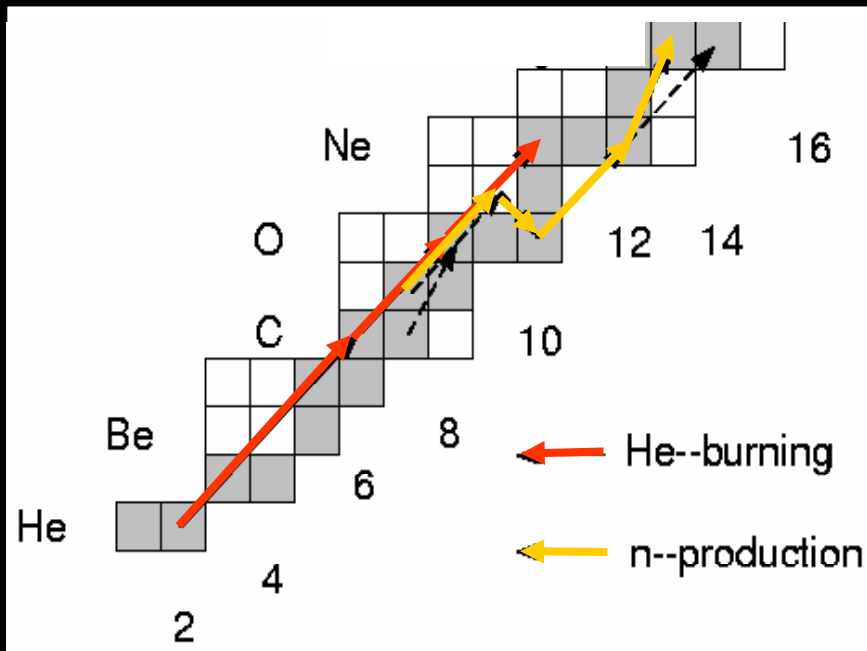


He-Burning in massive Stars

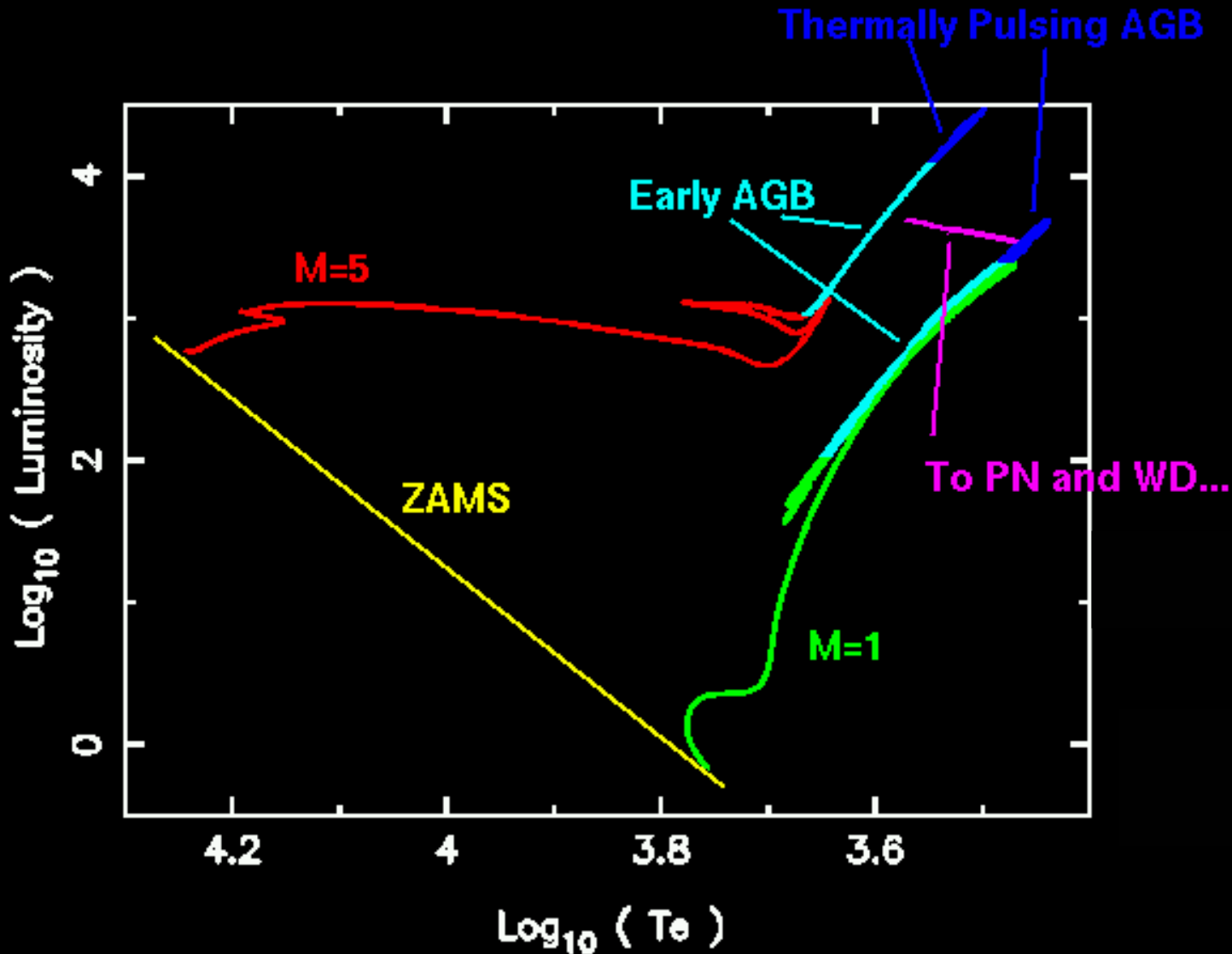
He-burning is ignited on the ${}^4\text{He}$ and ${}^{14}\text{N}$ ashes of the preceding hydrogen burning phase!



Most important reaction
-triple alpha process –
 $3\alpha \Rightarrow {}^{12}\text{C} + 7.96 \text{ MeV}$



Red Giant Evolution in HR diagram



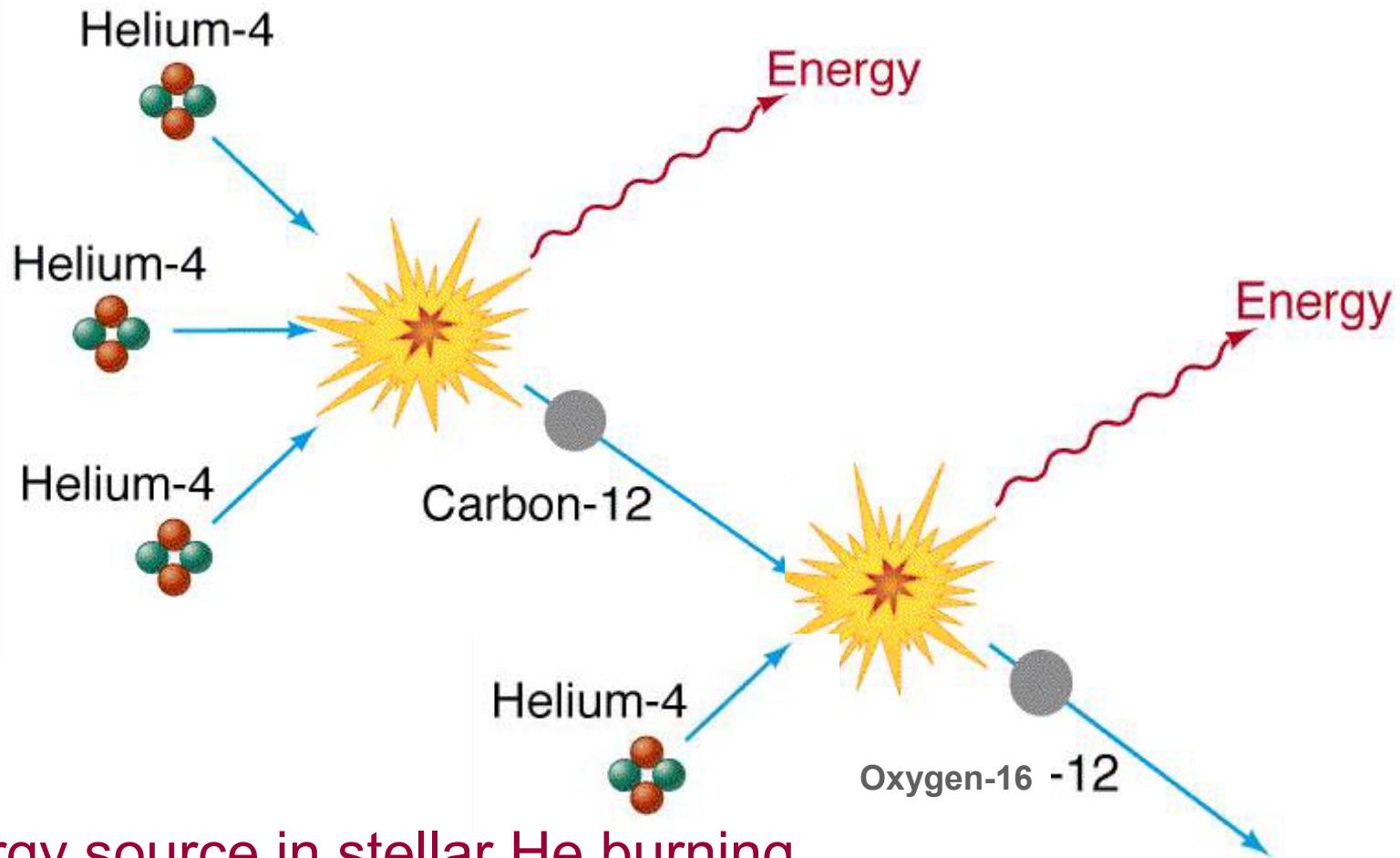
[H+HRDhmovie.gif](#)

[Hemovie.gif](#)

[H+HRDhmovie.gif](#)

[CNOmovie.gif](#)

Critical Reactions in He-burning



Energy source in stellar He burning

Energy release determined by associated reaction rates

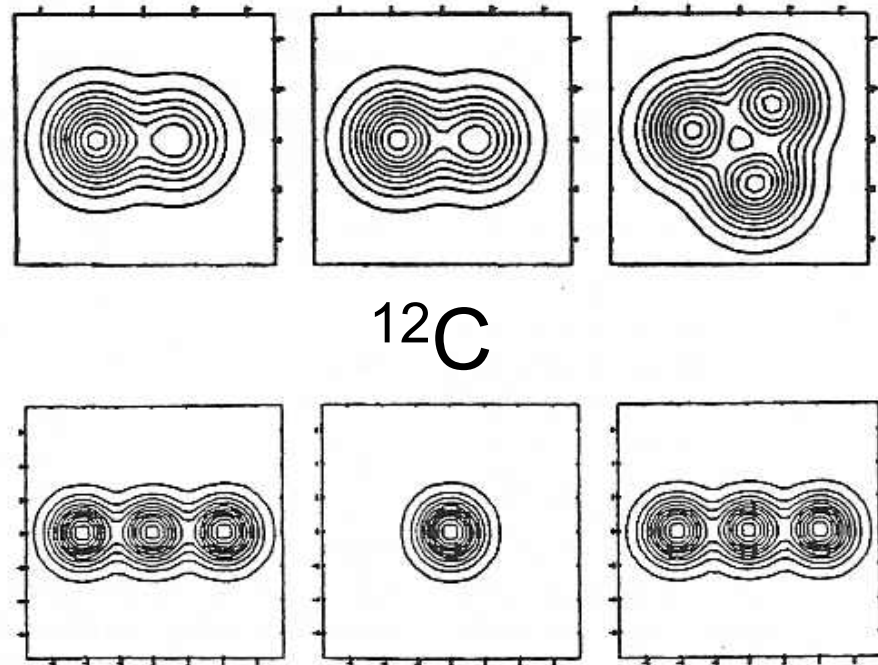
Nuclear Structure Aspects for evaluating and interpreting α capture reaction rates

hydrogen burning depends on capture probability of a proton
 \Leftrightarrow single particle configuration of final state

$$\sigma(E) \propto \left| \langle \psi_f | H | \psi_t + \psi_p \rangle \right|^2 \propto \Theta_p$$

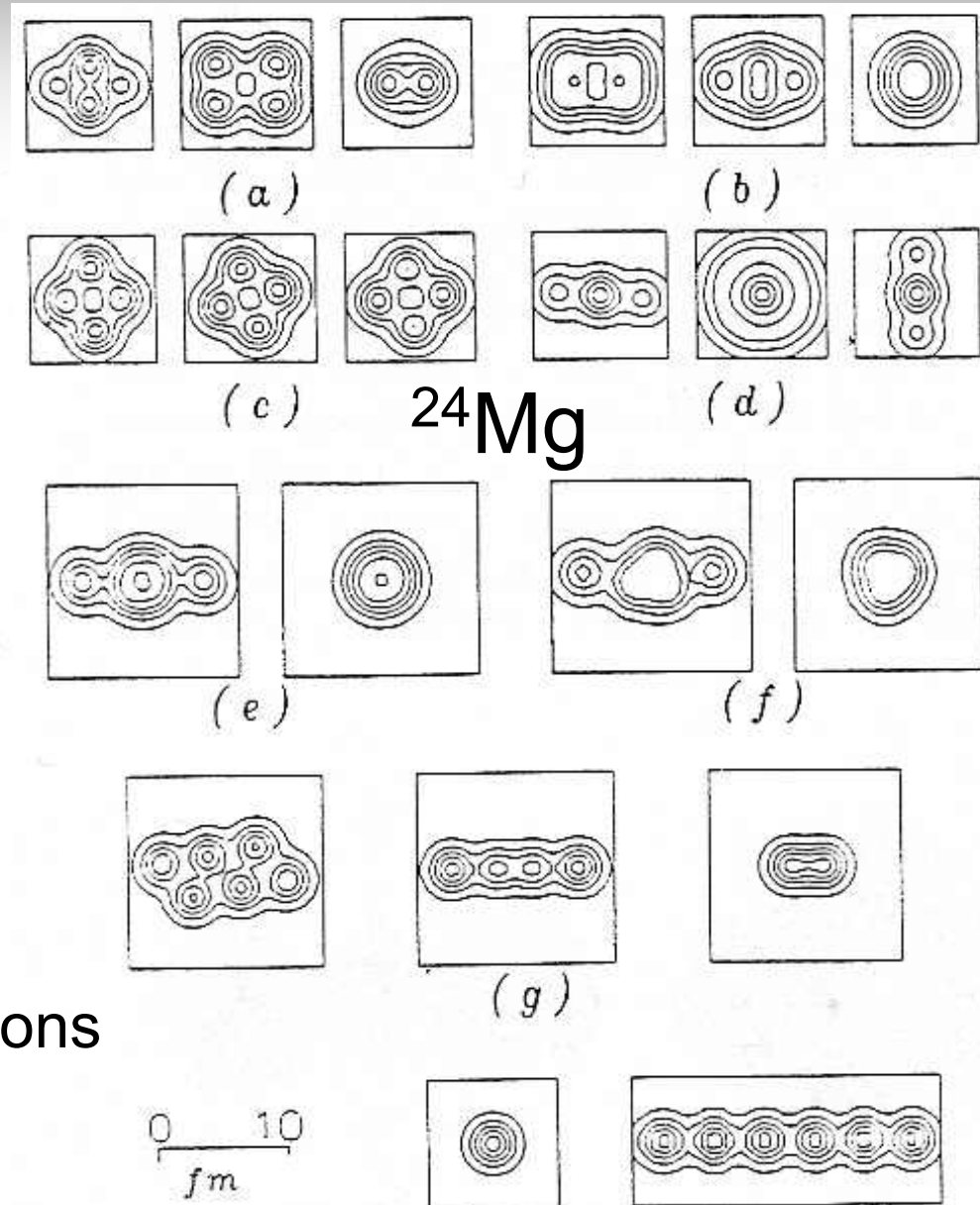
helium burning depends on capture probability of an α particle
 \Leftrightarrow α -cluster configuration of final state

α -cluster configurations in ^{12}C and ^{24}Mg



^{12}C

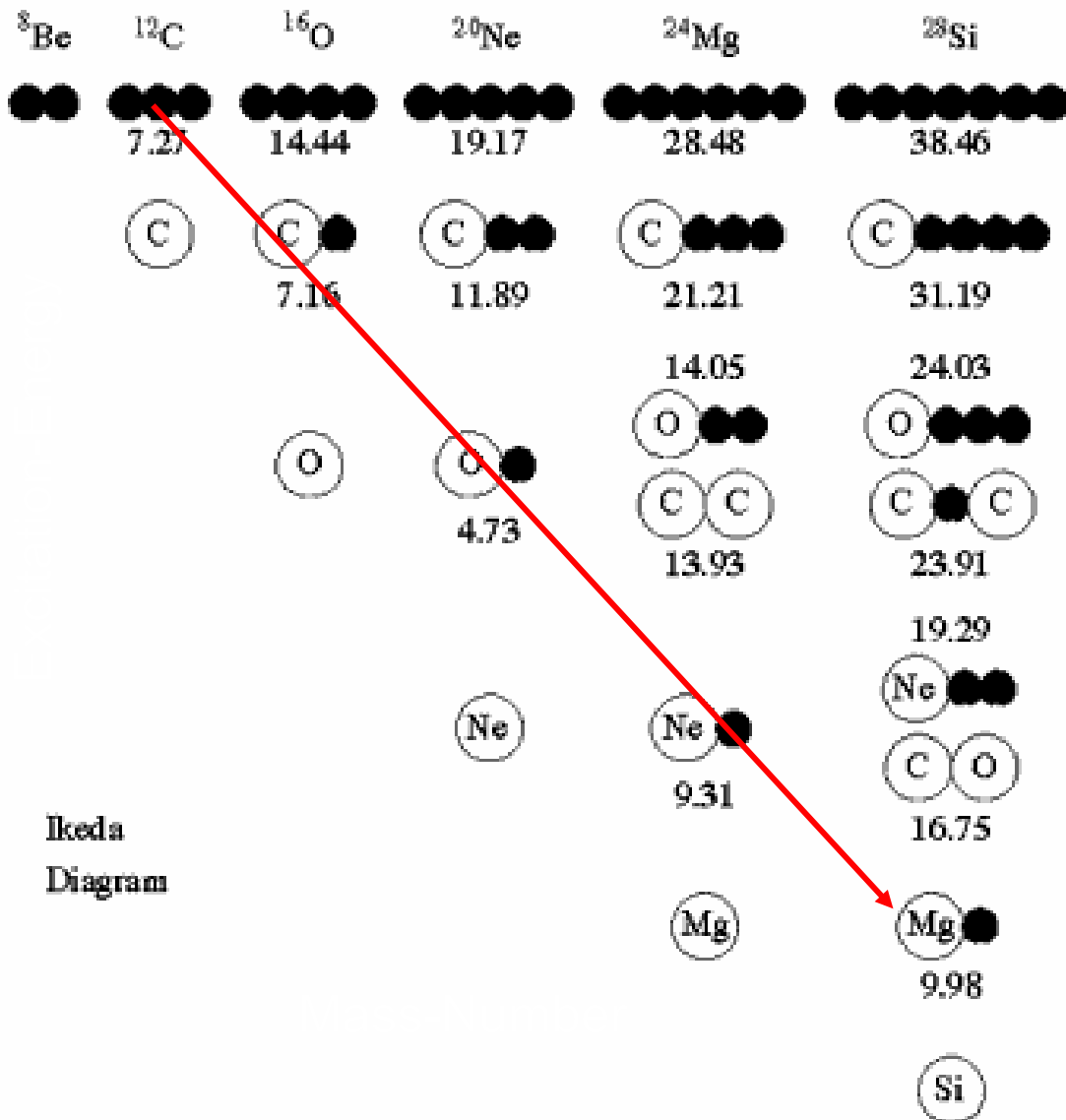
Cluster model predictions for
Cluster and shape configurations
For ^{12}C and ^{24}Mg $T=0$ nuclei



^{24}Mg

Alpha Cluster Structure in

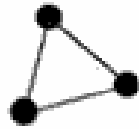
T=0,1 nuclei



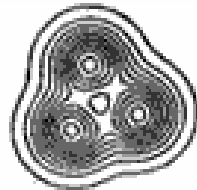
Pronounced
 α clustering
 In T=0 nuclei
 near α threshold
 $\theta_\alpha^2 > 0.1$

Ikeda
 Diagram

Geometry of cluster configurations



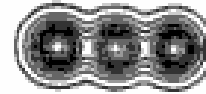
Equilateral Triangle



$^{12}\text{C}_{g.s.}$



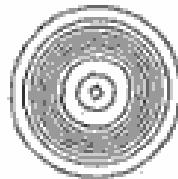
3 α - chain



$^{12}\text{C}^*$



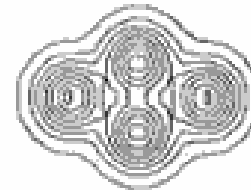
Regular Tetrahedron



$^{16}\text{O}_{g.s.}$



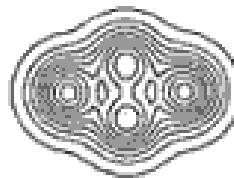
Kite



$^{16}\text{O}^*$



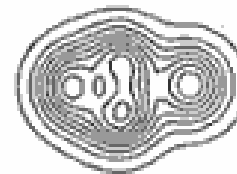
Trigonal Bipyramid



$^{20}\text{Ne}_{g.s.}$



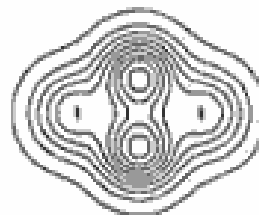
" $^{16}\text{O} - \alpha$ "



$^{20}\text{Ne}^*$



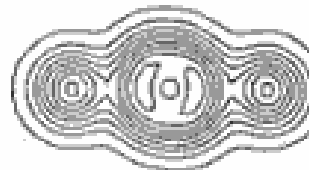
Bipyramid



$^{24}\text{Mg}_{g.s.}$



" $\alpha - ^{16}\text{O} - \alpha$ "



$^{24}\text{Mg}^*$

Resonant Reaction Rate

$$N_A \langle \sigma v \rangle = 1.54 \cdot 10^{11} \cdot \omega \gamma [MeV] \cdot \left(\frac{1}{\mu \cdot T_9} \right)^{3/2} \cdot e^{-\left(\frac{11.605 \cdot E_R [MeV]}{T_9} \right)}$$

$$\omega \gamma = \frac{(2J+1)}{(2j_p+1) \cdot (2j_T+1)} \cdot \frac{\Gamma_{in} \cdot \Gamma_{out}}{\Gamma_{tot}} \quad \Gamma_{tot} = \sum_i \Gamma_i$$

at low energy astrophysical conditions: $\Gamma_{in} \ll \Gamma_{out} \approx \Gamma_{tot}$

$$\Gamma_{in} = \Gamma_{\alpha} = T_{V_c, V_l} \cdot \Theta_{\alpha}$$

Network for stellar Helium burning

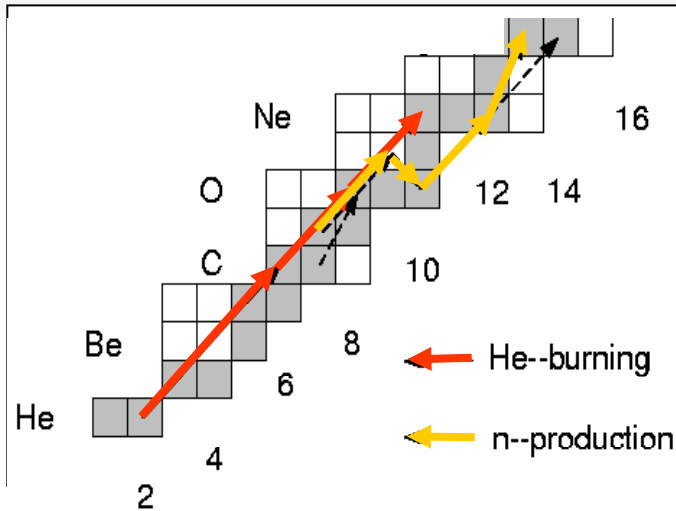
$$\begin{aligned} \frac{dY_{4\text{He}}}{dt} = & -Y_{4\text{He}}^3 \cdot \rho^2 \cdot N_A \langle \sigma v \rangle_{3\alpha} - Y_{12\text{C}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{12\text{C}(\alpha,\gamma)} \\ & - Y_{16\text{O}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{16\text{O}(\alpha,\gamma)} - Y_{14\text{N}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{14\text{N}(\alpha,\gamma)} \\ & \left(-Y_{18\text{O}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{18\text{O}(\alpha,\gamma)} - Y_{22\text{Ne}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{22\text{Ne}(\alpha,n)} \right) \end{aligned}$$

$$\frac{dY_{12\text{C}}}{dt} = -Y_{12\text{C}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{12\text{C}(\alpha,\gamma)} + Y_{4\text{He}}^3 \cdot \rho^2 \cdot N_A \langle \sigma v \rangle_{3\alpha}$$

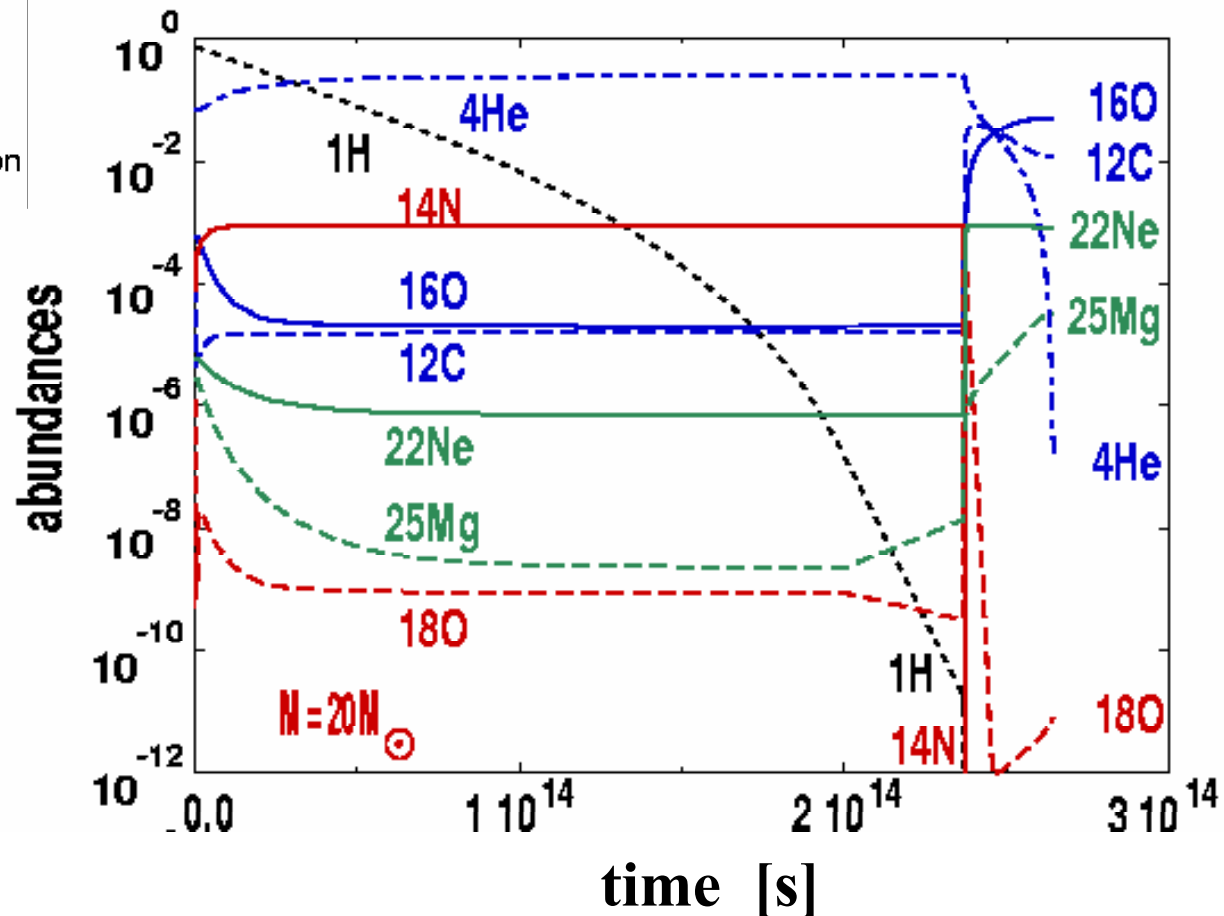
$$\frac{dY_{16\text{O}}}{dt} = -Y_{16\text{O}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{16\text{O}(\alpha,\gamma)} + Y_{12\text{C}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{12\text{C}(\alpha,\gamma)}$$

$$\frac{dY_{14\text{N}}}{dt} = -Y_{14\text{N}} \cdot Y_{4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{14\text{N}(\alpha,\gamma)}$$

Abundance evolution in stellar core

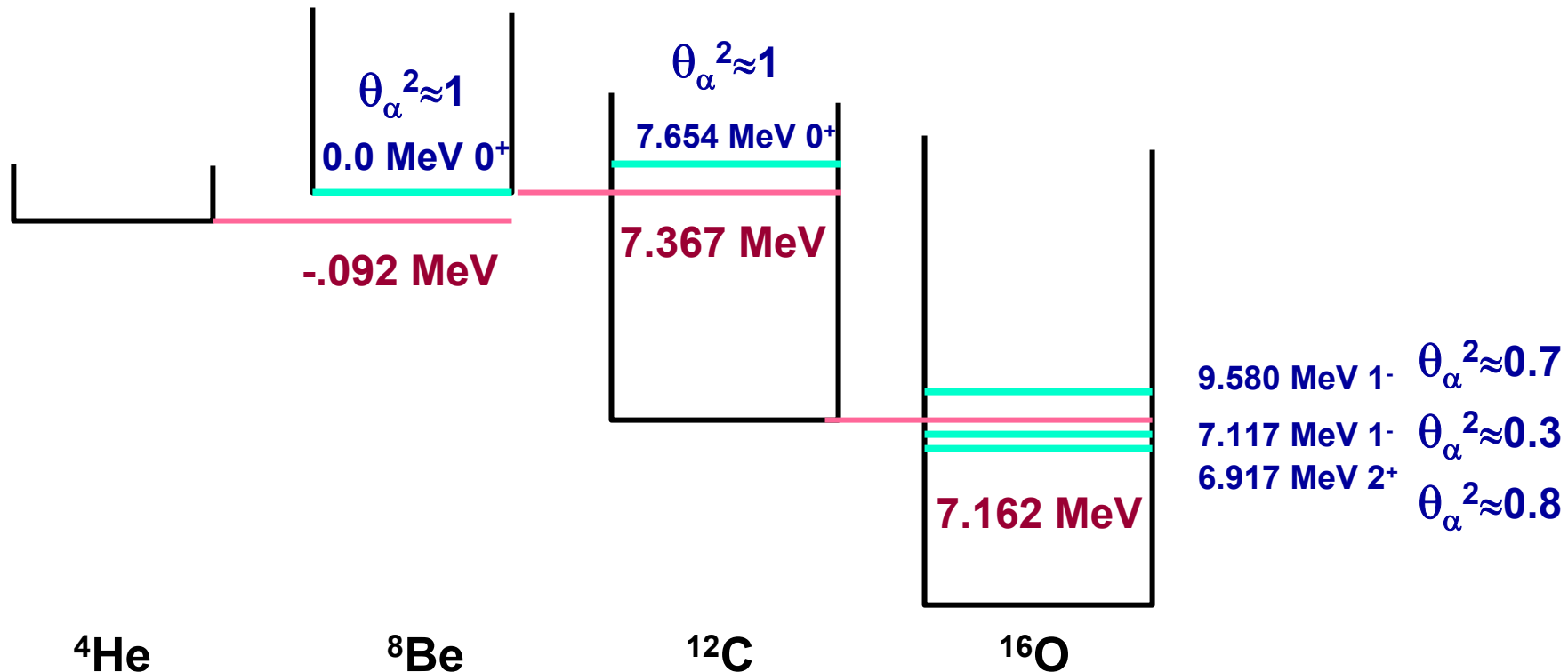


Decline of ${}^4\text{He}$
 (time-scale)
 increase in ${}^{12}\text{C}$, ${}^{16}\text{O}$
 \Rightarrow equilibrium ${}^{12}\text{C}/{}^{16}\text{O}$
 Rapid decline in ${}^{14}\text{N}$.



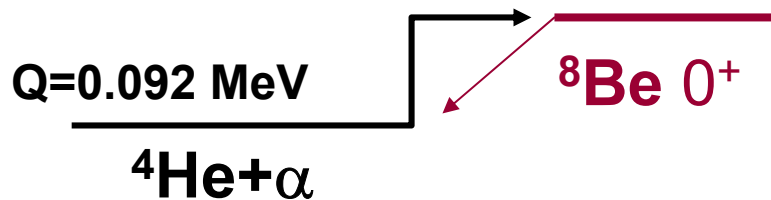
The case of: 3- α and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Reaction rates determined by α cluster state configurations providing strong resonances!



The ($\alpha\alpha\alpha$) Reaction as two step process

first step!



$$T_{1/2}({}^8\text{Be}) = 9.7 \cdot 10^{-17} \text{ s}$$

$$\Gamma_{\alpha} = 6.8 \text{ eV}$$

pure α cluster configuration

fast capture \Rightarrow equilibrium between capture and decay

$$\text{Interaction time: } t \approx \frac{2R_{\alpha}}{v_{\alpha}} = \frac{2 \cdot 1.3 \cdot A^{1/3}}{\sqrt{\frac{2E_{\alpha}^{cm}}{\mu}}} \approx \frac{4.17 \text{ fm}}{3.8 \cdot 10^{24} \text{ fm/s}} \approx 10^{-24} \text{ s} \ll \tau({}^8\text{Be})$$

Application of Saha Equation
For calculating ${}^8\text{Be}$ equilibrium:

$$N({}^8\text{Be}) = N_{\alpha}^2 \cdot \hbar^3 \cdot \left(\frac{2\pi}{\mu \cdot kT} \right)^{3/2} \cdot e^{\left(-\frac{Q}{kT} \right)}$$

Example for ${}^8\text{Be}$ equilibrium abundance:

Case of typical He-burning: $T=0.1\text{GK} \Rightarrow T_9=0.1$; $\rho=10^5\text{ g/cm}^3$

$$N({}^8\text{Be}) = 6 \cdot 10^{-35} \cdot N_\alpha^2 \cdot T_9^{-3/2} \cdot e^{\left(-\frac{1.068}{T_9}\right)}$$

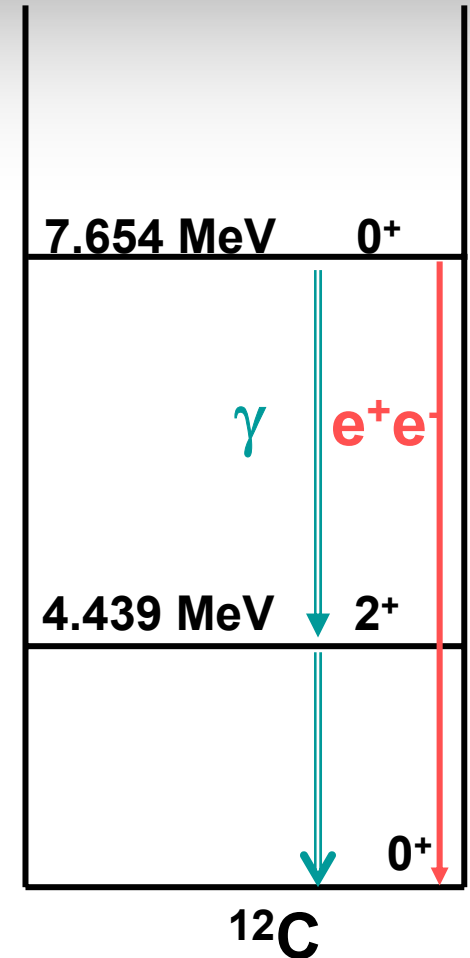
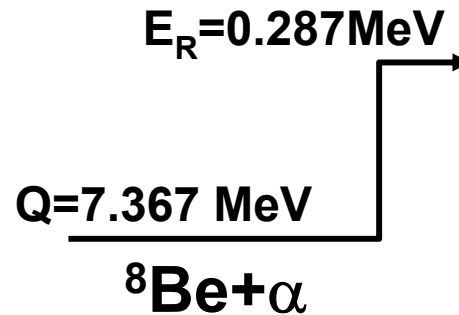
$$N({}^8\text{Be}) \approx 4.4 \cdot 10^{-38} \cdot N_\alpha^2$$

$$N = \rho \cdot N_A \cdot \frac{X_i}{A_i} \quad \Rightarrow \quad \frac{X({}^8\text{Be})}{X_\alpha^2} \approx 1.3 \cdot 10^{-9}$$

~ one ${}^8\text{Be}$ nucleus
for 10^9 α particles

Resonant capture on ^8Be

The Hoyle resonance!



$$N_A \langle \sigma v \rangle = 1.54 \cdot 10^{11} \cdot \omega \gamma \cdot \left(\frac{1}{\mu \cdot T_9} \right)^{3/2} \cdot e^{-\left(\frac{11.605 \cdot E_R}{T_9} \right)}$$

$$\omega \gamma = (2J + 1) \cdot \frac{\Gamma_{in} \cdot \Gamma_{out}}{\Gamma_{tot}}$$

Decay by sequential E2 γ transitions
or internal $e^+ e^-$ pair conversion

The Resonance Strength

$$\omega\gamma = \frac{\Gamma_\alpha \cdot (\Gamma_\gamma + \Gamma_{e^+e^-})}{\Gamma_\alpha + \Gamma_\gamma + \Gamma_{e^+e^-}}$$

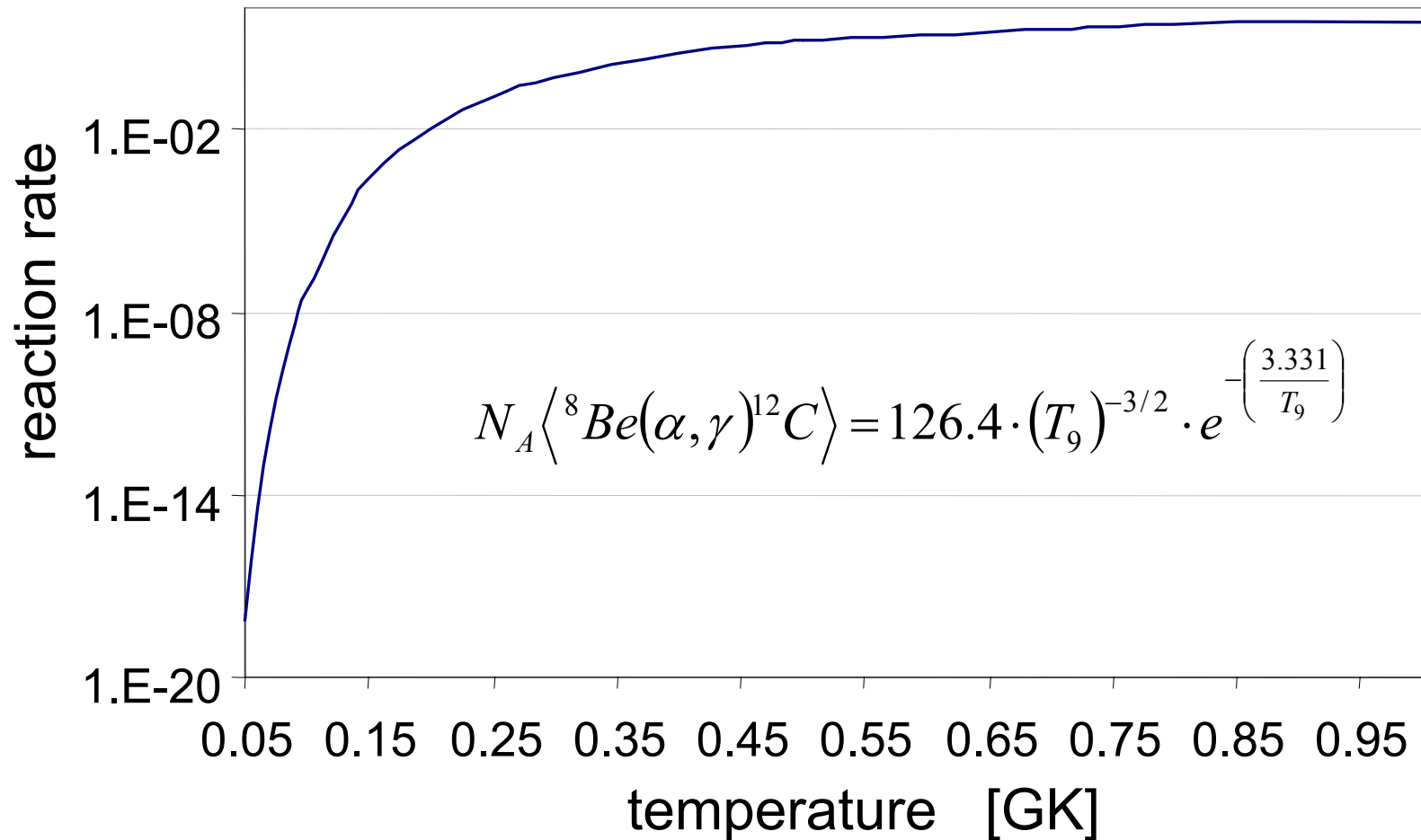
$$\Gamma_\alpha = 8.09 \pm 1.08 \text{ eV}$$

$$\Gamma_\gamma = 3.58 \pm 0.5 \text{ meV} \quad \frac{\Gamma_{rad}}{\Gamma_{tot}} = 4.12 \cdot 10^{-4}$$

$$\Gamma_{e^+e^-} = 60.6 \pm 3.9 \text{ } \mu\text{eV}$$

$$\omega\gamma = 3.58 \cdot 10^{-9} \text{ MeV} \quad \pm 12\%$$

The ${}^8\text{Be}+\alpha$ reaction rate



The total $\langle \alpha\alpha\alpha \rangle$ rate

$$r_{\alpha\alpha\alpha} = N_{^8\text{Be}} \cdot \rho \cdot \frac{X_\alpha}{A_\alpha} \cdot N_A \langle ^8\text{Be}(\alpha, \gamma)^{12}\text{C} \rangle$$

Step 1

Step 2

$$N(^8\text{Be}) = 6 \cdot 10^{-35} \cdot N_\alpha^2 \cdot T_9^{-3/2} \cdot e^{\left(-\frac{1.068}{T_9}\right)}$$

$$N_A \langle ^8\text{Be}(\alpha, \gamma)^{12}\text{C} \rangle = 126.4 \cdot (T_9)^{-3/2} \cdot e^{\left(-\frac{3.331}{T_9}\right)}$$

$$r_{\alpha\alpha\alpha} = \frac{1.26 \cdot 10^{-56}}{1 + \delta_{\alpha\alpha}} \cdot N_\alpha^3 \cdot T_9^{-3} \cdot e^{\left(-\frac{11.605 \cdot (0.092 + 0.278)}{T_9}\right)}$$

$$r_{\alpha\alpha\alpha} = 1.38 \cdot 10^{15} \cdot \rho^3 \cdot \left(\frac{X_\alpha}{4}\right)^3 \cdot T_9^{-3} \cdot e^{\left(-\frac{4.294}{T_9}\right)} \quad [cm^{-3}s^{-1}]$$

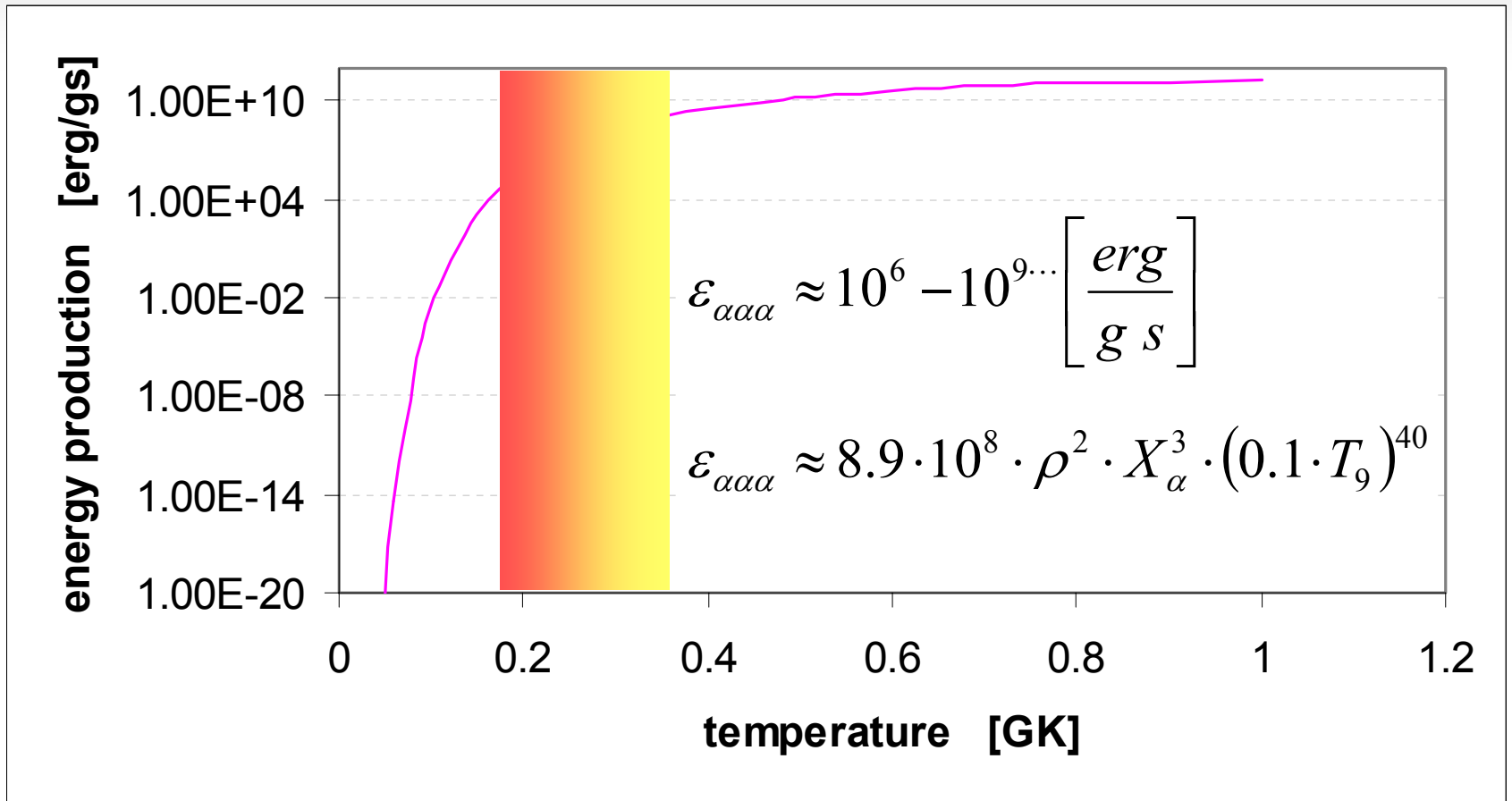
Energy production in He burning

$$\epsilon_{\alpha\alpha\alpha} = Q_{\alpha\alpha\alpha \Rightarrow {}^{12}\text{C}} \cdot \frac{r_{\alpha\alpha\alpha}}{\rho}$$

$$Q_{\alpha\alpha\alpha \Rightarrow {}^{12}\text{C}} = 7.274 \text{ MeV} = 1.16 \cdot 10^{-5} \text{ erg}$$

$$\epsilon_{\alpha\alpha\alpha} = 2.5 \cdot 10^8 \cdot \rho^2 \cdot X_{\alpha}^3 \cdot T_9^{-3} \cdot e^{-\frac{4.294}{T_9}} \left[\frac{\text{erg}}{\text{g s}} \right]$$

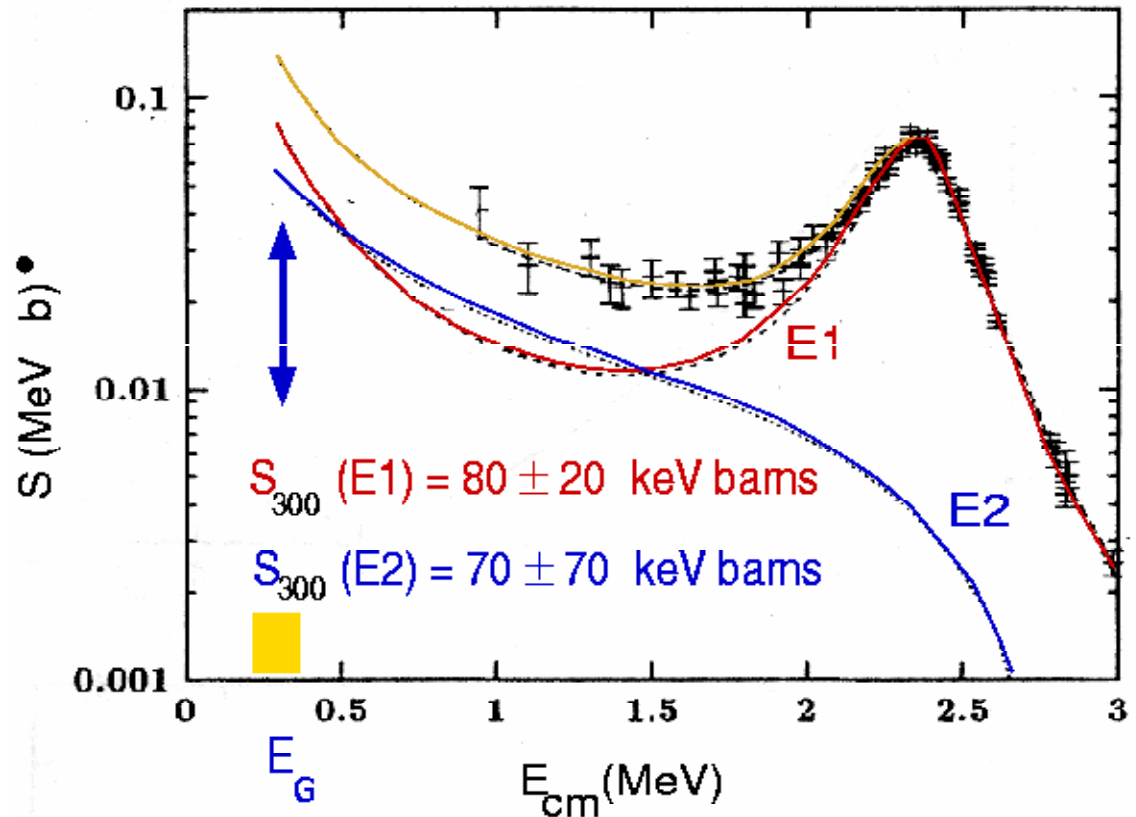
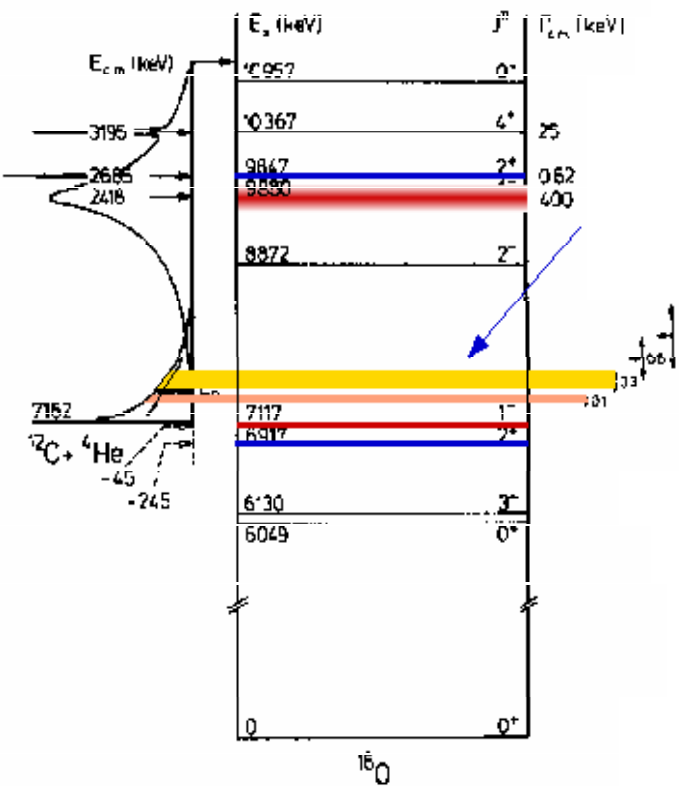
Example: $\rho=10^5 \text{ g/cm}^3$



T-dependent main energy source for stellar He-burning

$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, the Holy Grail

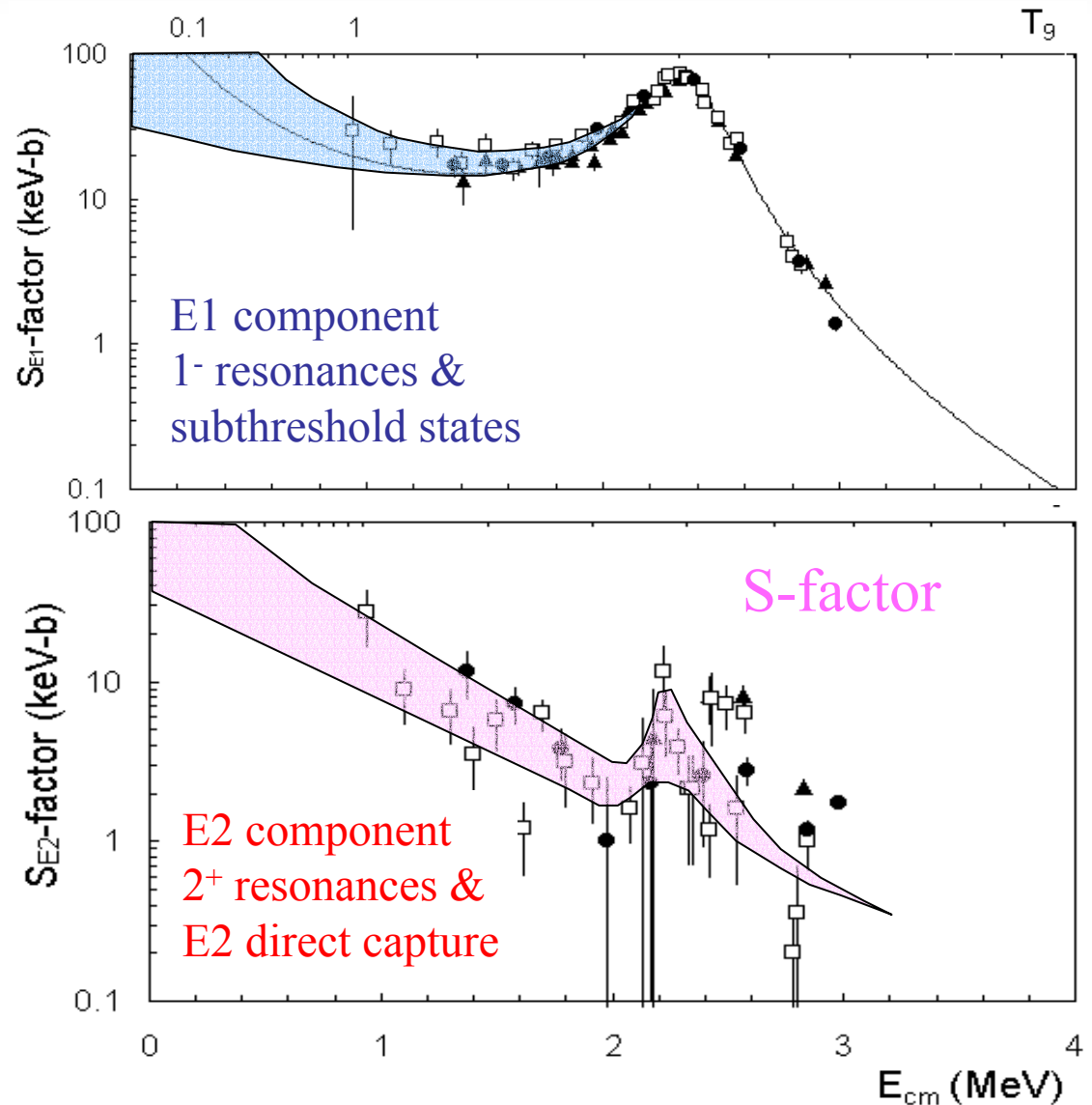
Level and Interference Structure



Uncertainty in low energy extrapolation

reaction contributions in $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$

Difficulty in the reliability of low energy extrapolation



R-matrix analysis

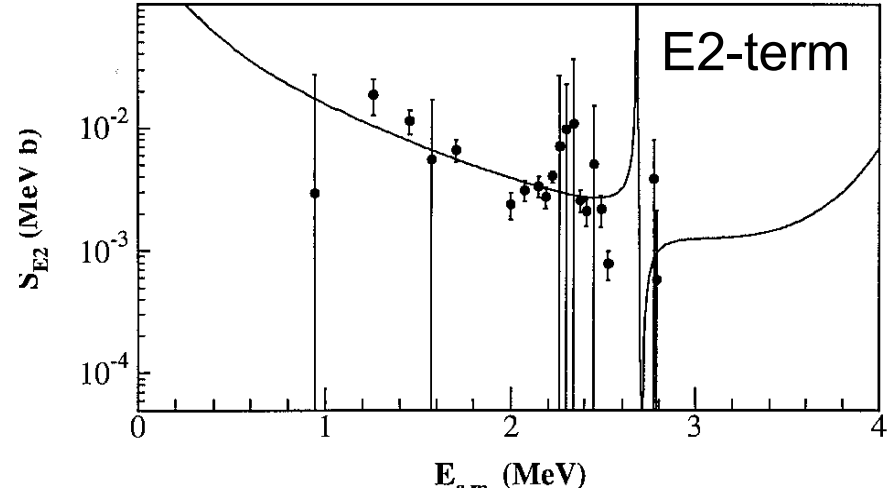
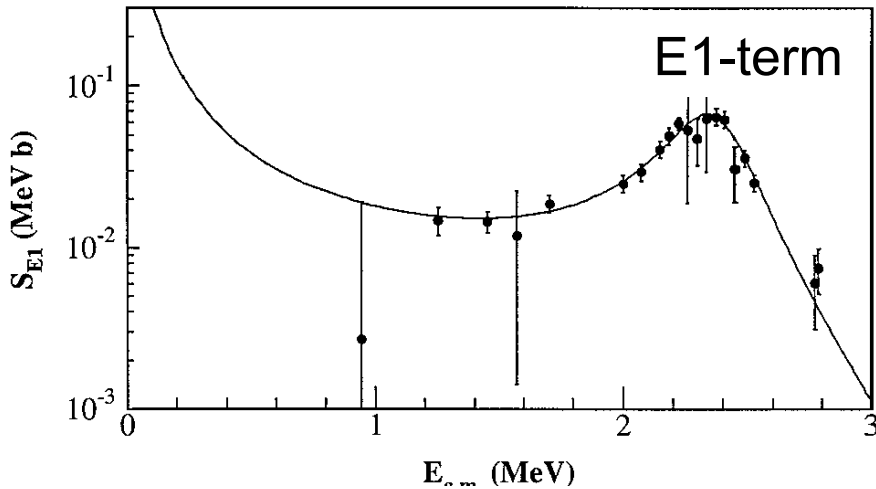
Complex resonance structure, interfering broad resonances

R-matrix analysis \Rightarrow R-matrix school

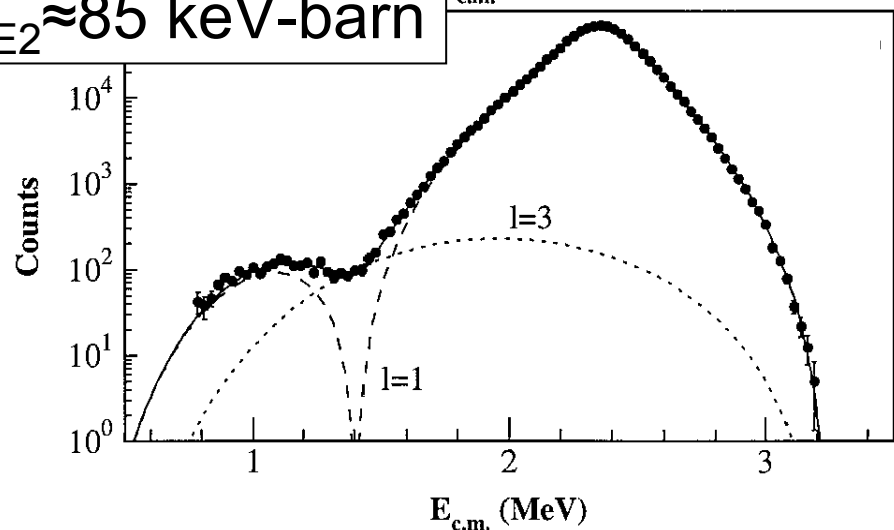
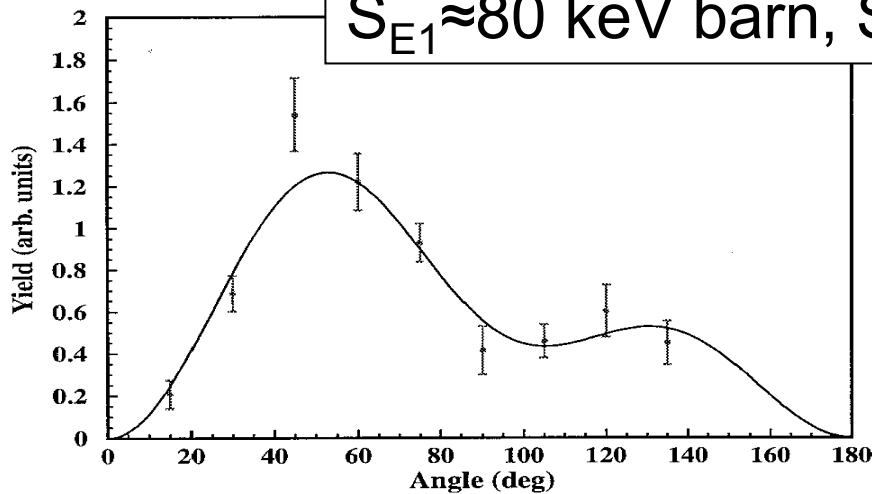
Parameters from probing ^{16}O compound nucleus through

- elastic scattering $^{12}\text{C}(\alpha, \alpha)^{12}\text{C}$
- β -delayed α -decay $^{16}\text{N}(\beta, \alpha)^{12}\text{C}$
- resonant α capture $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$
- α -transfer reaction $^{12}\text{C}(^7\text{Li}, t)^{16}\text{O}$

R-matrix fit examples



$S_{E1} \approx 80$ keV barn, $S_{E2} \approx 85$ keV-barn



From Kunz et al. PRL 86 (2004)

$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate

$$N_A \langle \sigma v \rangle = 6.9 \cdot 10^8 \cdot T_9^{-2/3} \cdot S_{eff} [\text{MeV} - b] \cdot e^{-\frac{32.11}{T_9^{1/3}}} \left[\frac{\text{cm}^3}{\text{s}} \right]$$

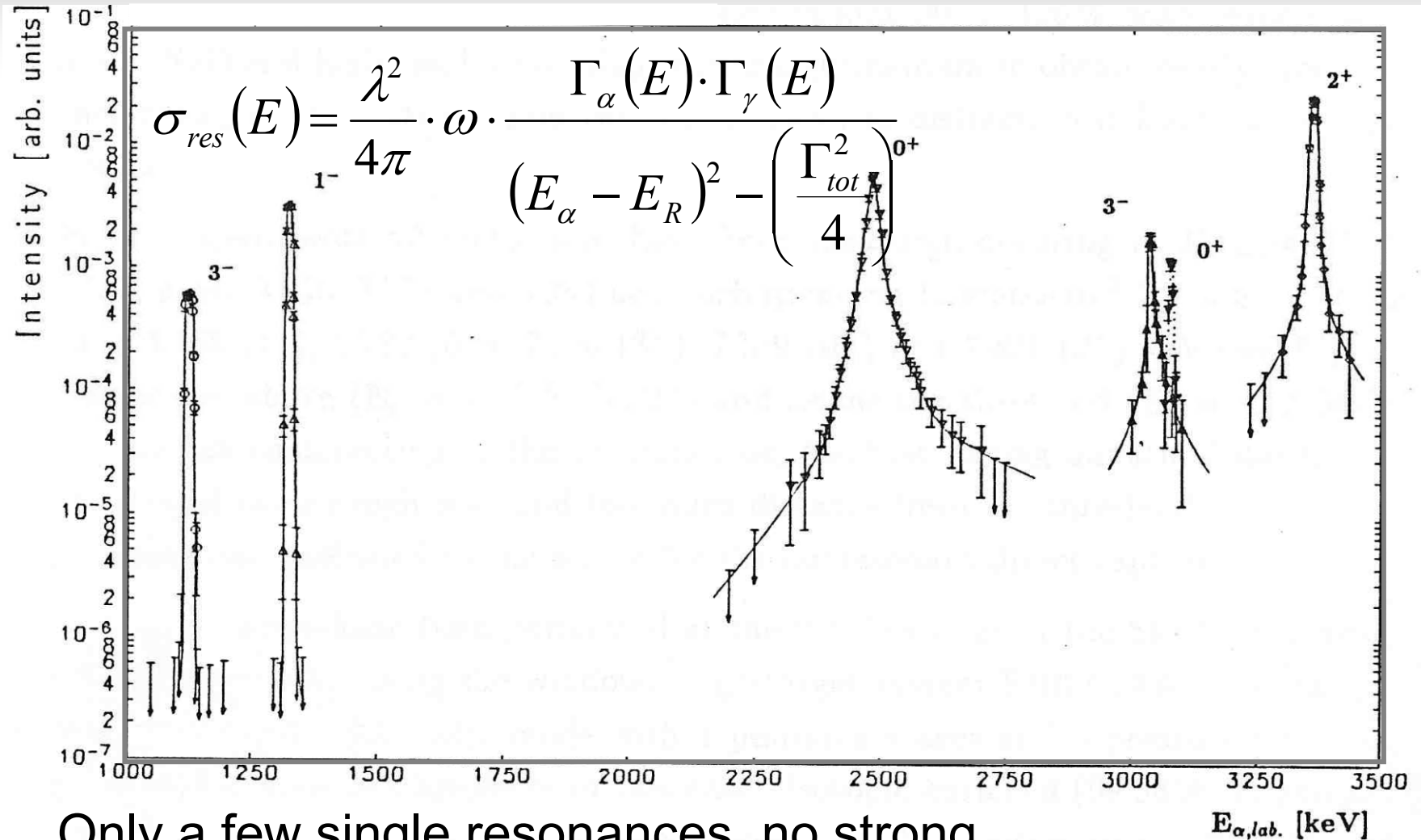
$$S_{eff} \approx 0.17 [\text{MeV} - b]$$

$$N_A \langle \sigma v \rangle \approx 1.2 \cdot 10^8 \cdot T_9^{-2/3} \cdot e^{-\frac{32.11}{T_9^{1/3}}} \left[\frac{\text{cm}^3}{\text{s}} \right]$$

Only very crude estimate!

E-T dependency needs to be considered!

The $^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne}$ reaction mechanism



Only a few single resonances, no strong non-resonant term observed in the excitation curve!

Direct capture contributions to the cross section of $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$

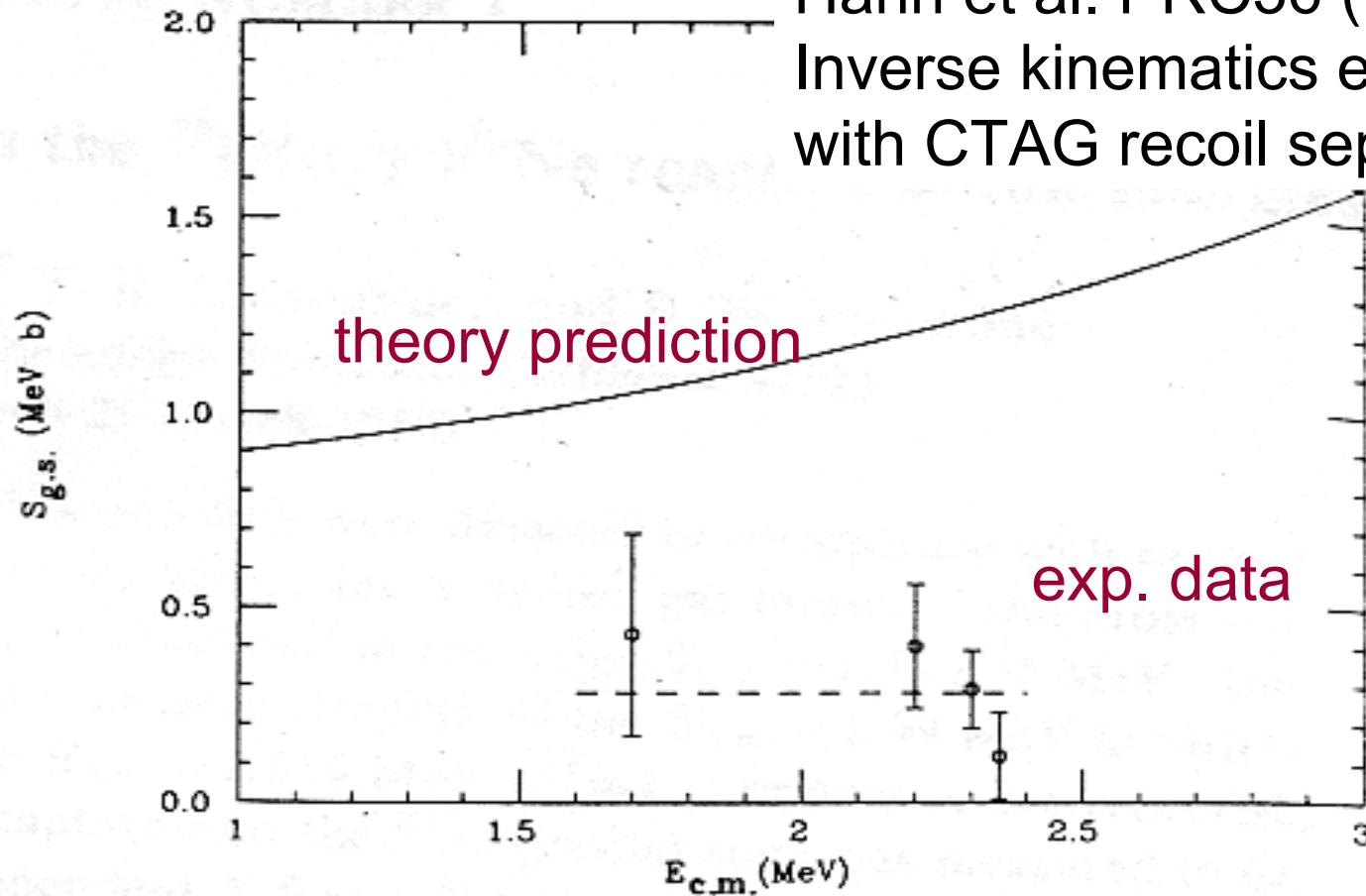
$$\sigma_{dc}(E1) \propto \mu^2 \cdot \left(\frac{Z_1}{M_1} - \frac{Z_2}{M_2} \right)^2 \frac{1}{\sqrt{\mu}} \frac{E_\gamma^3}{E_\alpha^{3/2}} \cdot \frac{(2J_f + 1)(2\ell_i + 1)}{(2J_t + 1)(2\ell_f + 1)} \cdot \left(\ell_i 010 | \ell_{f0} \right)^2 \int_0^\infty u_c(r) \cdot \Omega_{E1}(r) \cdot u_b(r) \cdot r^2 dr$$

For $^{16}\text{O} + \alpha$ $\frac{Z_1}{M_1} - \frac{Z_2}{M_2} = \frac{2}{4} - \frac{8}{16} = 0$ **No E1 dc-term**

$$\sigma_{dc}(E2) \propto \mu^4 \cdot \left(\frac{Z_1}{M_1^2} + \frac{Z_2}{M_2^2} \right)^2 \frac{1}{\sqrt{\mu}} \frac{E_\gamma^5}{E_\alpha^{3/2}} \cdot \frac{(2J_f + 1)(2\ell_i + 1)}{(2J_t + 1)(2\ell_f + 1)} \cdot \left(\ell_i 010 | \ell_{f0} \right)^2 \int_0^\infty u_c(r) \cdot \Omega_{E1}(r) \cdot u_b(r) \cdot r^2 dr$$

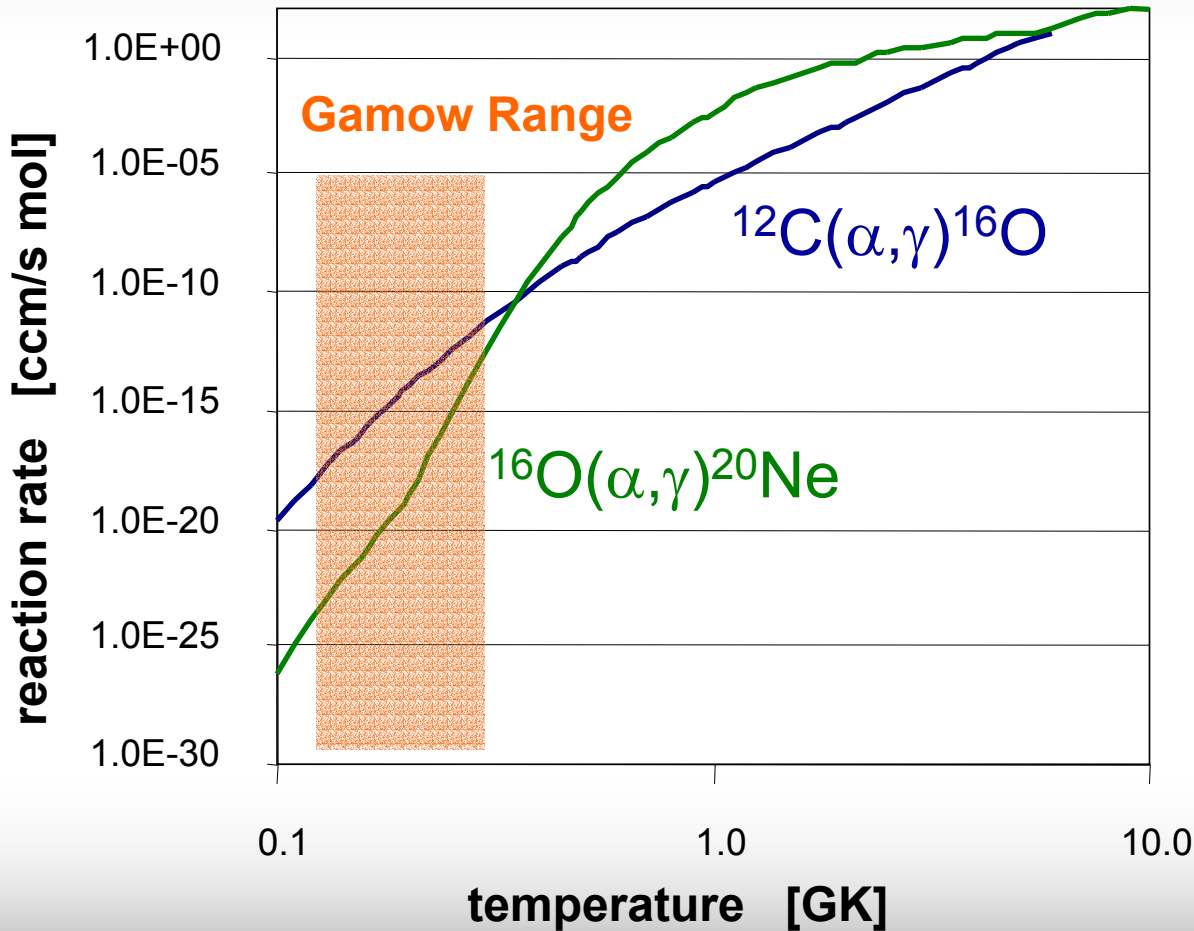
E2-dc S-factor term

Hahn et al. PRC36 (1987)
Inverse kinematics experiment
with CTAG recoil separator



No strong direct capture in E1 and E2 observed!

Impact of the $^{12}\text{C}, ^{16}\text{O}(\alpha, \gamma)$ rates

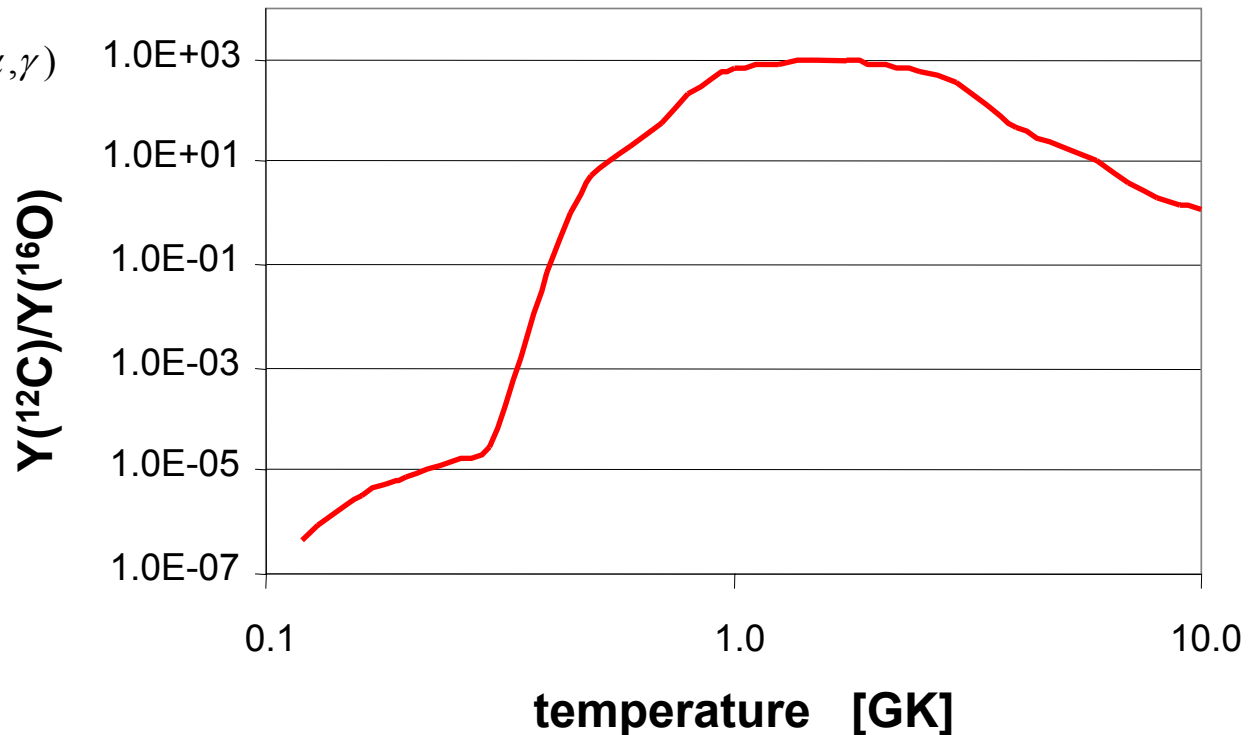


$^{12}\text{C}(\alpha, \gamma)$ rate dominates over the $^{16}\text{O}(\alpha, \gamma)$ rate at typical He-burning temperatures $T \sim 0.1\text{-}0.3$ GK.

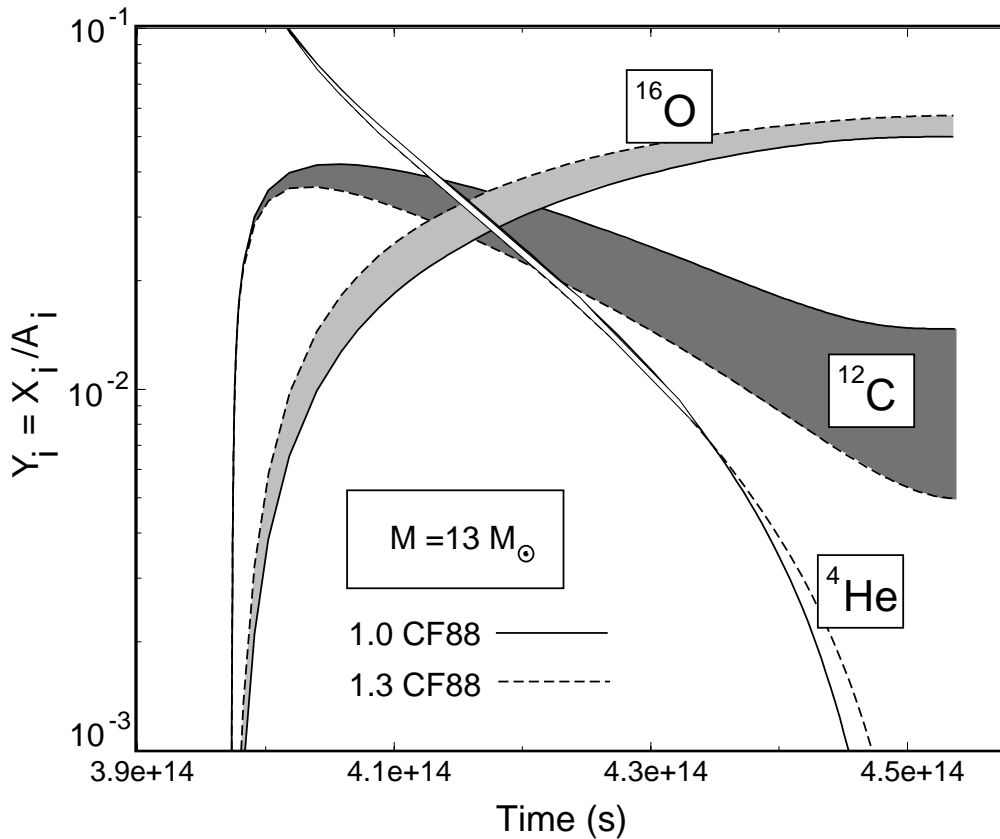
... the $^{16}\text{O}/^{12}\text{C}$ ratio in steady state

$$\frac{dY_{^{16}\text{O}}}{dt} = -Y_{^{16}\text{O}} \cdot Y_{^4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{^{16}\text{O}(\alpha,\gamma)} + Y_{^{12}\text{C}} \cdot Y_{^4\text{He}} \cdot \rho \cdot N_A \langle \sigma v \rangle_{^{12}\text{C}(\alpha,\gamma)} = 0$$

$$\frac{Y_{^{12}\text{C}}}{Y_{^{16}\text{O}}} = \frac{N_A \langle \sigma v \rangle_{^{16}\text{O}(\alpha,\gamma)}}{N_A \langle \sigma v \rangle_{^{12}\text{C}(\alpha,\gamma)}}$$



Consequences of $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$



- Late Stellar Evolution determines Carbon and/or Oxygen phase
- Type Ia Supernova central carbon burning of C/O white dwarf
- Type II Supernova shock-front nucleosynthesis in C and He shells of pre-supernova star

Summary

Several important experiments are being discussed

- ❑ Possible improvement in the accuracy of the 7.654 0^+ decay channel
- ❑ Improved low energy data (yield and angular distribution) for $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ R-matrix analysis
- ❑ Alternative approaches like $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$ to broaden data set and statistics for R-matrix analysis
- ❑ New resonance and E2 direct capture study for $^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}$