Convective Phase & Flame Triggering in SNIa

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Motivation

- Prior to the formation of the thermonuclear flame, slow, simmering reactions heat the core of the white dwarf, making it convectively unstable.

- Turbulent convection transports this energy from the core to the outer layers of the star.

- As the core heats up, the rate of energy generation increases and the convection becomes stronger.

- This pre-existing convective flow sets the conditions under which the flame is born.

- Recently, much effort has been made to simulate flame evolution, but less effort has been put into defining the proper initial conditions for these simulations.

White Dwarf Structure

Background State:

- Hydrostatic Equilibrium
- Isentropic
- Efficient convection

Density $$\rho_a(r) = \rho_o\left(1 - \frac{r^2}{2a^2}\right)$$

Gravity $$g(r) = \frac{4\pi}{3} G\rho_o r\left(1 - \frac{3r^2}{10a^2}\right)$$

Adiabatic background temperature $$T_a(r) \approx T_o\left(1 - \frac{r^2}{\Lambda^2}\right)$$

$\rho_o \approx 2 \cdot 10^9\, g/cm^3$ $\qquad T_o \approx 7 \cdot 10^8\,\mathrm{K}$

$a \approx 400\,\mathrm{km}$ $\qquad \Lambda \approx 750\,\mathrm{km}$

Convection Dynamics

Fluctuations: $T(\mathbf{r},t) \equiv T_a(r) + \delta T(\mathbf{r},t)$

$$\frac{\delta\rho(\mathbf{r},t)}{\rho_a(r)} \equiv -\delta_p \frac{\delta T(\mathbf{r},t)}{T_a(r)}$$

Dynamics: $\left(\partial_t + \mathbf{u} \bullet \nabla - \kappa(T)\nabla^2\right)\delta T(\mathbf{r},t) = Q(r,\delta T)$

Nuclear energy generation rate: $Q(r,\delta T) = \frac{T_o}{\tau}\left(1 - \frac{r^2}{\Lambda^2} + \frac{\delta T}{T_o}\right)^{23}$

T_o = Background central temperature
$\tau(T_o)$ = Nuclear reaction timescale
Λ = Stellar Structure Parameter

Lagrangian Path ***r(t):*** $\partial_t \delta T(\mathbf{r}(t)) = Q(r,\delta T)$

Convection Model

<u>Burning Zone</u>
Significant heating (Q large)
Buoyancy weak (g small)

r_b = Radius
δT_b = Excess Temperature
t_b = Residence time
U = Flow velocity

<u>Convection zone</u>
Minimal heating (Q small)
Buoyancy strong (g large)

R = Radius (fixed)
U = Flow velocity

- Cool fluid enters burning zone with velocity U, is heated, and carries heat to outer layer (convection zone).

- Process repeats (re-circulating flow).

Flow Pattern

A persistent “jet” of constant mean velocity *U* flowing through the stellar core is assumed.

1. Cold fluid is swept into jet (by mass conservation) from convection zone at one pole.
2. Jet carries cold fluid through the central burning zone, where it is heated.
3. Upon reaching top of convection zone, flow circulates and returns to jet inflow at opposite pole. Heat is deposited along top of convection zone.

Velocity scale *U* of jet is determined by buoyancy of hot bubbles rising to top of convection zone.

Alternate flow patterns with nearly constant radial velocity would yield similar predictions.

Burning Zone Dynamics

Lagrangian path through center of star: $r(t) = -r_b + Ut$

$$2r_b = Ut_b$$

Neglect non-linear effect of δT: $\partial_t \delta T \approx \frac{T_o}{\tau}\left(1 - \frac{r^2(t)}{\Lambda^2}\right)^{23}$

$$\frac{\delta T(r_b)}{T_o} \approx \frac{t_b}{\tau}\int_0^1 \left(1 - \frac{r_b^2}{\Lambda^2} z^2\right)^{23} dz \approx \frac{t_b}{\tau}$$

Assume $\delta T_b \approx \delta T(r_b)$; implies $\frac{\delta T_b}{T_o} \approx \frac{t_b}{\tau}$

Significant heating (Q large): $1 - \frac{r^2}{\Lambda^2} + \frac{\delta T_b}{T_o} > 1$ $\quad \frac{r_b^2}{\Lambda^2} \approx \frac{\delta T_b}{T_o}$

Convection velocity U: $U^2 \approx -g(R)R\frac{\delta\rho}{\rho_o} \approx G\beta\rho_o R^2 \frac{\delta T_b}{T_o}$

Results

$$\frac{\delta T_b}{T_o} \approx \left(\frac{\Lambda}{R}\right)\left(G\delta_p\rho_o\tau^2\right)^{-1/2} \qquad \frac{t_b}{\tau} \approx \left(\frac{\Lambda}{R}\right)\left(G\delta_p\rho_o\tau^2\right)^{-1/2}$$

$$\frac{r_b}{R} \approx \left(\frac{\Lambda}{R}\right)^{3/2}\left(G\delta_p\rho_o\tau^2\right)^{-1/4} \qquad U \approx \frac{R}{\tau}\left(\frac{\Lambda}{R}\right)^{1/2}\left(G\delta_p\rho_o\tau^2\right)^{1/4}$$

As background temperature T_o increases:

- Nuclear timescale τ shortens dramatically
- Burning zone gets larger (r_b increases)
- Excess temperature δT_b (buoyancy) increases
- Convection velocity U increases

(Assumes constant convective radius R.)

Heat Flux

Efficient convection: flux balances energy generation

Heat flux due to jet of area *A*: $$F \approx UA\delta T$$

Core energy generation rate: $$L \approx 4\pi \int_0^{r_b} Qr^2 dr \approx \frac{T_o r_b^3}{\tau}$$

Solve energy balance *F*=*L* for jet: $$A \approx r_b^2 \left(\frac{T_o}{\delta T_b} \frac{r_b}{U\tau} \right) \approx r_b^2$$

Jet area *A* scales with size of burning region r_b^2.

Location of Ignition Point

Heating of fluid element along path through core (*r=0*):

$$\frac{\delta T(r)}{\delta T(r_b)} \approx \frac{1}{2}\left(1+\frac{r}{r_b}\right)$$

(Heating continues on outbound trajectory!)

Hottest point: maximize total temperature:

$$T(r) = T_a(r) + \delta T(r)$$

$$r_{\max} \approx \left(\frac{\delta T(r_b)}{4\delta T_b}\right) r_b \approx r_b$$

Hottest point in path occurs off-center. Radius of hottest point (ignition point) scales with r_b.

Conditions for Ia Supernovae

<u>Estimated conditions near ignition</u>

- Convection parameter $G\delta_p \rho_o \tau^2 \approx 1500$
- Central temperature $T(0) \approx 7.8 \cdot 10^8 K$
- Max temperature $T_{\max} \approx 8.1 \cdot 10^8 K$
- Burning zone radius $r_b \approx 100 - 200 \text{ km}$
- Jet velocity $U \approx 50 - 100 \text{ km/s}$

- Ignition would occur off-center, in the outer part of the burning zone, within the jet of velocity *U*.
- Multiple Ignition points (due to fluctuations) might be found clustered within the jet of area r_b^2.
- Alternate flow patterns could have a different distribution of ignition points
- Ignition radius and jet velocity might vary among SNIa's.

Impact of Initial Conditions

Ignition point at radius r_b launches a burning bubble with initial velocity U and ash density deficit $\Delta\rho$.

Simplified Sharp-Wheeler (RT) model for bubble rise:

$$\rho_o \frac{d^2 r}{dt^2} \approx 2\alpha\Delta\rho \mathrm{g}(\mathrm{r})$$

Integrate to find time t_R to rise to radius R:

$$t_R \approx \tau_g \ln\left(\frac{2R}{r_b + U\tau_g}\right) \qquad \tau_g \equiv \sqrt{\frac{3}{8\pi\alpha G\Delta\rho}}$$

Volume of star burned before reaching surface:

$$\text{Volume} \approx \frac{4\pi}{3}\left(u_f t_R\right)^3$$ (u_f = flame speed)

Questions for future work

- What is the evolution of the central temperature T_o and the convection radius R?
- Do variations in the ignition radius and jet velocity make a difference in the explosion?
- Can one relate these variations in the initial conditions to differences in "observations?"
- Does rotation influence the initial conditions?
- Do multiple ignition points affect the explosion?
- Does the flow pattern affect the explosion?