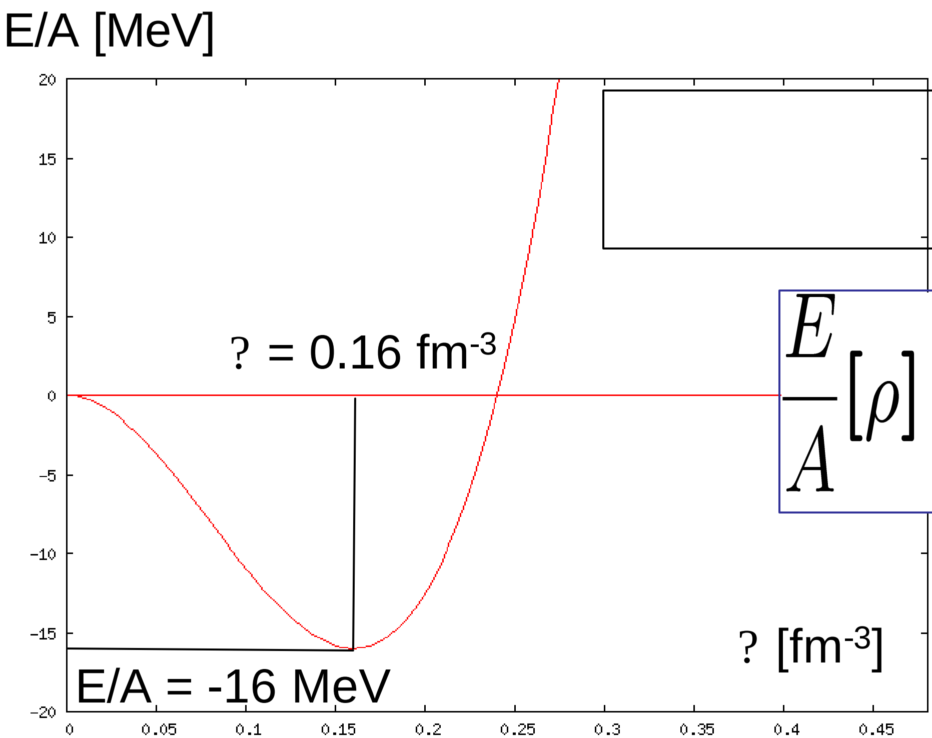
The nuclear matter incompressibility K_8 from different giant resonances and different model analysis

G. Colò

JINA Workshop, Notre Dame University - 14-15 July 2005

The nuclear matter (N = Z and no Coulomb interaction) incompressibility coefficient, K_8 , is a very important physical quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collisions, since it is directly related to the curvature of the nuclear matter (NM) equation of state (EOS), E = E[?].

$$K_\infty = k_f^2 \frac{d^2(E/A)}{dk_f^2}|_{k_{f0}} = 9\rho^2 \frac{d^2(E/A)}{d\rho^2}|_{\rho_0}$$



$$\frac{E}{A}[\rho] = \frac{E}{A}[\rho_0] + \frac{1}{18} K_\infty \left(\frac{\rho - \rho_0}{\rho_0}\right)^2$$

Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

The clearest example of compressional mode is certainly the Isoscalar Giant Monopole Resonance (ISGMR).

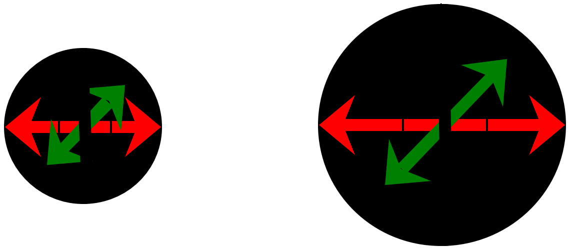
Its first evidences date back to the early 1970s. More data collected in the 1980s already showed that:

- the ISGMR manifests itself systematically in nuclei, and
- it corresponds to a well-defined single peak (~80 $A^{-1/3}$ MeV) in heavy nuclei like Sn or Pb and is more fragmented in lighter systems like Ca or Ni.

Recent data from Texas A&M University have better precision than all previous ones (± 2% on the moments of the strength function distribution).

Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare



There is a subtle, debated relationship between the measurements in finite nuclei and the nuclear matter incompressibility. We can first eliminate the main A-dependence of the ISGMR energy, which is a size-dependence, by defining a finite nucleus incompressibility K_A as in J.P. Blaizot, Phys. Rep. 64 (1980) 171:

$$E_{ISGMR} = \sqrt{\frac{\hbar^2 K_A}{m < r^2 >}}$$

In the past, the "macroscopic approach" has been used.
This means: attempts have been made to fit the coefficients of

$$K_A = K_\infty + K_{surf} A^{-1/3} + K_t d^2 + K_{Coul} Z^2 A^{-4/3} \quad (d=(N-Z)/A)$$

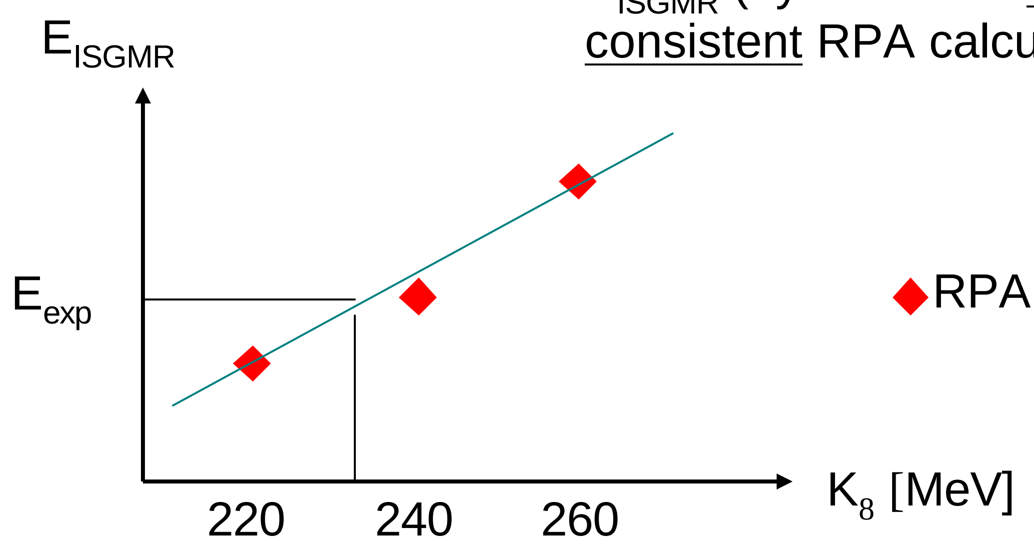
M. Pearson [Phys. Lett. B271 (1991) 12] has shown that the attempt to perform the fit by using the ISGMR data is statistically meaningless and would leave K_∞ undetermined (100-400 MeV). Cf. also: S. Shlomo and D. Youngblood, Phys. Rev. C47 (1993) 529.

Microscopic link E(ISGMR) ? nuclear incompressibility

Nowadays, we give credit to the idea that the link should be provided microscopically. The key concept is the Energy Functional E[?].

IT PROVIDES AT THE SAME TIME

- K_8 in nuclear matter (analytic)
- E_{ISGMR} (by means of self-consistent RPA calculations)



Extracted value of K_8

Skyrme

Gogny

RMF

In the past we did NOT have a fully self-consistent RPA. We still omitted some terms of the residual interaction in our codes.

In order to obtain “proper” results we defined

$$E_{\mathrm{ISGMR}} = \sqrt{\frac{m(1)}{m(-1)}}$$

and we obtained

m(1) from the double-commutator sum rule

m(-1) from contrained HF calculations (dielectric theorem).

$$H' = H + \lambda r^2,$$

$$m(-1) = -\frac{1}{2}\frac{\delta\langle r^2\rangle}{\delta\lambda} = \frac{1}{2}\frac{\delta^2\langle H\rangle}{\delta\lambda^2}$$

NOW WE HAVE IMPLEMENTED A FULLY SELF-CONSISTENT RPA. ALL THE TERMS IN THE RESIDUAL INTERACTION ARE INCLUDED, IN PARTICULAR THE TWO-BODY COULOMB AND TWO-BODY SPIN-ORBIT.

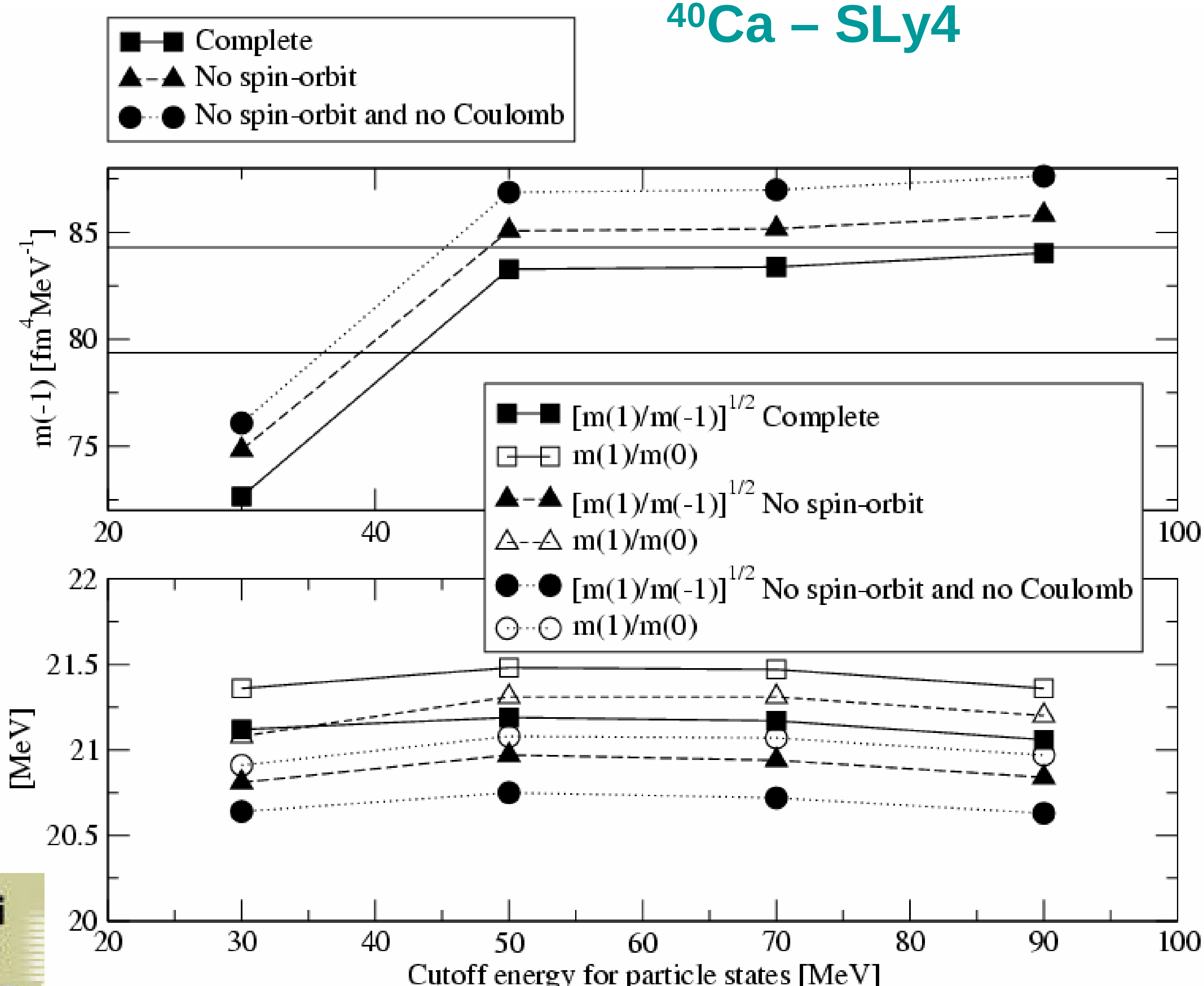
Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

^{40}Ca – SLy4

Test against CHF

Effect on centroid energies

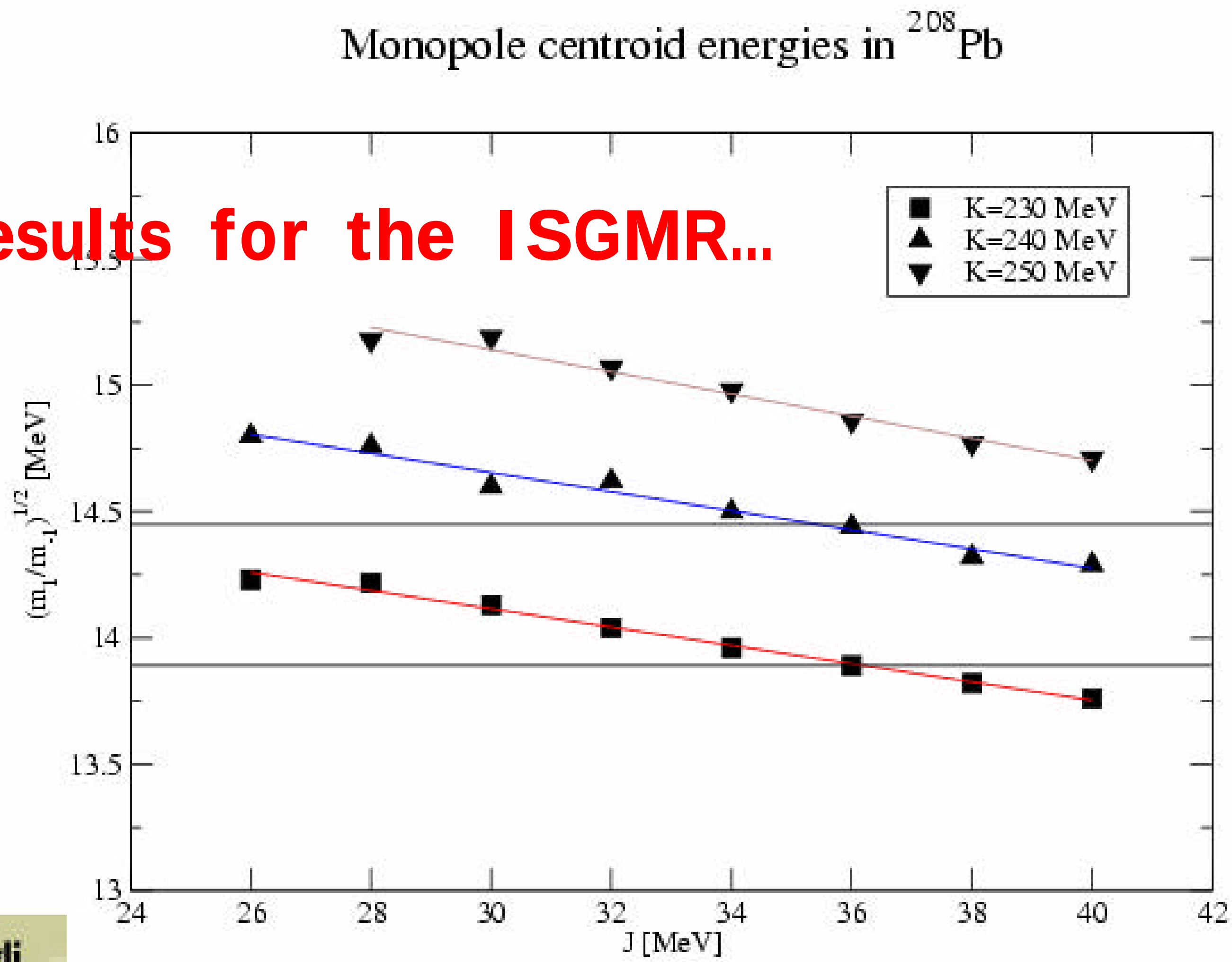


Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

SLy4 protocol, a=1/6

Results for the ISGMR…



K_8 around 230-240 MeV.

Università degli Studi di Milano

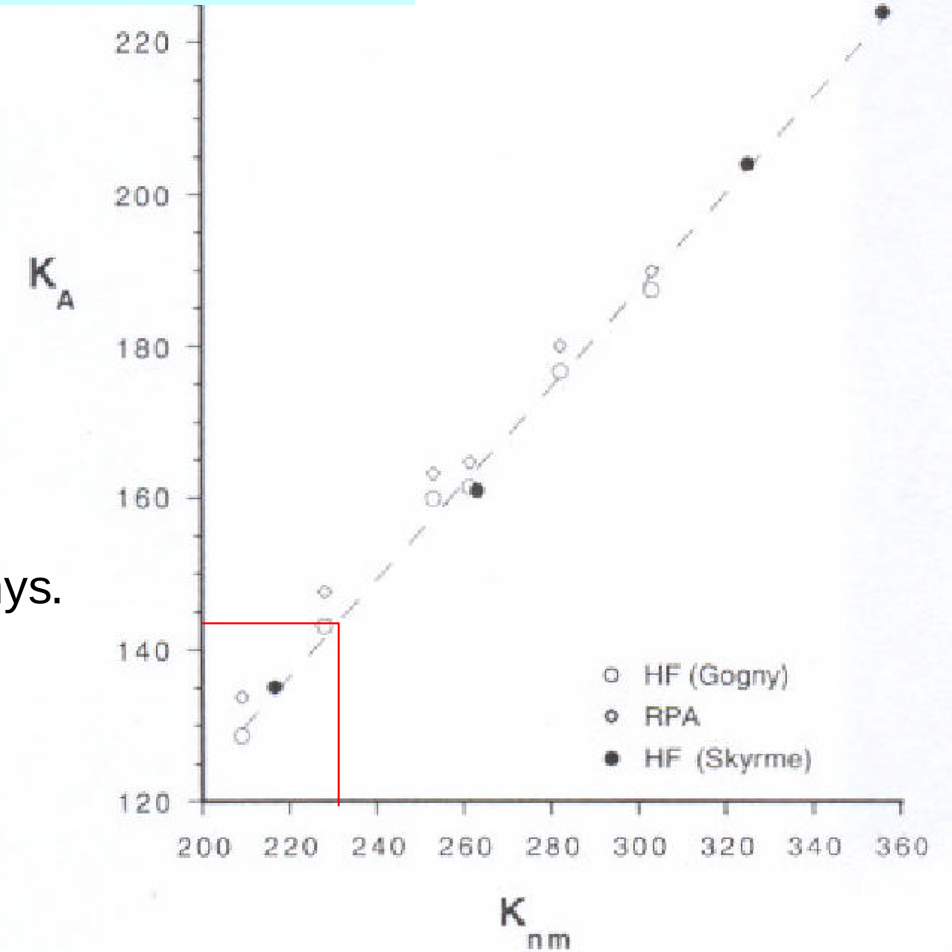
INFN Istituto Nazionale di Fisica Nucleare

CHF and RPA using Gogny

In ^{208}Pb, by inserting the data, one obtains K_8 = 230 MeV.

$$E_{ISGMR} = \sqrt{\frac{\hbar^2 K_A}{m < r^2 >}}$$

J.P. Blaizot *et al.*, Nucl. Phys. A591 (1995) 435.

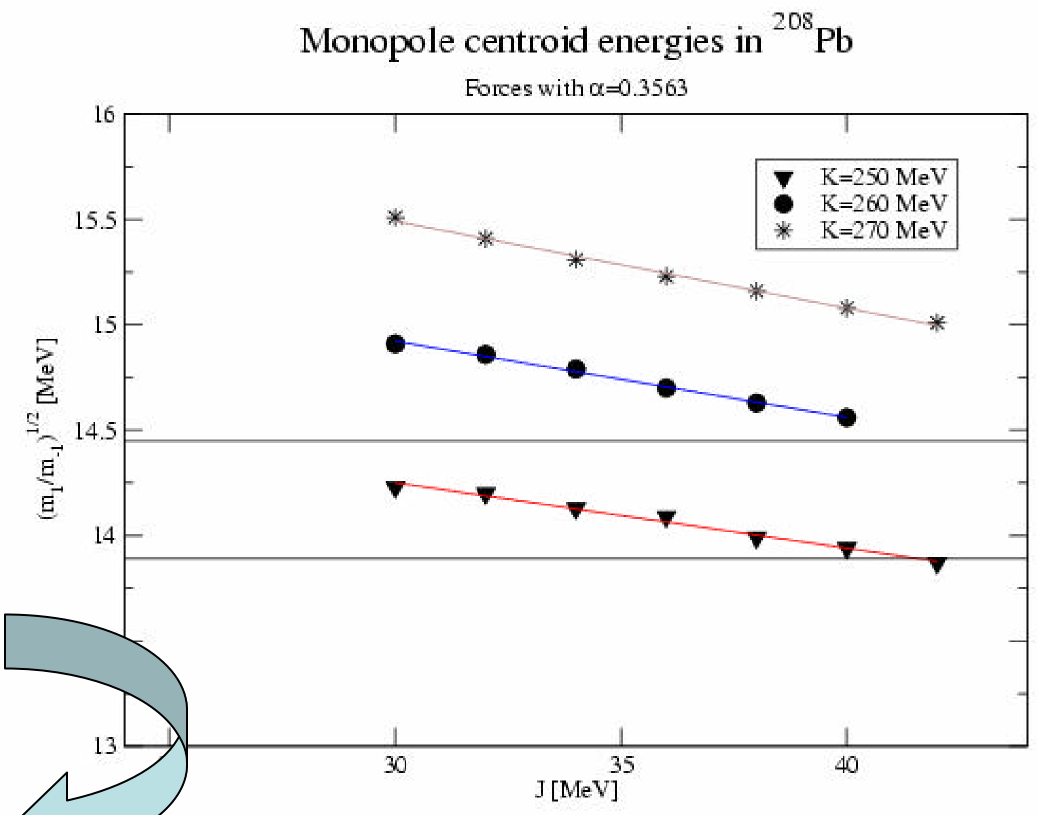


Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

FULL AGREEMENT BETWEEN THE NON-RELATIVISTIC FUNCTIONALS

- a=0.3563,
- neglect of the Coulomb exchange and center-of-mass corrections in the HF mean field.



We have increased the exponent in the density dependence of the Skyrme force

We have also increased the density dependence of the symmetry energy (K_t)

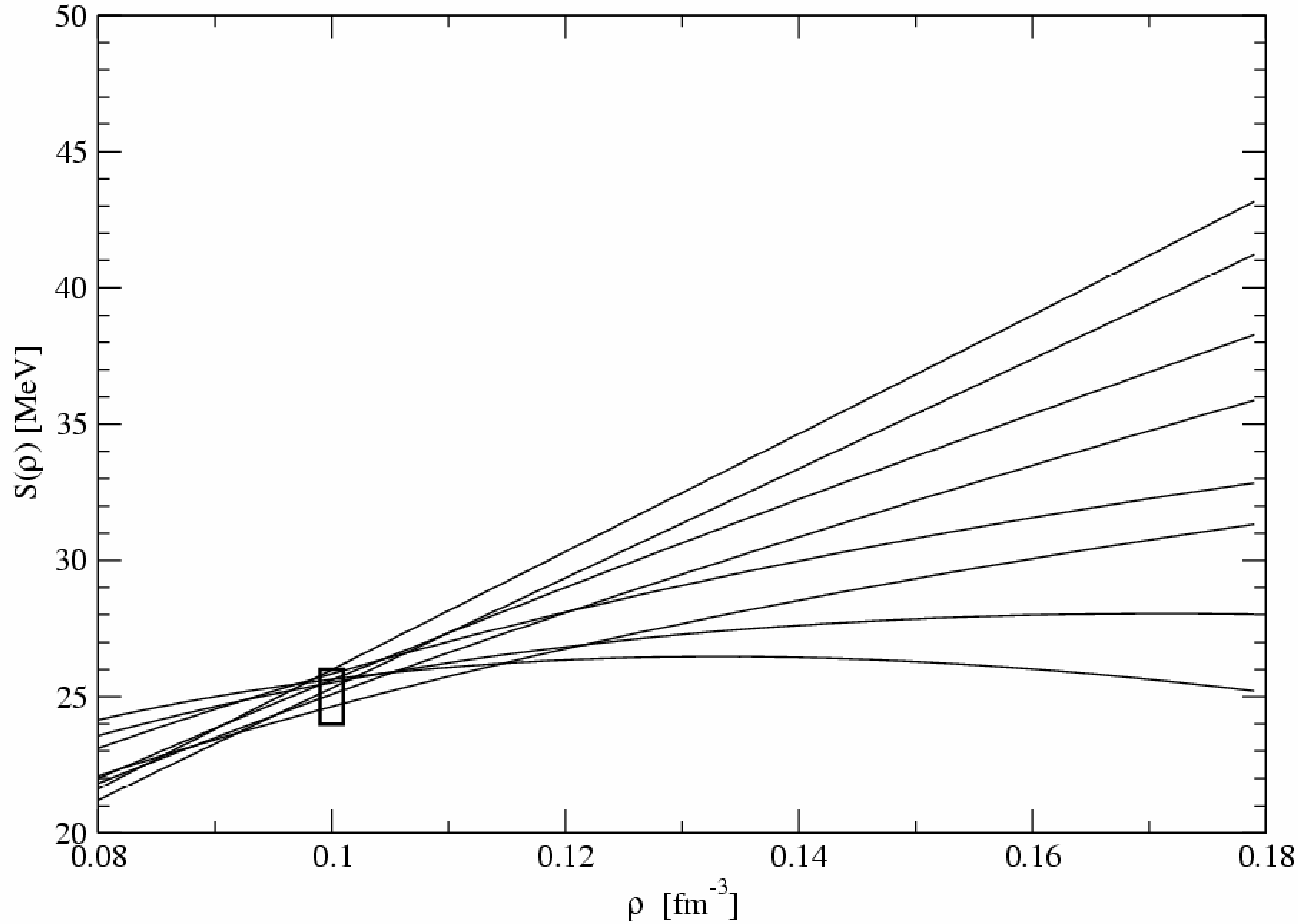
By-product: decrease of m*

K_t from ~ -300 to -380 MeV

The result of B.J. Agrawal *et al.*, is consistent with this plot !

The symmetry energy (E_{sym} or S)

$$\mathcal{E}(\varrho,\varrho_-) = \mathcal{E}_0(\varrho) + \varrho S(\varrho)\left(\frac{\varrho_-}{\varrho}\right)^2$$



All these forces fit finite nuclei: with different values of J and of the derivatives of S

CONCLUSION FROM THE ISGMR

Fully self-consistent calculations of the ISGMR using Skyrme forces lead to $K_8 \sim 230\text{-}240$ MeV.

Relativistic mean field (RMF) plus RPA: lower limit for K_8 equal to 250 MeV.

It is possible to build *bona fide* Skyrme forces so that the incompressibility is close to the relativistic value.

? $K_8 = 240 \pm 10$ MeV.

To reduce this uncertainity one should fix the density dependence of the symmetry energy.

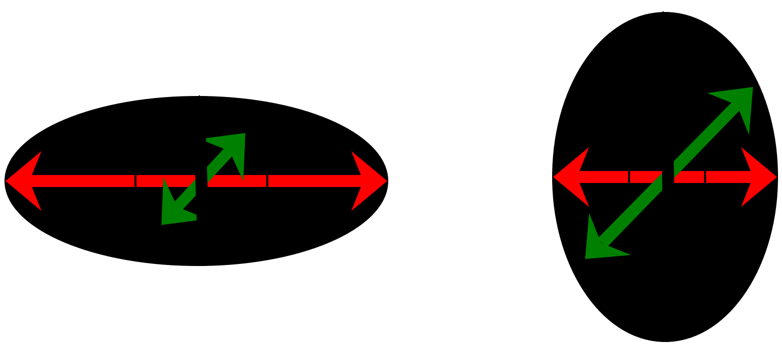
The Isoscalar Giant Dipole Resonance (ISGDR)

It is a compressional wave which travels along a given direction (say, the z-axis).

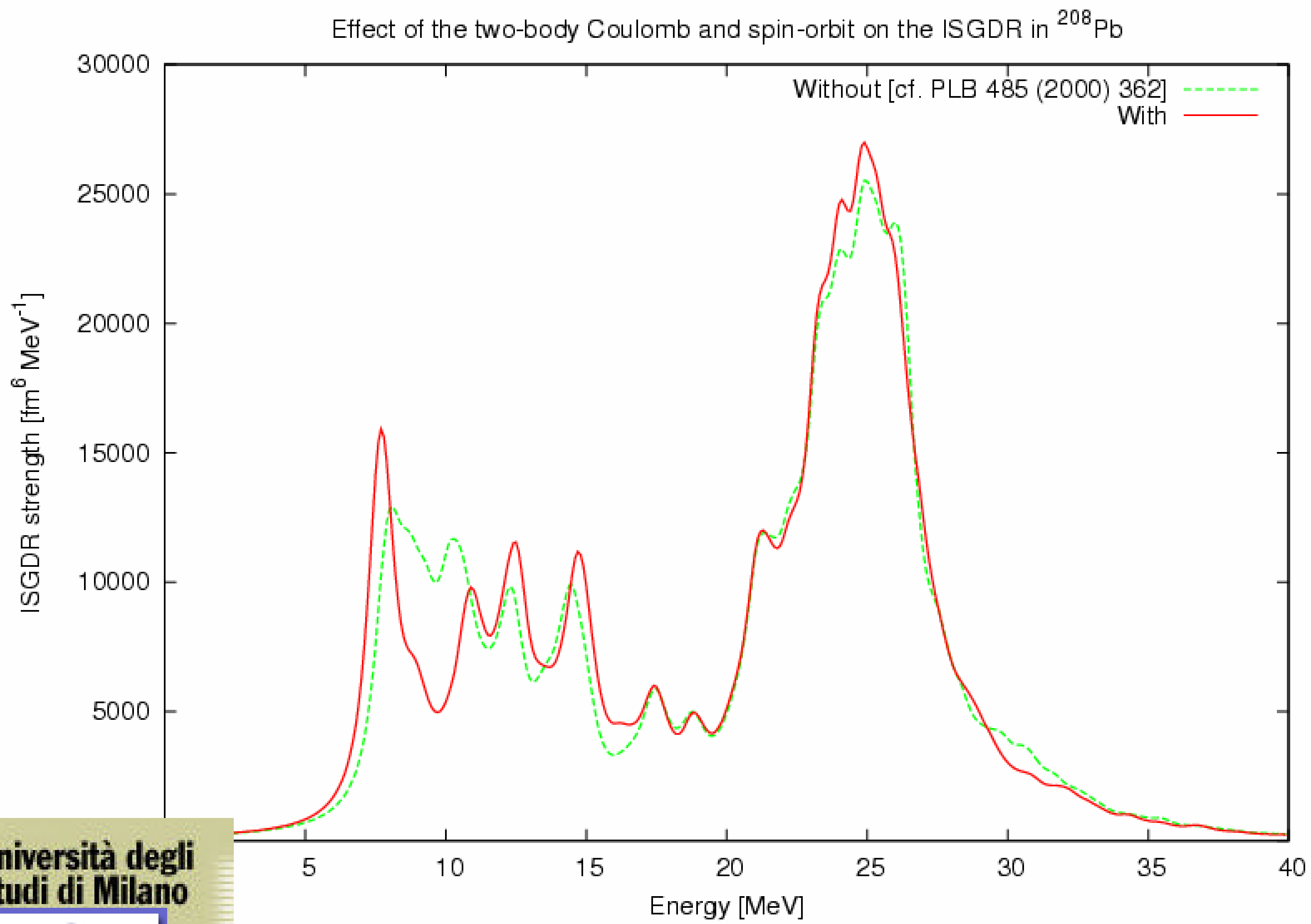
In principle, it provides an alternative way to extract K_8 .

Problem: presence of non-collective strength.

Exp.: disentangle various multipoles.
Theory: spurious state mixing (lying at E?0 due to approximations)



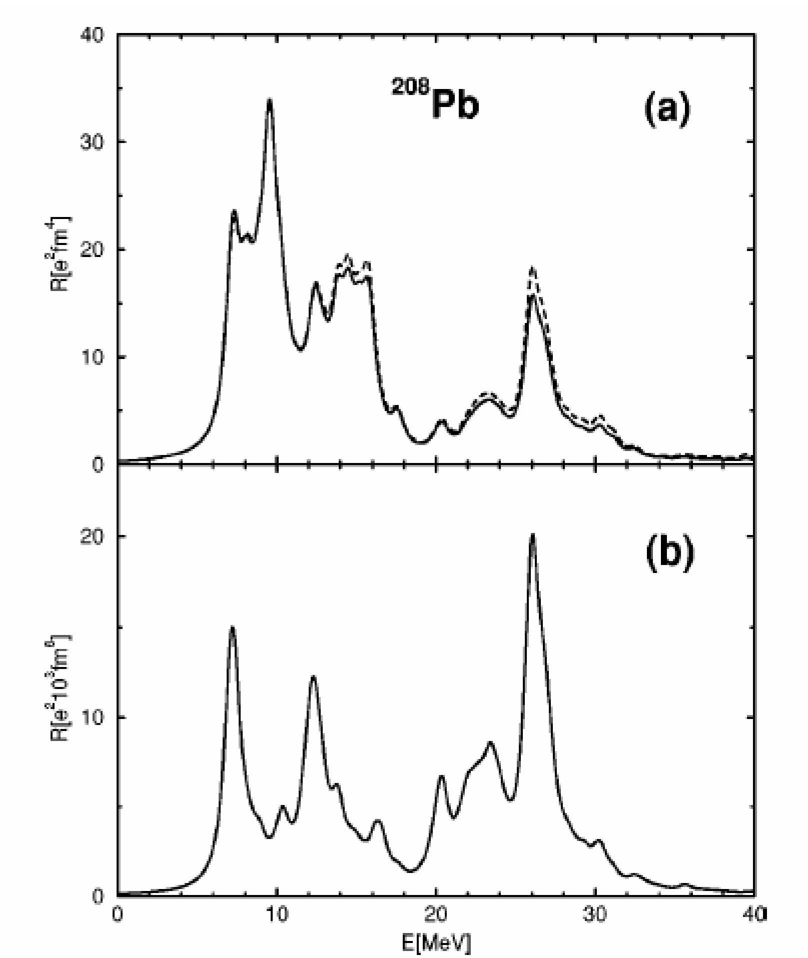
$$\hat{D} = r^3 Y_{1m}$$



Effect of the two-body Coulomb and spin-orbit on the ISGDR in 208Pb
Without [cf. PLB 485 (2000) 362]
With
ISGDR strength [fm6 MeV-1]
30000
25000
20000
15000
10000
5000
5
10
15
20
25
30
35
40
Energy [MeV]

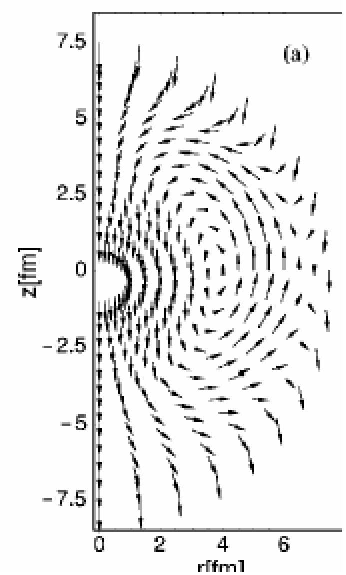
D. Vretenar et al., Phys. Rev. C65 (2002) 021301:

The low-lying dipole strength is a toroidal mode



(b) r^3Y_{1m}

(a) $\tilde{N}\times(r\times\tilde{N})r^3Y_{1m}$ *(vector operator coupled to the current)*

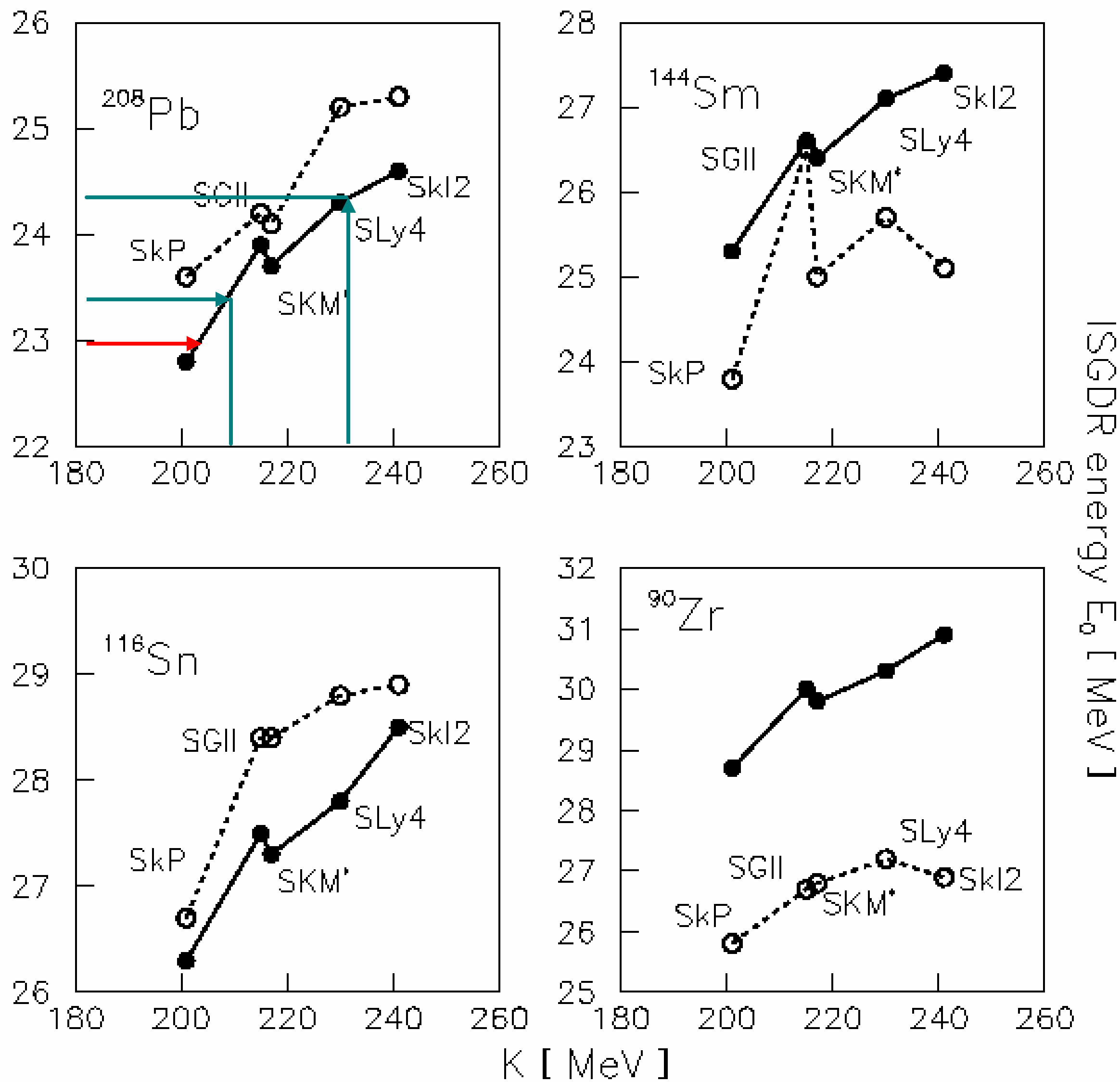


^{208}Pb:

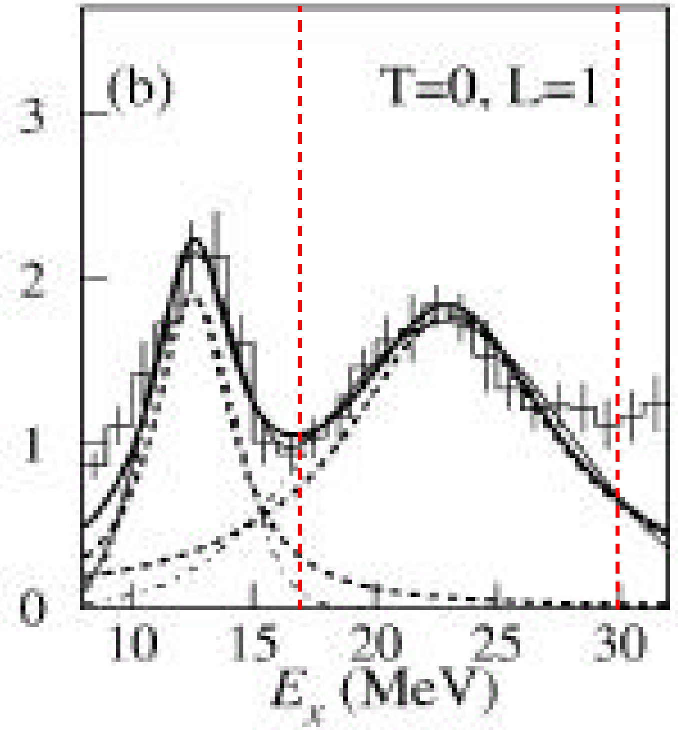
Uchida *et al.* 23.0 ± 0.3

Youngblood *et al.* 22.2 ± 0.3

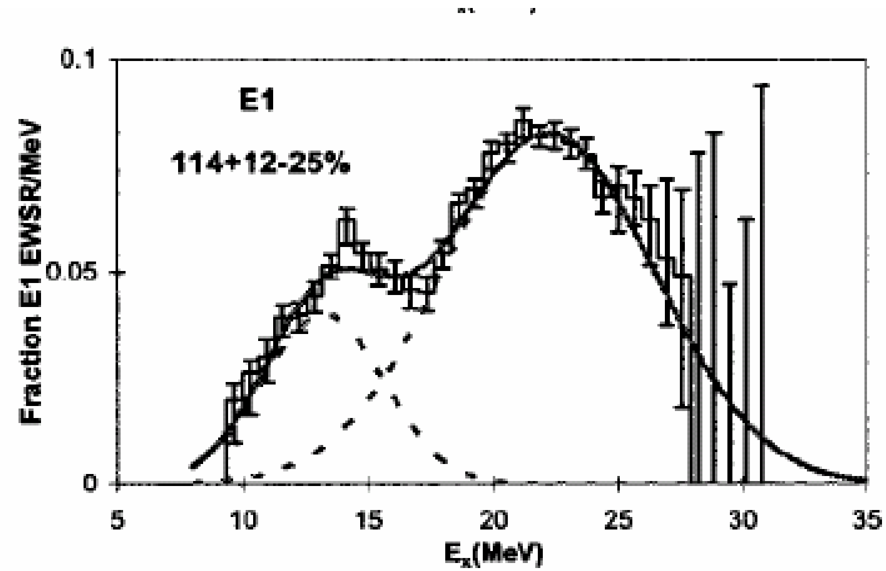
G.Colò, N. Van Giai, P.F. Bortignon, M.R. Quaglia



Why do we seem to extract a lower value for K_8 in this case (compared to the ISGMR) ?



M. Uchida *et al*., PLB 557 (2003) 12

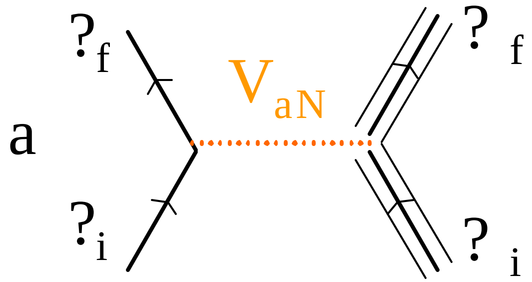


D.H. Youngblood et al, PRC 69 (2004) 034315

Hard to fix the amount of strength at high energy ?

Form factors are independent on E: this approximation is more doubtful if it is used to determine the strength on a broad interval

Hadron excitation of giant resonances



$$\frac{d\sigma^{DWBA}}{d\Omega} = (\frac{\mu}{2\pi\hbar^2})^2 \frac{k_f}{k_i}|T_{fi}|^2$$

$$T_{fi} = \langle\chi_f^{(-)}\Psi_f|V|\chi_i^{(+)}\Psi_i\rangle = \langle\chi_f^{(-)}|U_{tr}|\chi_i^{(+)}\rangle, \;\; U_{tr} \sim \int \Psi_f^* V \Psi_i.$$

Theorists: calculate transition strength S(E) within HF-RPA using a simple scattering operator $F \sim r^L Y_{LM}$:

$$S(E) = \sum_n |\langle\Psi_0|F|\Psi_n\rangle|^2 \delta(E - E_n)$$

Experimentalists: calculate cross sections within Distorted Wave Born Approximation (DWBA):

$$\frac{d\sigma(E)}{d\Omega} = (\frac{\mu}{2\pi\hbar^2})^2 \frac{k_f}{k_i}|\langle\chi_f^{(-)}|U_{tr}|\chi_i^{(+)}\rangle|^2$$

$$U_{tr} \sim \frac{\partial U g.s.}{\partial r}$$

CONCLUSION FROM THE ISGDR

The discrepancy between the ISGMR and the ISGDR seems more relevant than that between the various extractions of K_8 from the ISGMR.

Warnings:

- are we allowed to rely on the ISGDR as if it were a "single mode" ?
- what are the real uncertainities about the dipole strength at high-E ?

Acknowledgment

Collaboration on K_8 from the ISGMR:

J. Meyer and K. Bennaceur (IPN-Lyon), N. Van Giai (IPN-Orsay), P. Bonche (Saclay).

Inclusion of the two-body spin-orbit:

S. Fracasso (Milano).

What is the error on the determination of K_8 ?

$$E \sim \sqrt{K_\infty} \rightarrow \frac{\delta K_\infty}{K_\infty} = 2\frac{\delta E}{E}.$$

Rule of thumb: if we use the ^{208}Pb monopole energy, ± 150 keV of uncertainity on this quantity gives about ± 5 MeV uncertainity on K_∞.

The experimental measurement gives 14.17 $\pm$ 0.28 MeV [D. Youngblood, H.L. Clark and Y.-W. Lui, Phys. Rev. Lett. 82, 691 (1999)].

$$\rightarrow \delta K^{\mathrm{exp.}} \sim \pm 10 \text{ MeV}.$$

Theoretically, the best way to extract the centroid energy is by means of CHF. Errors on m_{-1} are again of the order of $\pm 3\%$.

$$E = \sqrt{\frac{m_1}{m_{-1}}} \rightarrow \delta K^{\mathrm{th.}} \sim \pm 7 \text{ MeV}.$$

The two errors are independent and should be added quadratically

…so how serious are the discrepancies ?

Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare

How to experimentally discriminate between models ?

$E \sim A^{-1/3}$

$dE/E = dA/3A$

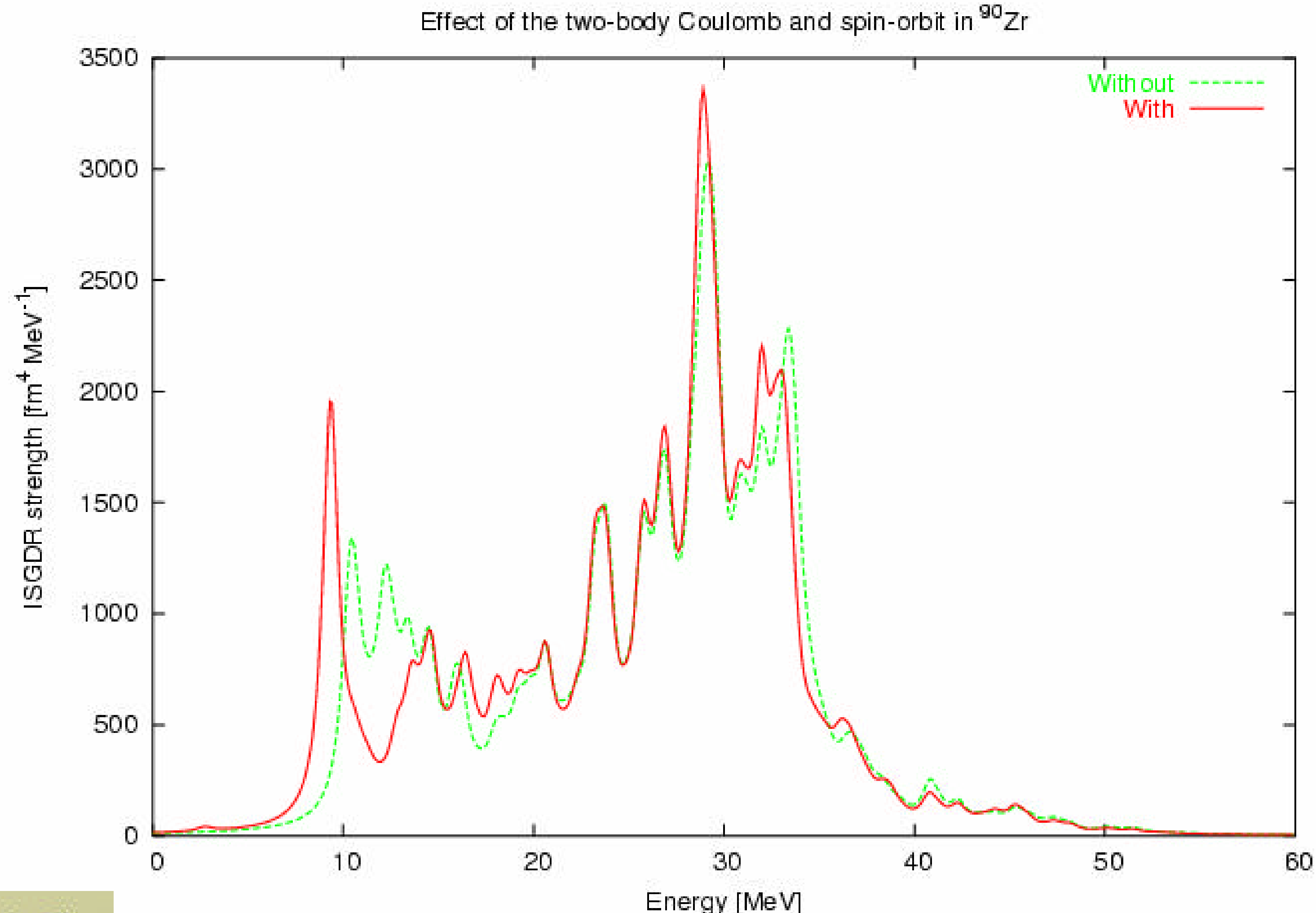
Even if we take a long isotopic chain of stable, spherical isotopes:

Sn ?   dE/E is of the order of 3%, that is, 0.45 MeV (˜ $2s_{exp}$).

Calculations should be made at the same level of accuracy.

Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare



Università degli Studi di Milano

INFN Istituto Nazionale di Fisica Nucleare