Notre Dame LECTUR 3.

CONFIGURATION SPACE of the NUCLEAR PROBLE

follow Vogt (1962)

EXTERNAL REGION

Show connection between various, σ, cross sections and collision matrix components

Division of Configuration Space into External and Internal regions.

External wave functions

$$\Psi = \sum_c \psi_c \phi_c$$

the other bits — the radial wave function

$$\psi_c \equiv \frac{1}{r_c} \Phi_\alpha \sum_{m_\ell} \sum_{m_s} (\ell s m_\ell m_s | J M_J) i^\ell Y_{\ell m_\ell} X_{s m_s}$$

$c \equiv (\alpha, \ell, s, J, M_J)$

Φ_α represents the state of internal excitation of the two particles in the channel.

$X_{s m_s}$ pertains to the coupled internal spins $(\vec{s} = \vec{I} + \vec{i})$

The ψ_c are a set of unit vectors for the external channel space. At the nuclear surface for each channel $(a_c = r)$ we get a piece, S_c, of the configuration space channel surface, S

$$S \equiv \Sigma_c S_c$$

In any integration over S we must integrate over all the angles of that piece, S_c.

The clear separation into internal and external parts of configuration space holds only if

$$\int_S \psi_c \psi_{c'}^* dS = \delta_{cc'}$$

This orthogonality condition exerts some constraints on the choice of channel radii.

we now follow the same steps we took for potential scattering

"INSIDE" BITS (inside $\mathcal{V}$)

$\times X_\lambda^*$

$\times^{c.c.} \Psi$

$$H\Psi = E\Psi$$

inside w.f. resonances

$$\begin{cases} H X_\lambda = E_\lambda X_\lambda \\ r_c \, dX_\lambda / dr_c = b_c X_\lambda \Big|_{S_c} \end{cases}$$

$$\Psi = \Sigma_\lambda C_\lambda X_\lambda$$ Harmonic Analysis

$$C_\lambda \equiv \int_{\mathcal{V}} X_\lambda^* \Psi \, d\mathcal{V}$$

Subtract & Integrate over $\mathcal{V}$

$$(E_\lambda - E)\int_{\mathcal{V}} X_\lambda^* \Psi \, d\mathcal{V} = \sum_c \int_{S_c} \frac{\hbar^2}{2m_c r_c} \left[X_\lambda^* \Psi' - \Psi X_\lambda^{*\prime}\right] dS_c$$

$$= \sum_c \left(\frac{\hbar^2}{2m_c r_c}\right)^{1/2} \gamma_{\lambda c} (\phi_c' - b_c \phi_c)$$

where $\gamma_{\lambda c} \equiv \left(\frac{\hbar^2}{2m_c r_c}\right)^{1/2} \int \psi_c^* X_\lambda \, dS$

Therefore:

$$C_\lambda = (E_\lambda - E)^{-1} \sum_c \gamma_{\lambda c} (\phi_c' - b_c \phi_c) \left(\hbar^2 / 2m_c r_c\right)^{1/2}$$

REDUCTION OF THE PROBLEM

$$\Gamma_{\lambda c} \equiv 2P_c\,\gamma_{\lambda c}^2 = \underset{\uparrow}{S_{\lambda c}}\,\Gamma_{\lambda c}^{sp}$$

SPECTROSCOPIC FACTOR

$\Gamma_{\lambda c}^{SP}$ = SINGLE-PARTICLE LEVEL WIDTH FOR ELASTIC SCATTERING IN THE "MEAN FIELD" APPROPRIATE TO CHANNEL C.

THE "WAVE PHYSICS" IS THAT OF THE ONE-DIMENSIONAL PROBLEM OF THE "MEAN FIELD".

For a square well

$$\Gamma_{\lambda c}^{sp} = 2\underbrace{P_c}_{\frac{k_c r_c}{F_c^2+G_c^2}}\;\underbrace{\frac{\hbar^2}{m_c r_c^2}}_{\text{square well reduced width}}$$

For the mean field (Saxon-Woods)

$$\Gamma_{\lambda c}^{sp} = 2P_c\,\underbrace{f}_{\text{reflection factor}}\,\frac{\hbar^2}{m_c r_c^2}$$

Evaluating the Harmonic Analysis on S, we find

$$(\hbar^2/2m_c r_c)^{1/2}\phi_c = \sum_{c'} R_{cc'}\left[\phi'_{c'} - b_{c'}\phi_{c'}\right](\hbar^2/2m_{c'}r_{c'})^{1/2}$$

where

$$\boxed{R_{cc'} \equiv \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}}$$

EXTERNAL BITS

$$\Psi = \sum_c \frac{1}{\sqrt{v_c}}(A_c I_c - B_c O_c)\psi_c$$

where $I_c^* = O_c \equiv \exp i[k_c r_c - \frac{1}{2}\ell_c\pi - \eta_c \ln 2k_c r_c]$

and $B_c \equiv \sum_{c'} U_{cc'} A_{c'}$

multiplication by ψ_c^* and integration over S yields:

$$\phi_c = \frac{1}{\sqrt{v_c}}\left(A_c I_c - \sum_{c'} U_{cc'} A_{c'} O_c\right)$$

Taking the derivative and matching internal solution yields

$$U = (kr)^{1/2} O^{-1}[1 - RL]^{-1}[1 - RL^*] I (kr)^{-1/2}$$

with $L_c \equiv O'_c O_c^{-1} - b_c \equiv S_c + i P_c$

$$P_c = \frac{k_c r_c}{F_c^2 + G_c^2} \qquad S_c = -b_c + \frac{F_c F_c' + G_c G_c'}{F_c^2 + G_c^2}$$