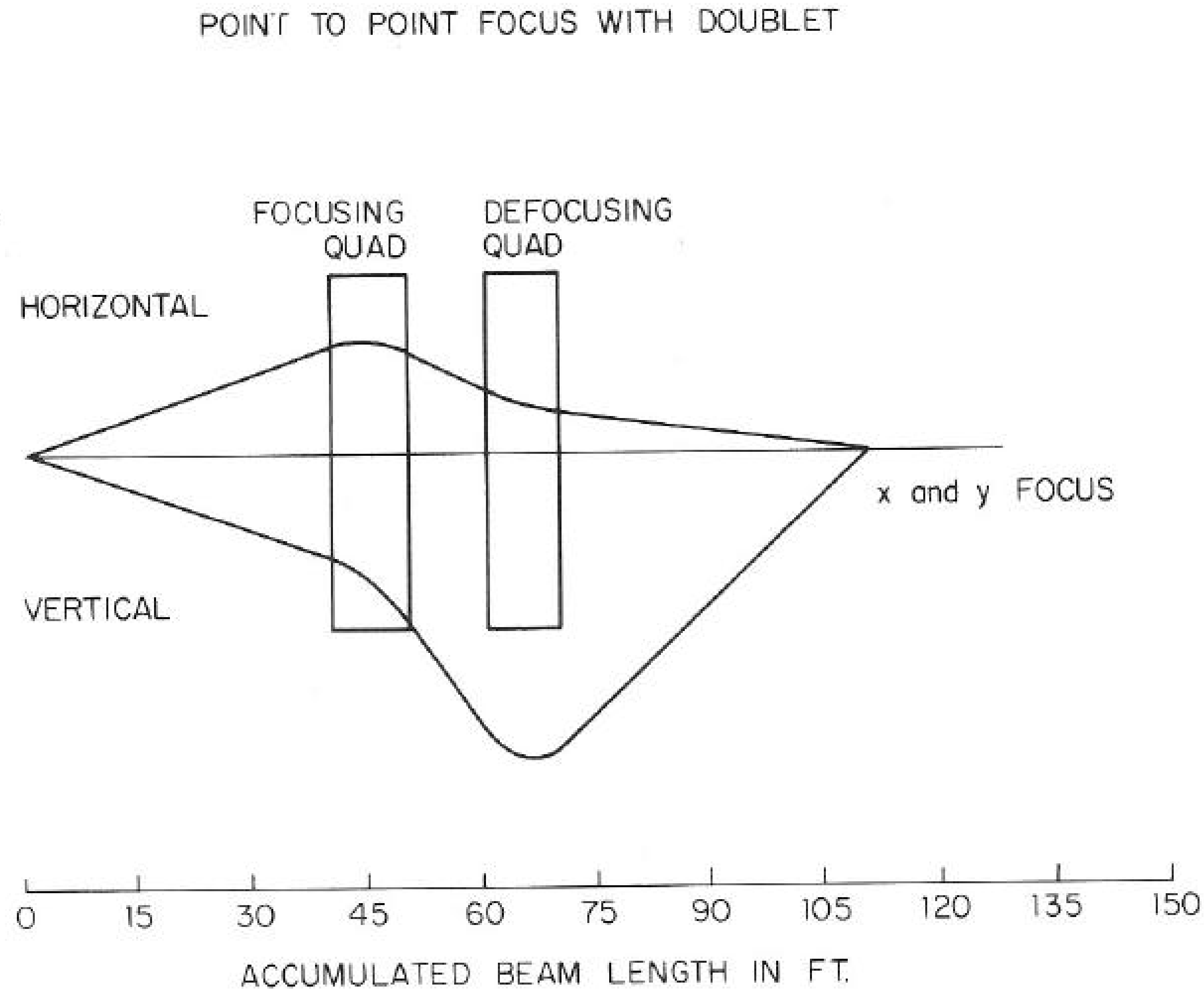


Figure 1.9 Point-to-point focusing with a quadrupole doublet. The two trajectories shown are in the horizontal and vertical planes respectively.

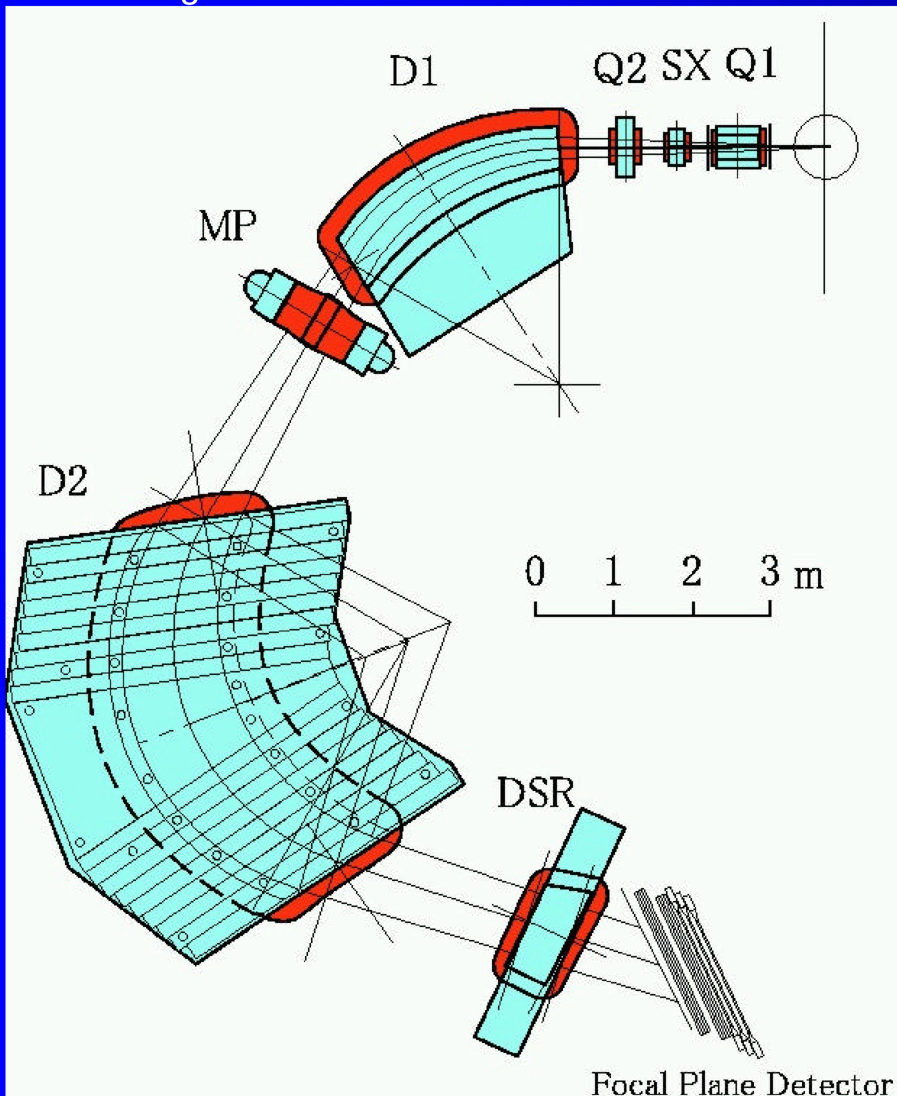
Q & A

- Question now? ASK!
- Any topic you want to hear and I haven't talked about? Let me know!

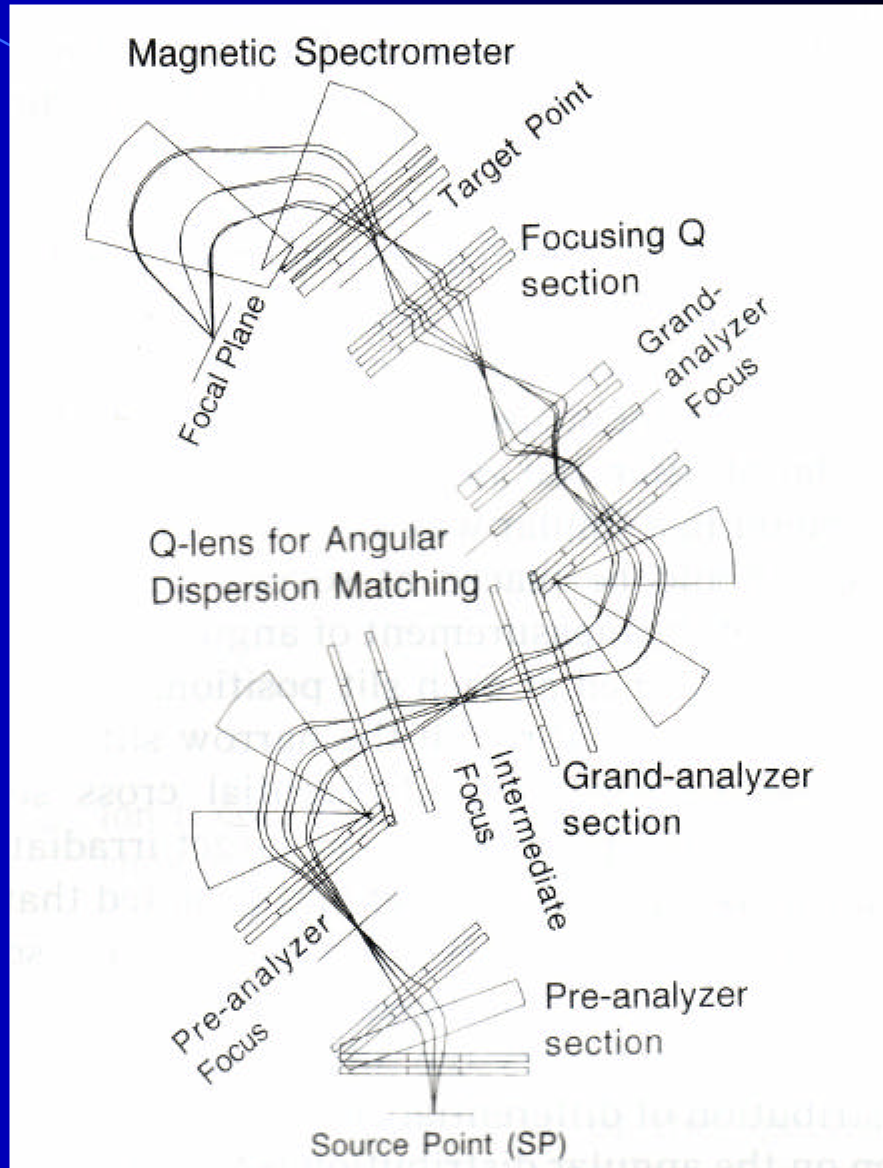
End Lecture 1

Grand Raiden High Resolution Spectrometer

Max. Magn. Rigidity: 5.1 Tm
Bending Radius: 3.0 m
Solid Angle: 3 msr

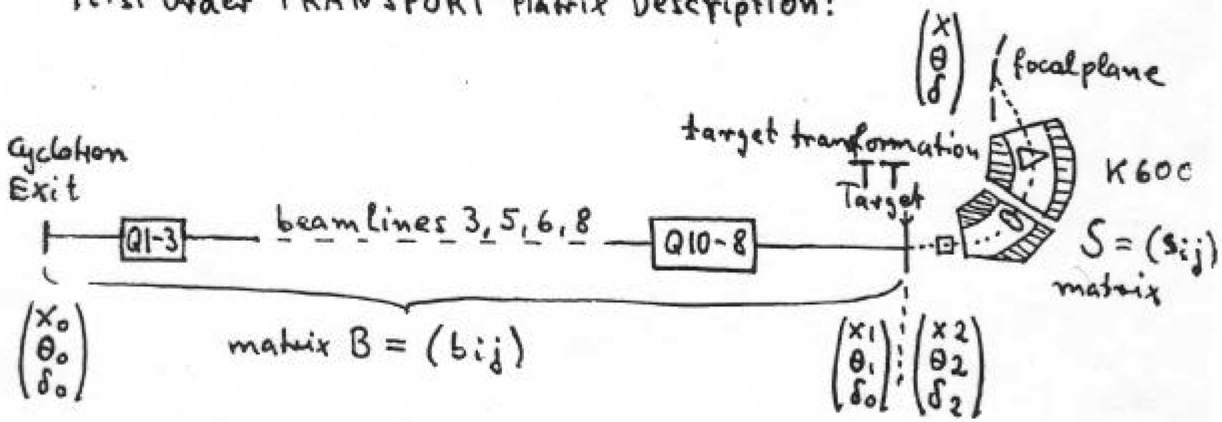


Beam Line/Spectrometer fully matched



Matching between beam line and spectrometer

First Order TRANSPORT Matrix Description:



$$B = \begin{pmatrix} b_{11} & b_{12} & b_{16} \\ b_{21} & b_{22} & b_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

$$S = \begin{pmatrix} s_{11} & s_{12} & s_{16} \\ s_{21} & s_{22} & s_{26} \\ 0 & 0 & 1 \end{pmatrix}$$

TT:

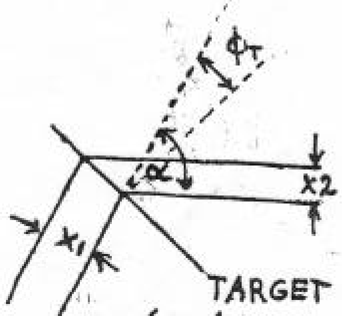
$x_2 = T\, x_1$

$\theta_2 = \Theta + \theta_1$

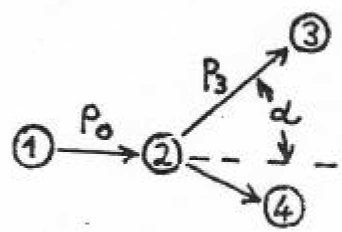
$\theta_2 =$ ~~random~~ angle within acceptance of spectrom.

$\delta_2 = K(\theta_2 - \theta_1) + C\,\delta_0$

Θ is random



$$T = \frac{\cos(\alpha - \phi_T)}{\cos \phi_T}$$



Reaction Kinematics

$p_3 = p_3(p_0, \alpha, Q)$

$$K = \frac{\partial p_3}{\partial \alpha}\frac{1}{p_3} \qquad C = \frac{\partial p_3}{\partial p_0}\frac{p_0}{p_3}$$

$$\begin{pmatrix} x \\ \theta \\ \delta \end{pmatrix} = S \cdot TT \cdot B \cdot \begin{pmatrix} x_0 \\ \theta_0 \\ \delta_0 \end{pmatrix}$$

Solution of first order Transport and Complete Matching

The transformation (without assuming $s_{12} = -s_{16}K$) in the bending plane from the cyclotron exit to the focal plane is given as:

$$x_{f.p.} = x_0\,(s_{11} b_{11} T + s_{12} b_{21})$$

$$\theta_0\,(s_{11} b_{12} T + s_{12} b_{22})$$ → kin. defoc. equ. (1)

$$\delta_0\,(s_{11} b_{16} T + s_{12} b_{26} + s_{16} C)$$ → disp. matching

$$\theta\,(s_{12} + s_{16} K)$$ → kin. correction (kin. displac.)

$$\theta_{f.p.} = x_0\,(s_{21} b_{11} T + s_{22} b_{21})\,.$$

$$\theta_0\,(s_{21} b_{12} T + s_{22} b_{22})$$ equ. (2)

$$\delta_0\,(s_{21} b_{16} T + s_{22} b_{26} + s_{26} C)$$ → angular disp. matching

$$\theta\,(s_{22} + s_{26} K)$$

$$\delta_{f.p.} = K \cdot \theta + C\,\delta_0$$

For details see: Y. Fujita et al., NIM B 126 (1997) 274

Complete Matching

For best **Resolution** in the focal plane, minimize the coefficients of all terms in the expression of **x** **f.p.**

For best **Angle Resolution** Minimize Coefficients of d_0 in expression of U **f.p.**

Note: Also the beam focus **b_{12}** on target is important (**b_{12}** = 0 for kinem. k = 0)

Spacial and Angular Dispersion Matching

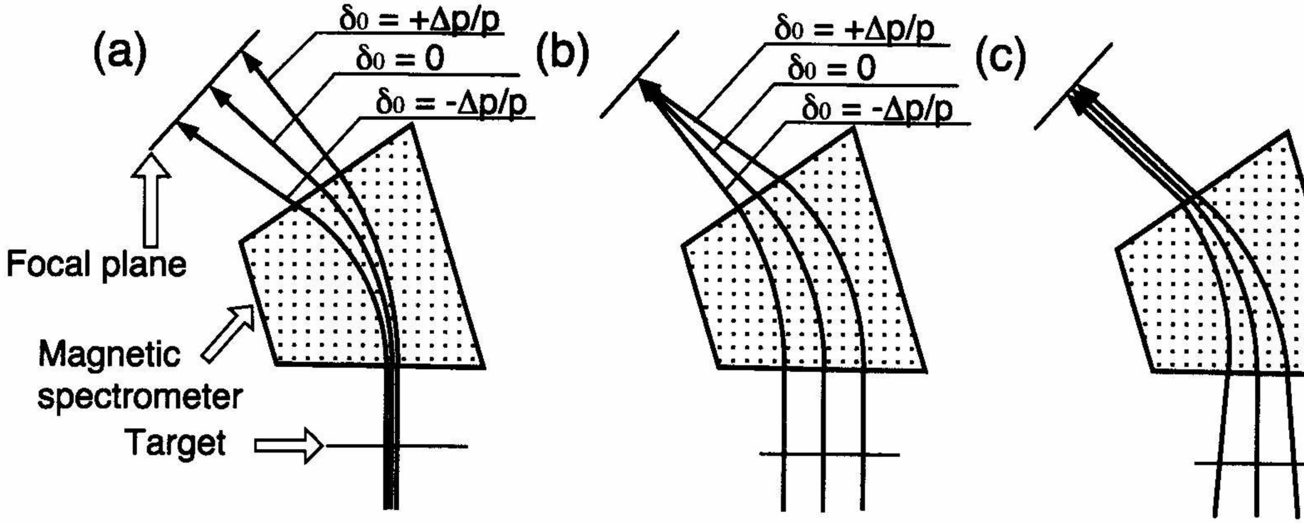
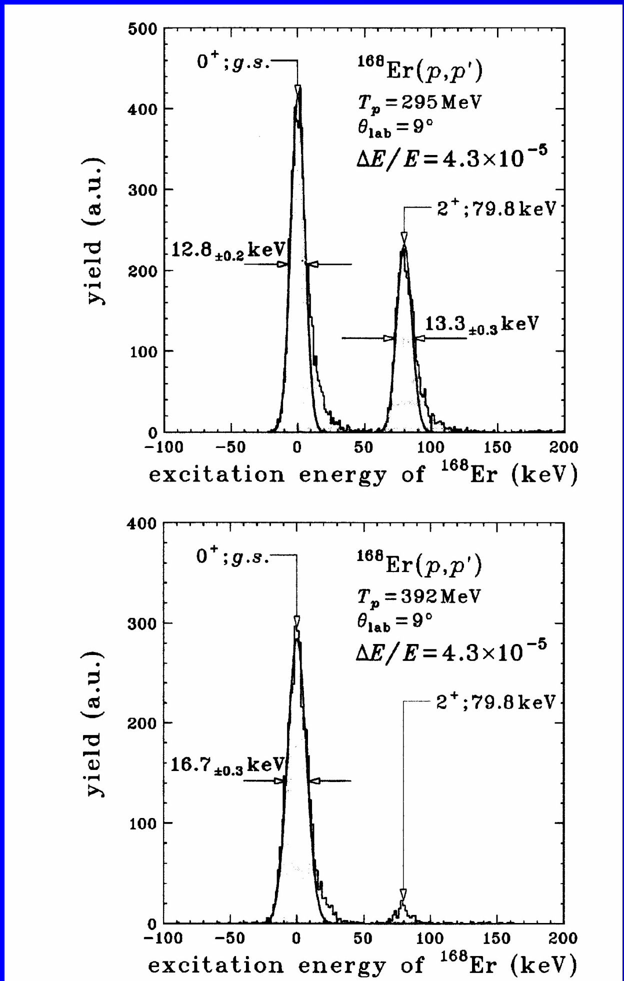


Figure 2.2: Schematic ion trajectories under different matching conditions of a beam line

High Momentum and Angular Resolution

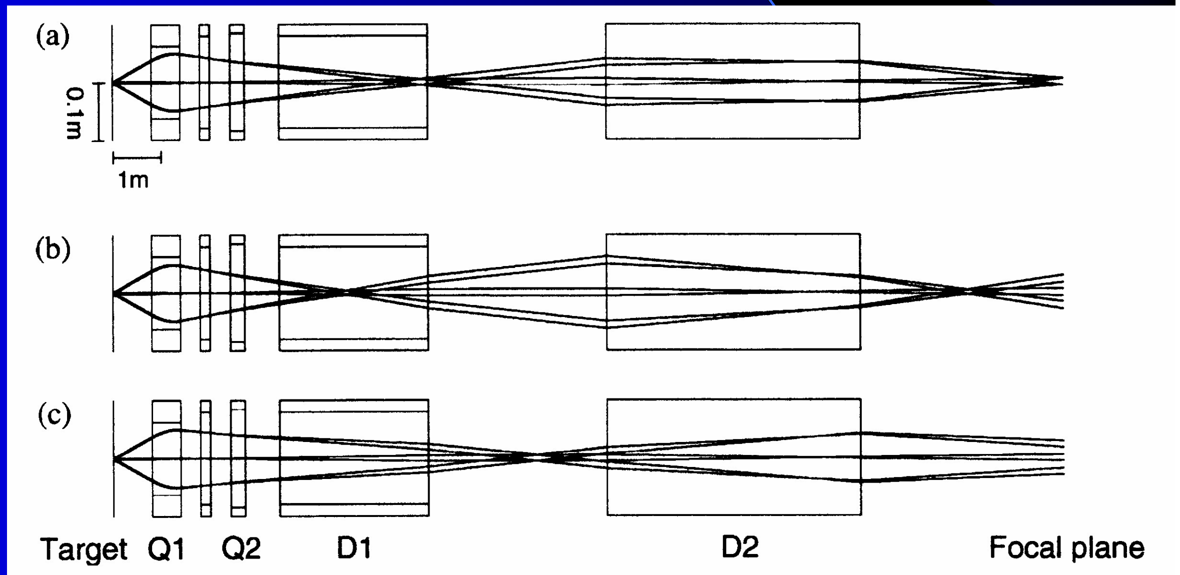
Spacial & Angular Dispersion Matching & Focus Condition allows

Energy Resolution: $\Delta E/E = 4.3 * 10^{-5}$, $\Delta p/p = 2.5*10^{-5}$, despite beam spread: $\Delta E = 4–6*10^{-4}$



Angular resolution: $\Delta Y_{scatt} = SQRT(\Delta Y^2_{hor} + \Delta\Phi^2) = 4 - 8$ msr

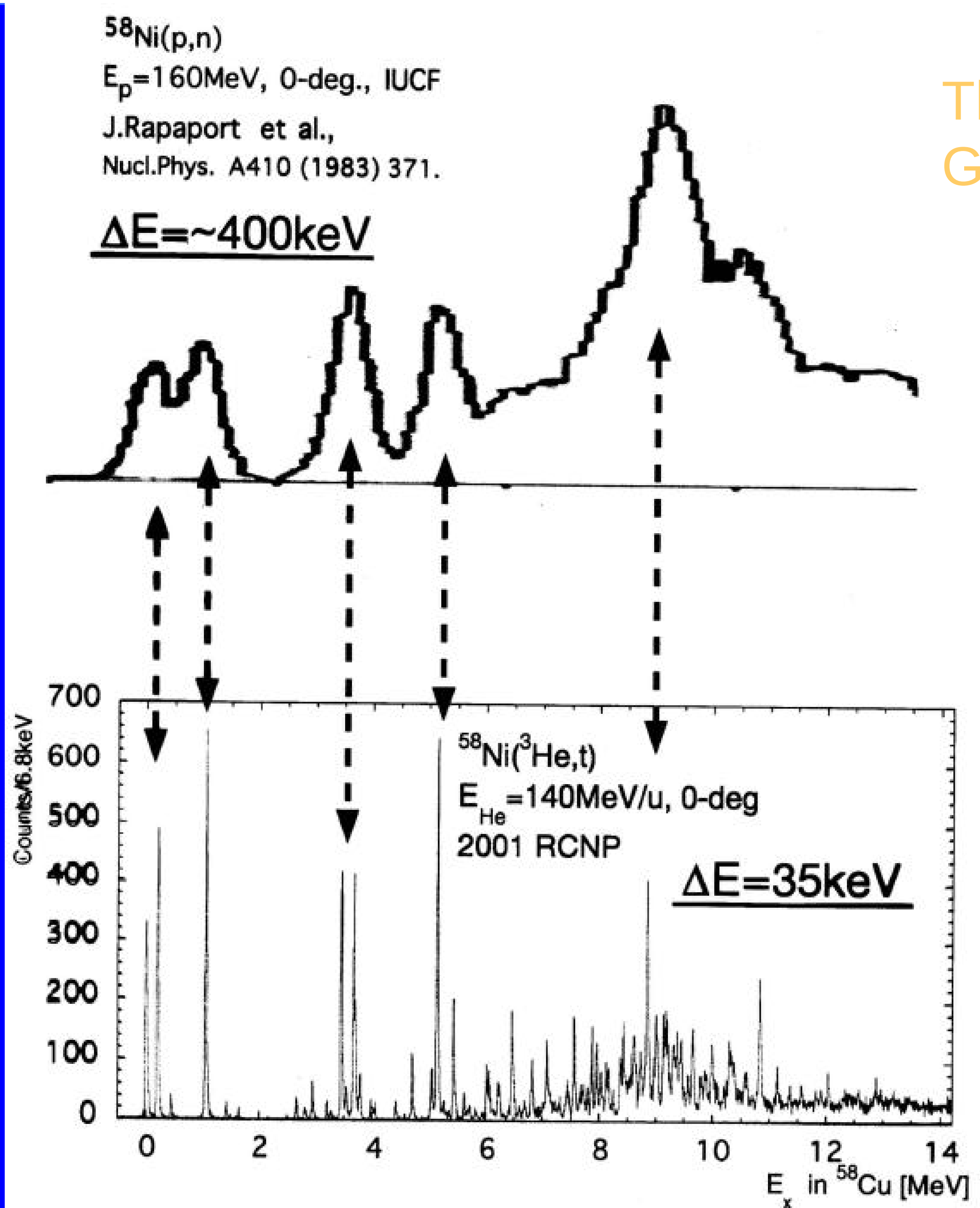
At angles close to beam (e.g. 0 deg) vert. angle component is needed → Overfocus mode, small target dimension, because (y|y) is large,
Limitation: multiple scattering in detector



Refs.: Y.Fujita et al, NIM B126(1997)274, H.Fujita et al. NIM A 469(2001)55, T.Wakasa et al, NIM A482(2002)79

The fine structure of Gamow-Teller Strength

eering (p,n) TOF measurements
CF established GT Giant Resonances



Now fine structure studied at RCNP in high resolution (^{3}He,t) Reactions at 0 deg.

Note: The beam energy spread of the cyclotron is at best 150 – 200 keV.

However the spectral resolution is 35 keV due to Dispersion Matching.

- 0 deg: magn. separation of beam
- Resolution better than beam spread
- Angle reconstructed despite 2 cm horiz. target spot, due to dispersion