Behavior of Nuclear Reaction Networks

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QSE: Quasi-statistical equilibrium

$$1)\sum_i A_i Y_i = 1$$

$$2)\sum_i Z_i Y_i = Y_e$$

$$3)\sum_{i, Z_i \geq 6} Y_i = Y_h$$

Define

$$g_1 = \sum_i A_i Y_i - 1$$

and

$$g_2 = \sum_i Z_i Y_i - Y_e$$

and

$$g_3 = \sum_i Y_i - Y_h$$

then

$$d(f - \lambda_1 g_1 - \lambda_2 g_2 - \lambda_3 g_3) = 0$$

so

$$\sum_i \left(\mu_i - \lambda_1 A_i - \lambda_2 Z_i - \lambda_3\right) dY_i = 0$$

Neutrons:

$$\mu_n - \lambda_1 = 0 \Rightarrow \lambda_1 = \mu_n$$

Pr*otons*:

$$\mu_p - \lambda_1 - \lambda_2 = 0 \Rightarrow \lambda_2 = \mu_p - \mu_n$$

Others:

$$\mu_i - \lambda_1 A_i - \lambda_2 Z_i = 0 \Rightarrow \mu_i = A_i \mu_n + Z_i(\mu_p - \mu_n)$$

$$\Rightarrow \mu_i = Z_i \mu_p + N_i \mu_n$$

$$(Z_i < 6)$$

$$\mu_i - \lambda_1 A_i - \lambda_2 Z_i - \lambda_3 = 0 \Rightarrow \mu_i = A_i \mu_n + Z_i(\mu_p - \mu_n) + \lambda_3$$

$$\Rightarrow \mu_i = Z_i \mu_p + N_i \mu_n + \lambda_3$$

$$\mu_i^{QSE} - \mu_i^{NSE} = \lambda_3 + Z_i(\mu_p^{QSE} - \mu_p^{NSE}) + N_i(\mu_n^{QSE} - \mu_n^{NSE})$$

so

$$\ln\left(\frac{Y_i^{QSE}}{Y_i^{NSE}}\right) = \frac{\lambda_3}{kT} + Z_i \ln\left(\frac{Y_p^{QSE}}{Y_p^{NSE}}\right) + N_i \ln\left(\frac{Y_n^{QSE}}{Y_n^{NSE}}\right)$$

thus

$$R_i = e^{\lambda_3 / kT} R_p^{Z_i} R_n^{N_i}$$

$\mathrm{Re}\,member$

$$d(f-\lambda_1 g_1-\lambda_2 g_2-\lambda_3 g_3)=0$$

$$\Rightarrow df=\lambda_1 dg_1+\lambda_2 dg_2+\lambda_3 dg_3$$

$$\Rightarrow df=\mu_n\sum_i dX_i+(\mu_p-\mu_n)dY_e+\lambda_3 dY_h$$

$$\Rightarrow df=(\mu_p-\mu_n)dY_e+\mu_h dY_h$$

(n,gamma)-(gamma,n) equilibrium

Define

$$g_1 = \sum_i A_i Y_i - 1$$

and

$$g_2 = \sum_i Z_i Y_i - Y_e$$

and

$$g_Z = \sum_A Y(Z, A) - Y_Z$$

then

$$d(f - \lambda_1 g_1 - \lambda_2 g_2 - \sum_Z \lambda_Z Y_Z) = 0$$

so

$$\sum_i \left(\mu_i - \lambda_1 A_i - \lambda_2 Z_i\right) dY - \sum_Z \lambda_Z dY_Z = 0$$

For a given Z

$$\mu_i - \mu_i^{NSE} = \lambda_Z + Z(\mu_p - \mu_p^{NSE}) + N_i(\mu_n - \mu_n^{NSE})$$

so

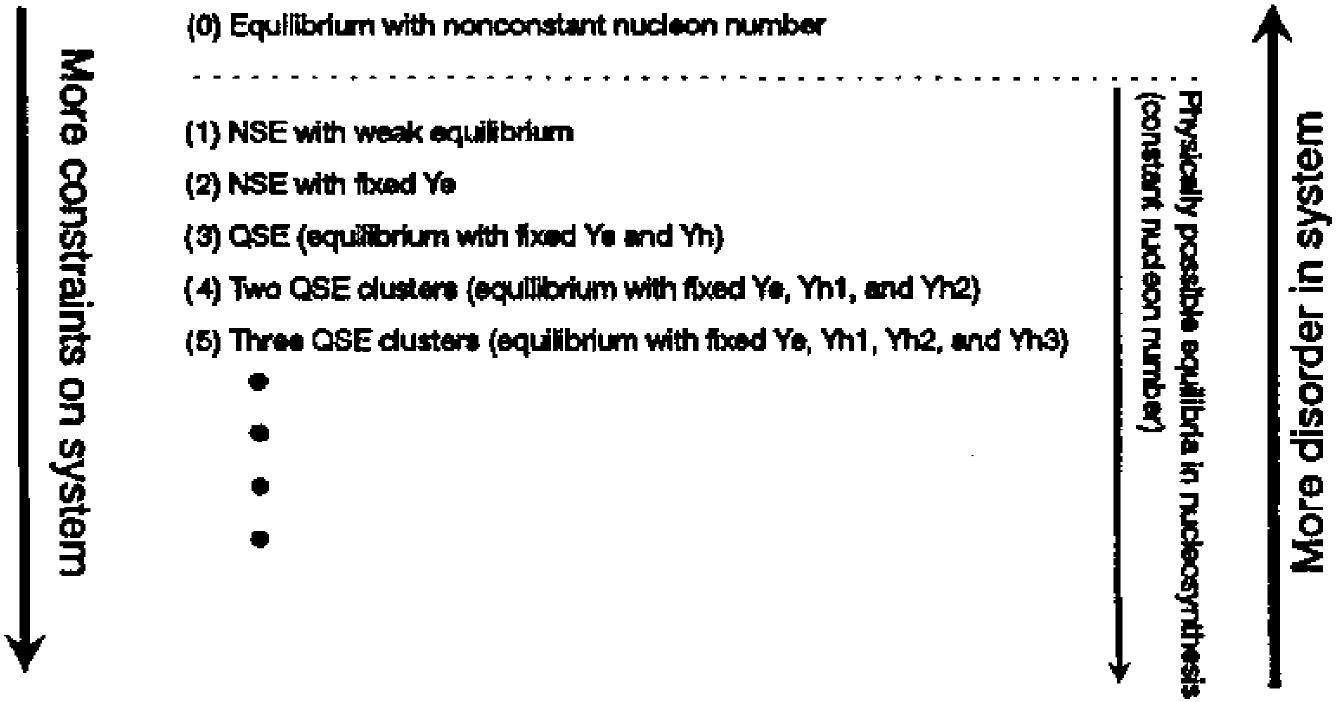
$$\ln\left(\frac{Y_i}{Y_i^{NSE}}\right) = \frac{\lambda_Z}{kT} + Z\ln\left(\frac{Y_p}{Y_p^{NSE}}\right) + N_i \ln\left(\frac{Y_n}{Y_n^{NSE}}\right)$$

thus

$$R_i = e^{\lambda_Z / kT} R_p^Z R_n^{N_i}$$

$$\Rightarrow \frac{R(Z, A+1)}{R(Z, A)} = R_n$$

Hierarchy of Statistical Equilibria in Nucleosynthesis



(0) Equilibrium with nonconstant nucleon number

(1) NSE with weak equilibrium

(2) NSE with fixed Ye

(3) QSE (equilibrium with fixed Ye and Yh)

(4) Two QSE clusters (equilibrium with fixed Ye, Yh1, and Yh2)

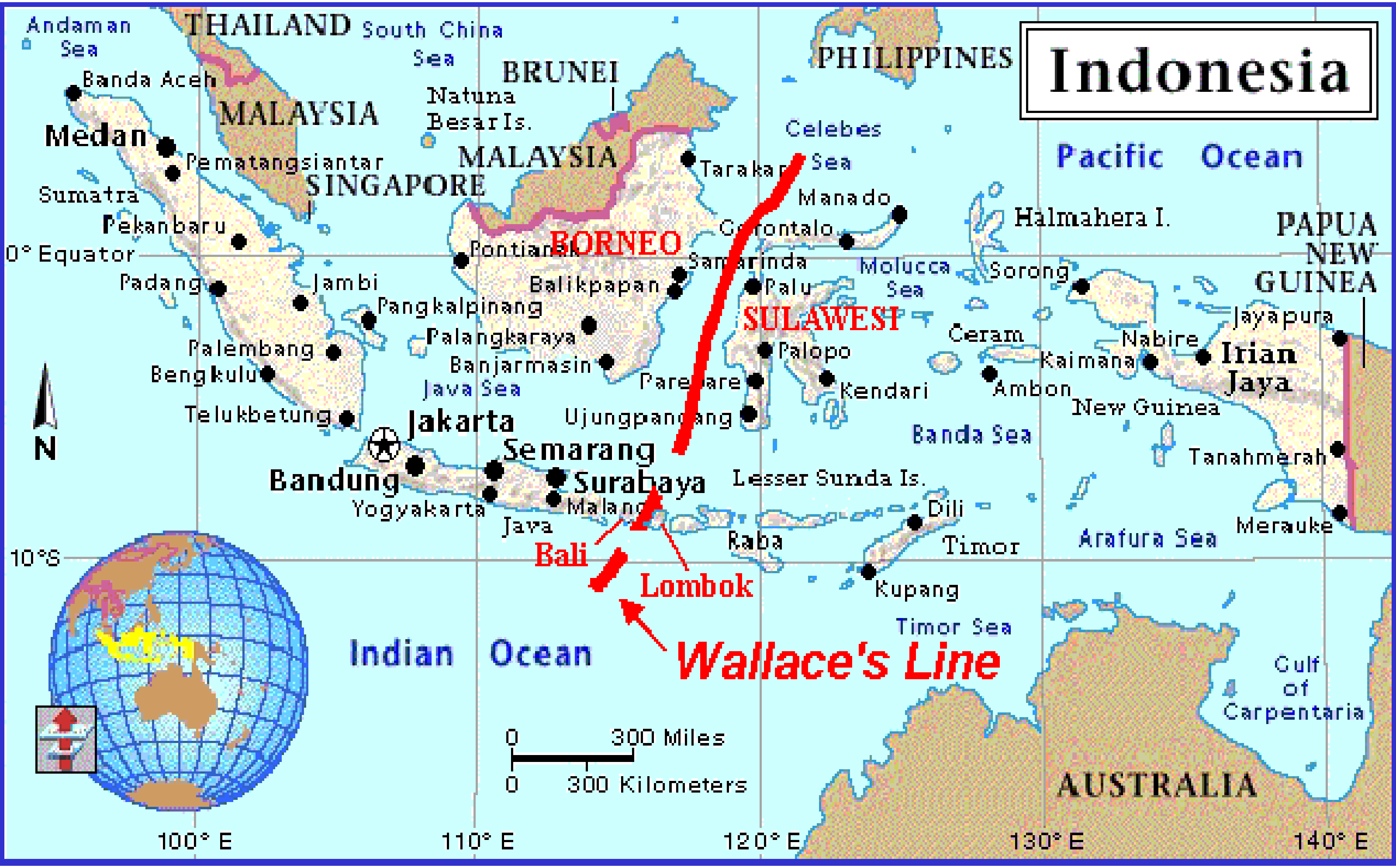
(5) Three QSE clusters (equilibrium with fixed Ye, Yh1, Yh2, and Yh3)

- ●
- ●
- ●
- ●

More constraints on system

Physically possible equilibria in nucleosynthesis (constant nucleon number)

More disorder in system


Indonesia
Wallace's Line
Bali
Lombok
BORNEO
SULAWESI
Andaman Sea
THAILAND
South China Sea
BRUNEI
PHILIPPINES
Banda Aceh
MALAYSIA
Natuna Besar Is.
Medan
Pematangsiantar
MALAYSIA
SINGAPORE
Celebes Sea
Pacific Ocean
Tarakan
Manado
Sumatra
Halmahera I.
Pekanbaru
Gorontalo
PAPUA NEW GUINEA
0° Equator
Pontianak
Samarinda
Molucca Sea
Sorong
Padang
Jambi
Balikpapan
Palu
Pangkalpinang
Jayapura
Palangkaraya
Ceram
Nabire
Palembang
Palopo
Kaimana
Irian Jaya
Bengkulu
Banjarmasin
Parepare
Kendari
Ambon
Java Sea
New Guinea
Telukbetung
Ujungpandang
Jakarta
Banda Sea
N
Semarang
Tanahmerah
Bandung
Surabaya
Lesser Sunda Is.
Yogyakarta
Malang
Dili
Java
Raba
Timor
Arafura Sea
Merauke
10°S
Kupang
Timor Sea
Indian Ocean
Gulf of Carpentaria
0 300 Miles
0 300 Kilometers
AUSTRALIA
100° E
110° E
120° E
130° E
140° E

Parameters

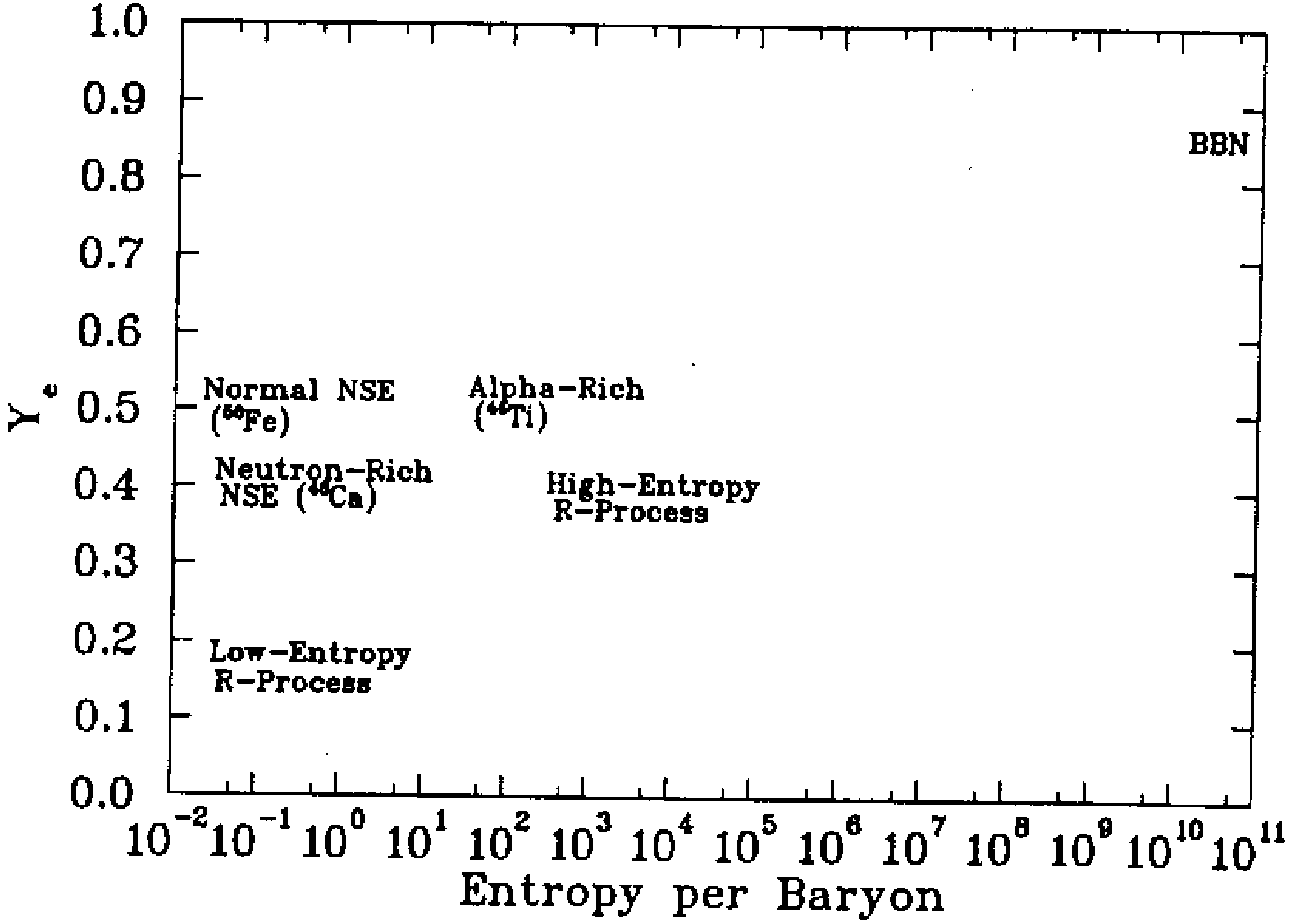
- Y_e :
  - net number of electrons per nucleon
  - fraction of all nucleons that are protons
  - E.g., $Y_e = 0.5$ means equal numbers of neuetron and protons. $Y_e = 0$ means all neutrons. $Y_e=1$ means all protons.

Parameters (cont.)

- Entropy per nucleon
  - Degrees of freedom per nucleon
  - In units of Boltzmann's constant k_B
  - E.g., all free nucleons: s/k_B = about 3
  - E.g., all nucleons in ^{56}Fe: s/k_B about 3/56
  - $s/k_B > 20$ typically means a lot of photons and electron-positron pairs around.

Parameters (cont.)

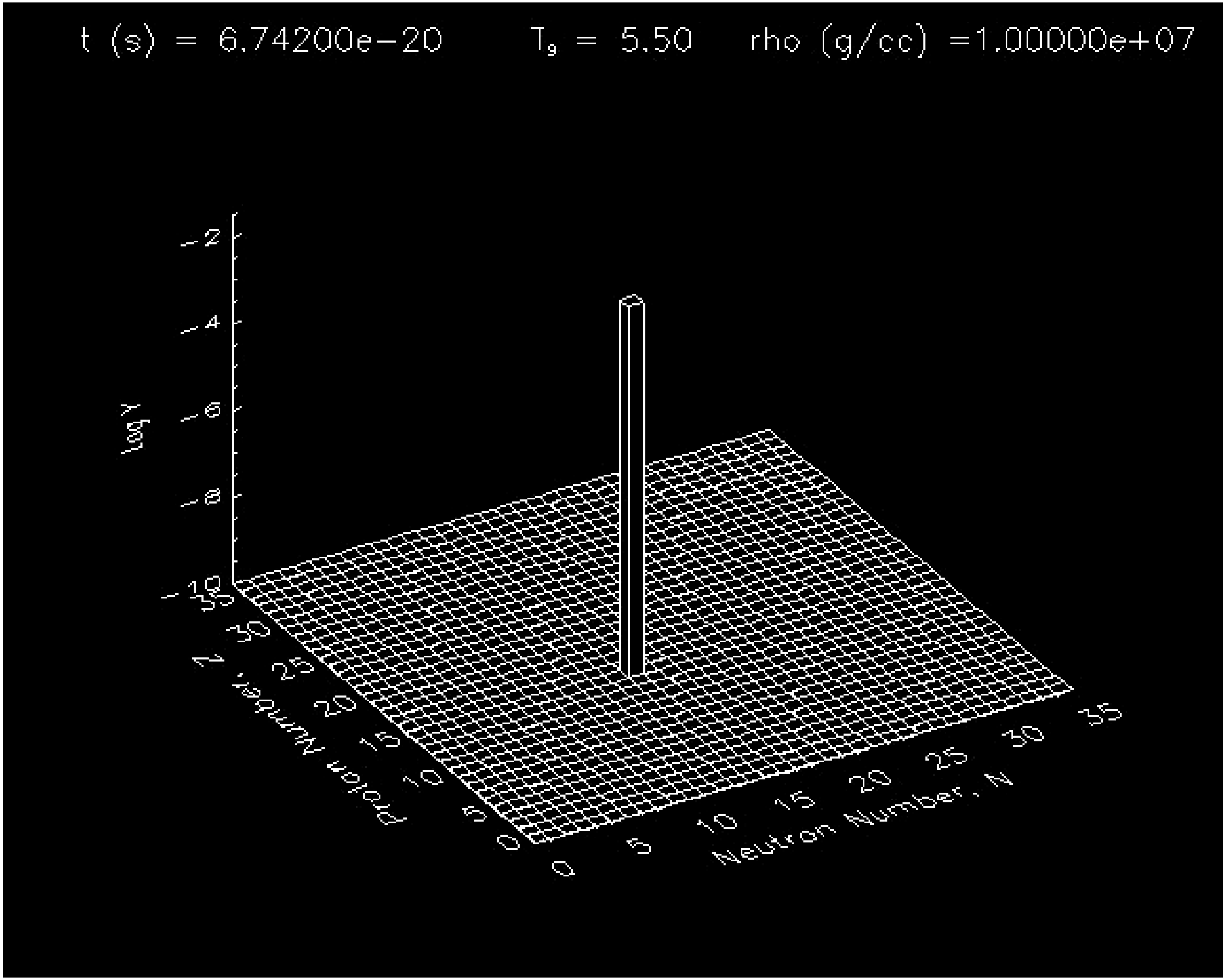
- Timescale tau:
  - Density expansion efolding timescale
  - Typically milliseconds to seconds

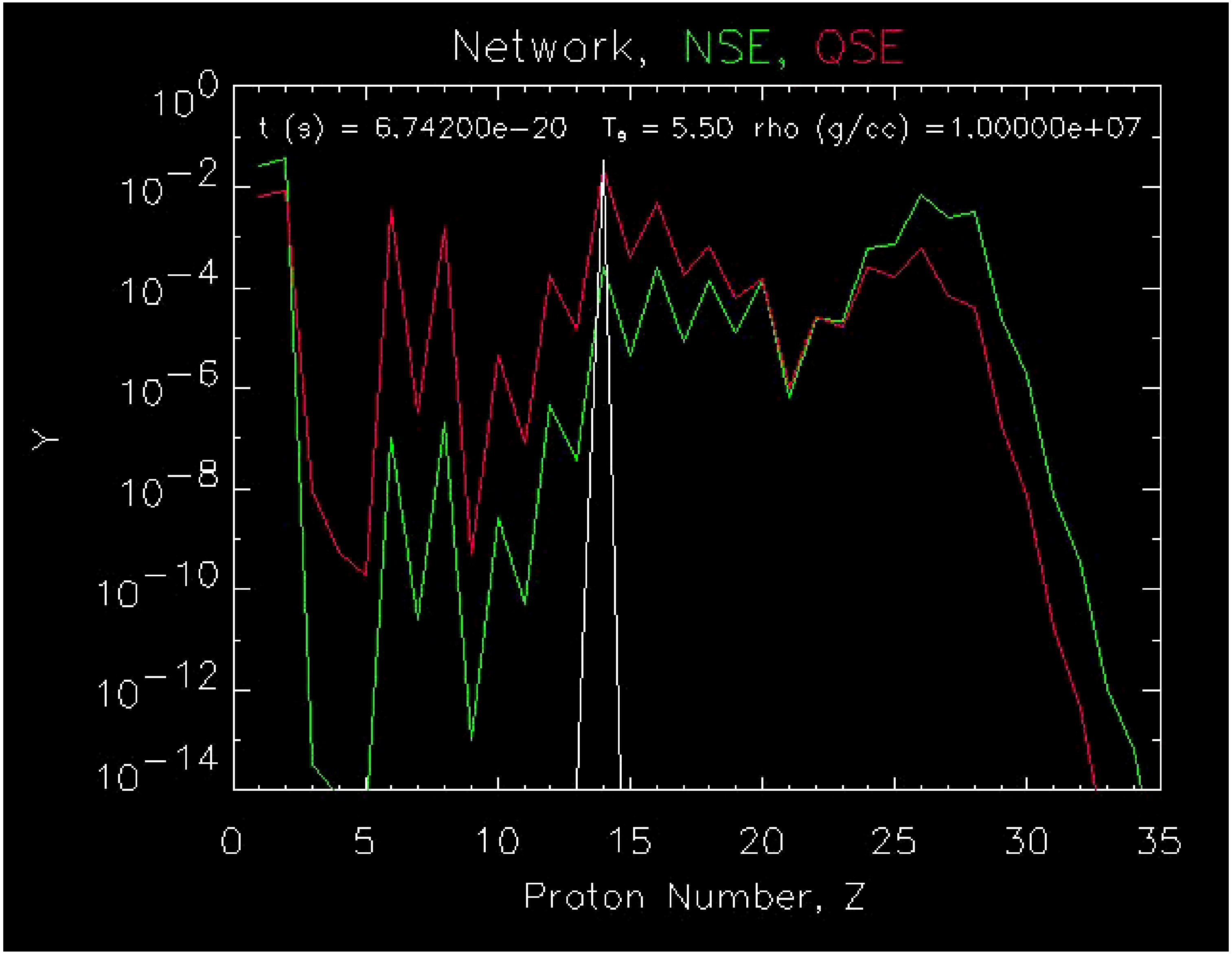


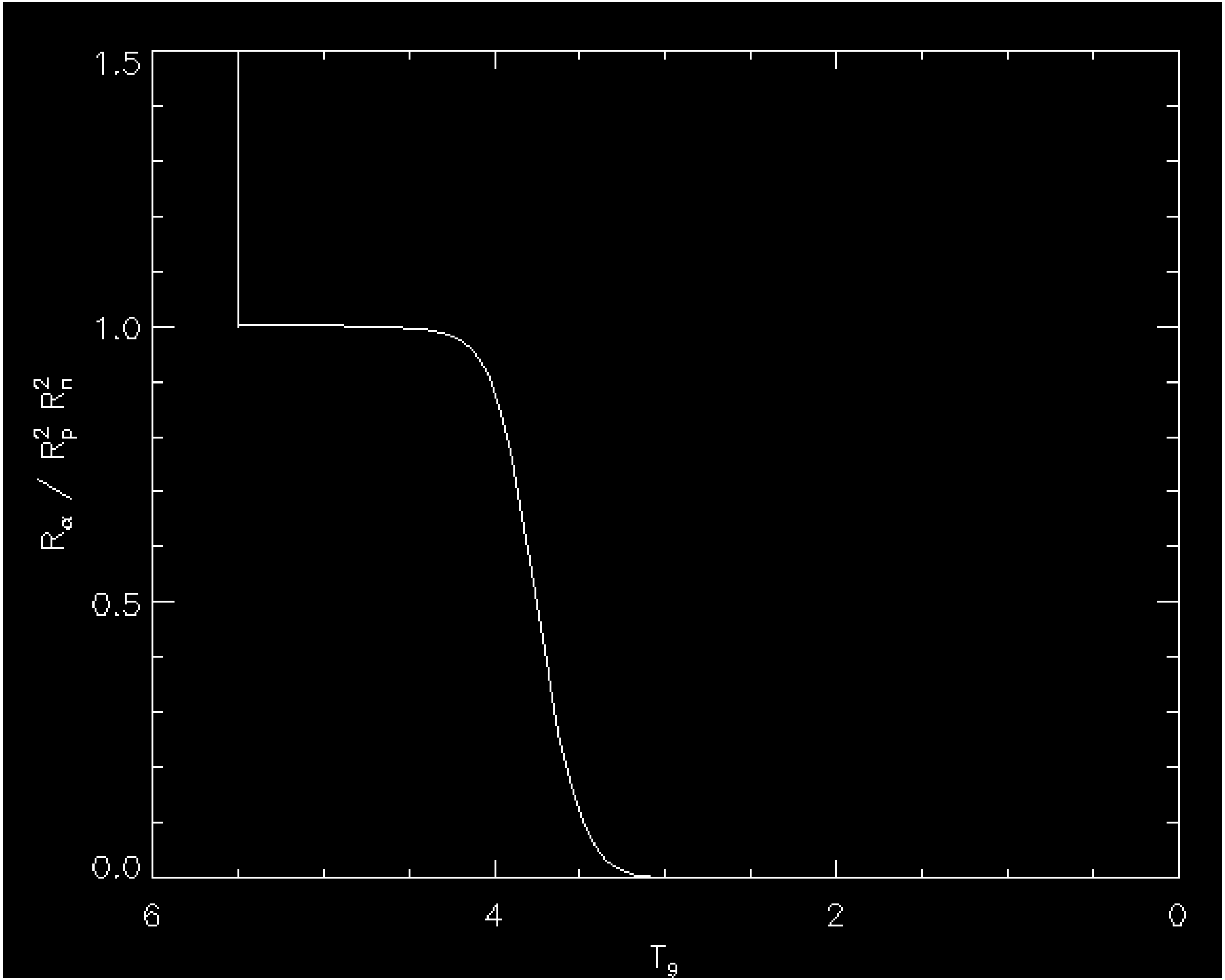
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Y_e
BBN
Normal NSE
(^{56}Fe)
Alpha-Rich
(^{44}Ti)
Neutron-Rich
NSE (^{48}Ca)
High-Entropy
R-Process
Low-Entropy
R-Process
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11}
Entropy per Baryon

The Alpha-Rich Freezeout

Explosive Silicon Burning

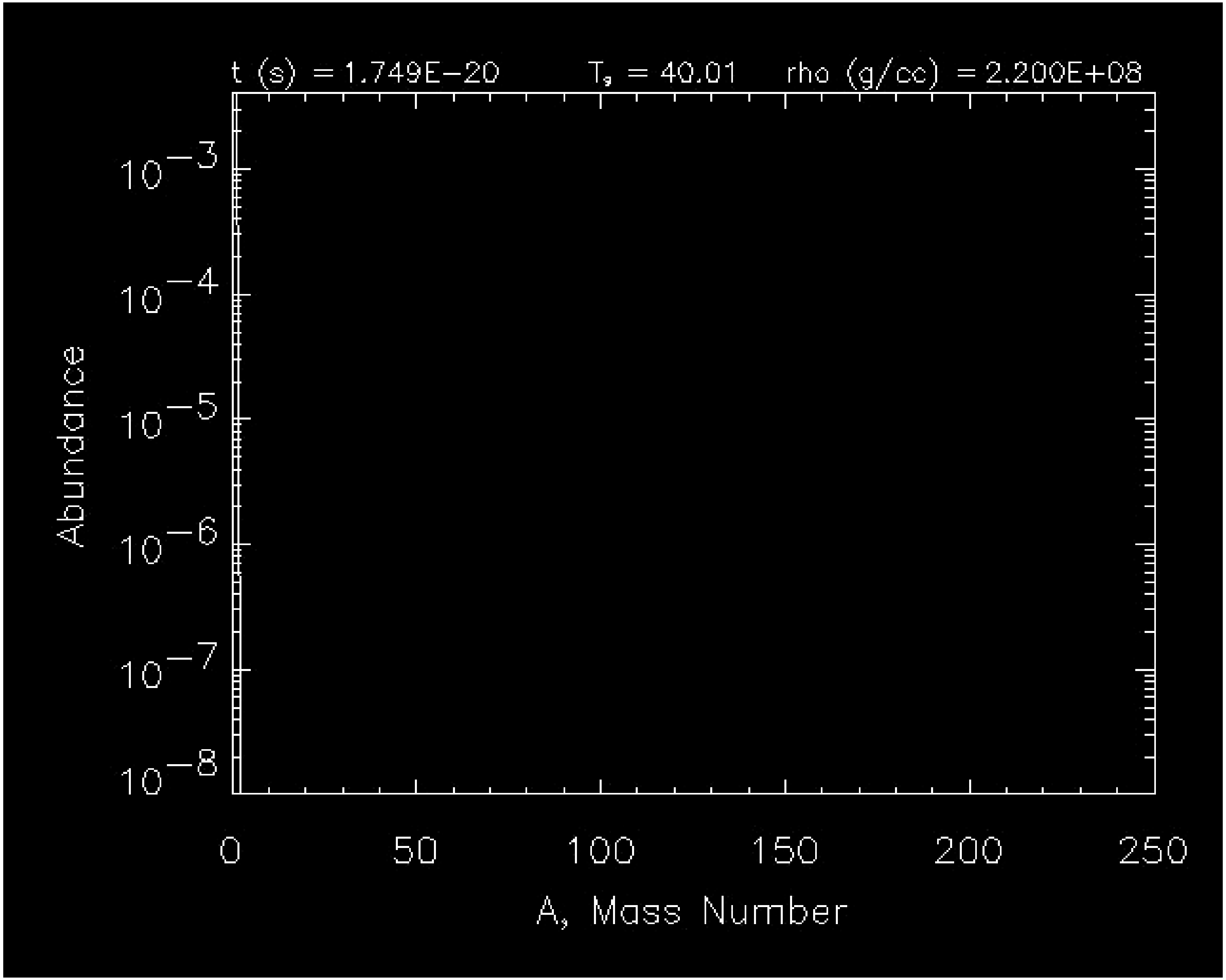

t (s) = 6.74200e-20
T_9 = 5.50
rho (g/cc) =1.00000e+07
log Y
-2
-4
-6
-8
-10
Proton Number, Z
0
5
10
15
20
25
30
35
Neutron Number, N
0
5
10
15
20
25
30
35

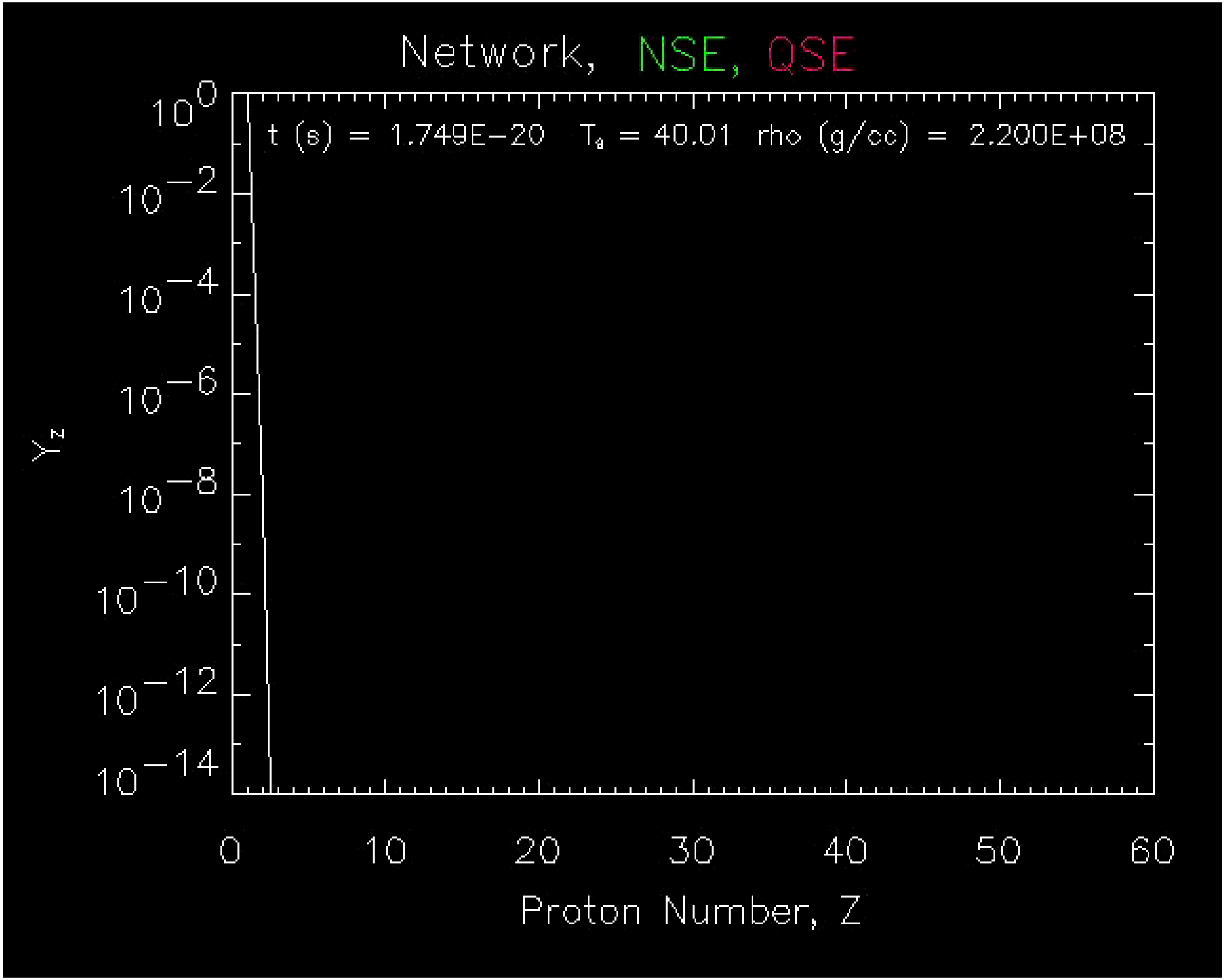

Network, NSE, QSE
t (s) = 6.74200e-20 T9 = 5.50 rho (g/cc) =1.00000e+07
Y
10^0
10^-2
10^-4
10^-6
10^-8
10^-10
10^-12
10^-14
0
5
10
15
20
25
30
35
Proton Number, Z

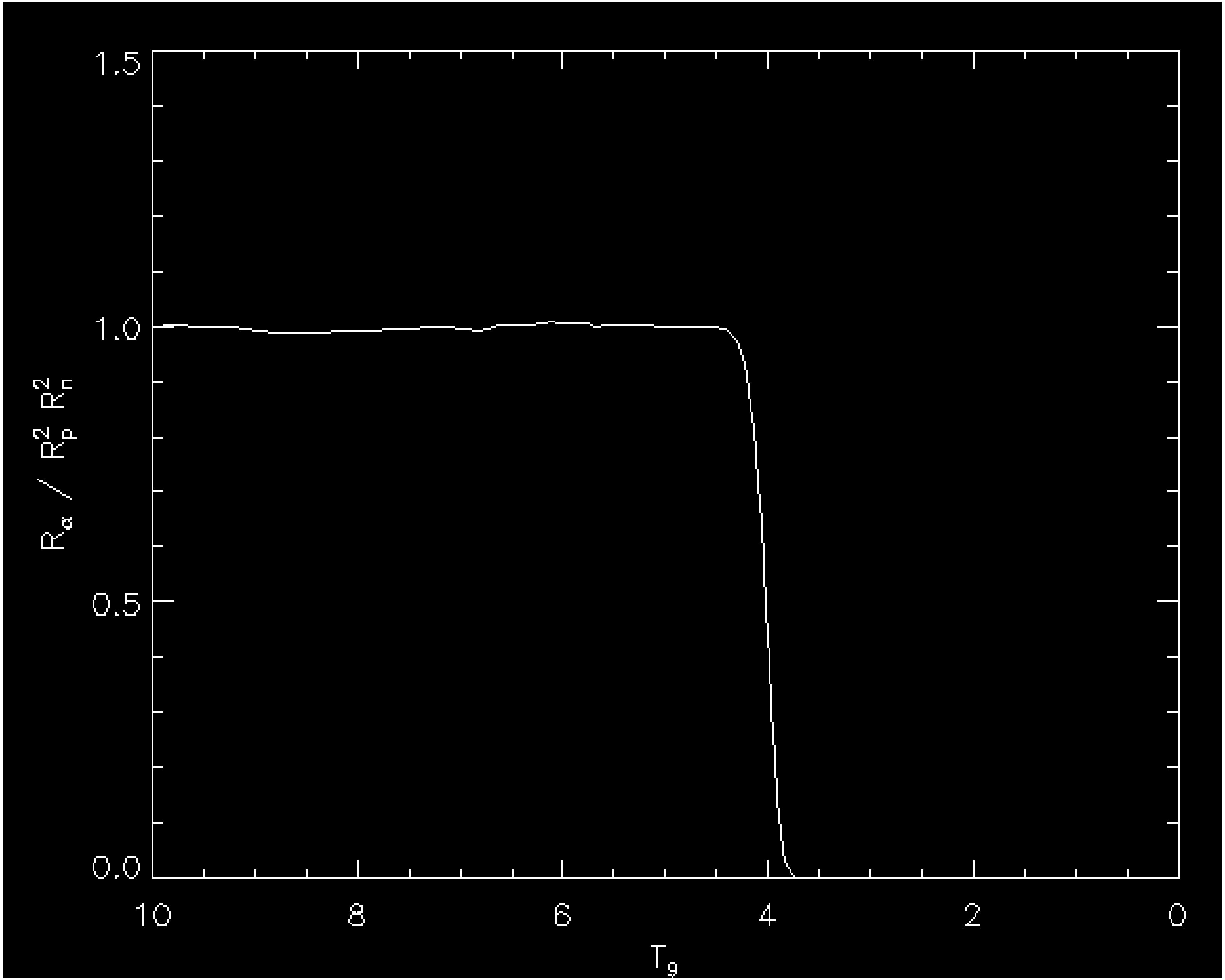


1.5
1.0
0.5
0.0
R_α / R_p^2 R_n^2
6
4
2
0
T_9

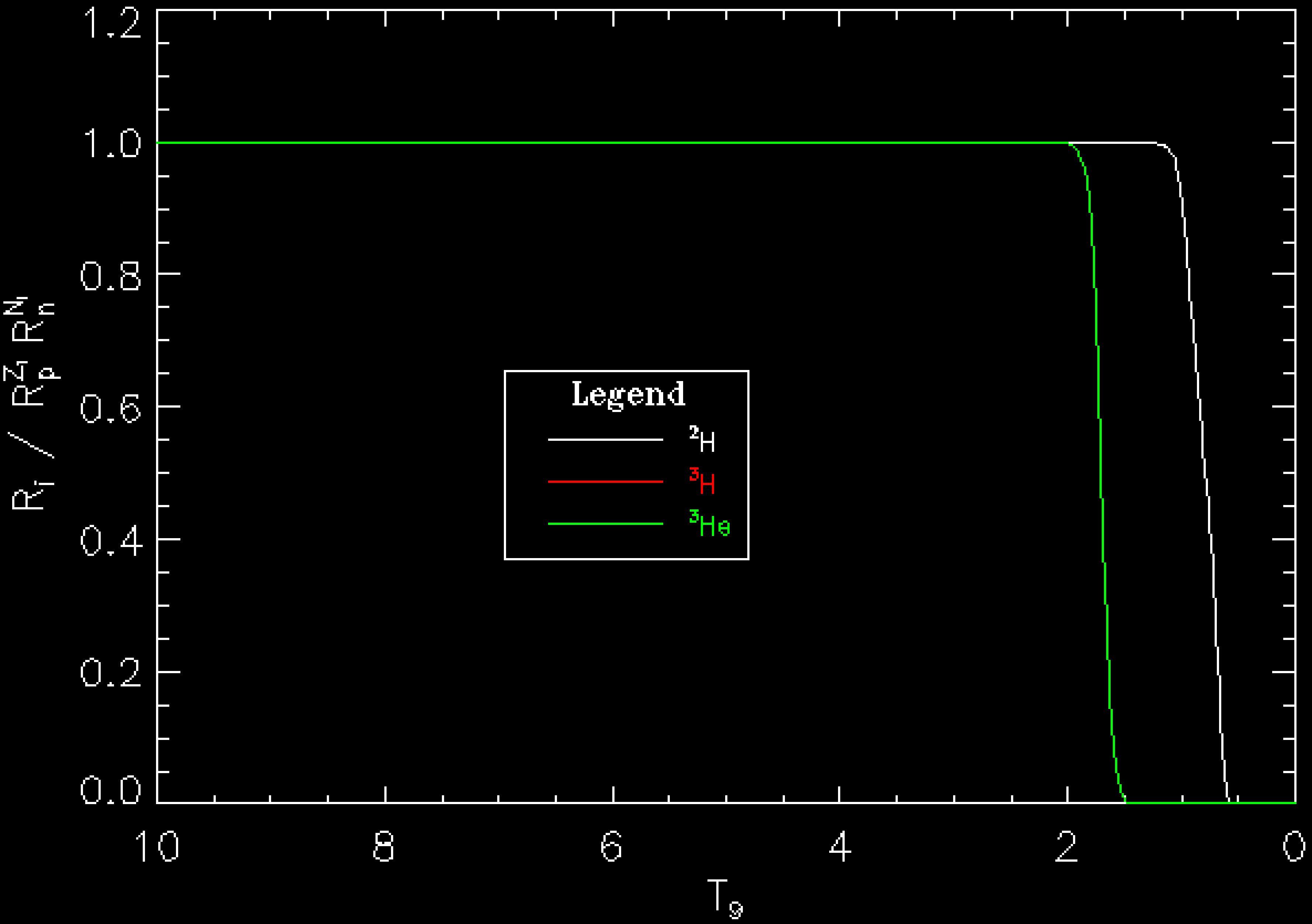
The r-Process (I)


t (s) = 1.749E-20
T_9 = 40.01
rho (g/cc) = 2.200E+08
Abundance
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
50
100
150
200
250
A, Mass Number

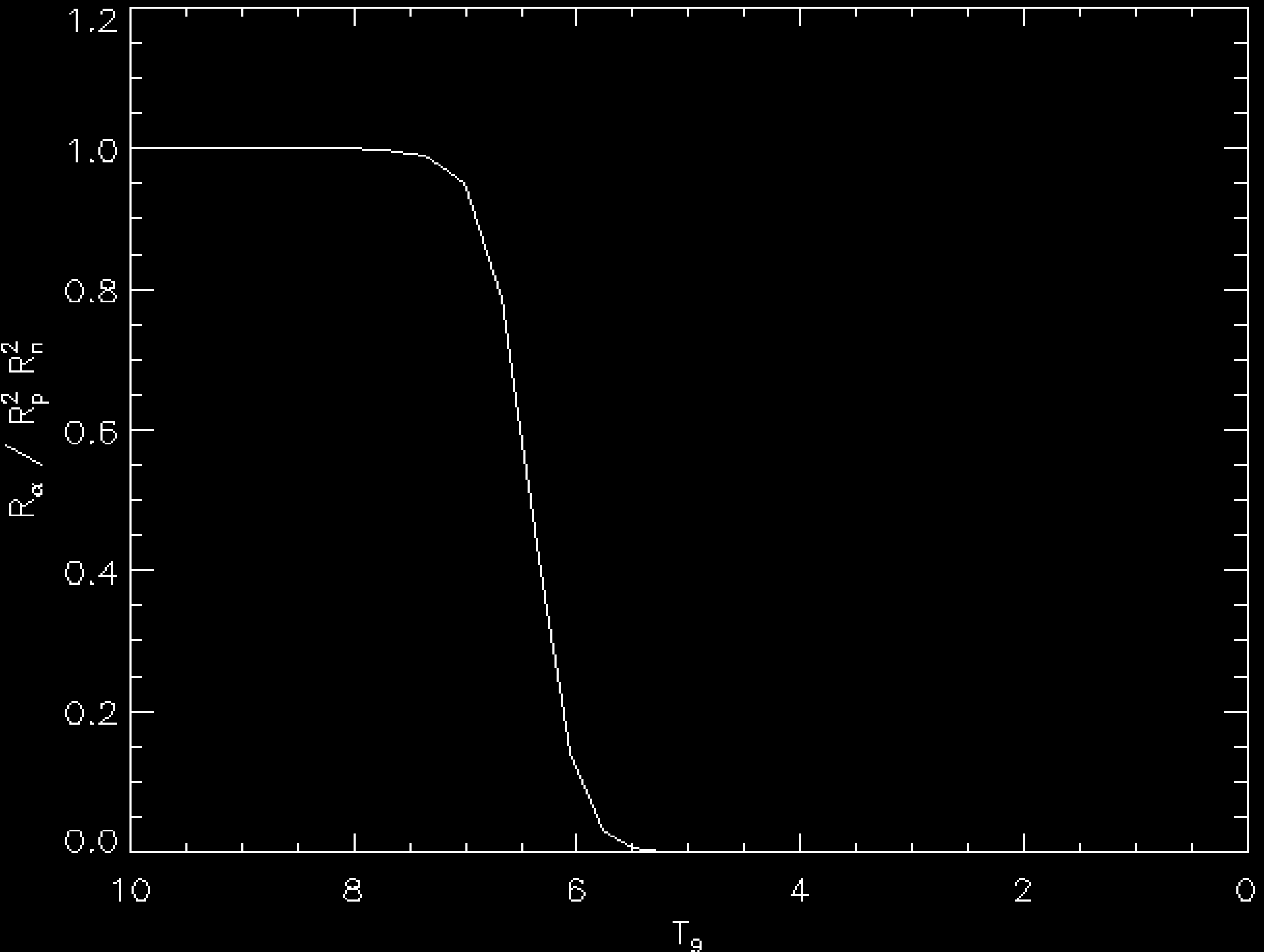

Network, NSE, QSE
t (s) = 1.749E-20 T_9 = 40.01 rho (g/cc) = 2.200E+08
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
Y_z
0
10
20
30
40
50
60
Proton Number, Z


1.5
1.0
0.5
0.0
R_α / R_p^2 R_n^2
10
8
6
4
2
0
T_9

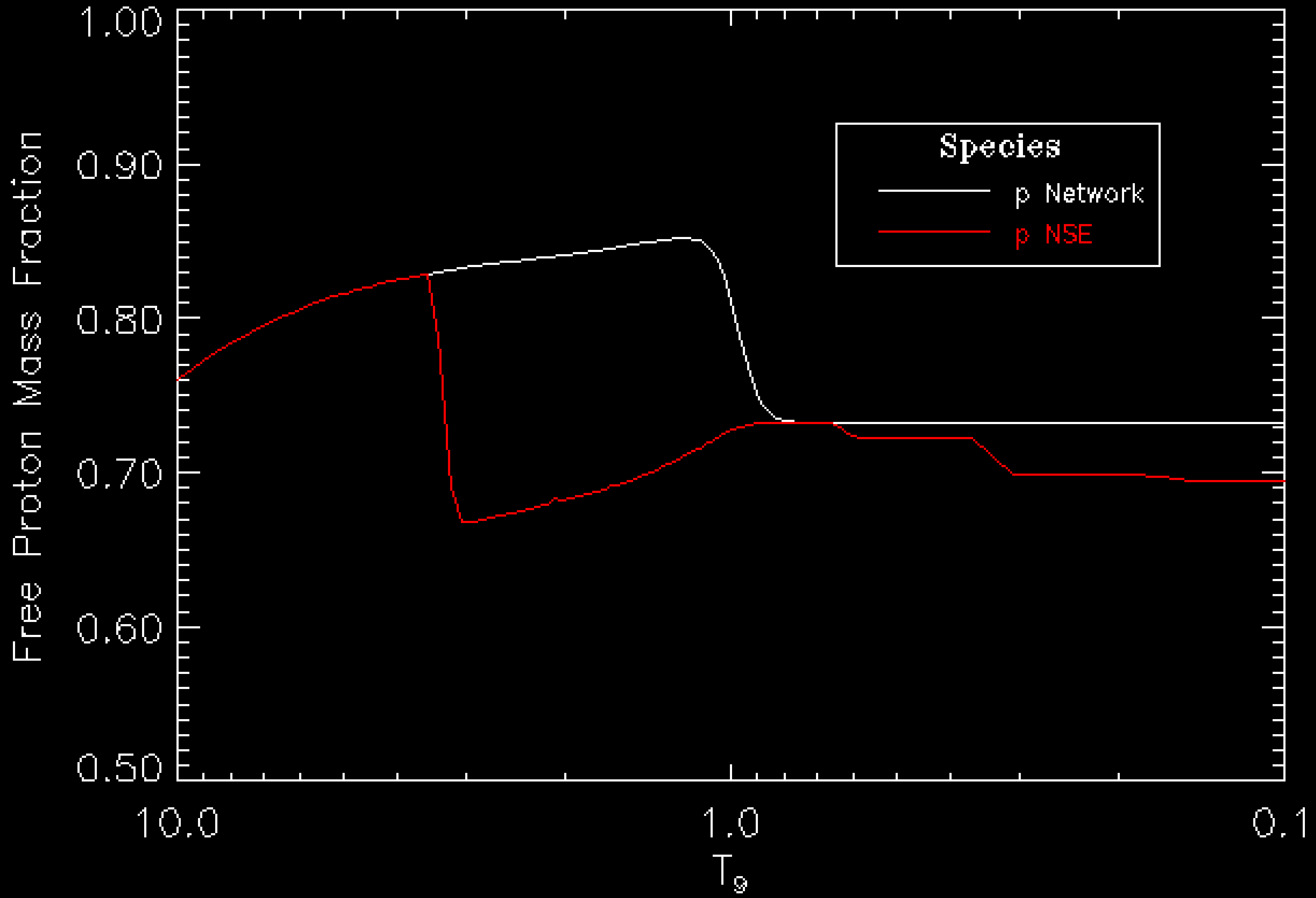
BBN

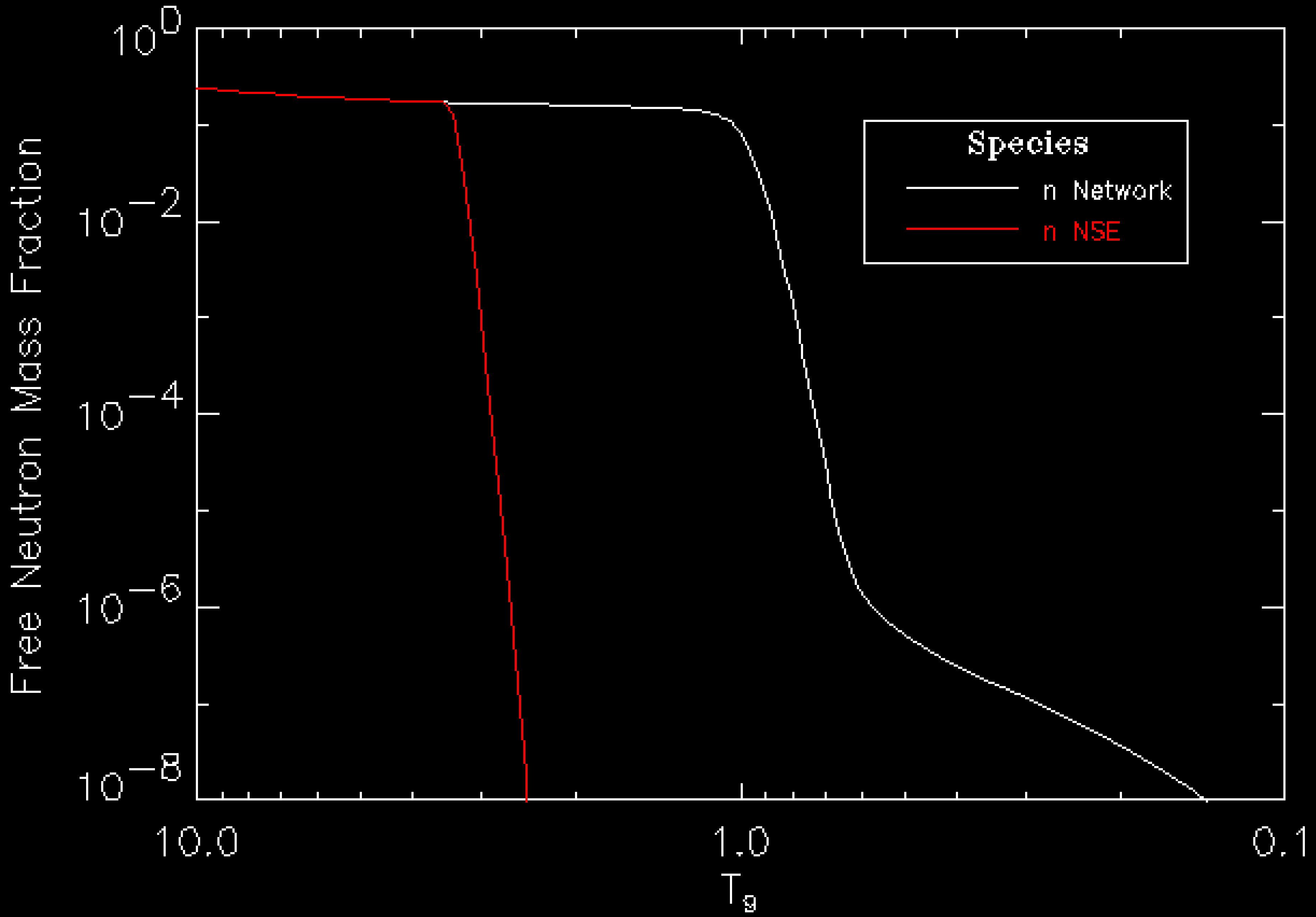


1.2
1.0
0.8
0.6
0.4
0.2
0.0
R_i / $R_p^Z R_n^N$
10
8
6
4
2
0
T_9
Legend
^{2}H
^{3}H
^{3}He

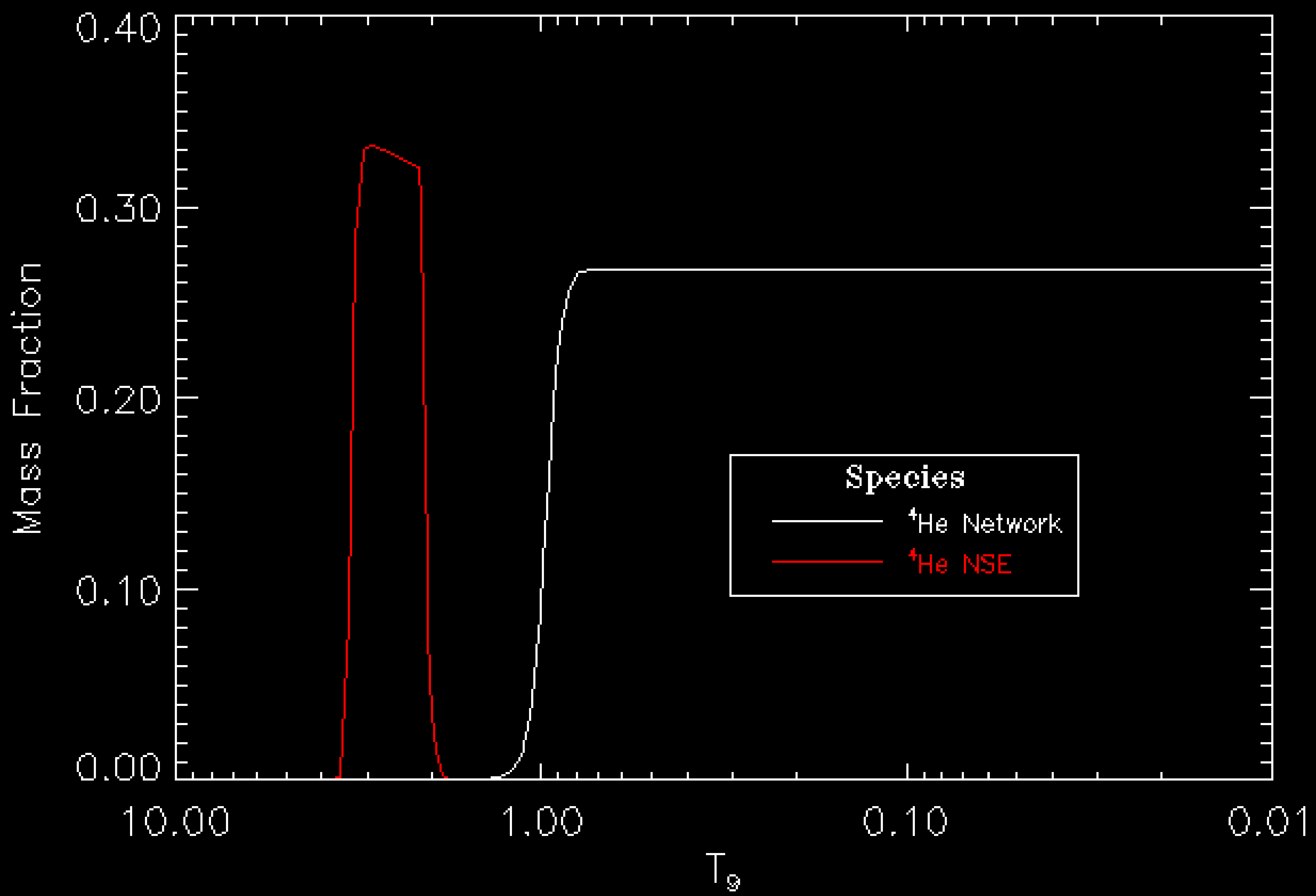


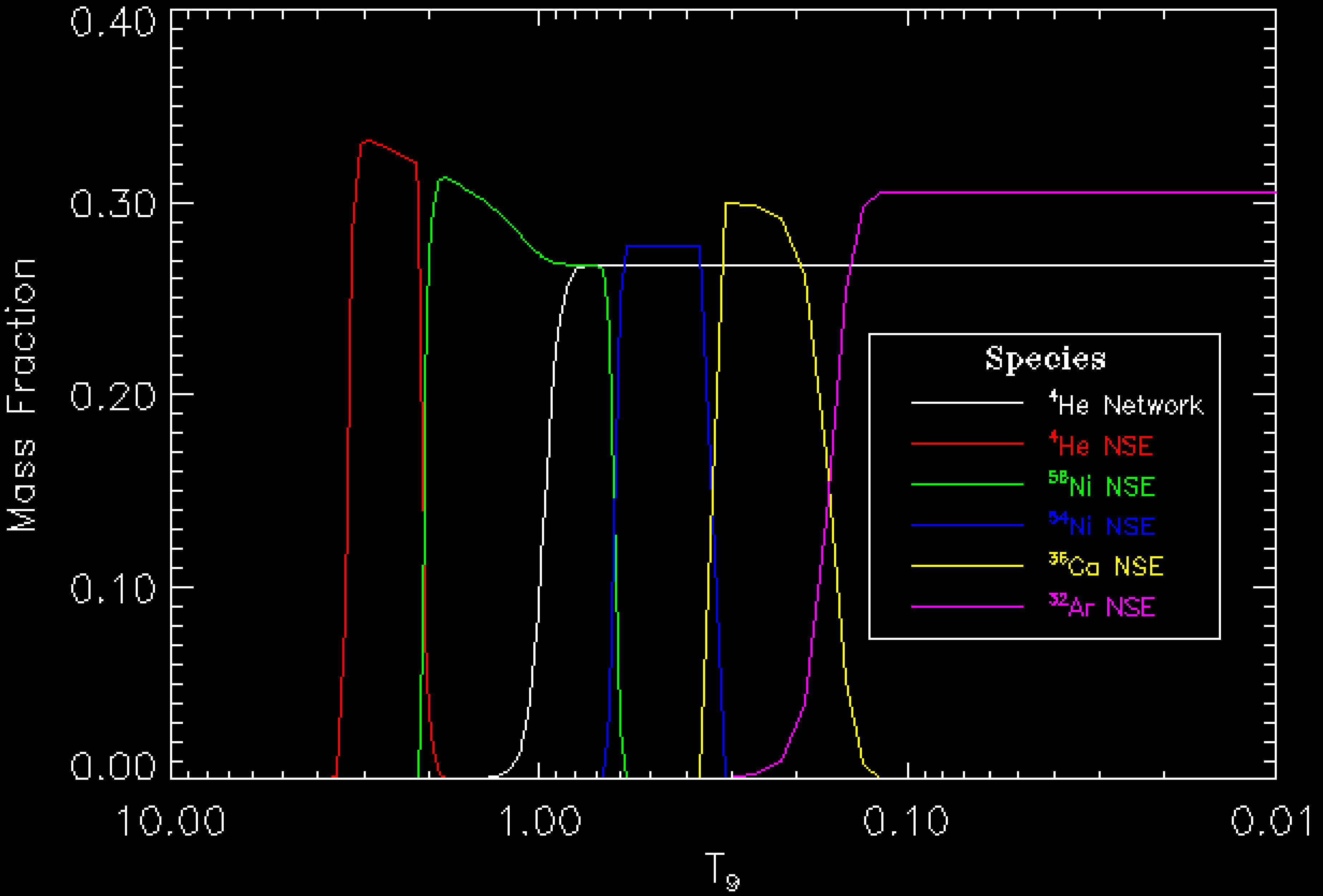
1.2
1.0
0.8
0.6
0.4
0.2
0.0
R_α / R_p^2 R_n^2
10
8
6
4
2
0
T_9


Free Proton Mass Fraction
1.00
0.90
0.80
0.70
0.60
0.50
10.0
1.0
0.1
T_9
Species
p Network
p NSE

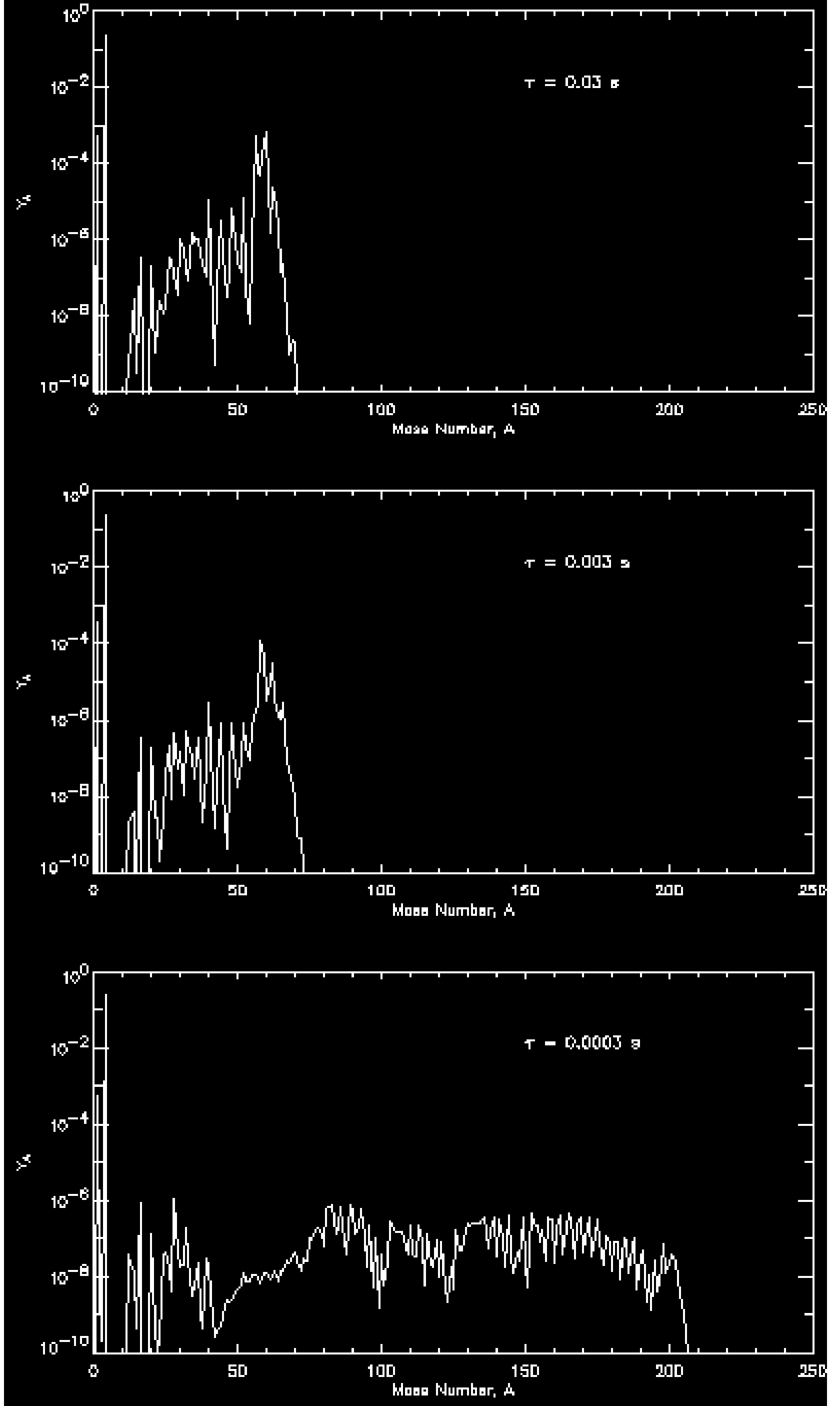


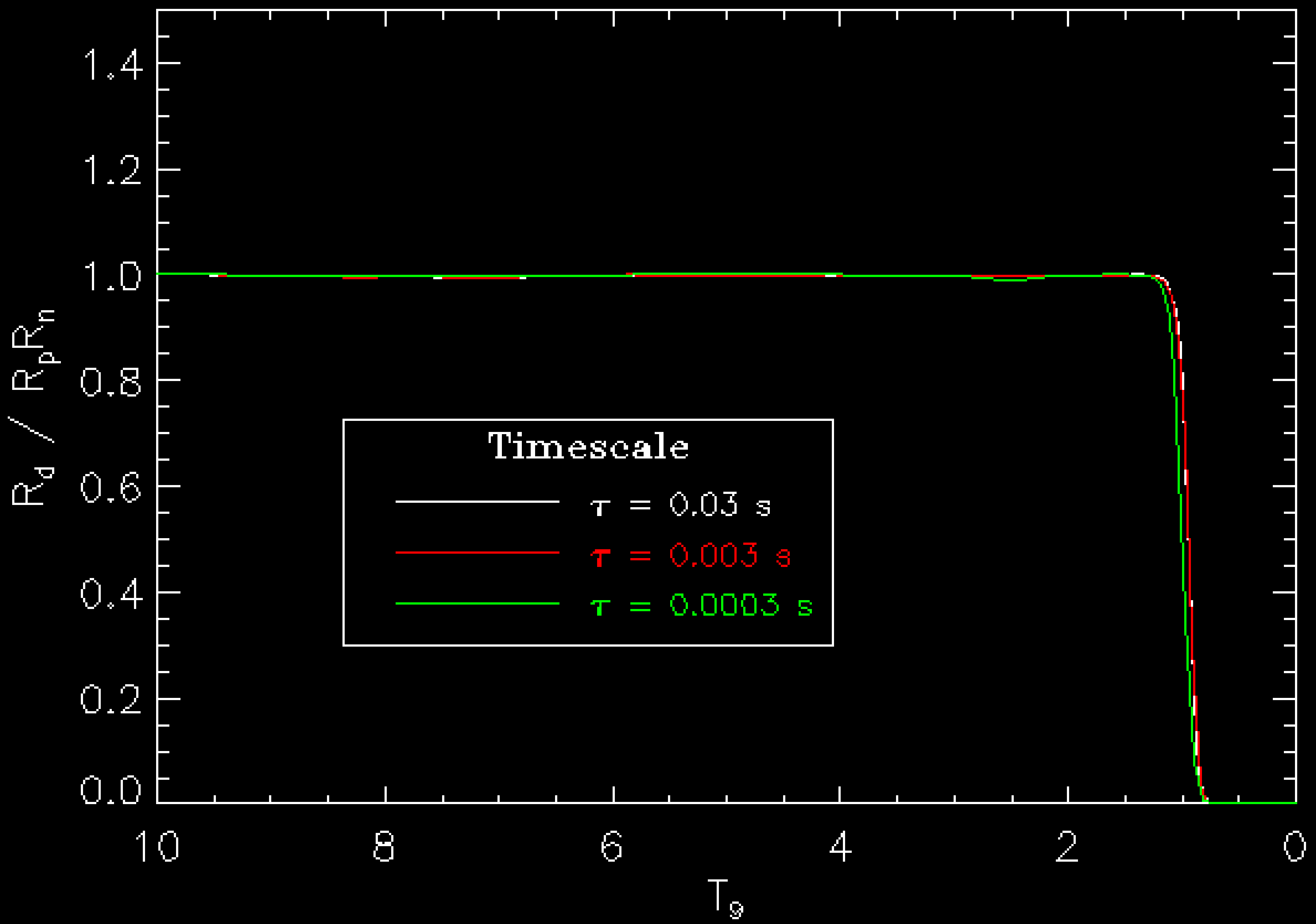
Free Neutron Mass Fraction
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10.0
1.0
0.1
T_9
Species
n Network
n NSE

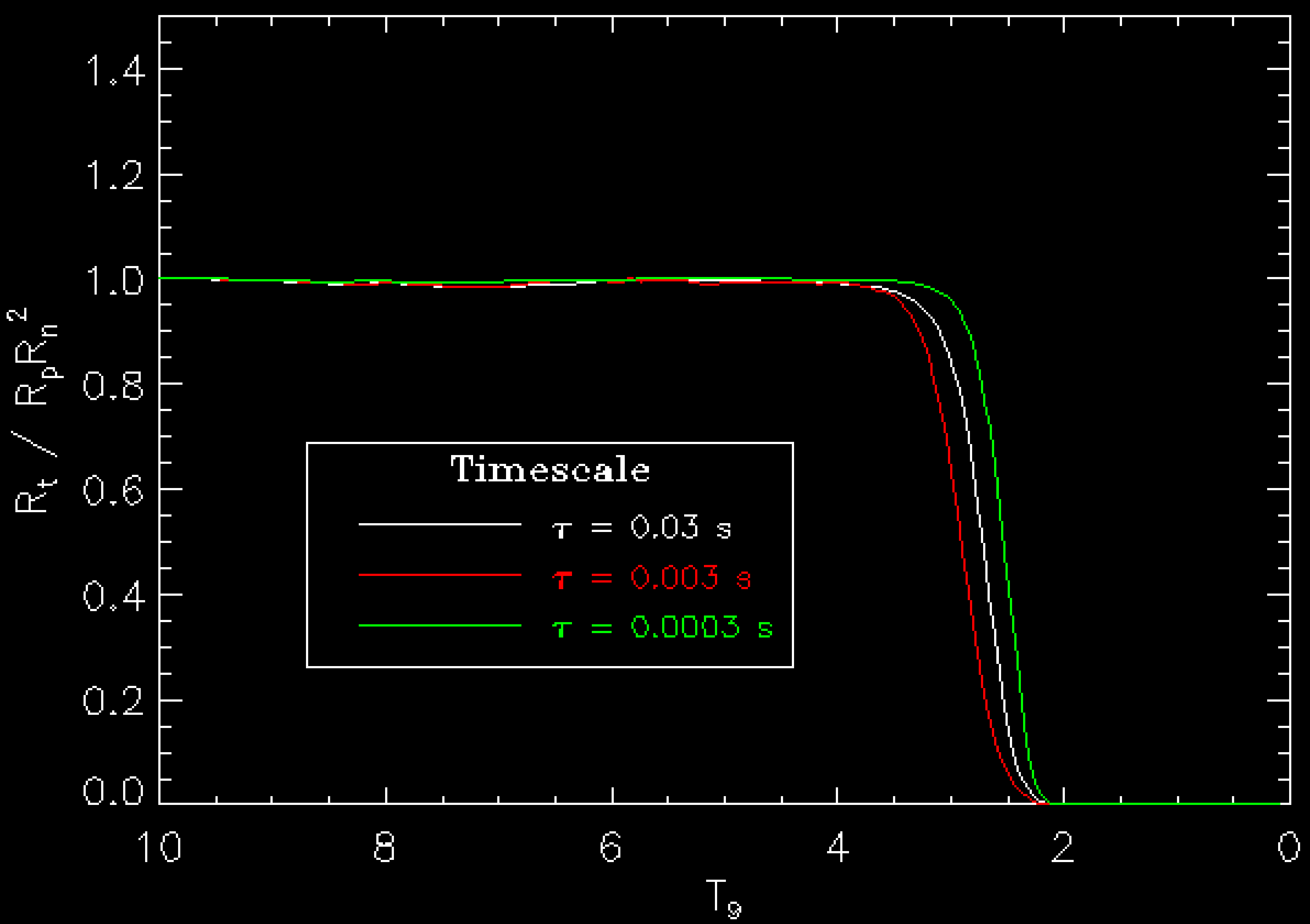

0.40
0.30
0.20
0.10
0.00
Mass Fraction
10.00
1.00
0.10
0.01
T_9
Species
^{4}He Network
^{4}He NSE

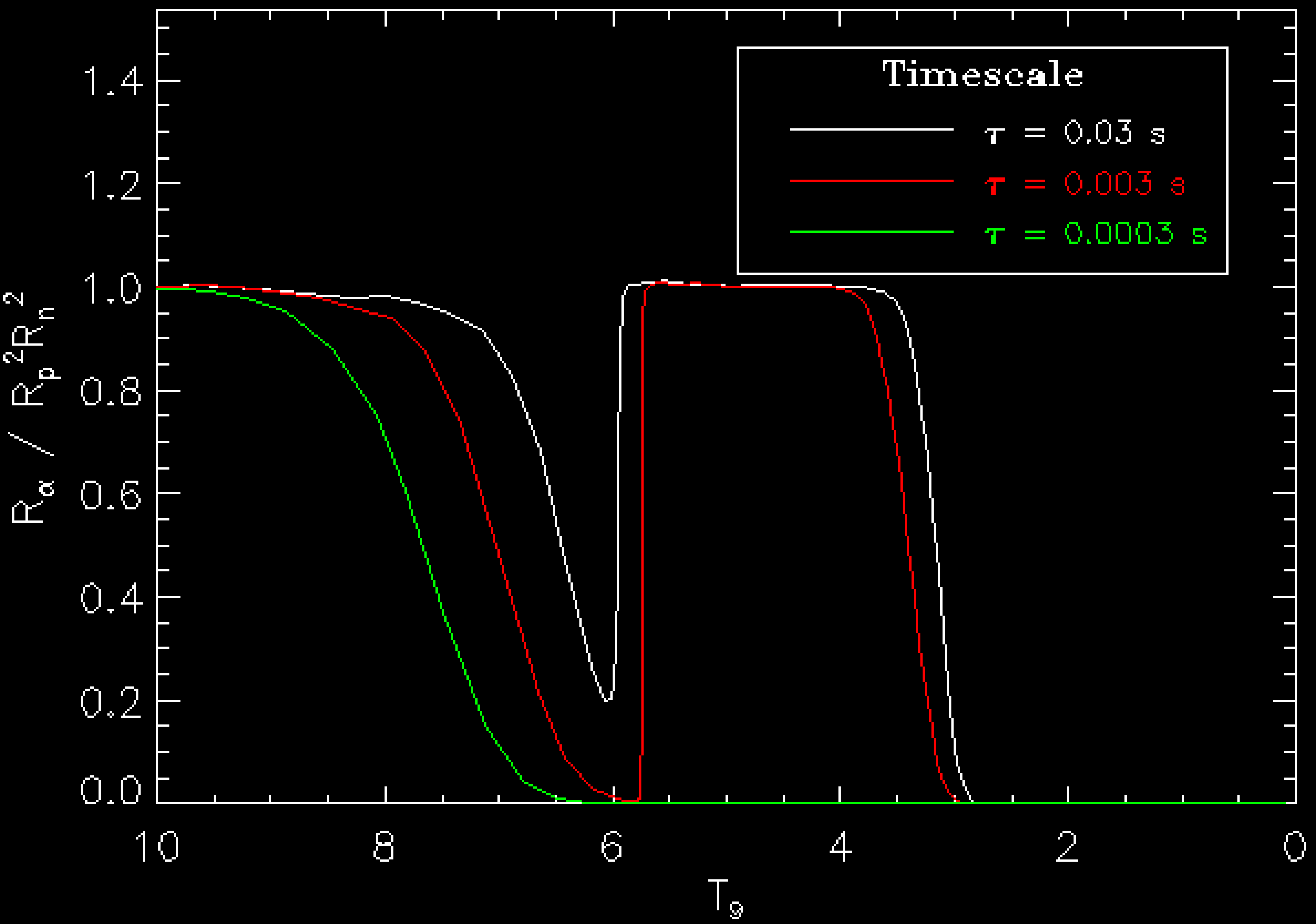

Mass Fraction
0.40
0.30
0.20
0.10
0.00
10.00
1.00
0.10
0.01
T_9
Species
^{4}He Network
^{4}He NSE
^{56}Ni NSE
^{54}Ni NSE
^{36}Ca NSE
^{32}Ar NSE

The r-Process (II)


τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s
Y_A
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
0
50
100
150
200
250
Mass Number, A

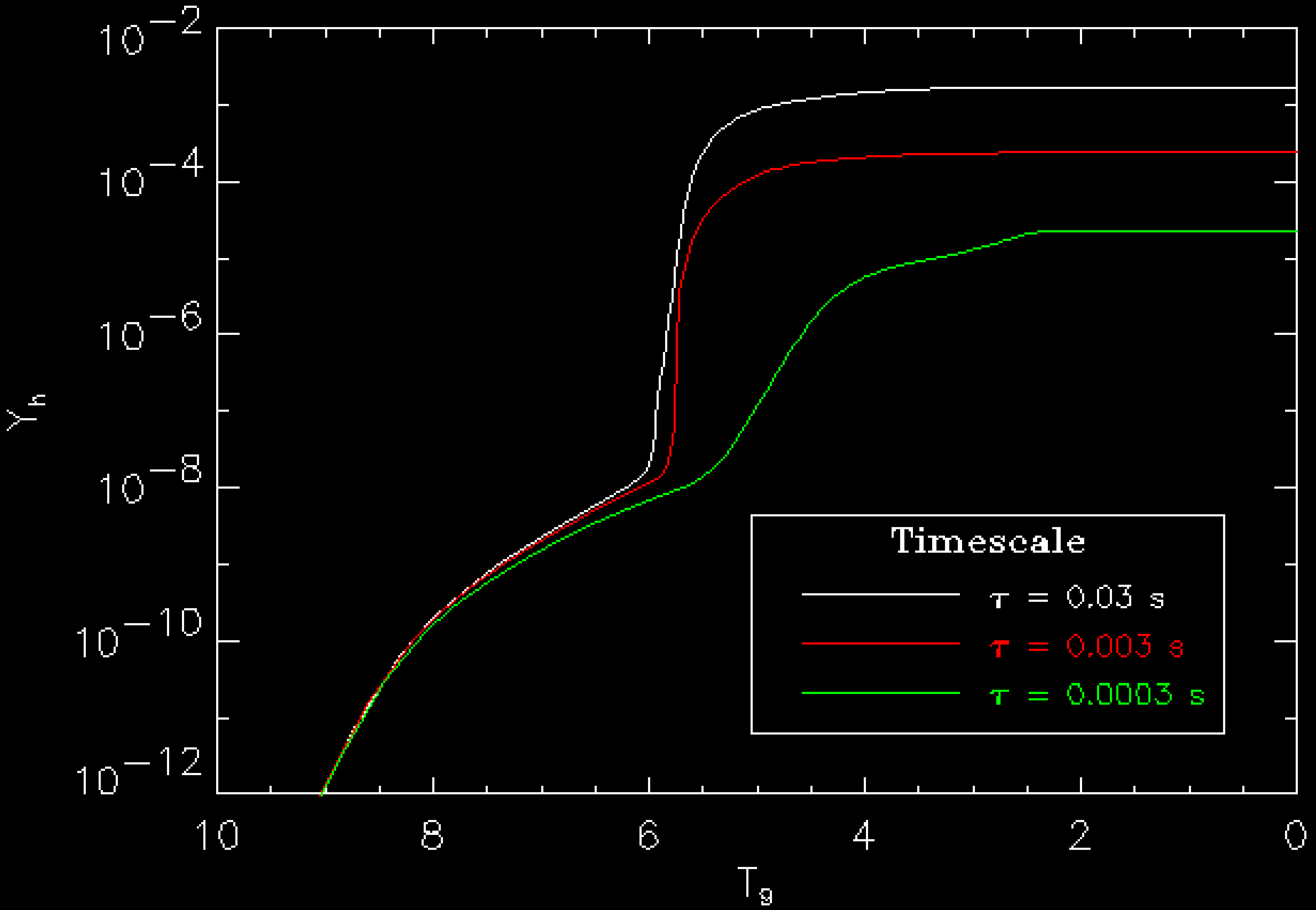

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
R_d / $R_p R_n$
10
8
6
4
2
0
T_9
Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s

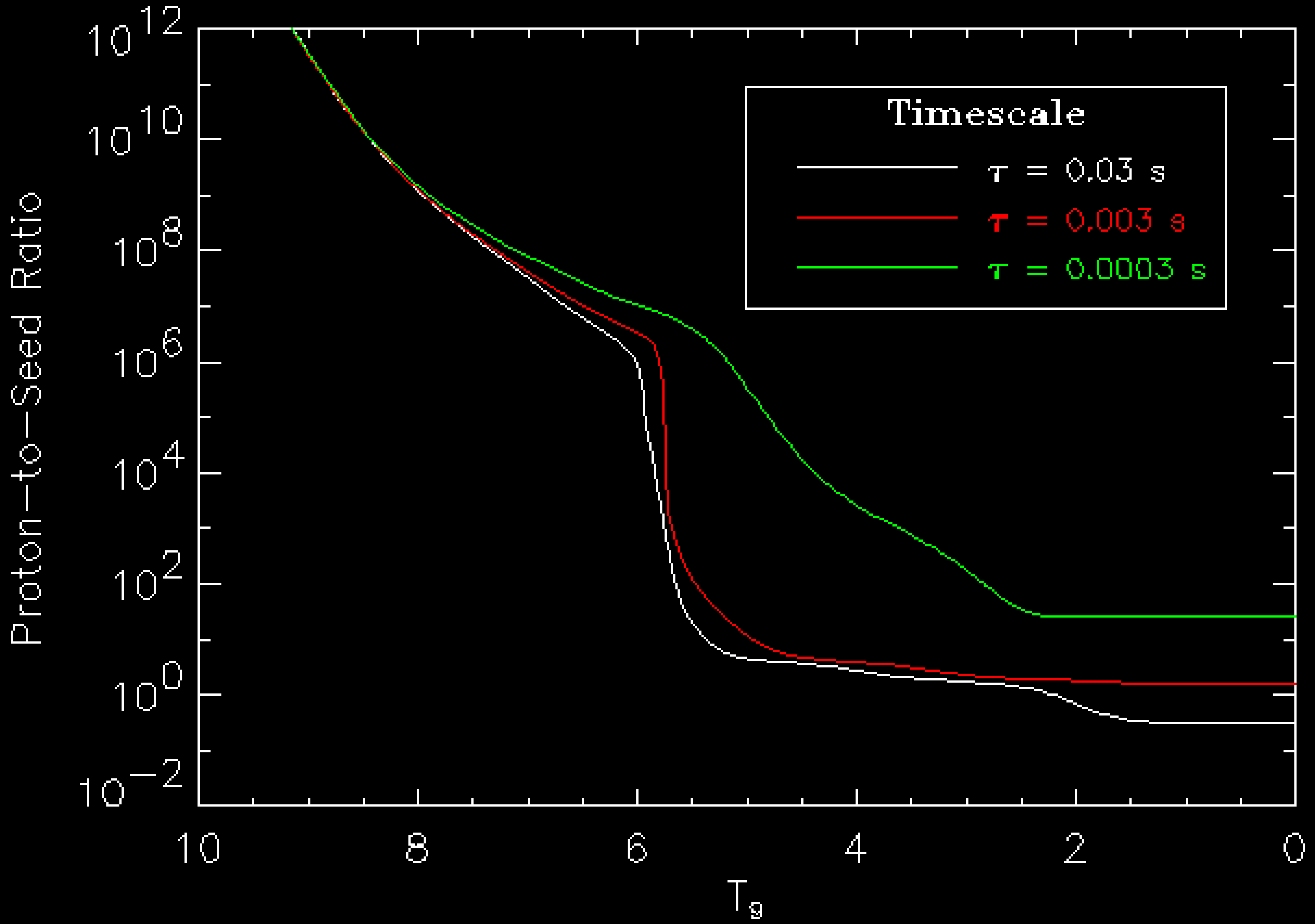

Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s
$R_t / R_p R_n^2$
T_9
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10
8
6
4
2
0

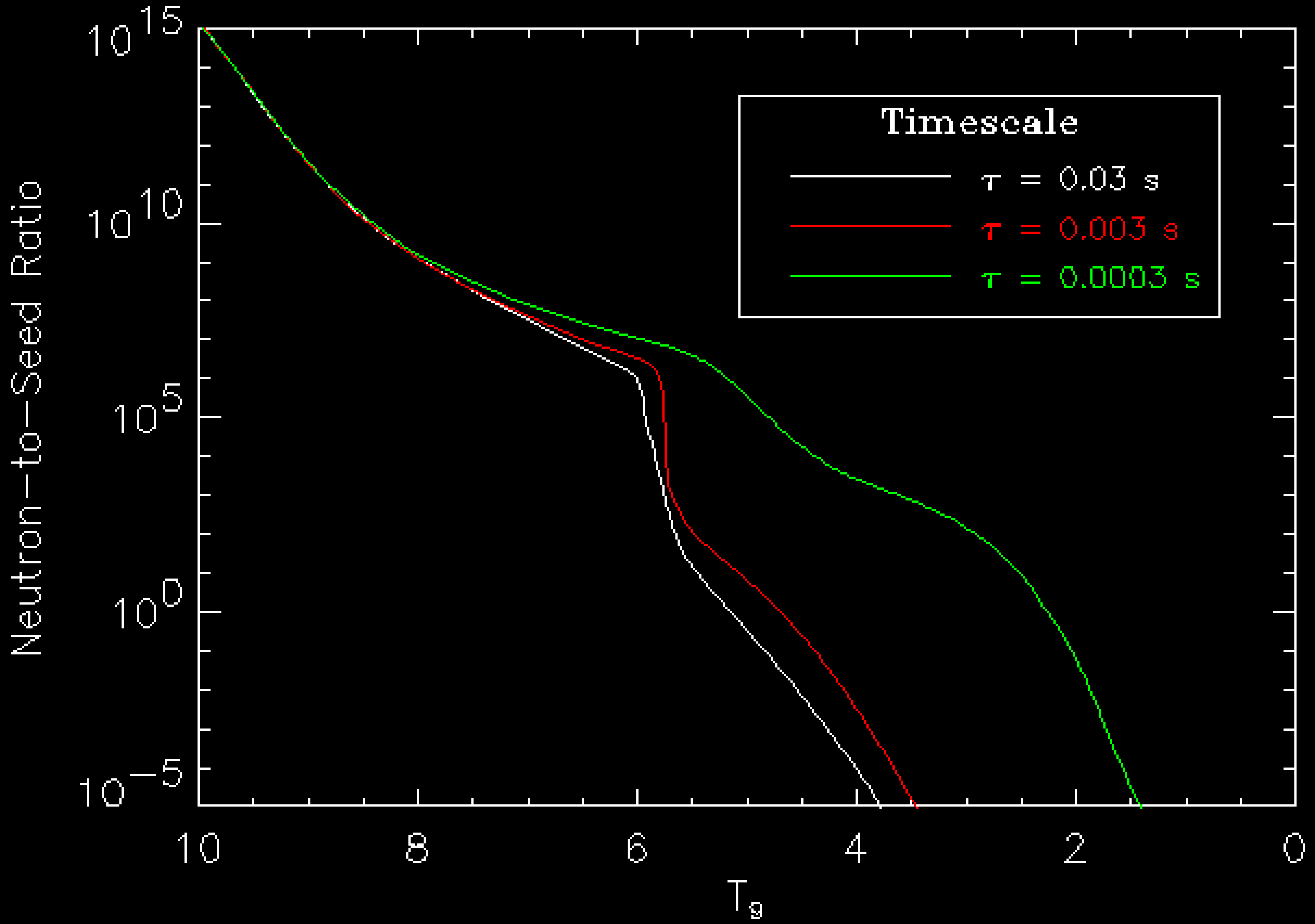

Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s
$R_\alpha / R_p^2 R_n^2$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10
8
6
4
2
0
T_9

Assembling ^{4}He from n,p

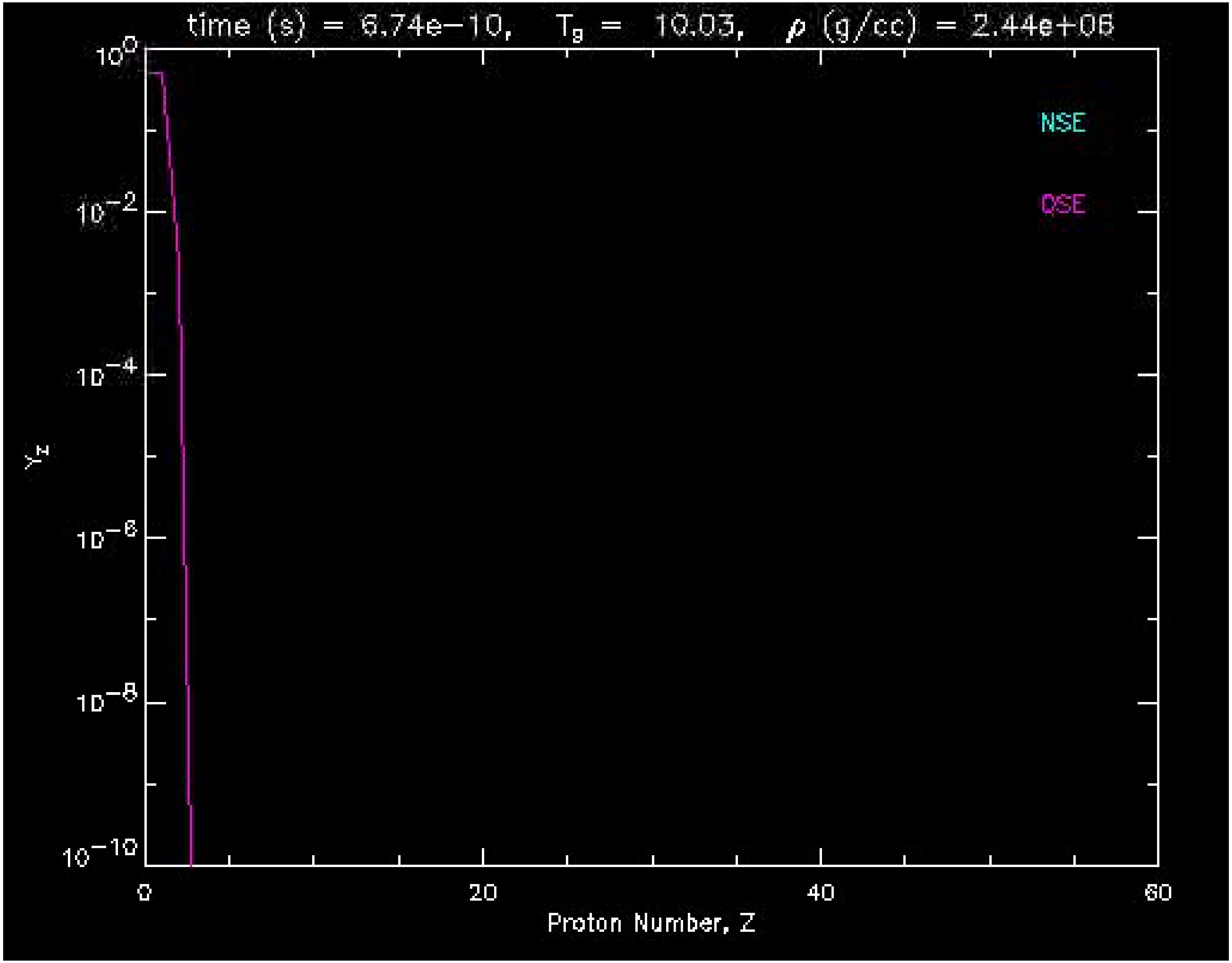
- Early: reaction sequences like $p(n,g)d(n,g)t(d,n)^{4}He$
- Later: catalysis by reaction cycles like $^{56}Fe(n,g)^{57}Fe(n,g)^{58}Fe(p,g)^{59}Co(p,^{4}He)^{56}Fe$

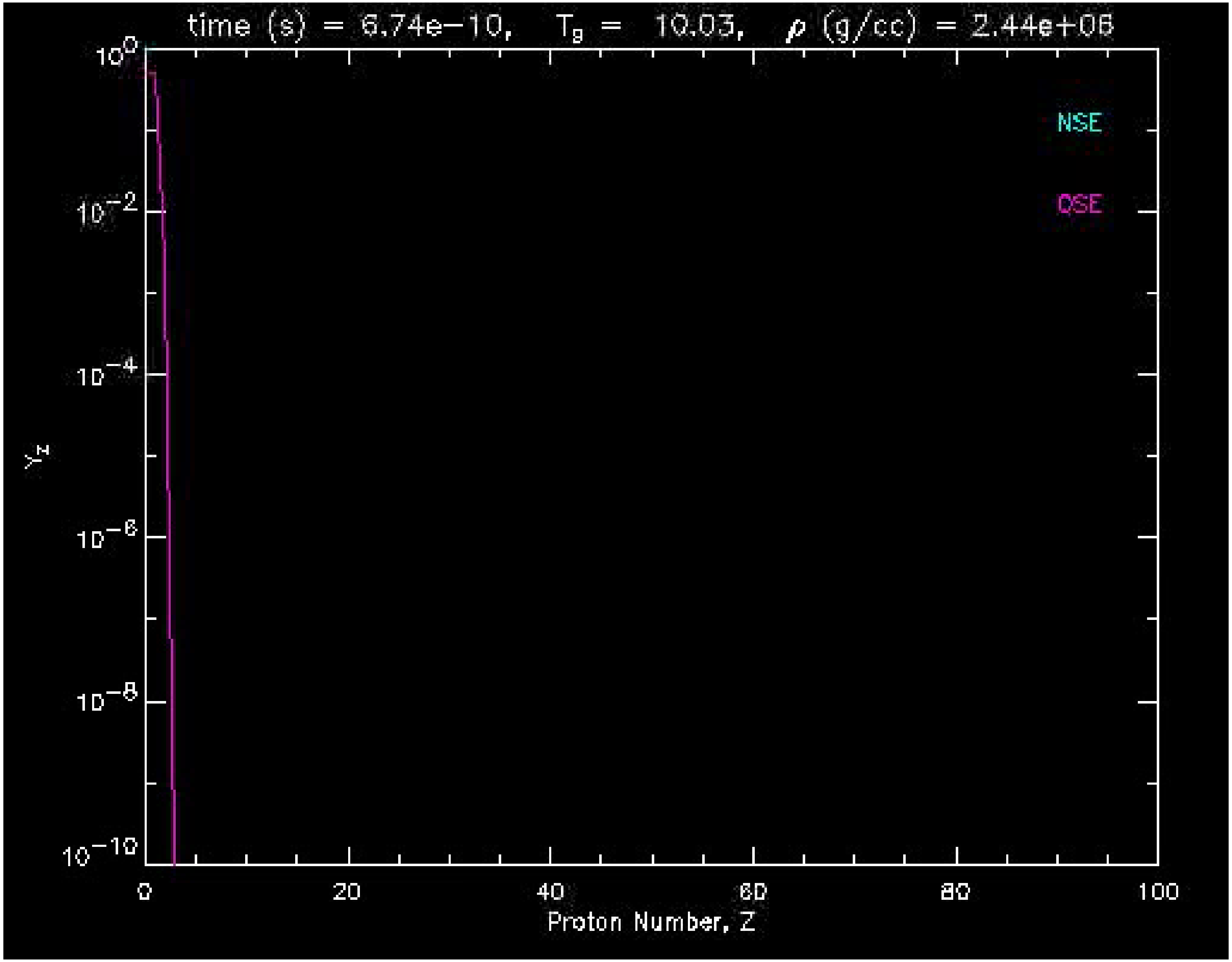

10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_h
10
8
6
4
2
0
T_9
Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s


Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s
Proton-to-Seed Ratio
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10
8
6
4
2
0
T_9



Neutron-to-Seed Ratio
10^{15}
10^{10}
10^{5}
10^{0}
10^{-5}
10
8
6
4
2
0
T_9
Timescale
τ = 0.03 s
τ = 0.003 s
τ = 0.0003 s


time (s) = 6.74e−10, T_9 = 10.03, ρ (g/cc) = 2.44e+06
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
Y_Z
0
20
40
60
Proton Number, Z
NSE
QSE


time (s) = 6.74e−10, T_9 = 10.03, ρ (g/cc) = 2.44e+06
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
Y_Z
NSE
QSE
0
20
40
60
80
100
Proton Number, Z

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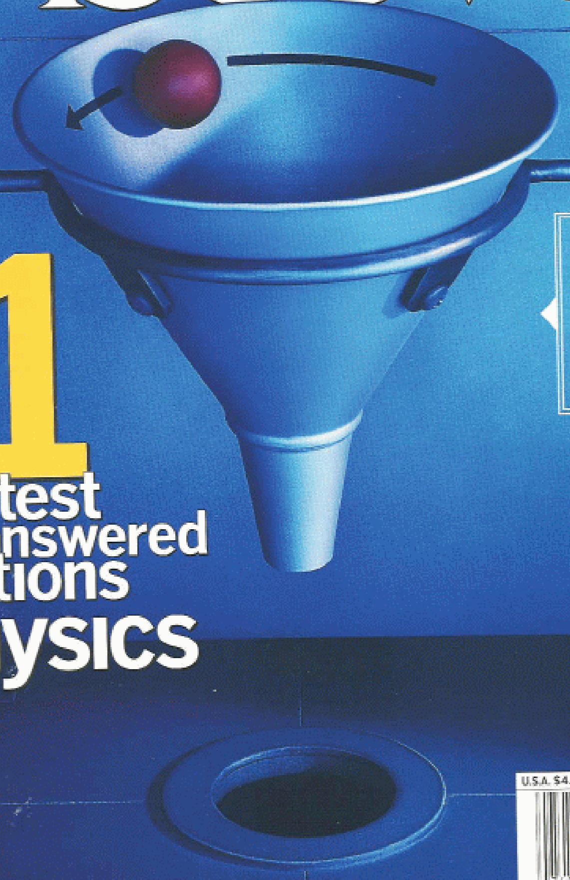

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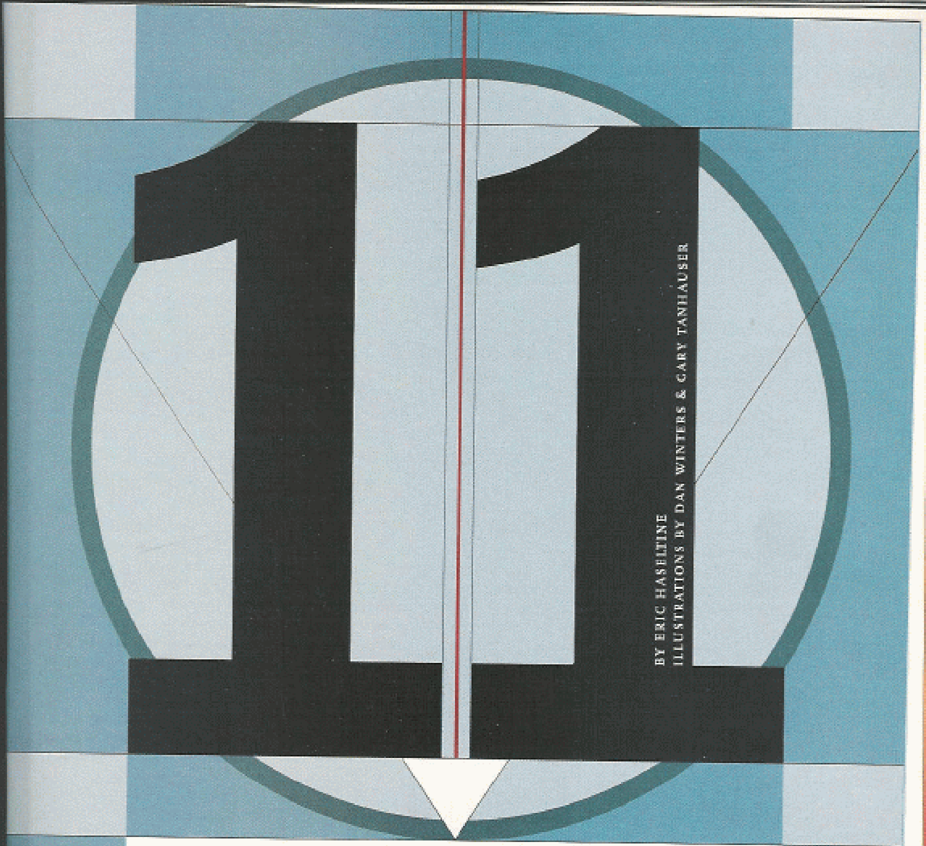


11

BY ERIC HASELTINE
ILLUSTRATIONS BY DAN WINTERS & CARY TANHAUSER

THE GREATEST UNANSWERED QUESTIONS OF PHYSICS

RESOLUTION OF THESE PROFOUND QUESTIONS COULD UNLOCK THE SECRETS OF EXISTENCE AND DELIVER A NEW AGE OF SCIENCE WITHIN SEVERAL DECADES

HERE'S A TALE OF MODERN PHYSICS: TWO SCIENTISTS work at the same university in different fields. One studies huge objects far from Earth. The other is fascinated by the tiny stuff right in front of him. To satisfy their curiosities, one builds the world's most powerful telescope, and the other builds the world's best microscope. As they focus their instruments on ever more distant and ever more minuscule objects, they begin to observe structures and behaviors never before seen—or imagined. They are excited but frustrated because their observations don't fit existing theories. →

OPPOSITE, QUESTION #8: All the atoms in the universe are built around an essential particle—the proton. But unified field theory predicts that time may eventually run out for protons, and they could decay into a spray of subparticles.

One day they leave their instruments for a caffeine break and happen to meet in the faculty lounge, where they begin to commiserate about what to make of their observations. Suddenly it becomes clear to both of them that although they seem to be looking at opposite ends of the universe, they are seeing the same phenomena. Like blind men groping a beast, one scientist has grasped its thrashing tail and the other its chomping snout. Comparing notes, they realize it's the same alligator.

This is precisely the situation particle physicists and astronomers find themselves in today. Physicists, using linear and circular particle accelerators as their high-resolution "microscopes," study pieces of atoms so small they can't be seen. Astronomers, using a dozen or so new supersize telescopes, also study the same tiny particles, but theirs are waiting for them in space. This strange collision of information means that the holy grail of particle physics—understanding the unification of all four forces of nature (electromagnetism, weak force, strong force, and gravity)—will be achieved in part by astronomers.

The implications are exciting to scientists because bizarre marriages of unrelated phenomena have created leaps of understanding in the past. Pythagoras, for example, set science spinning when he proved that abstract mathematics could be applied to the real world. A similar leap occurred when Newton discovered that the motions of planets and falling apples are both due to gravity. Maxwell created a new era of physics when he unified magnetism and electricity. Einstein, the greatest unifier of them all, wove together matter, energy, space, and time.

But nobody has woven together the tiny world of quantum mechanics and the big world we see when we look through a telescope. As these come together, physicists realize they are getting very close to a single "theory of everything" that accounts for the fundamental workings of nature, the long-sought unified field theory.

About two years ago, after a presentation by the National Research Council's board on physics and astronomy that showed the converging agendas of the two fields, NASA administrator Daniel Goldin suggested a special report that would detail how much astronomers and physicists could benefit from one another's insight. Recently, the council's committee on the physics of the universe released that report. It details 11 profound questions, some of which may be answered within a decade. If so, science is likely to make one of its greatest leaps in history.

But first, what we don't know.

QUESTION 1

What is dark matter?

All the ordinary matter we can find accounts for only about 4 percent of the universe. We know this by calculating how much mass would be needed to hold galaxies together and cause them to move about the way they do when they gather in large clusters. Another way to weigh the unseen matter is to look at how gravity bends the light from distant objects. Every measure tells astronomers that most of the universe is invisible.

It's tempting to say that the universe must be full of dark clouds of dust or dead stars and be done with it, but there are persuasive arguments that this is not the case. First, although there are ways to spot even the darkest forms of matter, almost every attempt to find missing clouds and stars has failed. Second, and more convincing, cosmologists can make very precise calculations of the nuclear reactions that occurred right after the Big Bang and compare the expected results with the actual composition of the universe. Those calculations show that the total amount of ordinary matter, composed of familiar protons and neutrons, is much less than the total mass of the universe. Whatever the rest is, it isn't like the stuff of which we're made.

The quest to find the missing universe is one of the key efforts that has brought cosmologists and particle physicists together. The leading dark-matter candidates are neutrinos or two other kinds of particles: neutralinos and axions, predicted by some physics theories but never detected. All three of these particles are thought to be electrically neutral, thus unable to absorb or reflect light, yet stable enough to have survived from the earliest moments after the Big Bang.

QUESTION 2

What is dark energy?

Two recent discoveries from cosmology prove that ordinary matter and dark matter are still not enough to explain the structure of the universe. There's a third component out there, and it's not matter but some form of dark energy.

The first line of evidence for this mystery component comes from measurements of the geometry of the universe. Einstein theorized that all matter alters the shape of space and time around it. Therefore, the overall shape of the universe is governed by the total mass and energy within it. Recent studies of radiation left over from the Big Bang show that the universe has the simplest shape—it's flat. That, in turn, reveals the total mass density of the universe. But after adding up all the potential sources of dark matter and ordinary matter, astronomers still come up two-thirds short.

The second line of evidence suggests that the mystery component must be energy. Observations of distant supernovas show that the rate of expansion of the universe isn't slowing as scientists had once assumed; in fact, the pace of the expansion is increasing. This cosmic acceleration is difficult to explain unless a pervasive repulsive force constantly pushes outward on the fabric of space and time.

Why dark energy produces a repulsive force field is a bit complicated. Quantum theory says virtual particles can pop into existence for the briefest of moments before returning to nothingness. That means the vacuum of space is not a true void. Rather, space is filled with low-grade energy created when virtual particles and their antimatter partners momentarily pop into and out of existence, leaving behind a very small field called vacuum energy.

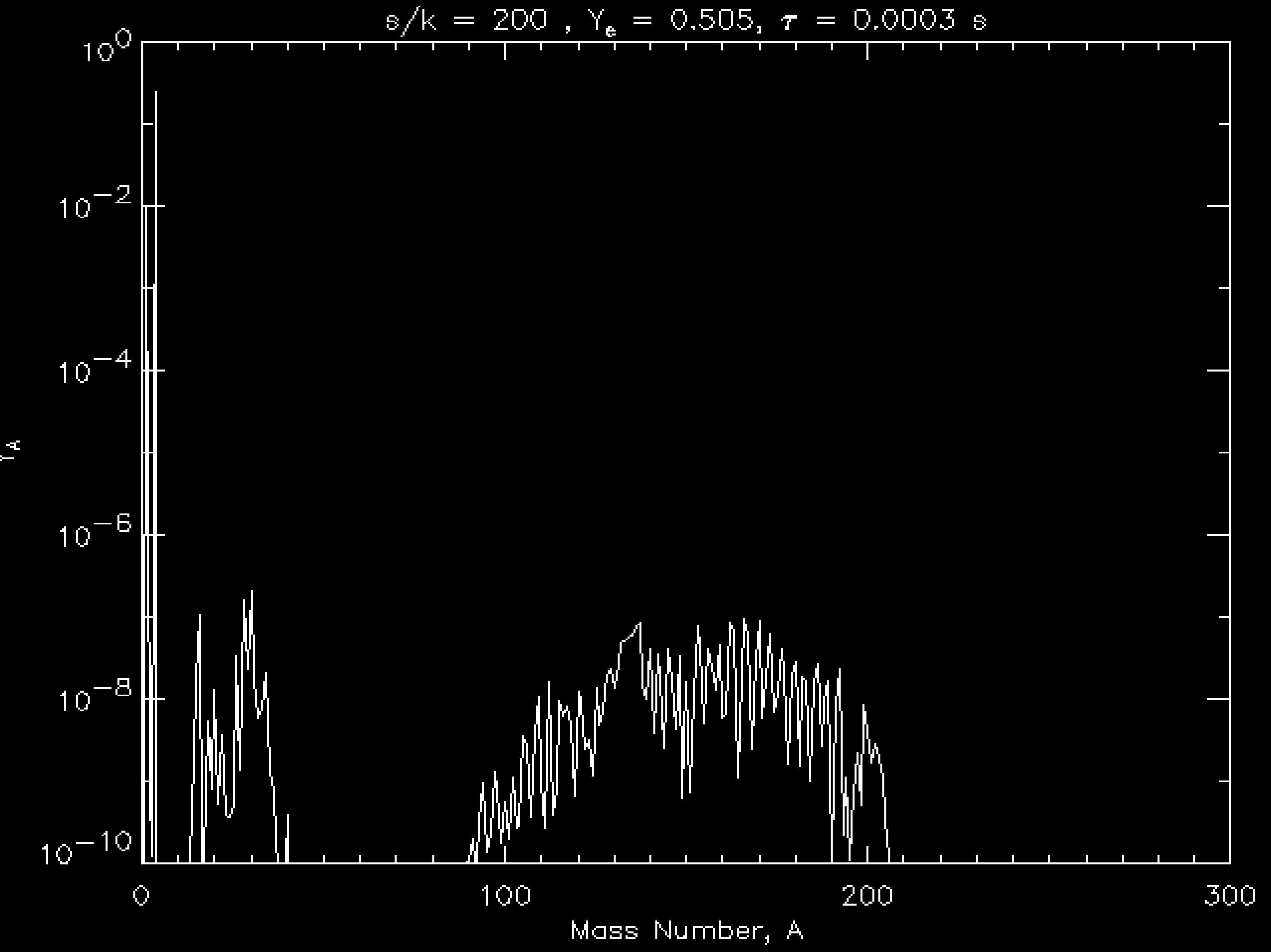
That energy should produce a kind of negative pressure, or

OPPOSITE, QUESTION #7: Add a little heat, and molecules can be easily transformed from solids into liquids and then gases. But what happens at extreme temperatures? Does matter break down into a soup of subatomic particles—called a quark-gluon plasma—and then into energy?

QUESTION 3

How were the heavy elements from iron to uranium made? Both dark matter and possibly dark energy originate from the earliest days of the universe, when light elements such as helium and lithium arose. Heavier elements formed later inside stars, where nuclear reactions jammed protons and neutrons together to make new atomic nuclei. For instance, four hydrogen nuclei (one proton each) fuse through a series of reactions into a helium nucleus (two protons and two neutrons). That's what happens in our sun, and it produces the energy that warms Earth.

But when fusion creates elements that are heavier than iron, it requires an excess of neutrons. Therefore, astronomers assume that heavier atoms are minted in supernova explosions, where there is a ready supply of neutrons, although the specifics of how this happens are unknown. More recently, some scientists have speculated that at least some of the heaviest elements, such as gold and lead, are formed in even more powerful blasts that occur when two neutron stars—tiny, burned-out stellar corpses—collide and collapse into a black hole.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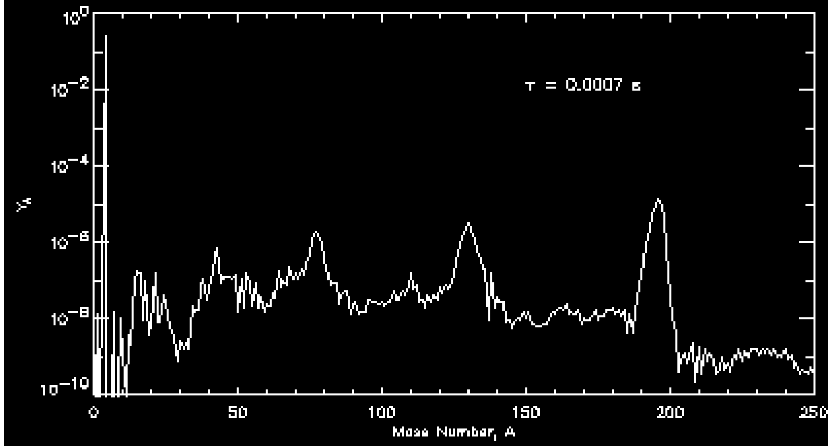
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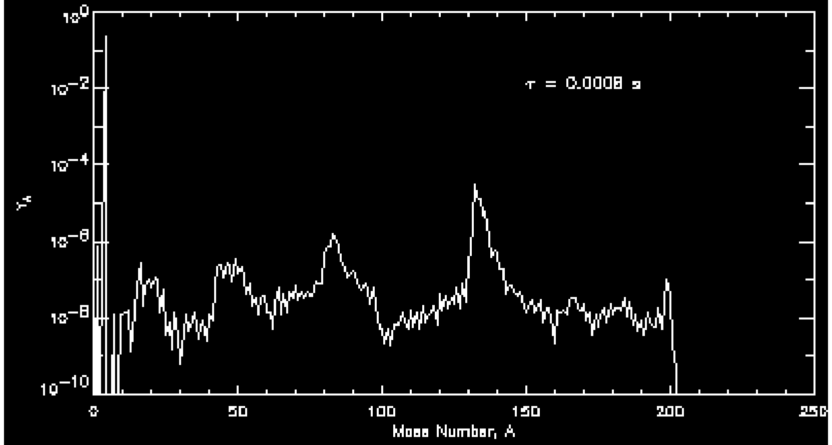
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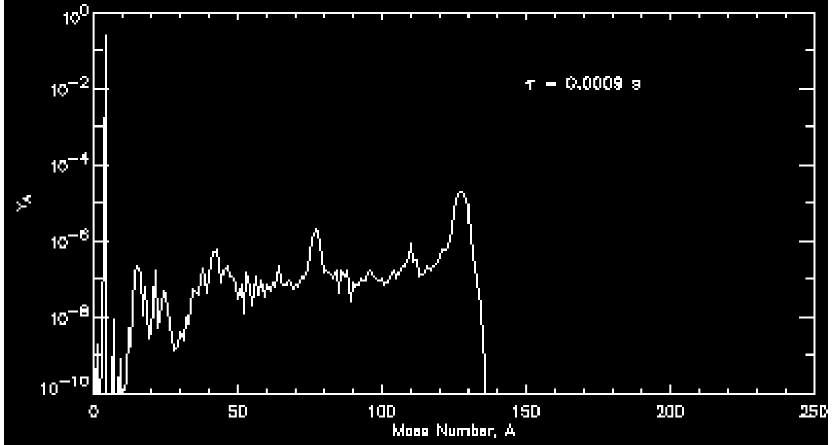
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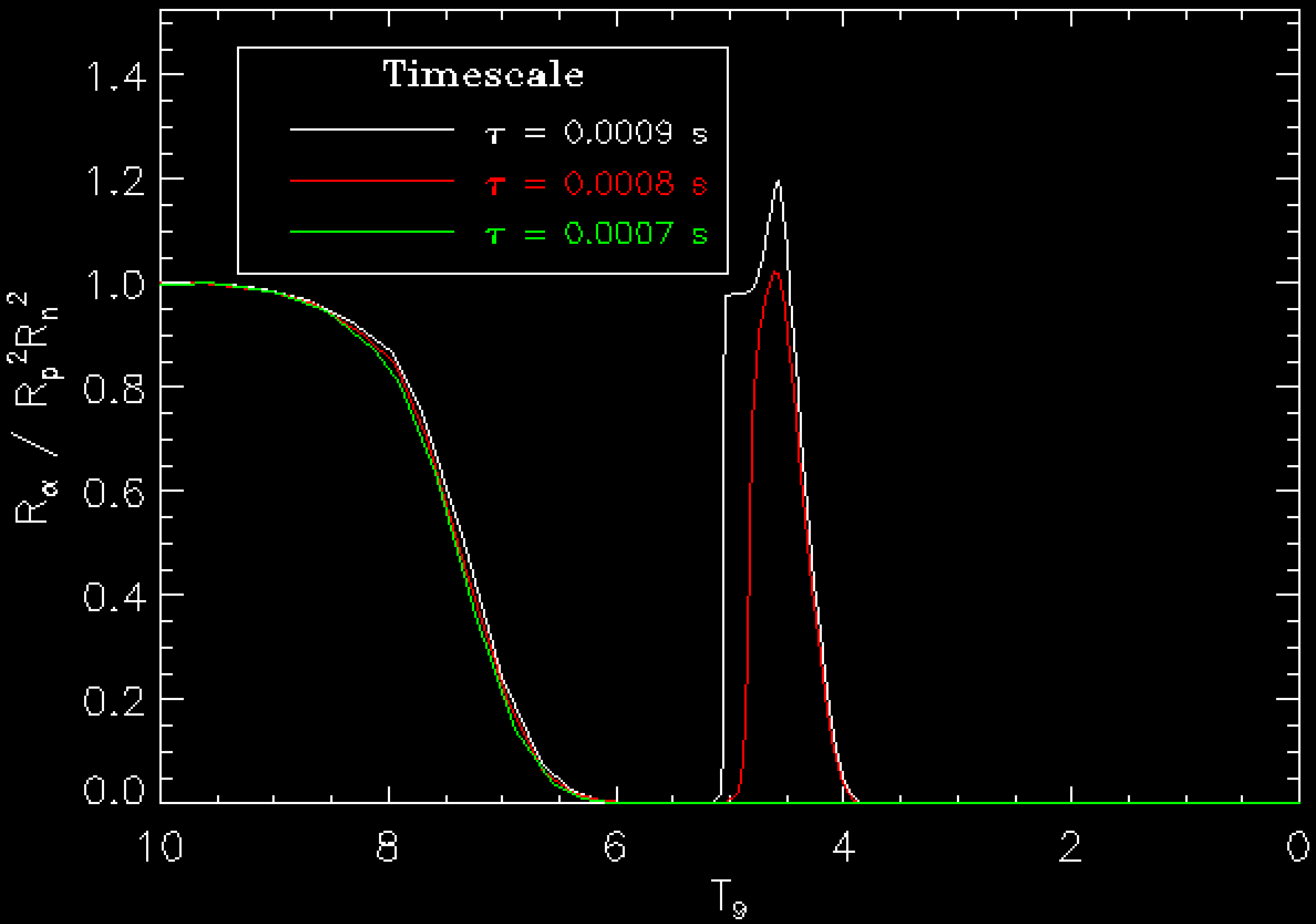
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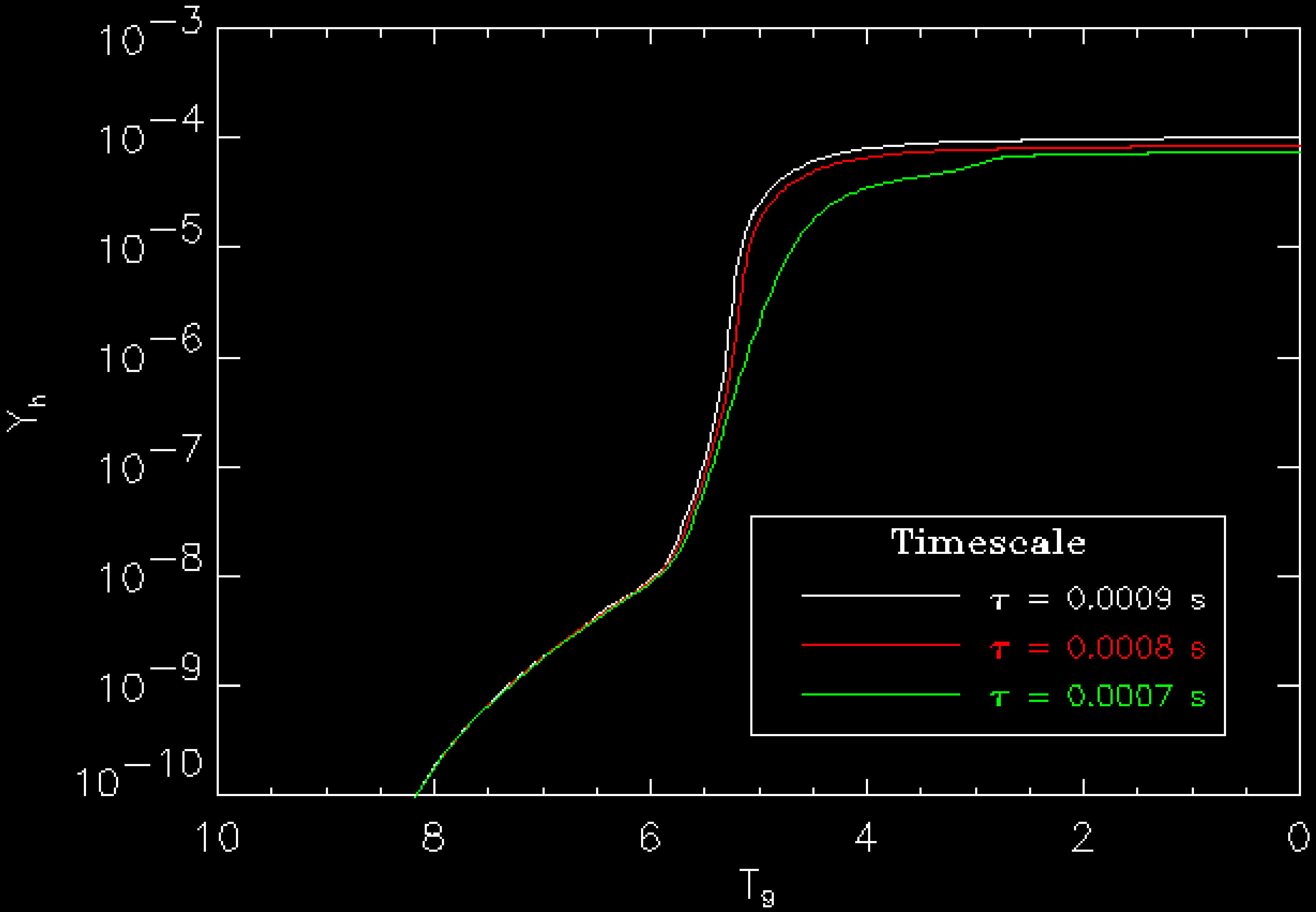
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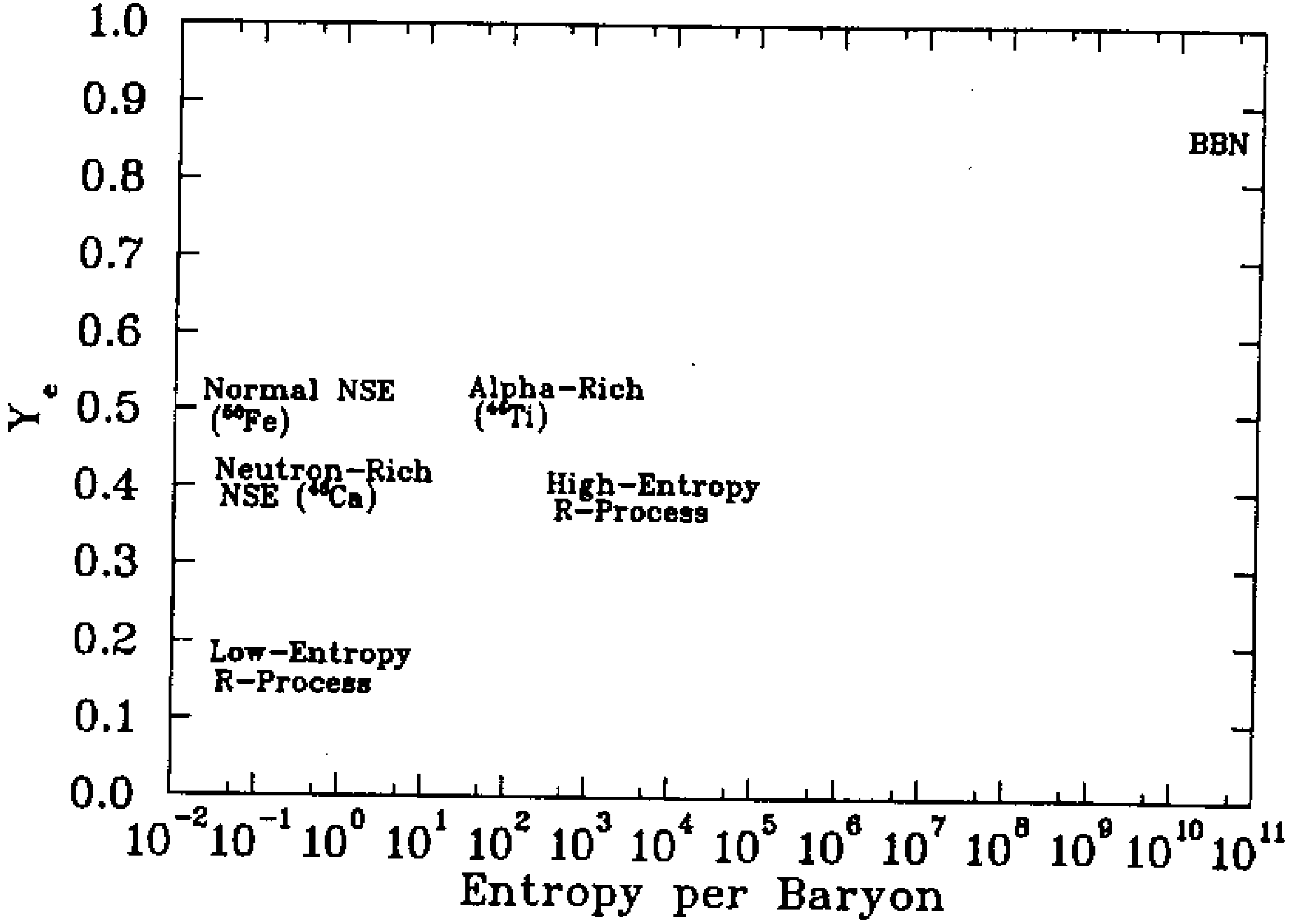

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Y_e
BBN
Normal NSE
(^{56}Fe)
Alpha-Rich
(^{44}Ti)
Neutron-Rich
NSE (^{48}Ca)
High-Entropy
R-Process
Low-Entropy
R-Process
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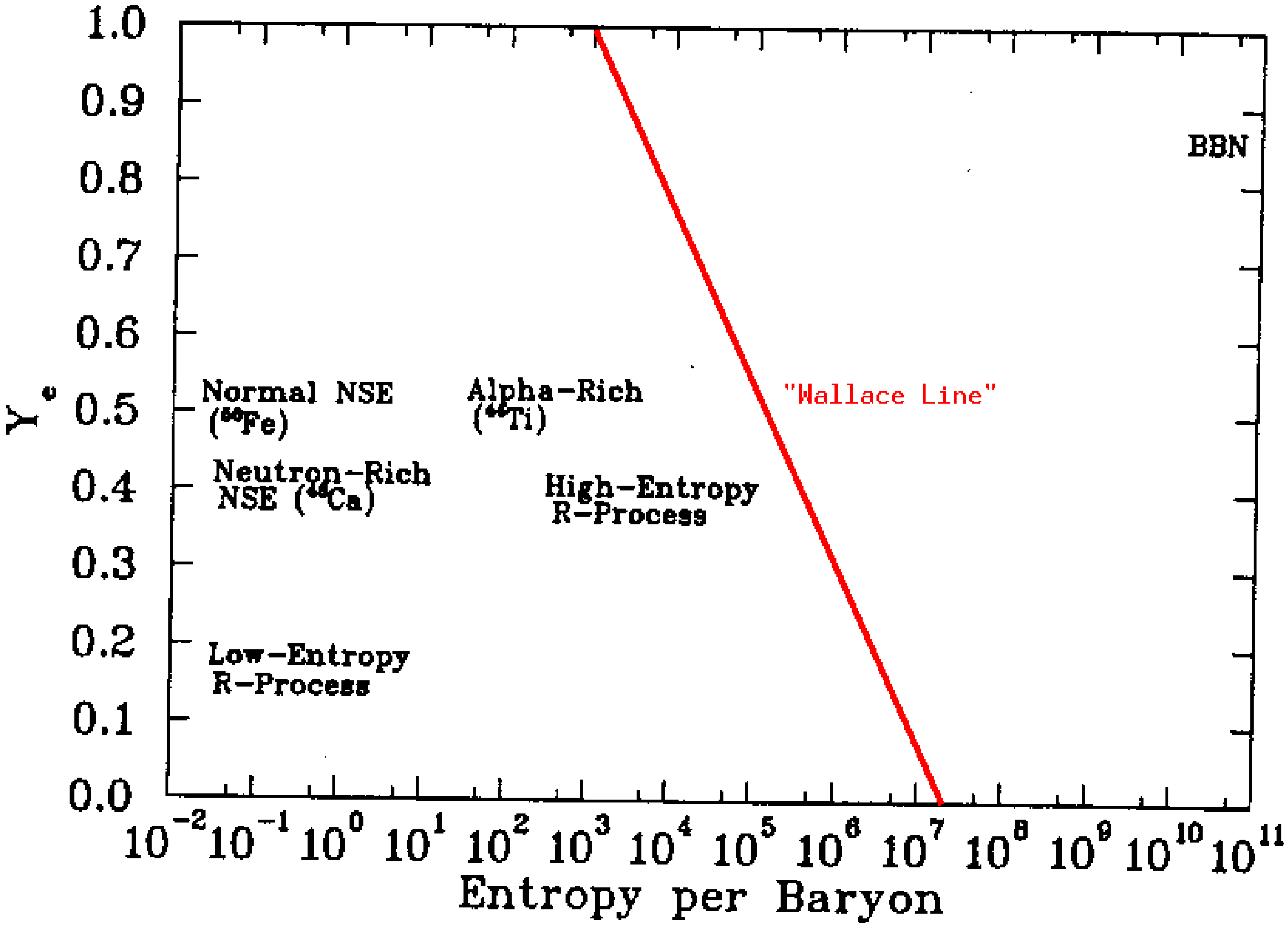
Entropy per Baryon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Normal NSE
(^{56}Fe)
Alpha-Rich
(^{44}Ti)
Neutron-Rich
NSE (^{48}Ca)
High-Entropy
R-Process
Low-Entropy
R-Process
"Wallace Line"
BBN
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11}
Entropy per Baryon

New FITS files to play with

- R process files: rprocess.html
- Big bang calculations: bbang1.fits (s/k=2x10^{10}), bbang2.fits (s/k=2x10^{9}), bbang3.fits (s/k=2x10^{11})

Take home message I:

Steady states, steady states, steady states

Take home message II:

Constrained equilibria,
constrained equilibria, constrained
equilibria