Behavior of Nuclear Reaction Networks

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For the ^{238}U decay chain

$$\frac{dY_{234}}{dt} = -\lambda_{234} Y_{234} + \lambda_{238} Y_{238}$$

$$\left(\frac{1}{\Delta t} + \lambda_{234}\right)\Delta Y_{234} - \lambda_{238}\Delta Y_{238} = -\lambda_{234} Y_{234} + \lambda_{238} Y_{238}$$

and

$$\frac{dY_{238}}{dt} = -\lambda_{238} Y_{238}$$

$$\left(\frac{1}{\Delta t} + \lambda_{238}\right)\Delta Y_{238} = -\lambda_{238} Y_{238}$$

Answer to Problem 2.1

$$\Delta Y_{238} = -\frac{\lambda_{238} Y_{238}}{\frac{1}{\Delta t} + \lambda_{238}}$$

and

$$\Delta Y_{234} = \frac{1}{\frac{1}{\Delta t} + \lambda_{234}} \left[-\lambda_{234} Y_{234} + \frac{\lambda_{238} Y_{238}}{1 + \lambda_{238} \Delta t} \right]$$

Consider

$$\Delta t >> 1/\lambda_{234}$$

but

$$\Delta t < 1/\lambda_{238},$$

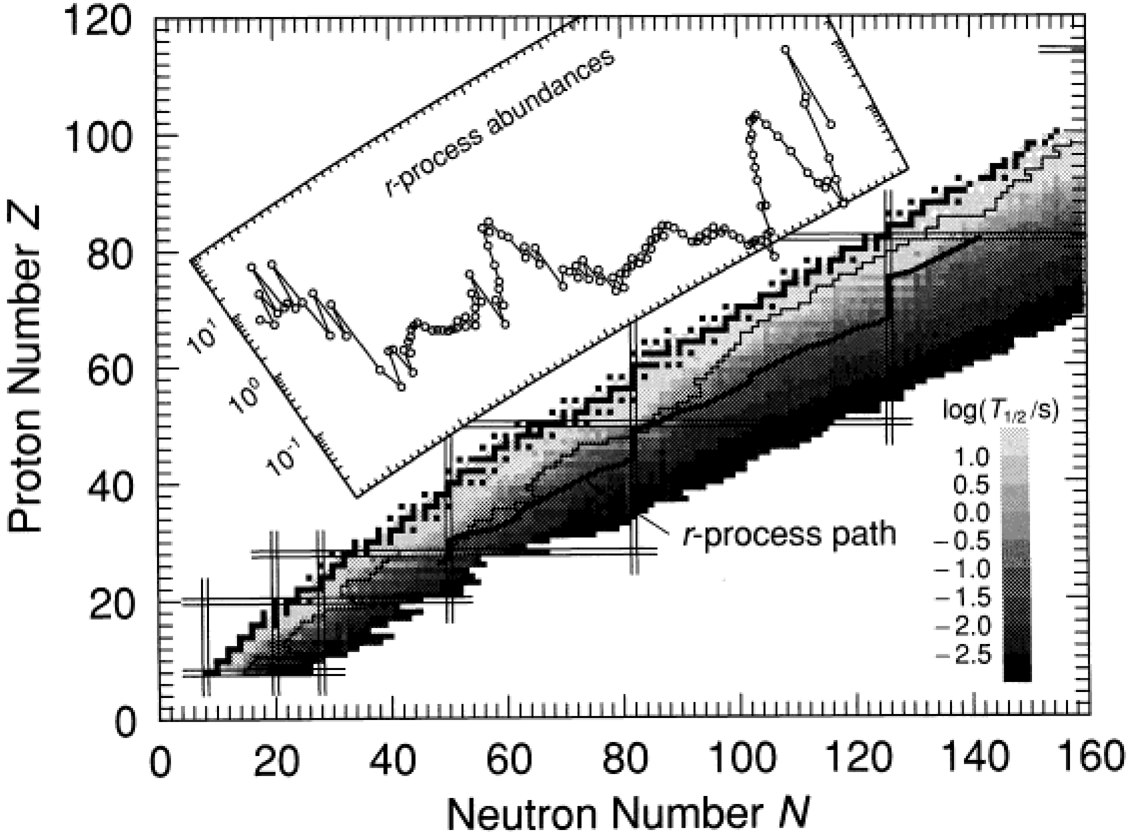
then

$$Y_{238}(t+\Delta t) = Y_{238} + \Delta Y_{238} = \frac{Y_{238}}{1+\lambda_{238}\Delta t}$$

and

$$Y_{234}(t+\Delta t) = Y_{234} + \Delta Y_{234} = \frac{\lambda_{238}}{\lambda_{234}} Y_{238}(t+\Delta t)$$

S Process



r-process abundances
10^1
10^0
10^-1
r-process path
log(T_{1/2}/s)
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
Proton Number Z
Neutron Number N
0
20
40
60
80
100
120
140
160

Neutron Sources

- $^{13}C(\alpha,n)^{16}O$ written as ^{13}C(alpha,n)^{16}O
  - ^{13}C made from protons diffusing into ^{12}C-rich region
- ^{22}Ne(alpha,n)^{25}Mg
  - ^{22}Ne made from ^{14}N left over from CNO:
  - ^{14}N(alpha,gamma)^{18}F(beta+)^{18}O(alpha,gamma)^{22}Ne

Sites of the s process

- Weak component: core helium burning in massive stars (production up to A=90)
- Main component: helium shell flashes in Asymptotic Giant Branch (AGB) stars (production up to A=209)

Steady states in s process

$$\frac{dY_{73Ge}}{dt} = -N_A < \sigma v >_{73} \rho Y_{73Ge} Y_n + N_A < \sigma v >_{72} \rho Y_{72Ge} Y_n \Rightarrow 0$$

$$\Rightarrow \frac{Y_{73Ge}}{Y_{72Ge}} = \frac{< \sigma v >_{72}}{< \sigma v >_{73}}$$

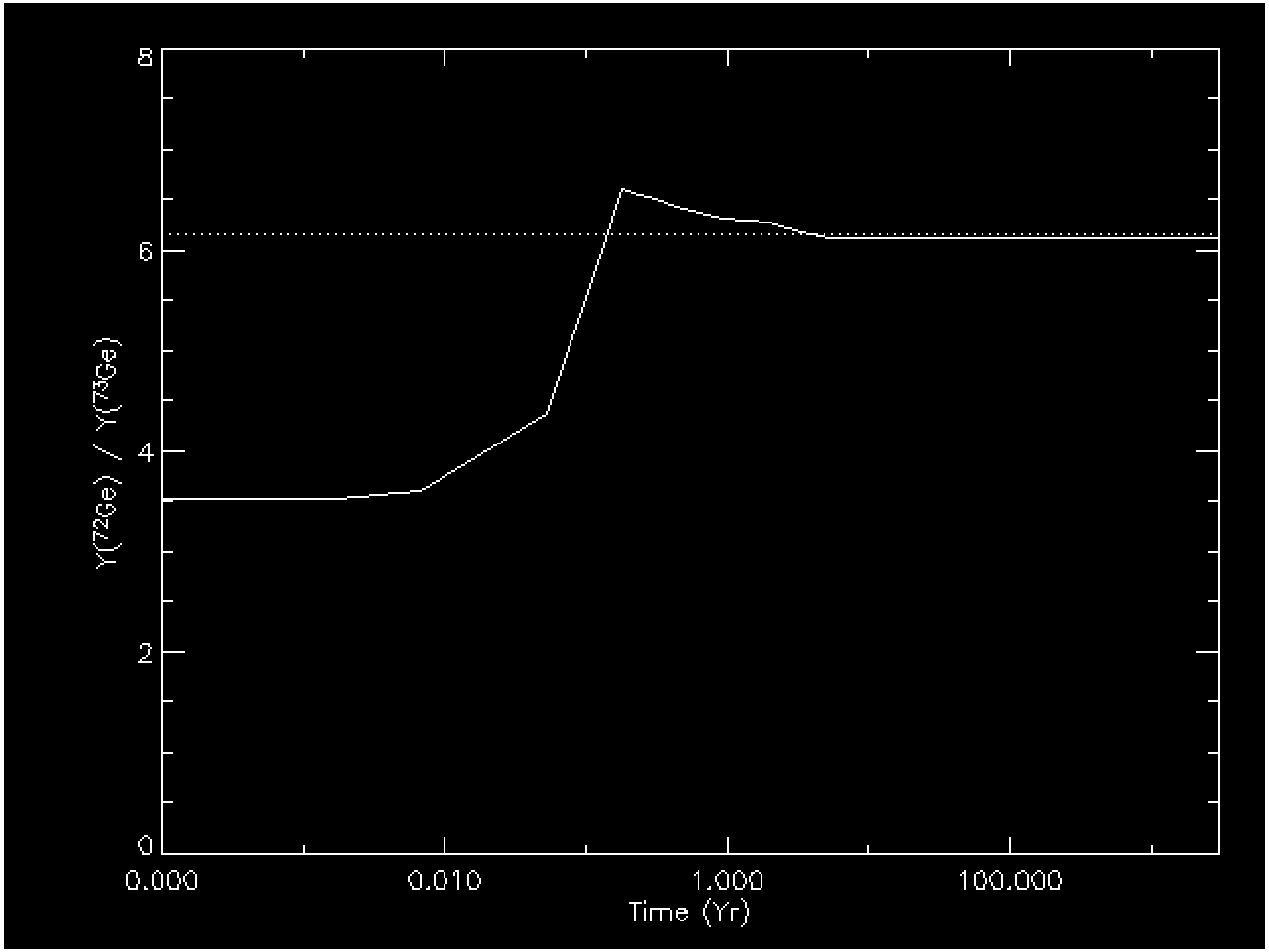
A simple helium burning + s-process example

- Start with abundances after H burning + some extra ^{13}C.
- T_9=0.15, density = 1000 g/cc
- The FITS file:

http://www.ces.clemson.edu/~mbradle/JINA/

Choose sprocess.fits. Notice also the reactions file sprocess_reactions.html

The steady state



Consider now equilibrium: p+n<->d

$$\frac{dY_p}{dt} = -N_A \langle \sigma v \rangle \rho Y_p Y_n + \lambda_\gamma Y_d$$

$$\Rightarrow \frac{Y_p^{eq} Y_n^{eq}}{Y_d^{eq}} = \frac{\lambda_\gamma}{N_A < \sigma v >}$$

More formally

$$df = -sdT - PdV + \sum_i \mu_i dY_i$$

Consider

$$dT = dV = 0$$

$$\Rightarrow df = \sum_i \mu_i dY_i$$

For p,n, and d network

$$df = \mu_p dY_p + \mu_n dY_n + \mu_d dY_d$$

but

$$dY_d = -dY_p = -dY_n$$

so

$$df = -(\mu_p + \mu_n - \mu_d)dY_p$$

For equilibrium

$$df = 0$$

$$so$$

$$\mu_p + \mu_n = \mu_d$$

General Nucleosynthesis Network

$$\mu(Z, A) = Z\mu_p + (A - Z)\mu_n$$

Nuclear Binding Energy

$$M(Z,A)c^2 = 931.478A + \Delta(Z,A) \quad Mev$$

$$B(Z,A) = Zm_p c^2 + (A-Z)m_n c^2 - M(Z,A)c^2$$

so

$$B(Z,A) = Z\Delta_p + (A-Z)\Delta_n - \Delta(Z,A)$$

and

$$B/A = B(Z,A)/A$$

The Nuclear Data Tool

http://nucleo.ces.clemson.edu/home/online_tools/nuclear_data/0.1

Y_e

$$Y_e = \sum_i Z_i Y_i$$

The NSE Calculator

http://nucleo.ces.clemson.edu/pages/nse/0.1

An alpha-rich freezeout example

- Start with Y_e=0.5 at T_9=10 and density = 10^8 g/cc
- Volume expands on a 0.1 second timescale, density proportional to T_9^3
- The FITS file:

http://www.ces.clemson.edu/~mbradle/JINA/

Choose alpha-rich.fits

Tasks for today

- S process (sprocess.fits)
  - Plot the ^{4}He abundance vs. time (in years: 1 year = 3.15e7 seconds)
  - Plot neutron number density vs. time
  - Plot abundances vs. A for several time records
  - Investigate steady states
- Nuclear Data
  - Determine the nucleus with the highest B/A (hint: it is between Z=20 and Z=30)
  - Determine the nucleus with the highest B/A but with Z=N (It also lies between Z=20 and Z=30. Is that nucleus stable against beta decay?)

Tasks for Today (cont.)

- NSE Calculator
  - Compute NSE for fixed T and varying density or varying T and fixed density. Plot abundances vs. the varying quantity. Try this similarly for varying Y_e.
- Alpha-rich freezeout (alpha-rich.fits)
  - Compare network abundances and NSE abundances at various time records in the calculation.
- Investigate the Oracle (next)

The Oracle of Bacon

http://oracleofbacon.org