Types of Equilibria

- Steady Flow of Reactions
- Chemical Equilibrium of Reactions
- Complete Chemical Equilibrium (NSE)
- Clusters of Chemical Equilbrium (QSE)
- QSE Clusters linked by Steady Flow

CNO(I)-Cycle in Steady Flow

The CNO-Cycles in Hydrogen Burning

cycle	reaction sequence
CNOI	$^{12}C(p,\gamma)^{13}N(e^+\nu)^{13}C(p,\gamma)^{14}N\,(p,\gamma)^{15}O(e^+\nu)^{15}N(p,\alpha)^{12}C$
CNOII	$^{15}N(p,\gamma)^{16}O(p,\gamma)^{17}F\,(e^+\nu)^{17}O(p,\alpha)^{14}N$
CNOIII	$^{17}O(p,\gamma)^{18}F(e^+\nu)^{18}O(p,\alpha)^{15}N$
CNOIV	$^{18}O(p,\gamma)^{19}F(p,\alpha)^{16}O$

$$\dot{Y}_1 = \rho N_A \langle 12,1\rangle Y_{12}Y_1 - \rho N_A \langle 13,1\rangle Y_{13}Y_1 - \rho N_A \langle 14,1\rangle Y_{14}Y_1 - \rho N_A \langle 15,1\rangle Y_{15}Y_1$$
$$= -4C_{CNO} = -4\rho N_A \langle 14,1\rangle Y_{14}Y_1 = -\frac{1}{\tau_{1,14}}Y_1$$

$$\dot{Y}_4 = \rho N_A \langle 15,1\rangle Y_{15}Y_1 = C_{CNO}$$

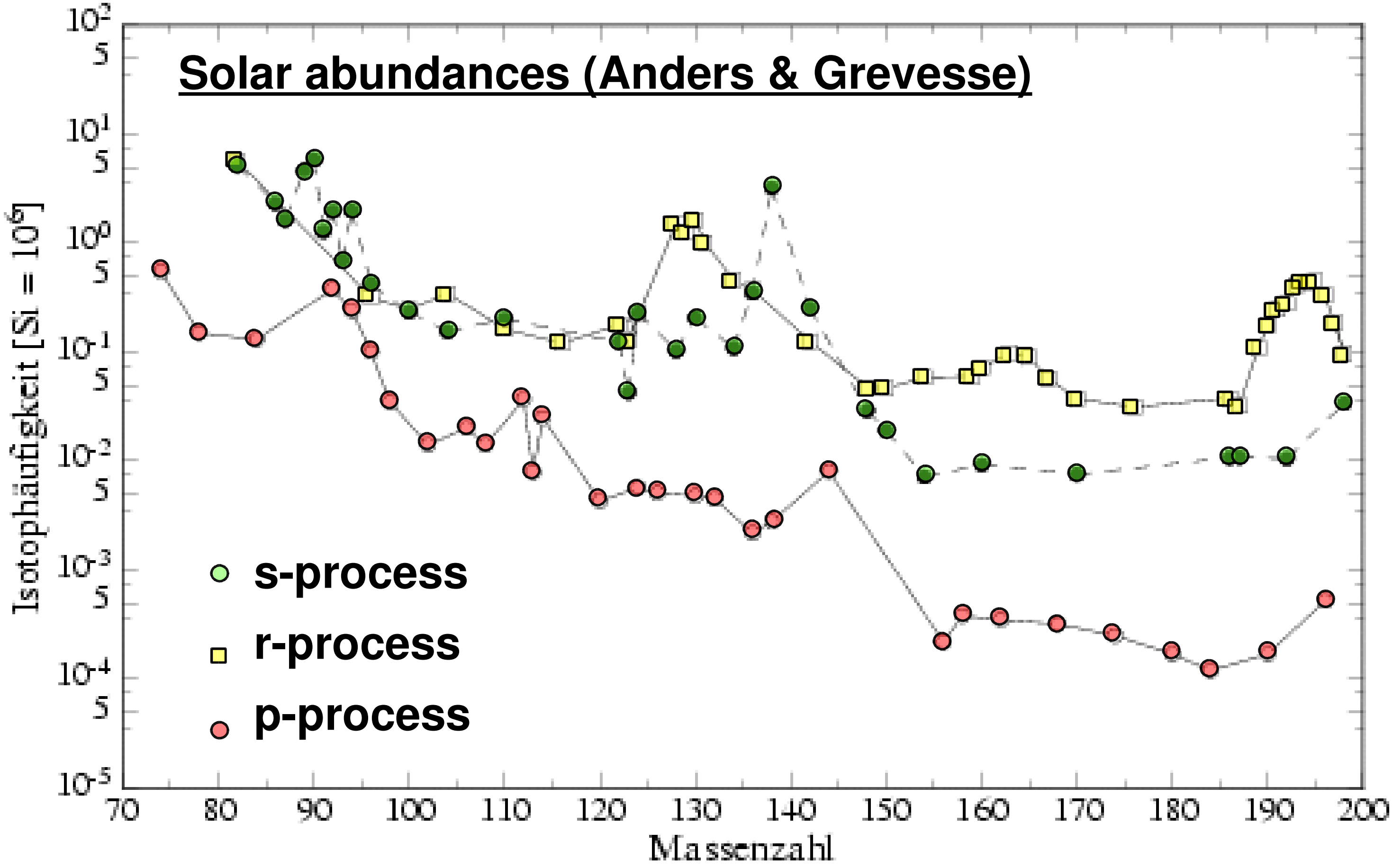
the network entry for nuclei with mass numbers A=12, 13, 14, 15 is governed in each case by a production reaction (proton reaction on A-1) and a distruction reaction (proton reaction on A). In case of a steady flow they cancel and lead to Y=0 for all A, linking all of these terms and identical to (A=14 is useful as this encounters the slowest reaction and essentially all mass assembles in ^{14}N)

$$C_{CNO} = \rho N_A \langle 14,1\rangle Y_{14}Y_1$$

$$Y_{14} \approx \frac{1.4\times 10^{-2}}{14}$$

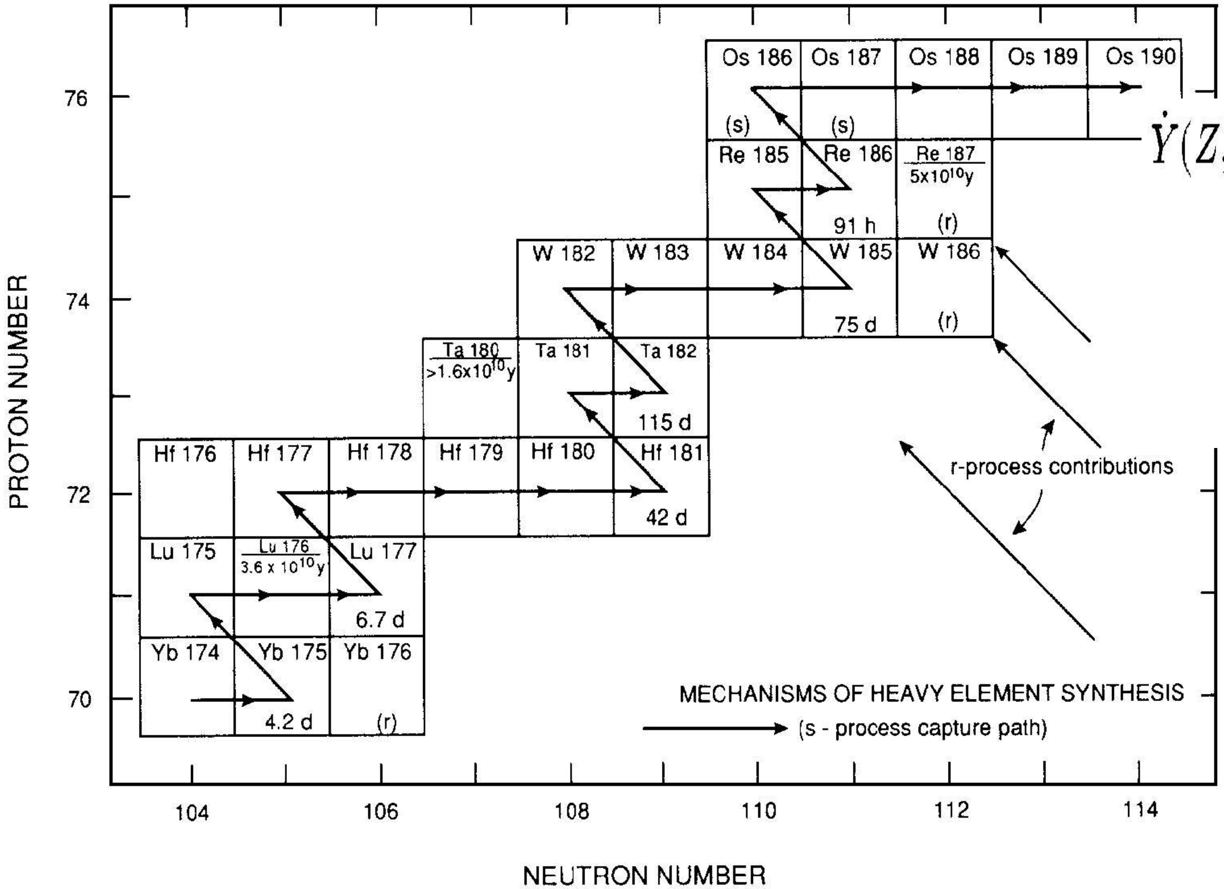
summing all mass fractions of CNO nuclei for solar metallicity

s-process and steady flow



shown are s-, r-, and p-only nuclei!

s-process and steady flow



possible destruction of nucleus (Z,A)

$$\dot{Y}(Z,A) = -\lambda_{\beta^-}(Z,A)Y(Z,A) - \rho N_A < \sigma v >_{n,\gamma} Y_n Y(Z,A)$$
$$= -\lambda_{\beta^-}(Z,A)Y(Z,A) - < \sigma v >_{n,\gamma} n_n Y(Z,A)$$
$$= -\frac{1}{\tau_\beta} Y(Z,A) - \frac{1}{\tau_{n,\gamma}} Y(Z,A).$$

$\tau_n > \tau_\beta$ beta-decay to (Z+1,A)

only one nucleus per A needs to be considered!

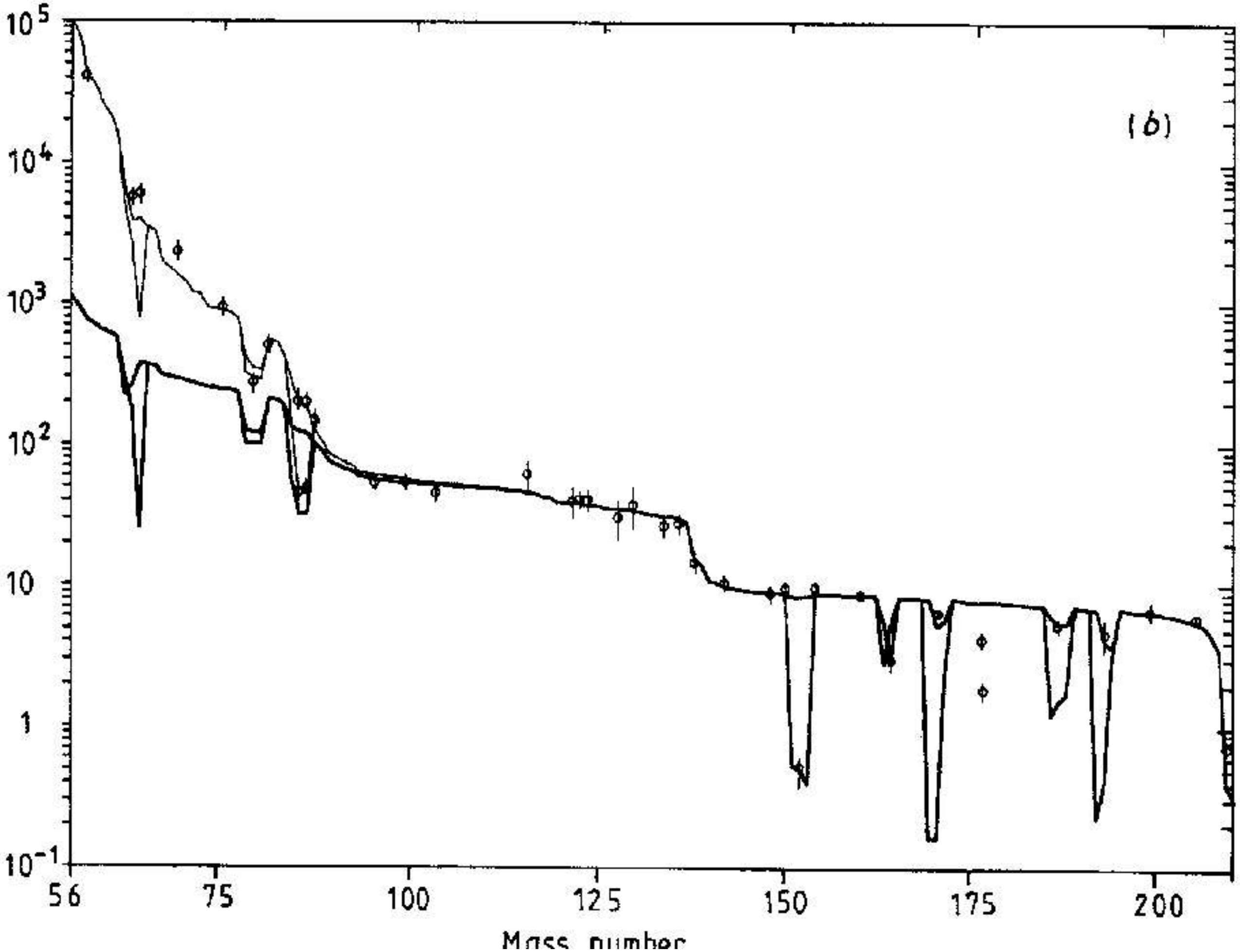
$$\dot{Y}(A) = n_n < \sigma v >_{n,\gamma} Y(A-1) - n_n < \sigma v >_{n,\gamma} Y(A)$$

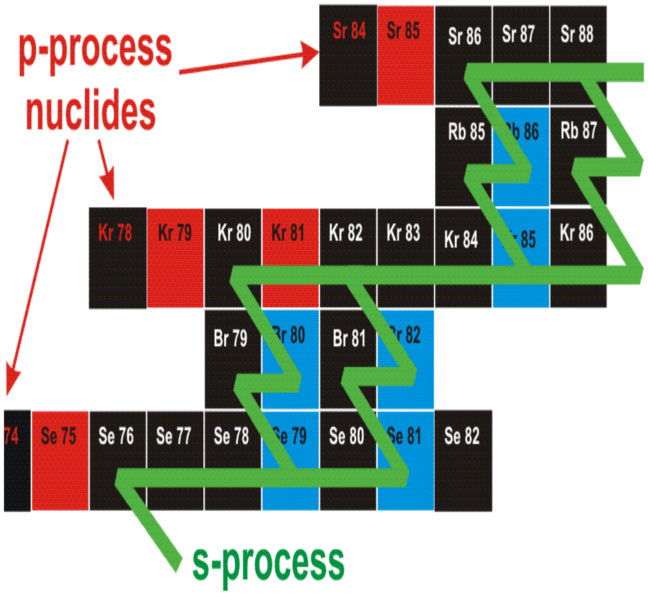
in case of steady flow =0

$\sigma \approx 1/v, < \sigma v > = \sigma(v) v$ therefore

$$\sigma(A-1, 30 \text{ keV}) Y(A-1) = \sigma(A, 30 \text{ keV}) Y(A)$$

The sigma*N-curve

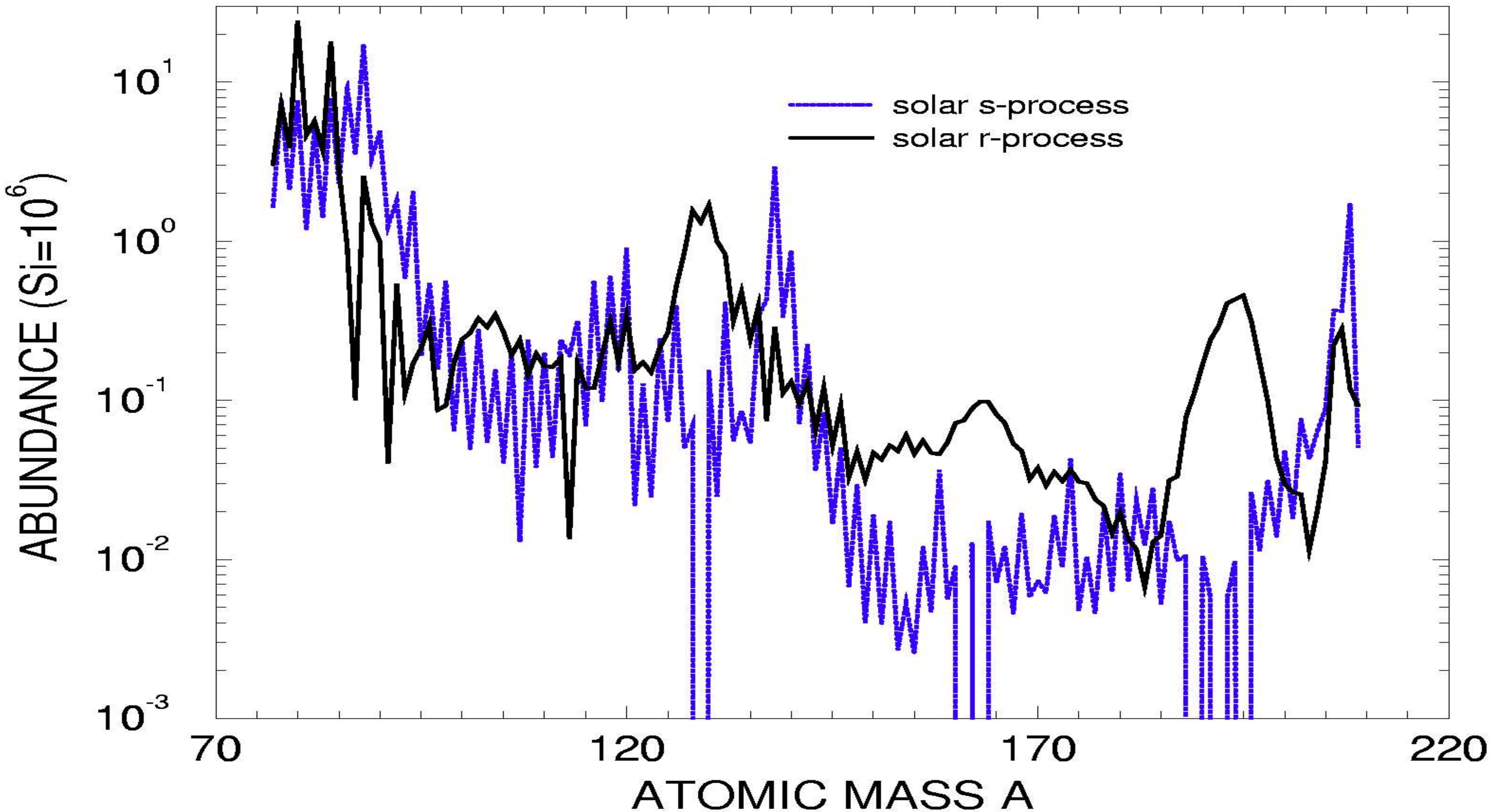




double values due to branchings

a complete steady flow is not given, but in between magic numbers (where the neutron capture cross sections are small) almost attained!

s- and r-decomposition



the almost constant sigma*N-curve leads to a large odd-even staggering in the abundances (due to the odd-even staggering in n-capture cross sections!

Steady flows and chem. equilibrium in stellar burning

pp-cycles and CNO-cycle lead to steady flows in H-burning

1. Hydrogen Burning $T = (1\text{-}4) \times 10^7$ K

   pp-cycles -> $^1H(p,e^+\nu)^2H$

   CNO-cycle -> slowest reaction $^{14}N(p,\gamma)^{15}O$

2. Helium Burning $T=(1\text{-}2) \times 10^8$ K

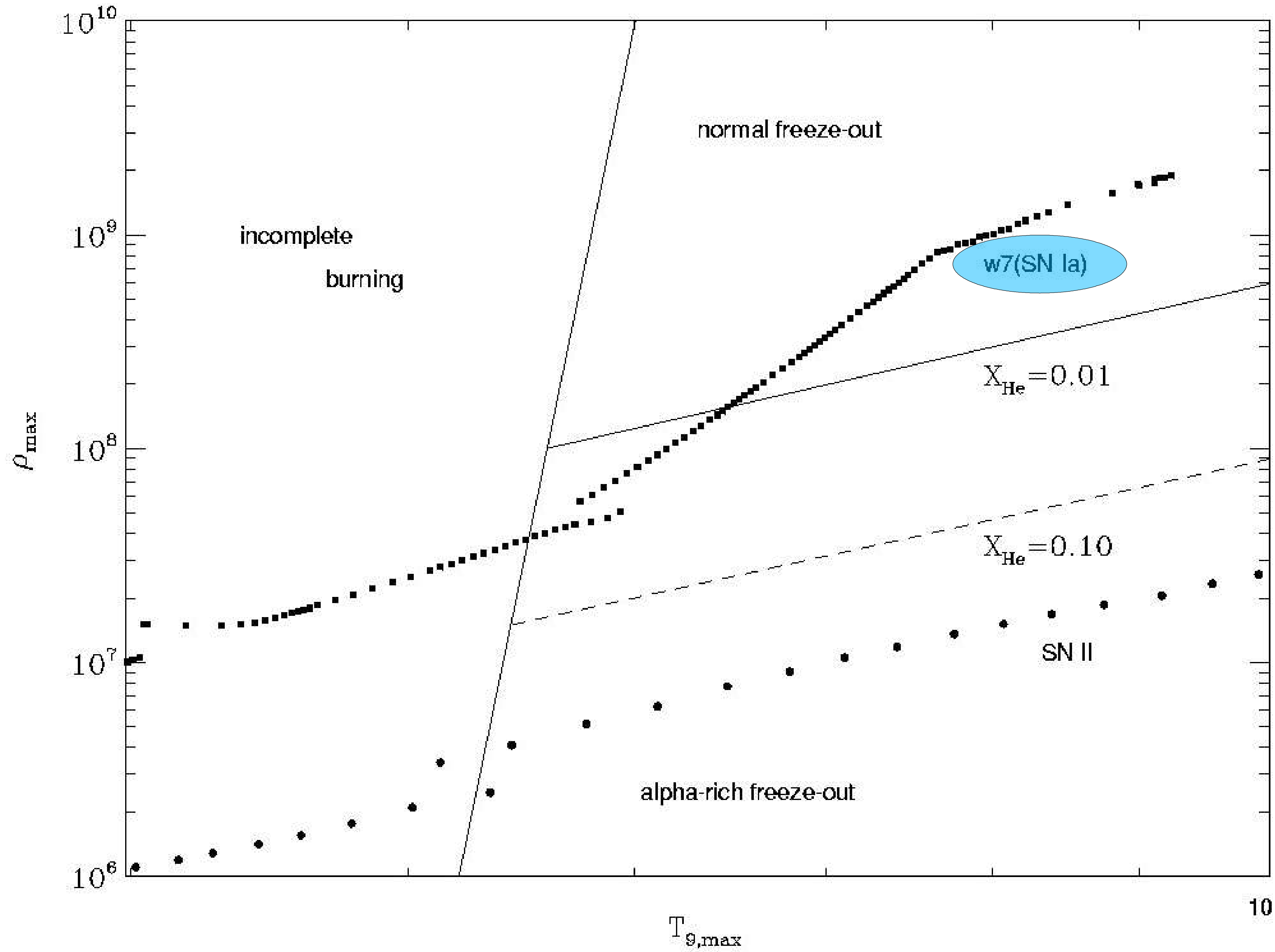
   $^4He + ^4He \Leftrightarrow {}^8Be$ $^8Be(\alpha,\gamma)^{12}C[(\alpha,\gamma)^{16}O]$

   $^{14}N(\alpha,\gamma)^{18}F(\beta^+)^{18}O(\alpha,\gamma)^{22}Ne(\alpha,n)^{25}Mg$

$^4He + ^4He \leftrightarrow {}^8Be$ is in chemical equilibrium

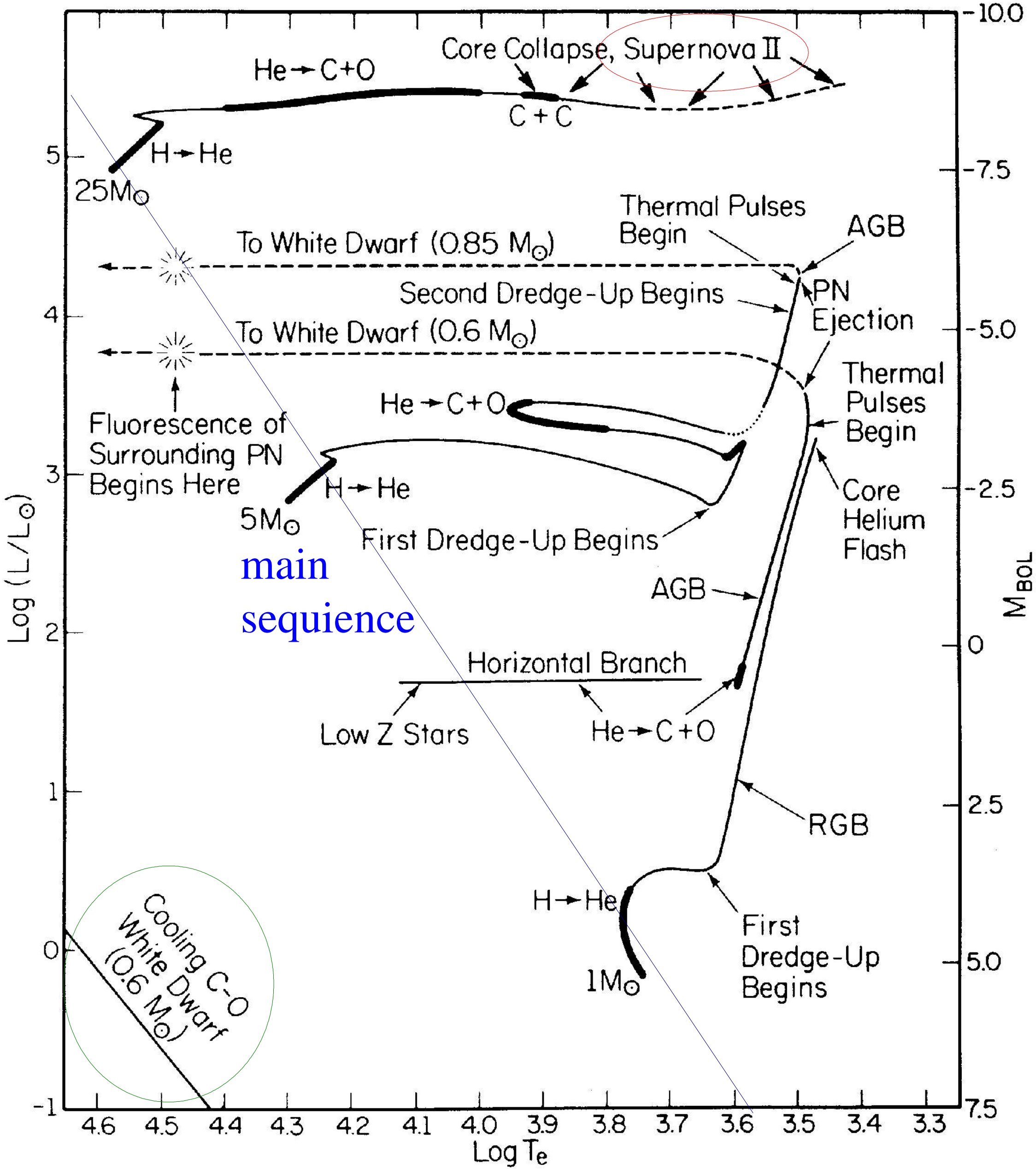
released neutrons lead to steady flow in neutron capture

Complete chem. equilibrium (NSE)



Si-burning in stellar evolution and expl. Si-burning at high densities lead to NSE!

Astrophysical Sites



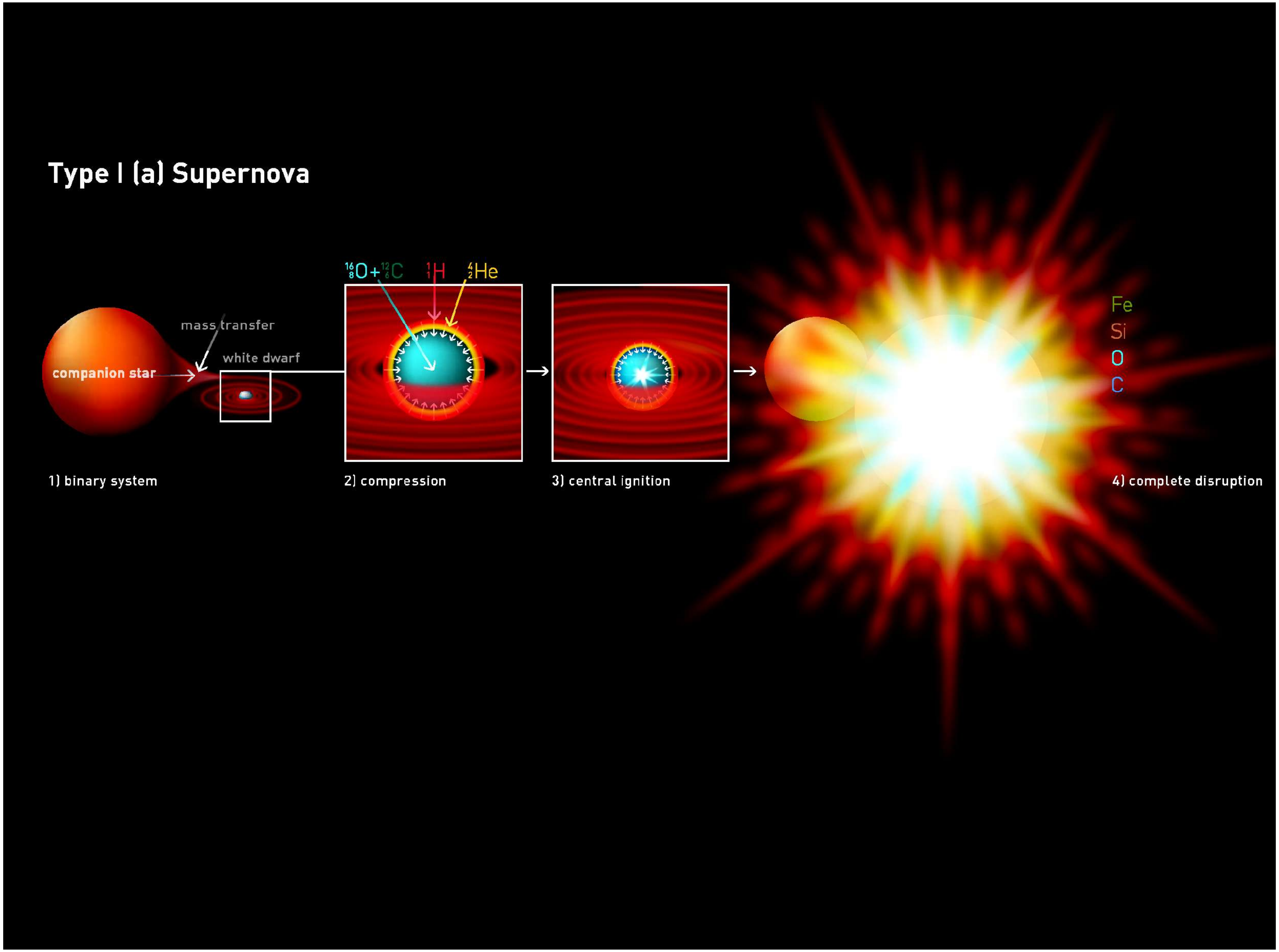
Hertzsprung-Russell Diagram of Stellar Evolution from Iben, showing as end stages

- white dwarfs

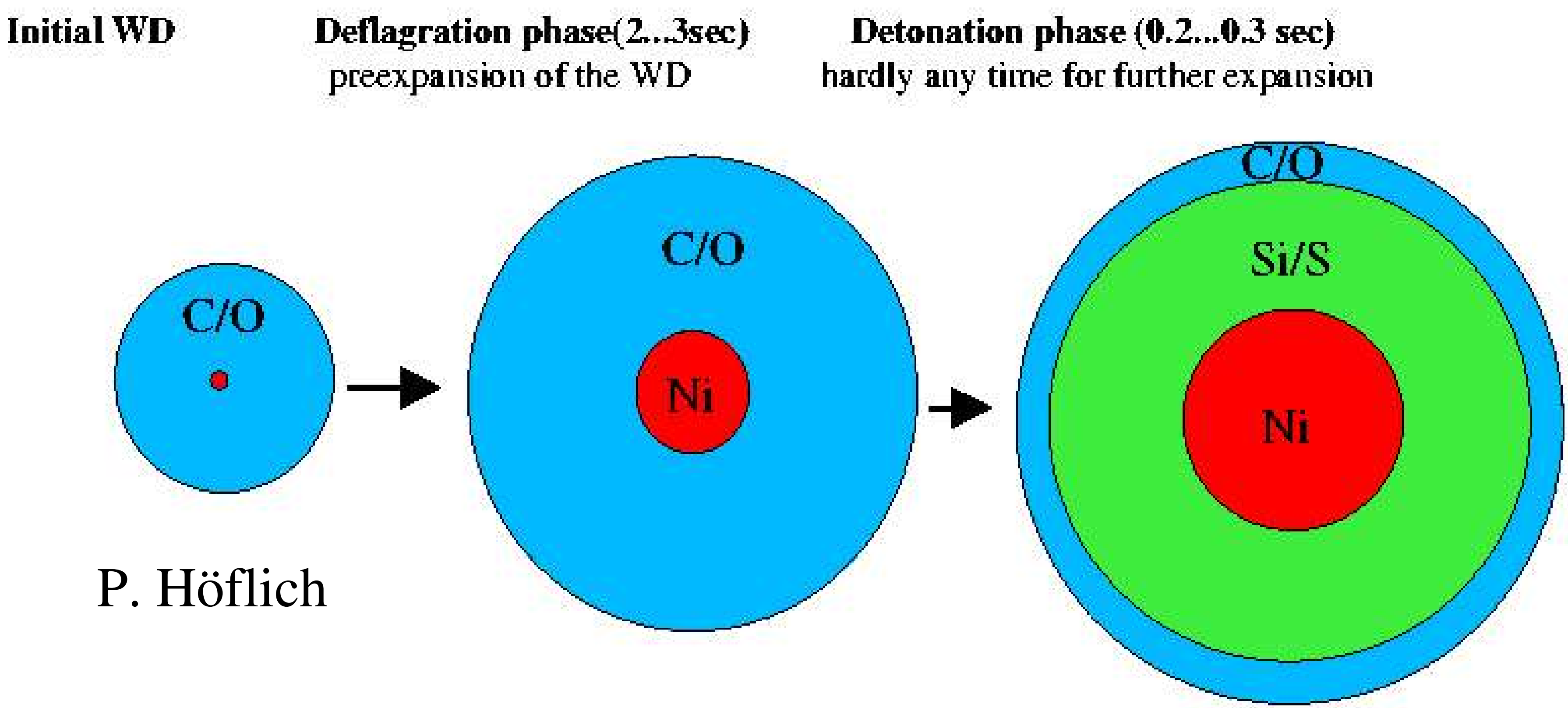
and

- (core collapse) supernovae

Type Ia Supernovae from Accretion in Binary Stellar Systems



Back of the Envelope SN Ia



$M_{ch} \approx 1.4 M_\odot$ of $^{12}C/^{16}O$=1 WD $\rightarrow$ 1.398776 $M_\odot$ ^{56}Ni

$\rightarrow 2.19\times10^{51}$ erg - $E_{grav} \approx (5-6)\times 10^{50}$ erg

reduction due to intermediate elements like Mg, Si, S, Ca

$\rightarrow 1.3\times10^{51}$ erg

Neutronization via electron capture
(high Fermi energies at central densities)

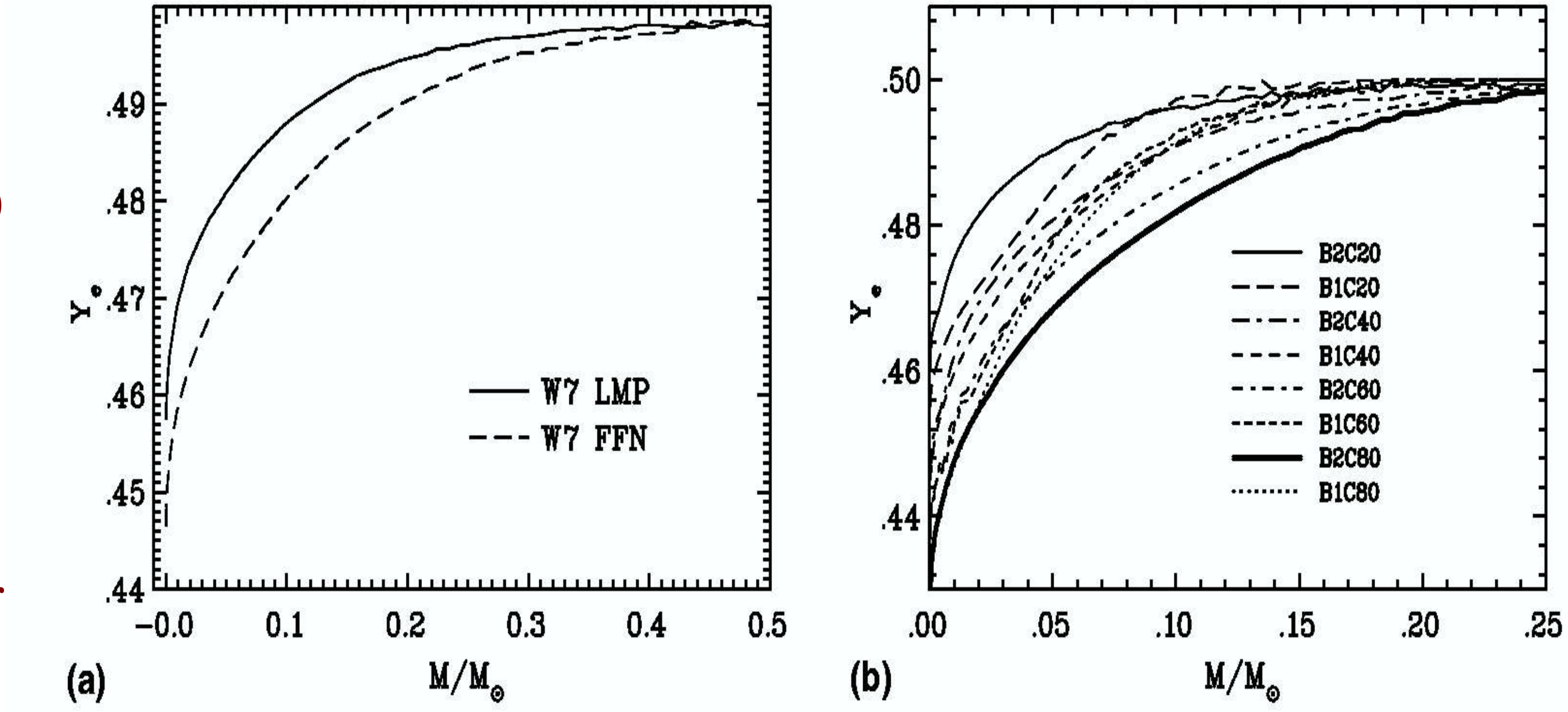
dominant NSE-abundance

^{56}Ni

^{54}Fe (^{58}Ni)

^{56}Fe

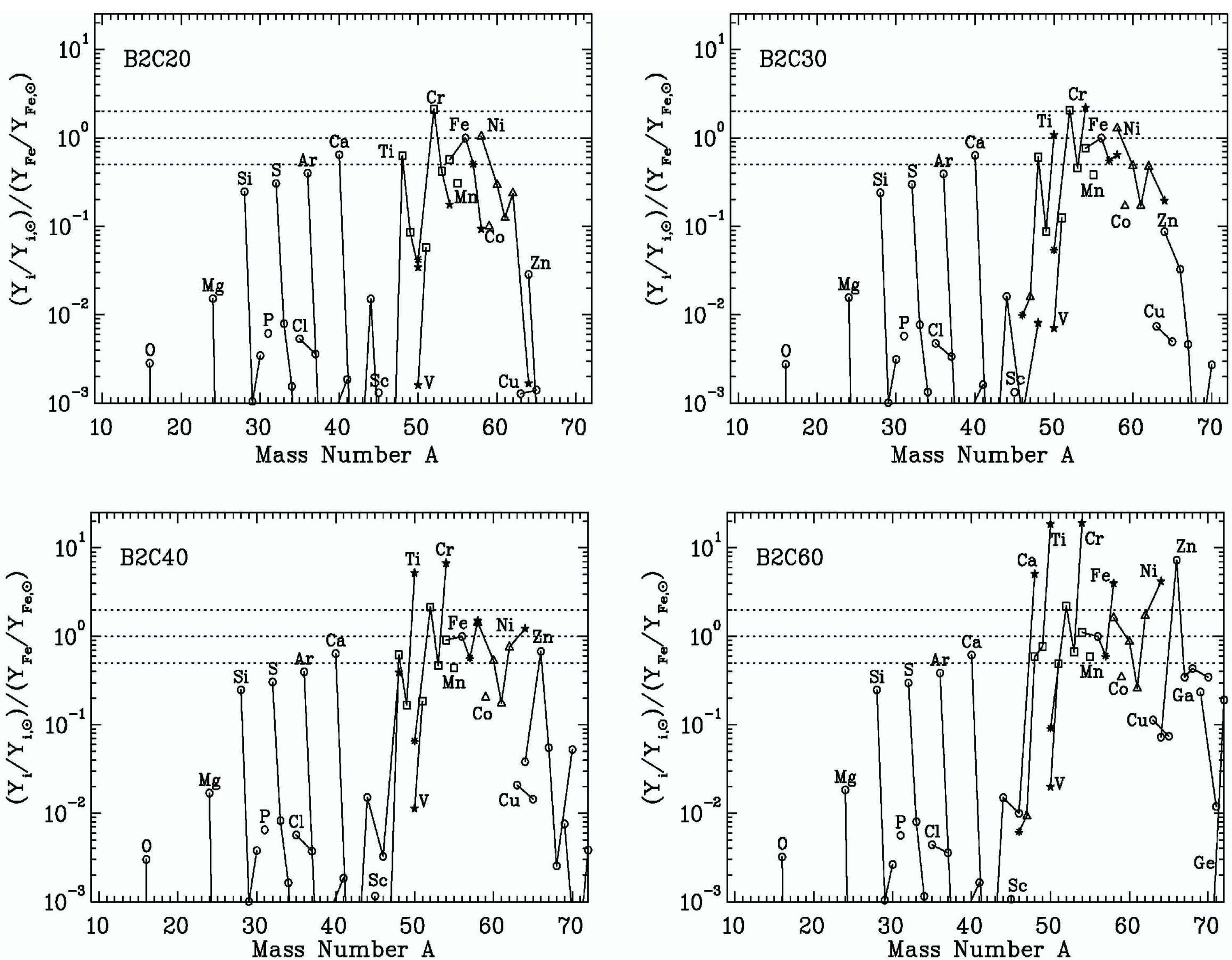
^{50}Ti, ^{54}Cr



(a) Test for influence of new shell model electron capture rates (LMP)

(b) Test for burning front propagation speed

Ignition density determines Ye and neutron-richness of (60-70% of) Fe-group



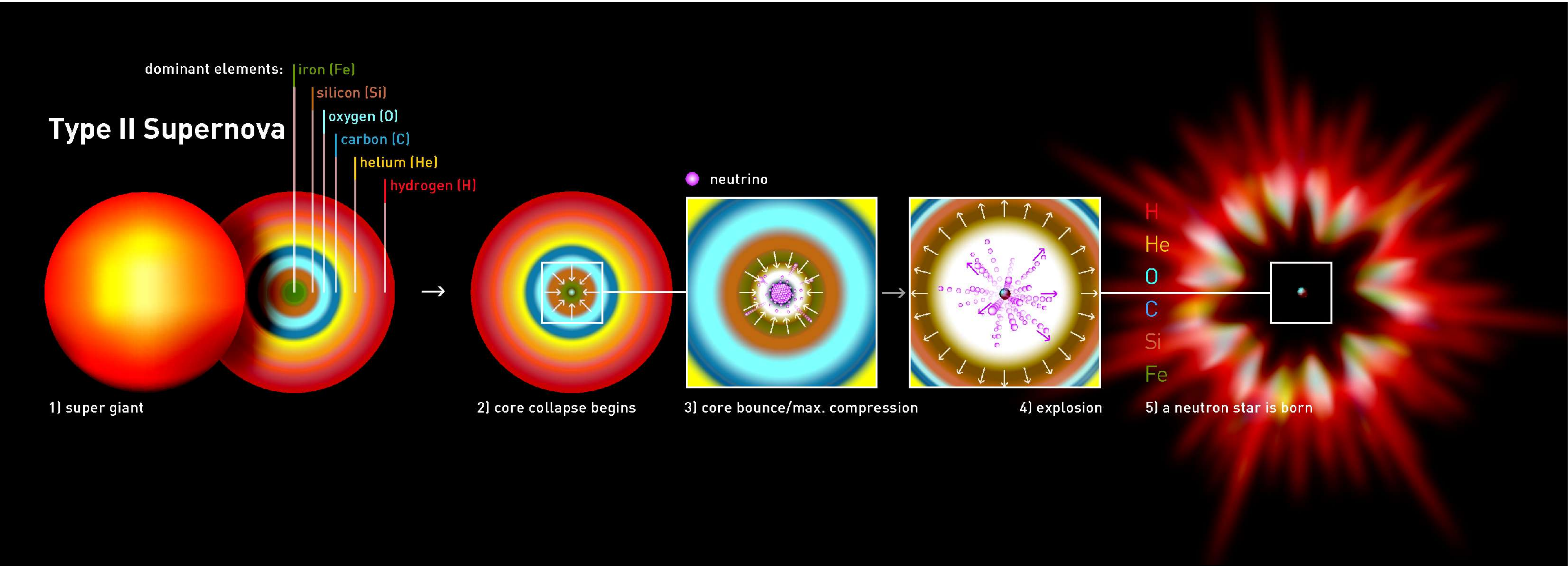
results of explosive C, Ne, O and Si-burning:
Fe-group to alpha-elements 2/1-3/1

SNe Ia dominate Fe-group, over-abundances by more than factor 2 not permitted

→

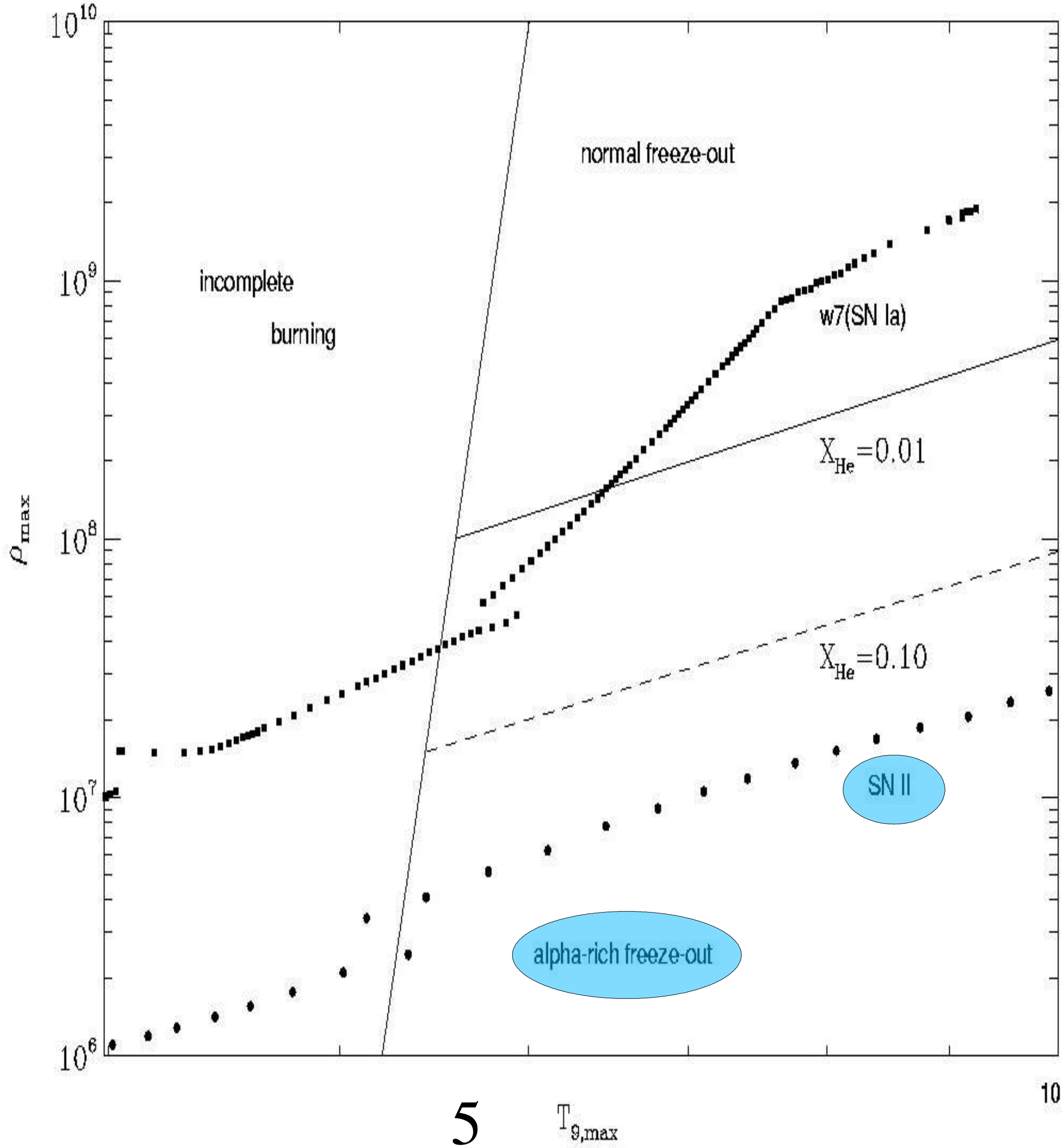
central density <4 10^9 gcm^{-3}

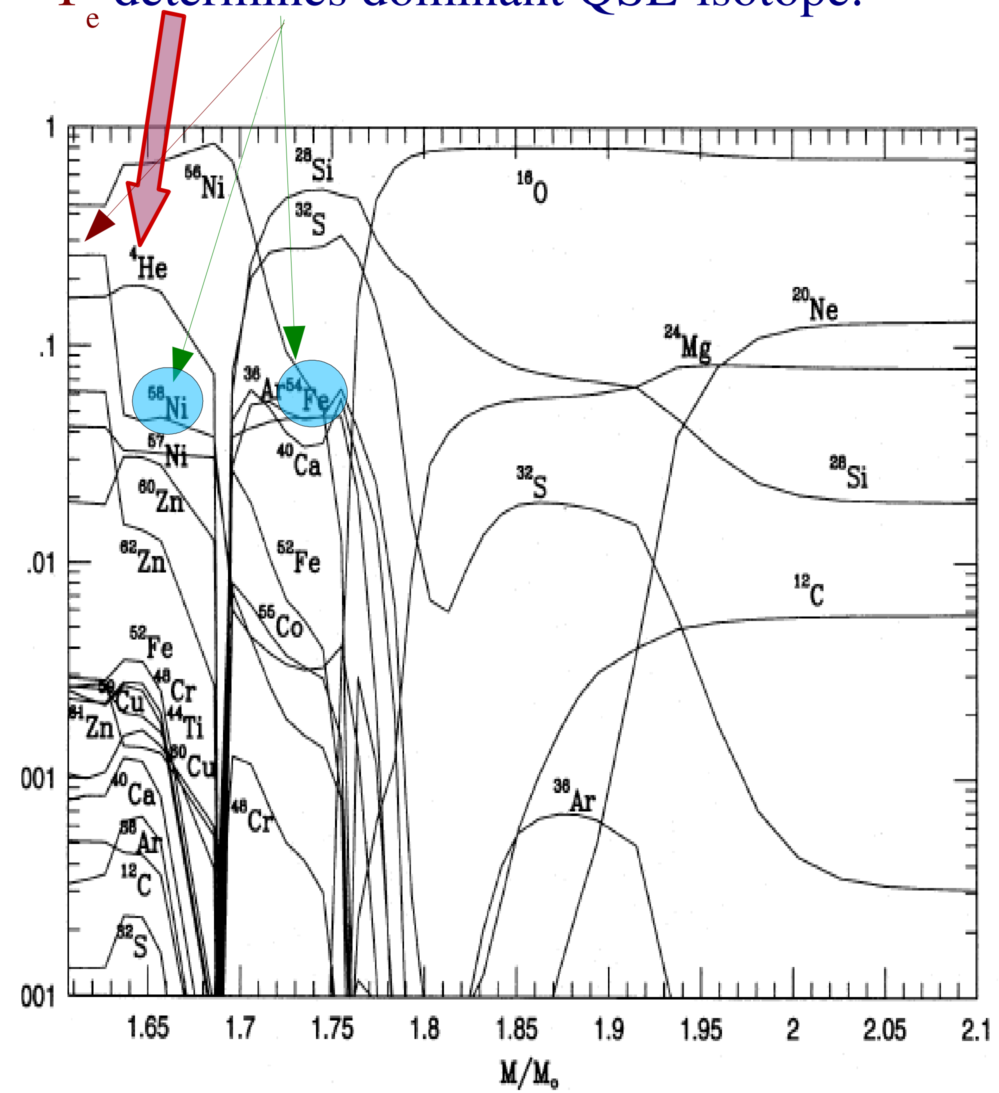
Core Collapse Supernovae from Massive Stars



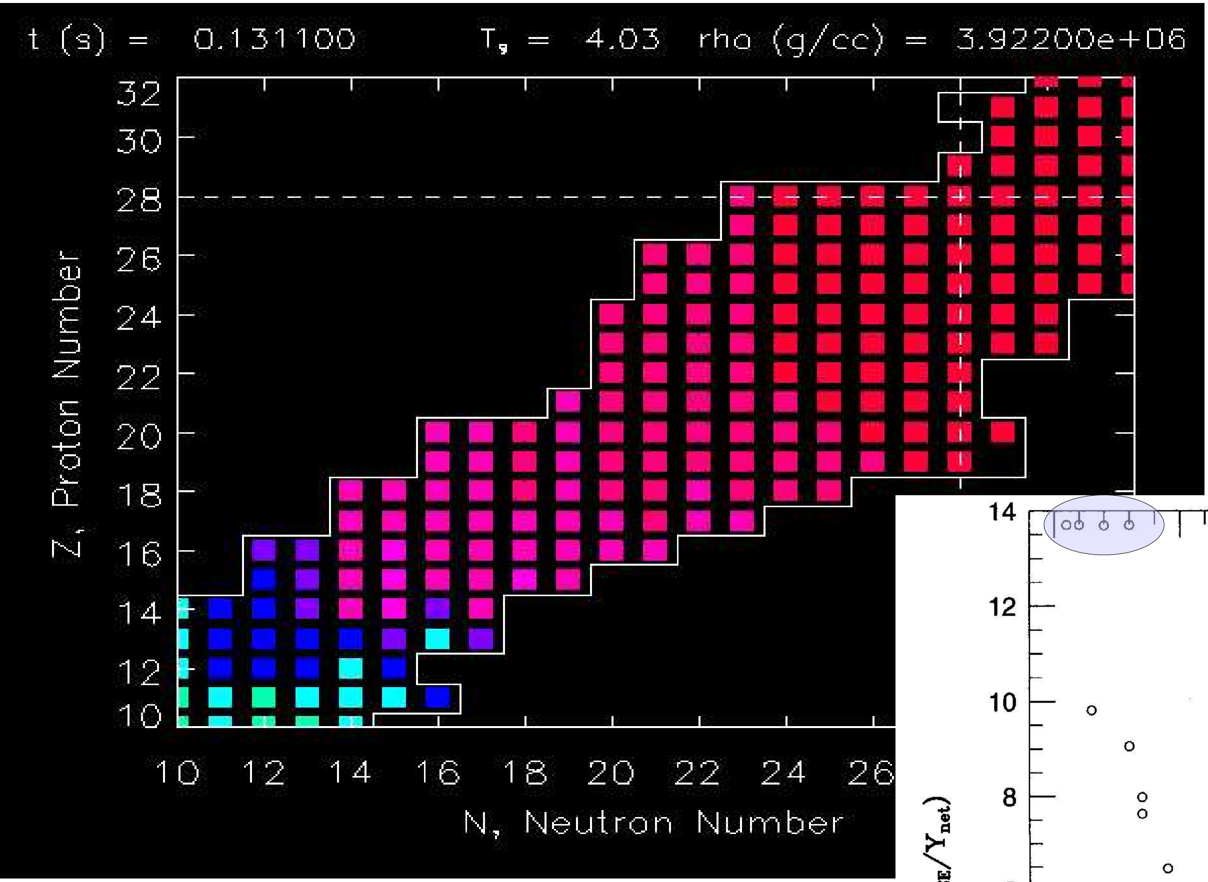
QSE in low density expl. Si-burning

disconnected light element (n,p,He) and Si-Fe QSE-cluster, high alpha-abundance prefers alpha-rich nuclei (^{58}Ni over ^{54}Fe), Y_e determines dominant QSE-isotope.

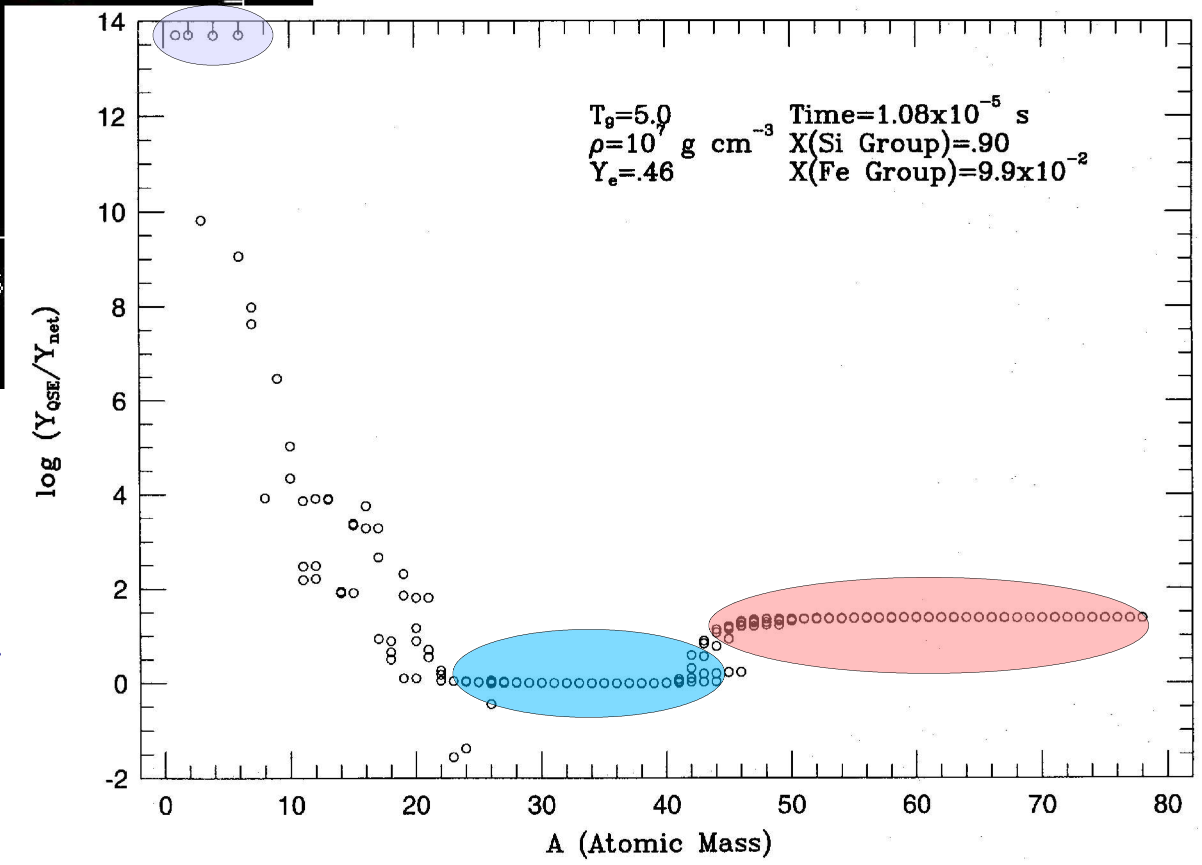




QSE in explosive Si-burning



full NSE is not attained, but there exist equilibrium groups around ^{28}Si, ^{56}Ni and n,p,^{4}He, which are separated by slow reactions



Sample Calculations from

- B.S. Meyer and
- Hix and Thielemann

small Q-values of reactions out of Z=20, N=20 cause small cross sections and hold up equilibrium

QSE Formalism

light group

$$Y_{NSE}(^AZ) = C(^AZ) Y_n^{\,N} Y_p^{\,Z}$$

Si-group

$$Y_{QSE,\mathrm{Si}}(^AZ) = \frac{C(^AZ)}{C(^{28}\mathrm{Si})} Y(^{28}\mathrm{Si}) Y_p^{Z-14} Y_n^{N-14}$$

Ni/Fe-group

$$Y_{QSE,\mathrm{Ni}}(^AZ) = \frac{C(^AZ)}{C(^{56}\mathrm{Ni})} Y(^{56}\mathrm{Ni}) Y_p^{Z-28} Y_n^{N-28}$$

$$C(^AZ) = \frac{G(^AZ)}{2^A} \left(\frac{\rho N_A}{\theta}\right)^{A-1} A^{\frac{3}{2}} \exp\left(\frac{B(^AZ)}{k_B T}\right)$$

$$Y_{NG} = \sum_{i \in Lt\ group} N_i Y_i + \sum_{i \in Si\ group} (N_i - 14) Y_i + \sum_{i \in Fe\ group} (N_i - 28) Y_i$$

$$Y_{ZG} = \sum_{i \in Lt\ group} Z_i Y_i + \sum_{i \in Si\ group} (Z_i - 14) Y_i + \sum_{i \in Fe\ group} (Z_i - 28) Y_i$$

$$Y_{SiG} = \sum_{i \in Si\ group} Y_i \,,$$

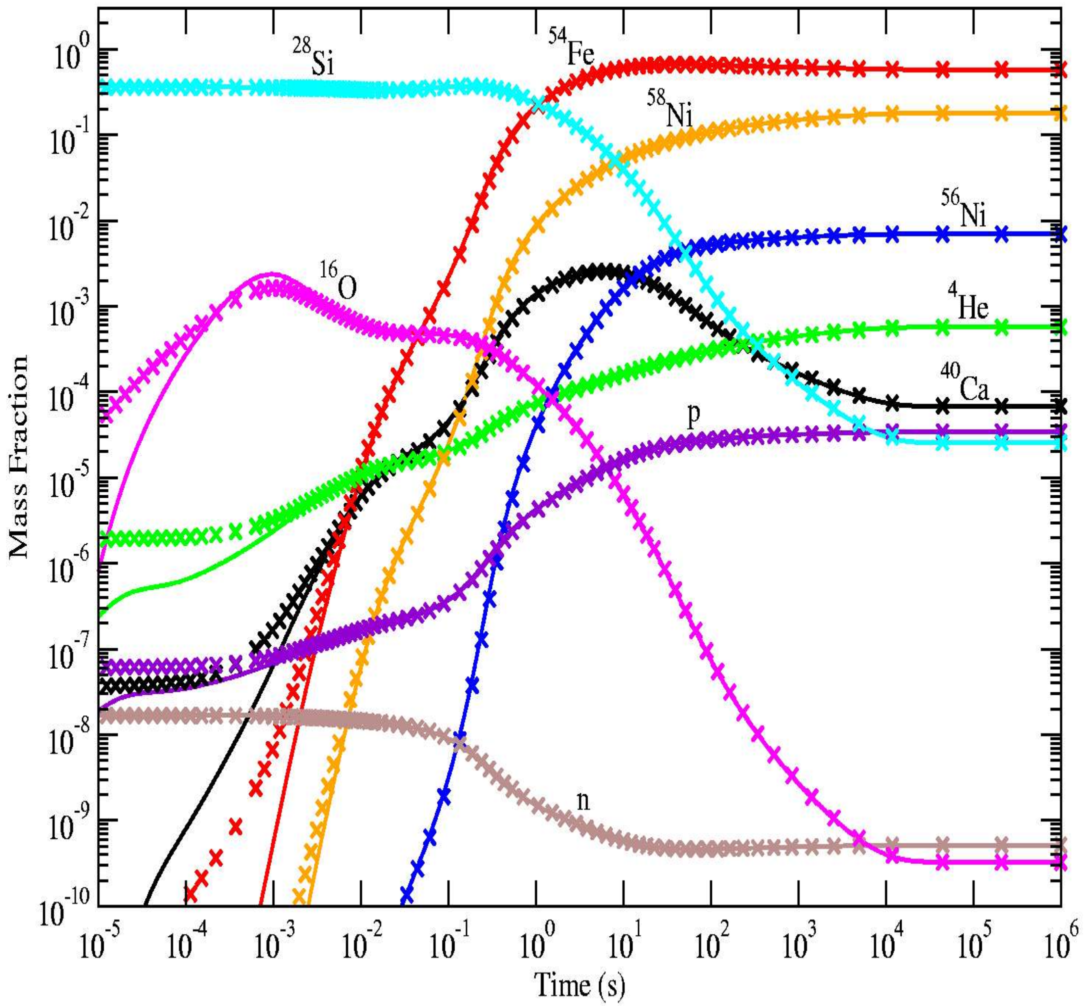
$$Y_{FeG} = \sum_{i \in Fe\ group} Y_i \,.$$

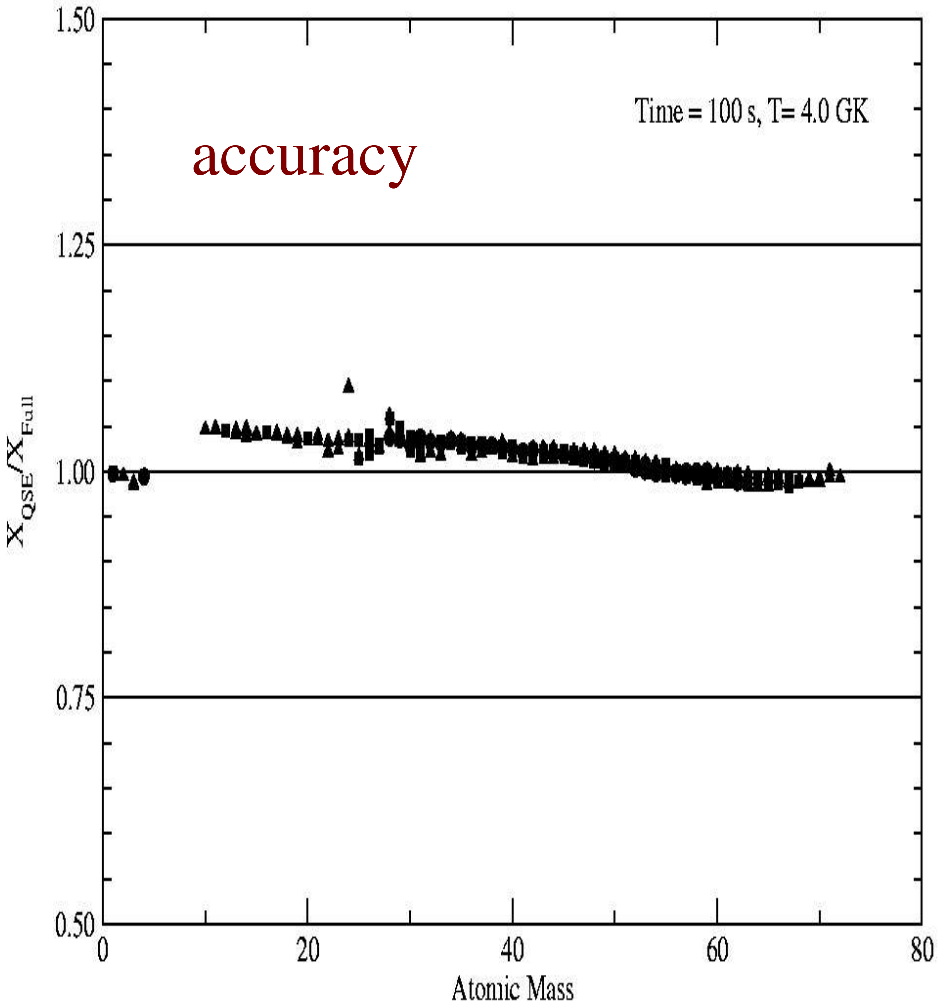
$$\theta = \left(\frac{m_u k_B T}{2\pi\hbar^2}\right)^{3/2}$$

binding energy differences, i.e. masses enter directly

time evolution for those quantities which are in equilibrium and the individual abundances of nuclei with slow reactions which link equilibrium groups (Hix, Parete-Koon, Freiburghaus, Thielemann 2007)

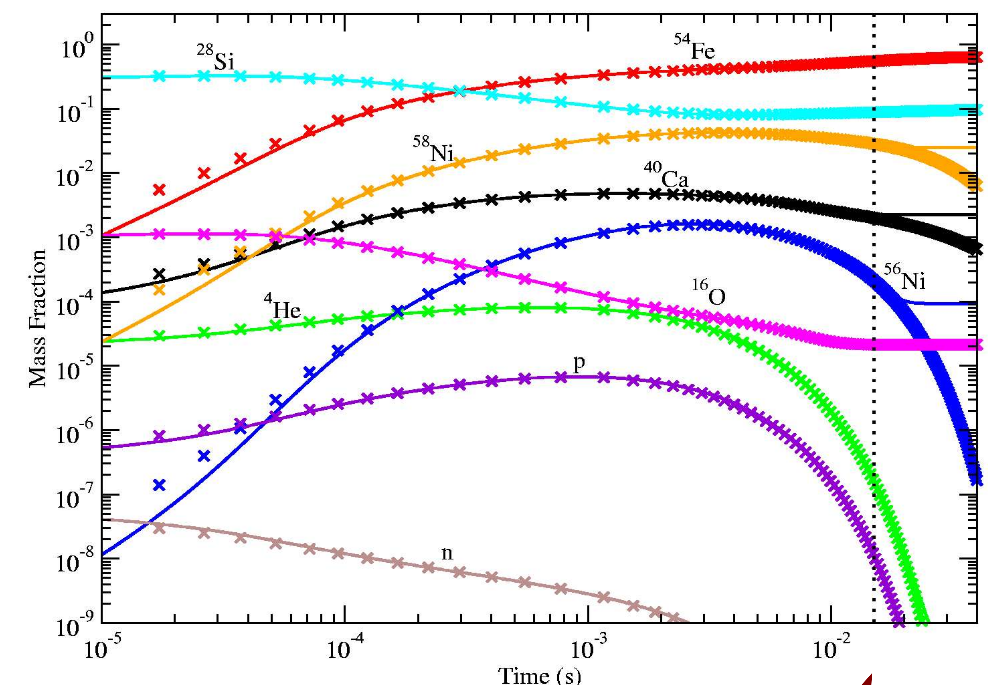
Obtaining quilibrium at high T

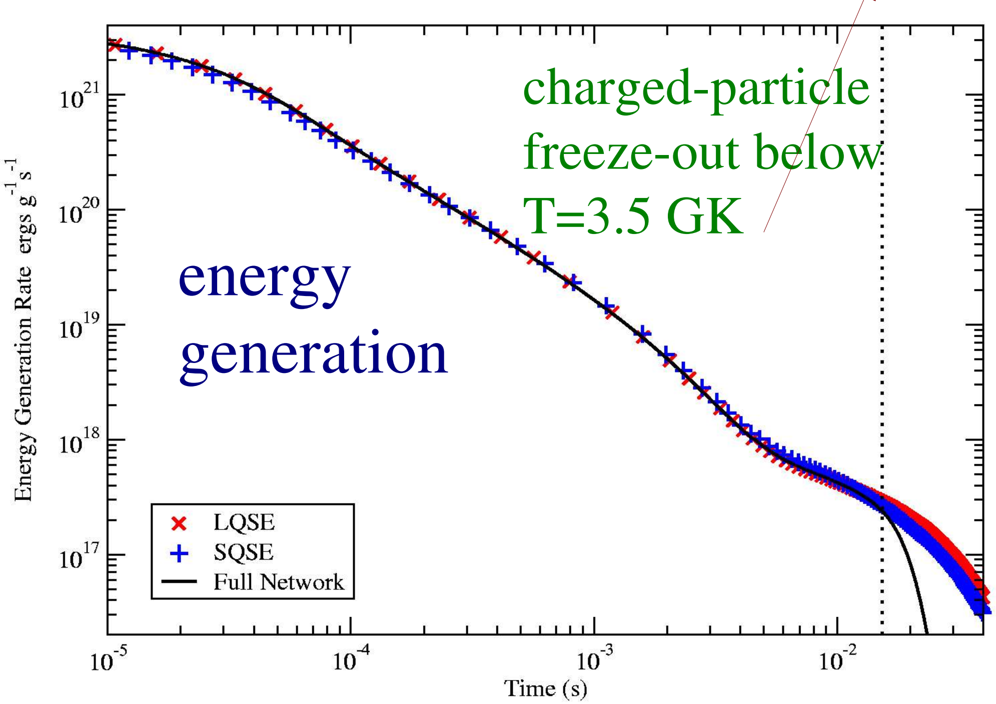


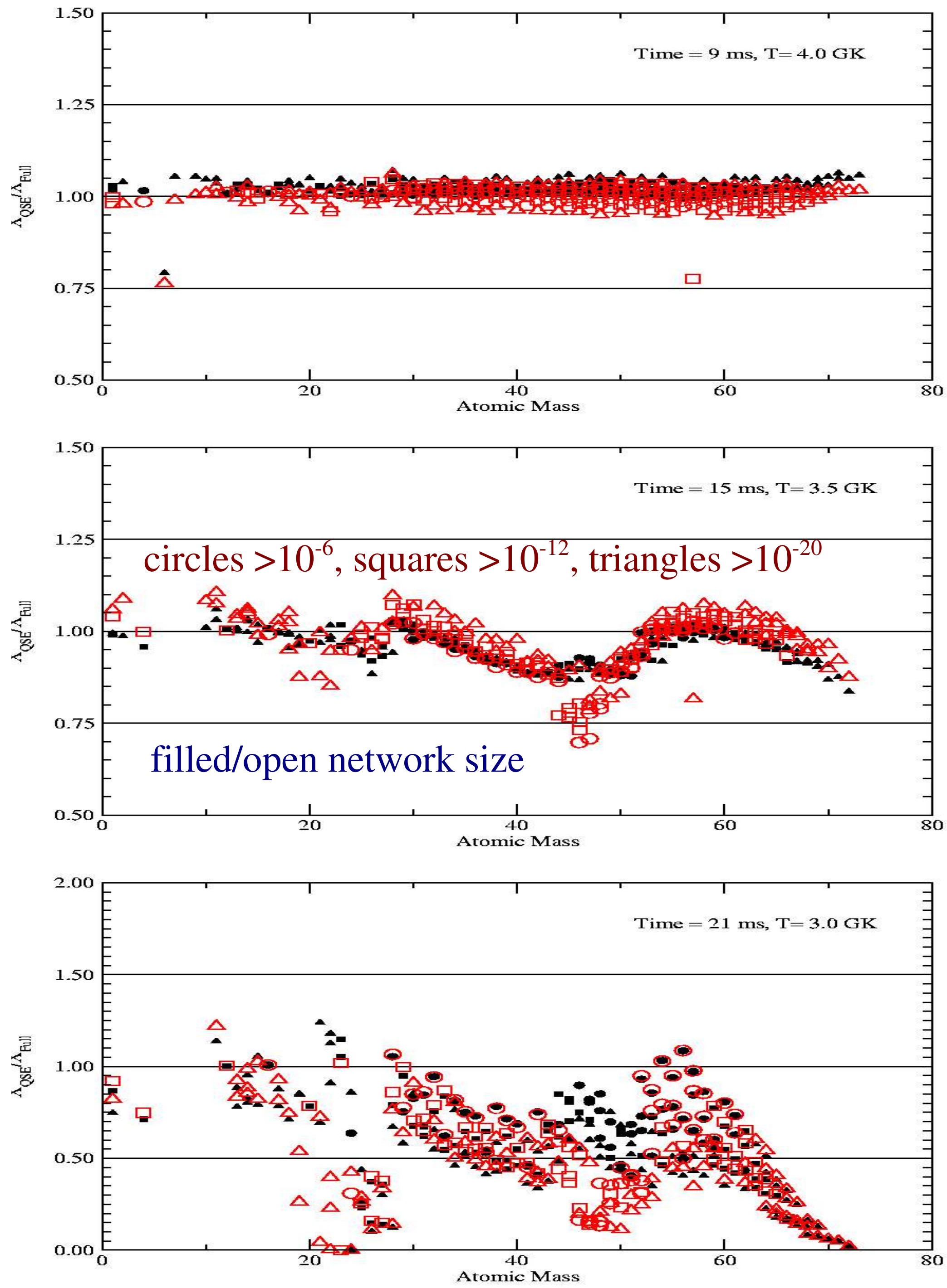


at T=4 GK the equilibrium description is correct after about 10-3 s!

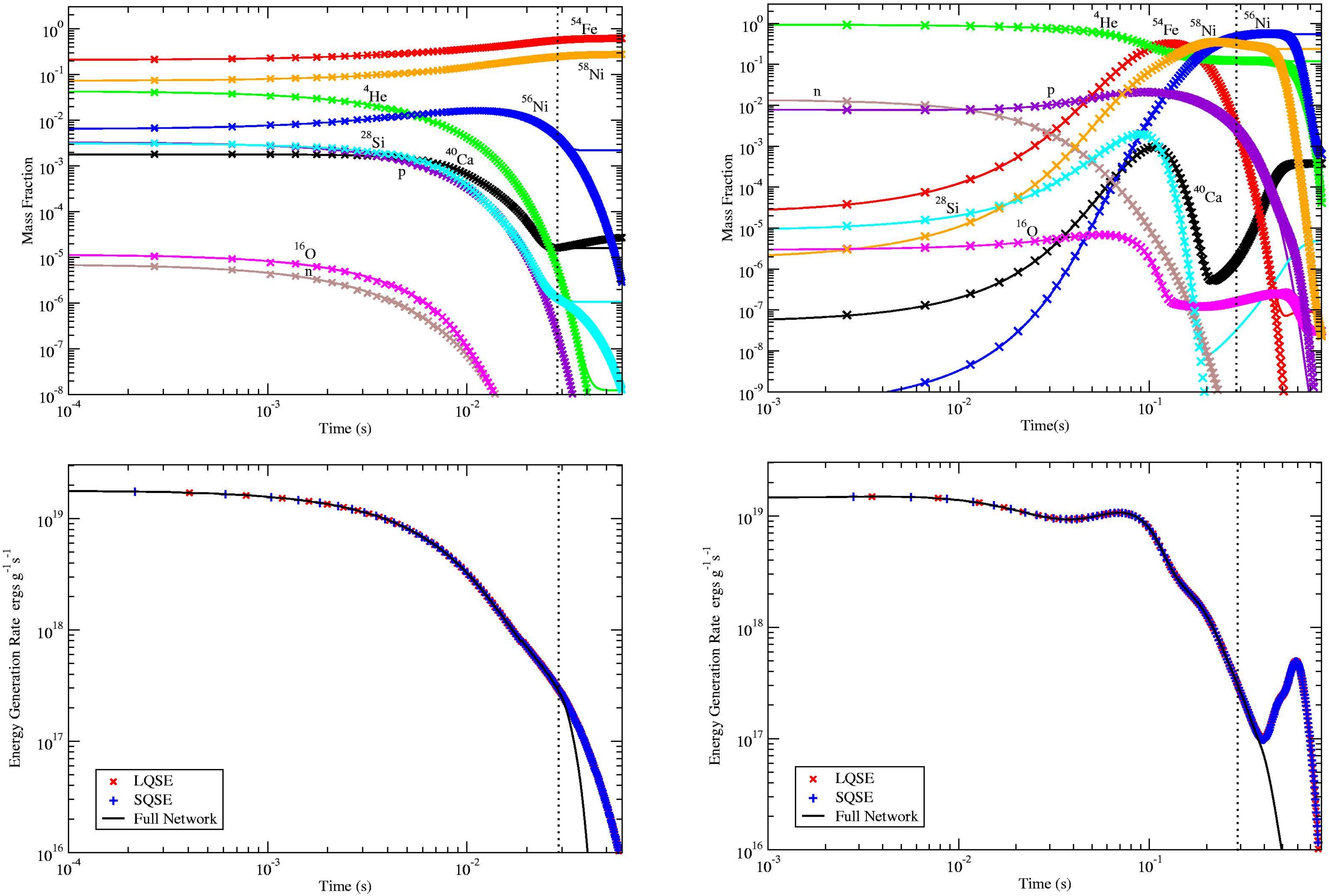
Incomplete Si-burning with freeze-out







Normal and alpha-rich freeze-out



Interim conclusions

- steady flows are approached in many hydrostatic burning stages during stellar evolution, including the s-process. They are determined by rates (often the smallest ones), which are/can be related to small Q-values.
- NSE/QSE equilibria are obtained in hydrostatic Si-burning and in explosive burning. Abundance distribution depends directly on mass differences, but for these applications mostly close to stability.
- How about QSE-equilibria linked by steady flows (and far from stability)?