Probing the EOS of Neutron-Rich Matter with Heavy-Ion Reactions

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- **EOS of neutron-rich matter**
- **Terrestrial lab probes**

  Two examples: Isospin diffusion as a probe at low densities

  Pion production as a probe at high densities
- **Summary**

EOS of Asymmetric Nuclear Mater

At density ρ and neutron-excess $\delta \equiv (\rho_n - \rho_p)/\rho$

$$e(\rho, \delta) \equiv E/A = T_F(\rho, \delta) + V_0(\rho) + \delta^2 V_2(\rho)$$

Empirical parabolic law:

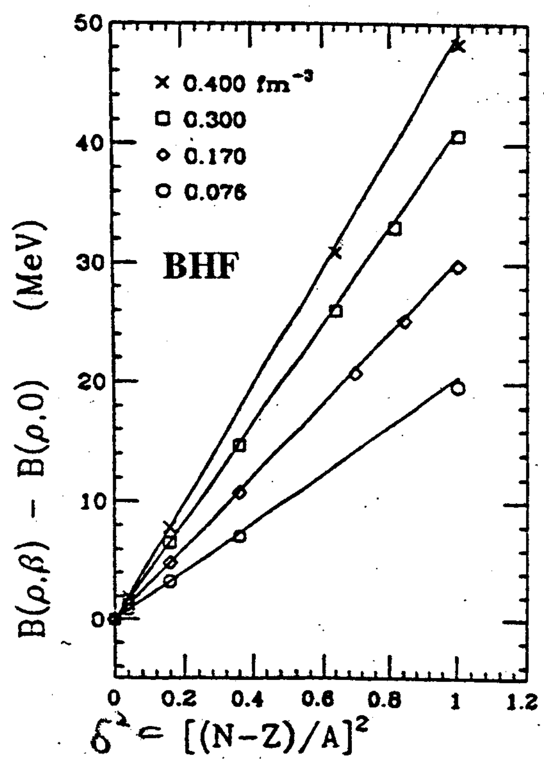
$$\underline{e(\rho, \delta) = e(\rho, 0) + e_{sym}(\rho)\delta^2 + \mathcal{O}(\delta^4)}$$

$\Longrightarrow e_{sym}(\rho) = e(\rho, \text{pure neutron matter}) - e(\rho, \text{symmetric nuclear matter})$

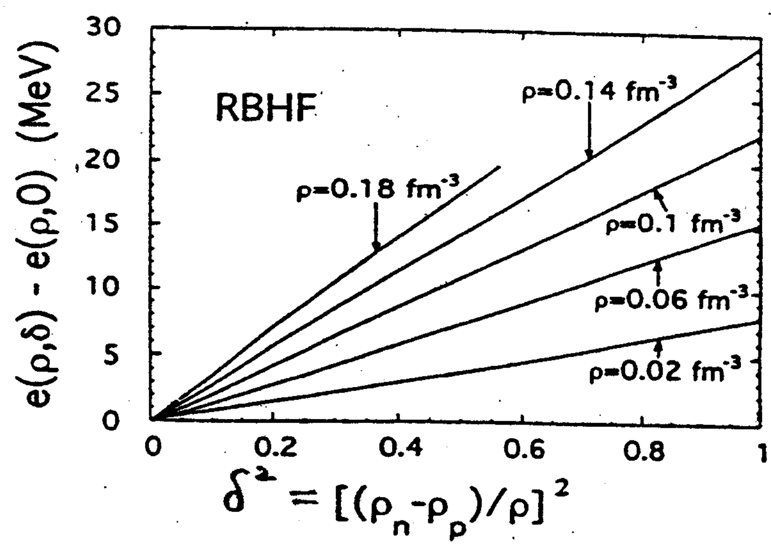
$\equiv \frac{1}{2}\left(\frac{\partial^2 e}{\partial \delta^2}\right)_\rho$

Theoretical evidence: essentially all many-body calculations

Two examples:



I. Bombaci and U. Lombardo, PRC **44** (1991) 1892.



H. Huber, F. Weber and M.K. Weigel, PLB **317** (1993) 485.

The latest microscopic many-body theory predictions

On the nuclear symmetry energy and the neutron skin in neutron rich nuclei

A.E.L. Dieperink,[1] Y. Dewulf,[2] D. Van Neck,[2] M. Waroquier,[2] and V. Rodin[3]

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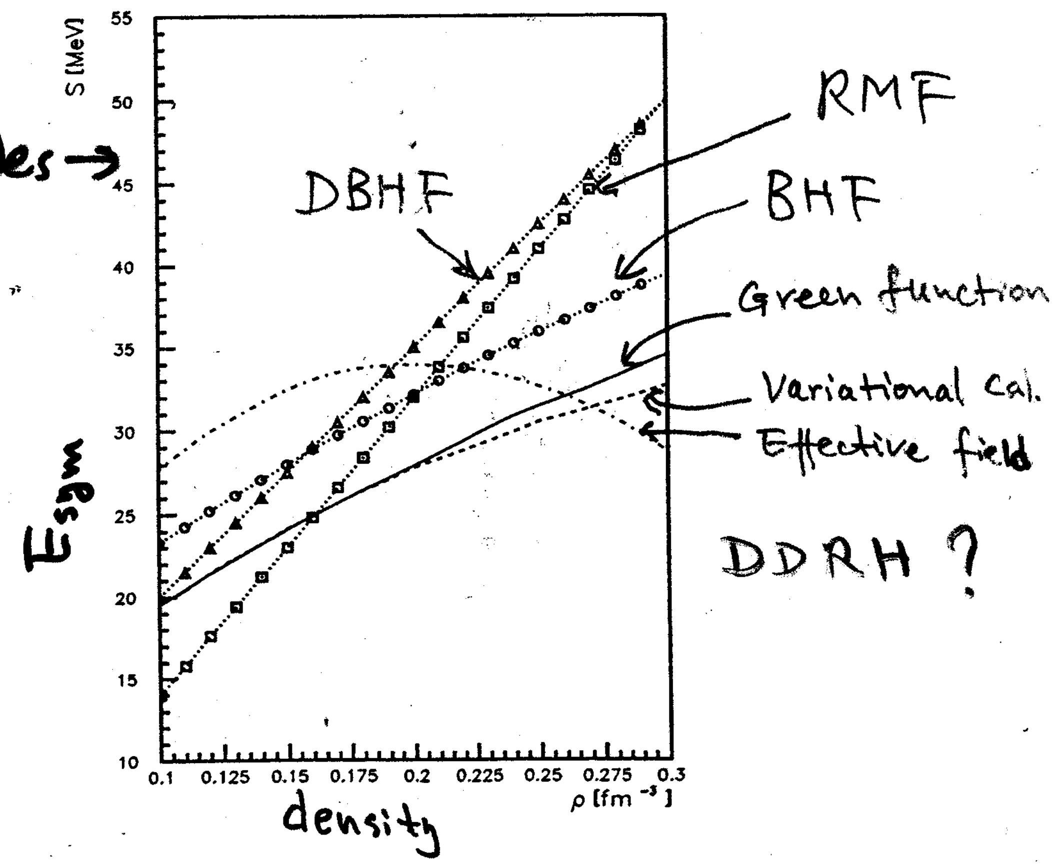
arXiv:nucl-th/0312031 v1 10 Dec 2003

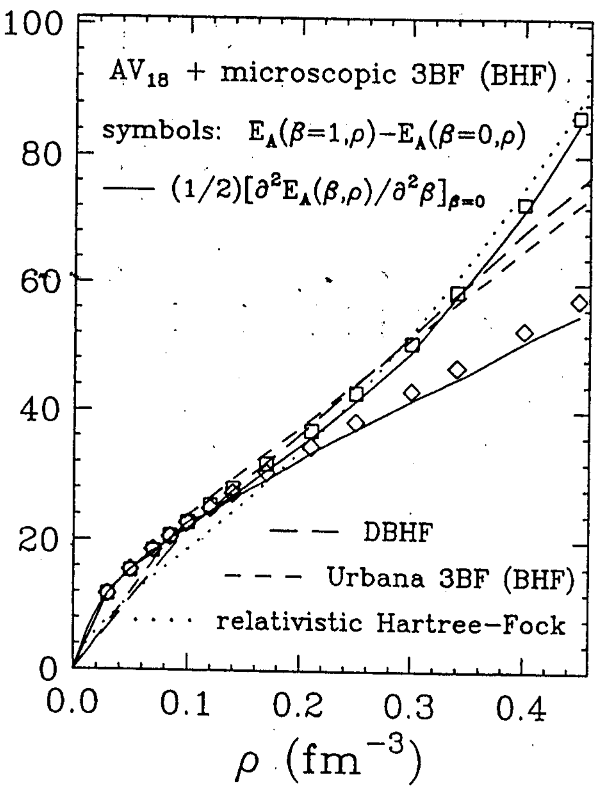
Several examples →

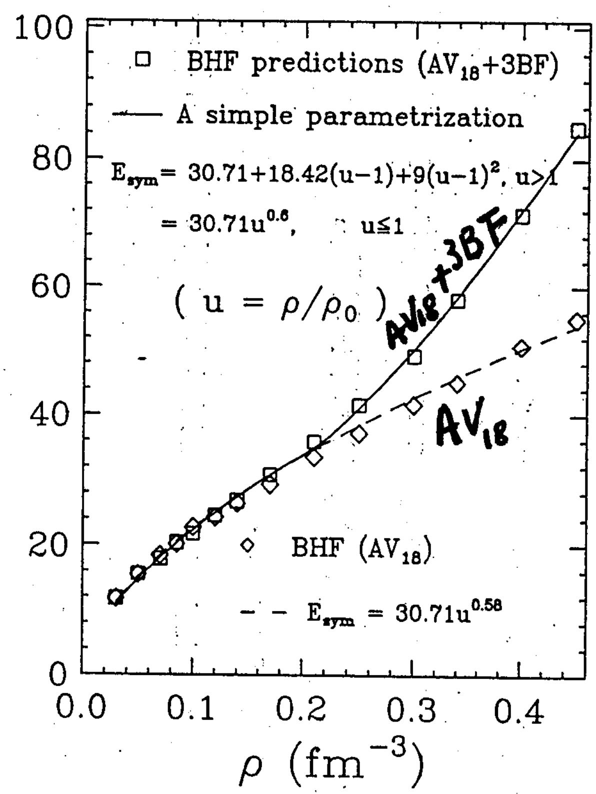


IG. 3: Overview of several theoretical predictions for the symmetry energy S: Brueckner-Hartree-Fock (continuous choice) ith Reid93 potential (circles), self-consistent Green function theory with Reid93 potential (full line), variational calculation om [17] with Argonne Av14 potential (dashed line), Dirac-Brueckner-Hartree-Fock calculation from [25] (triangles), relativistic iean-field model from [28] (squares), effective field theory from [30] (dash-dotted line).

Wei Zuo and Umberto Lombardo

extended BHF





Compressibility, effective mass and density dependence in Skyrme forces

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arXiv:nucl-th/0309012 v1 5 Sep 2003

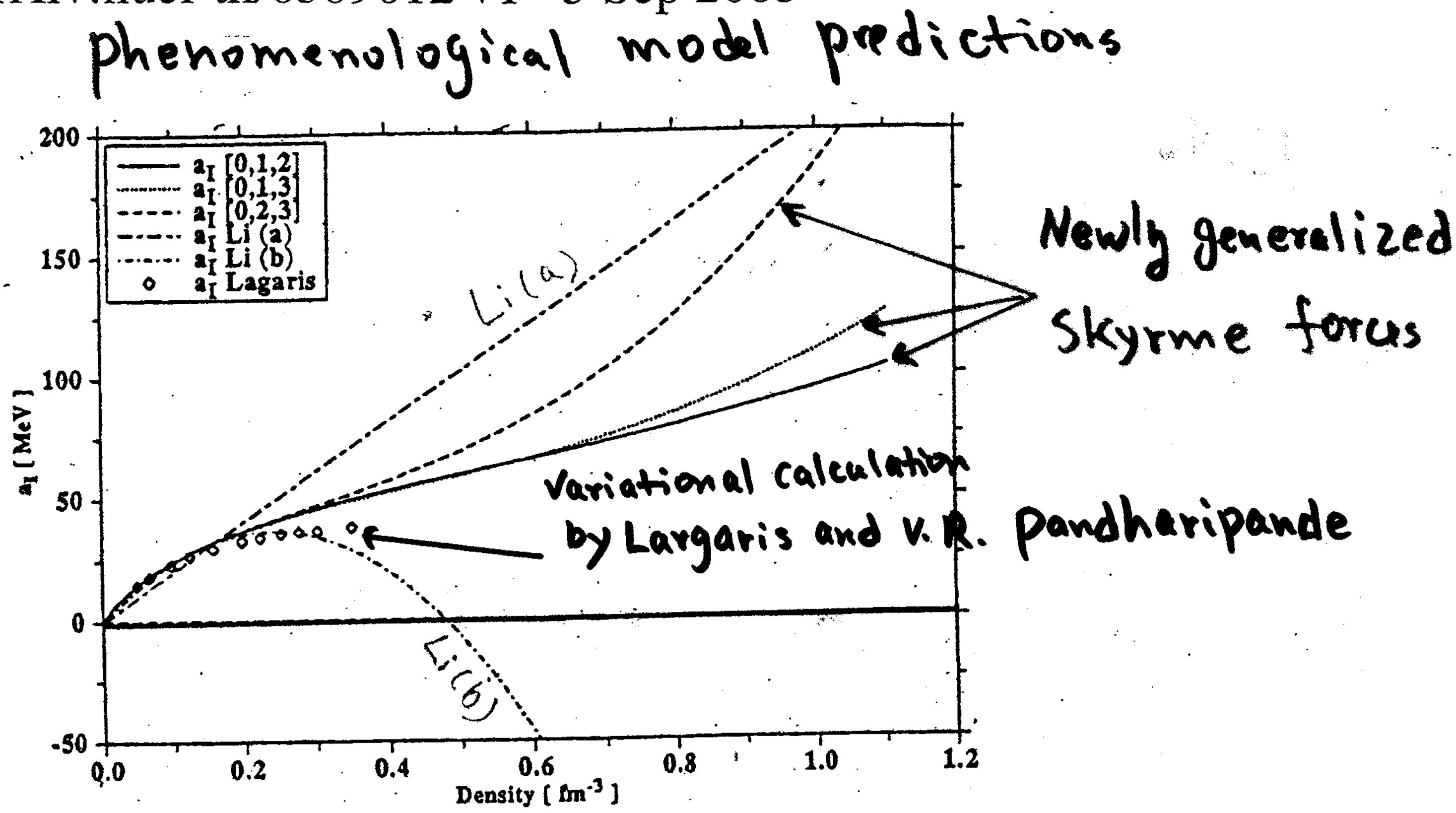


Figure 3. Symmetry energy in infinite matter as a function of the density. Li(a) and (b): phenomenological parametrizations by Bao-An Li [16], see text. Open diamonds: variational calculation of Lagaris *et al.* [17]. Inset: symmetry energy in the vicinity of equilibrium density of symmetric nuclear matter.

Promising Probes of the $E_{sym}(\rho)$ in Nuclear Reactions

(an incomplete list !)

(a) At low densities

- Sizes of n-skins of unstable nuclei from total reaction xsections
- **Parity violating electron scattering studies of the n-skin in ^{208}Pb**
- Multiplicity and spectrum of pre-equilibrium neutrons/protons
- **Isospin fractionation and isoscaling in multifragmentation**
- Isospin diffusion
- **Proton differential elliptic flow at high transverse momenta**
- Isospin dependence of transverse flow and balance energy
- **Neutron-proton correlation functions at low relative momenta**
- $t/^3He$ ratio
- **Deflection function**

(b) Towards high densities

- π^-/π^+ ratio in heavy-ion collisions induced by high-energy radioactive beams
- **Neutron-proton differential flow in heavy-ion collisions**
- Precursor of *isospin separation instability* in the excitation function of nuclear collective flow
- **Asymmetric nuclear matter induced unique ρ^0-ω mixing and its effects on dilepton and photon production**

*** **The most sensitive observables require simultaneous measurement of neutrons and protons, i.e., neutron-proton differential quantities.**

Status on the density-, momentum- and isospin-dependent single nucleon potential $U_\tau(k,\rho,\delta)$

(1) Microscopic many-body theories
A lot of work, $U_\tau(k,\rho,\delta)$ is normally plotted as a function of density at few ρ and δ in all published papers.

No analytical expression for $U_\tau(k,\rho,\delta)$ is given in any of the publications, the predictions are thus hard to be used in heavy-ion studies.

(2) Two phenomenological expressions for $U_\tau(k,\rho,\delta)$ exist.
Both are derived using the HF with Gogny forces as a guide

Both contains parameters to mimic microscopic many-body predictions on the $E_{sym}(\rho)$ and $M^*\tau$ (ρ,δ)

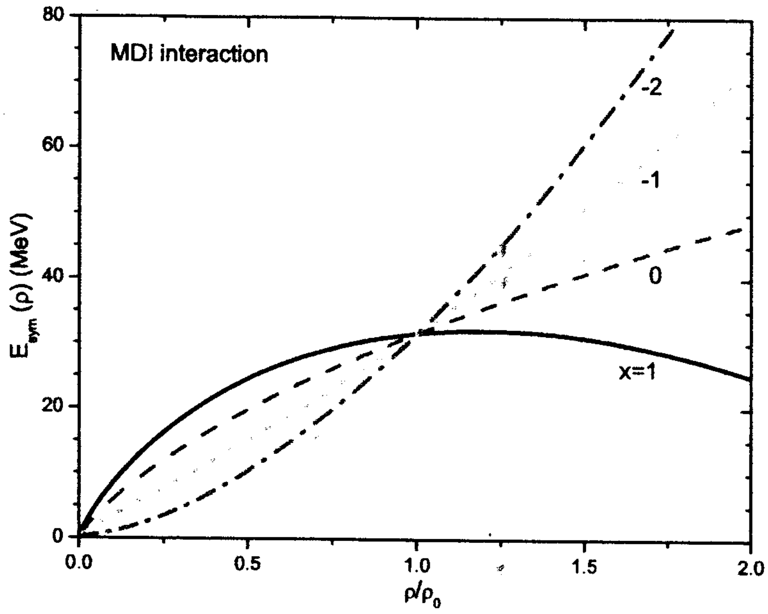
(A) Ignazio Bombaci
Two parameters (x_o and x_3) to mimic the $E_{sym}(\rho)$

Three parameters (z_1, x_o and x_3) to mimic the nucleon effective mass M^*n (ρ,δ) and M^*p (ρ,δ)

(B) Subal Das Gupta et al.
Three parameters (x, A_{like} and A_{unlike}) to mimic the $E_{sym}(\rho)$

M^*n (ρ,δ) and M^*p (ρ,δ) are fixed, they are independent of the parameters x, A_{like} and A_{unlike} since they are all in the density-dependent parts of the potential.

*** Akira Ono **uses one parameter x in the Gogny 2-body potential to vary the $E_{sym}(\rho)$ in AMD calculations**



Momentum and isospin dependent nucleon potential used in the IBUU transport model

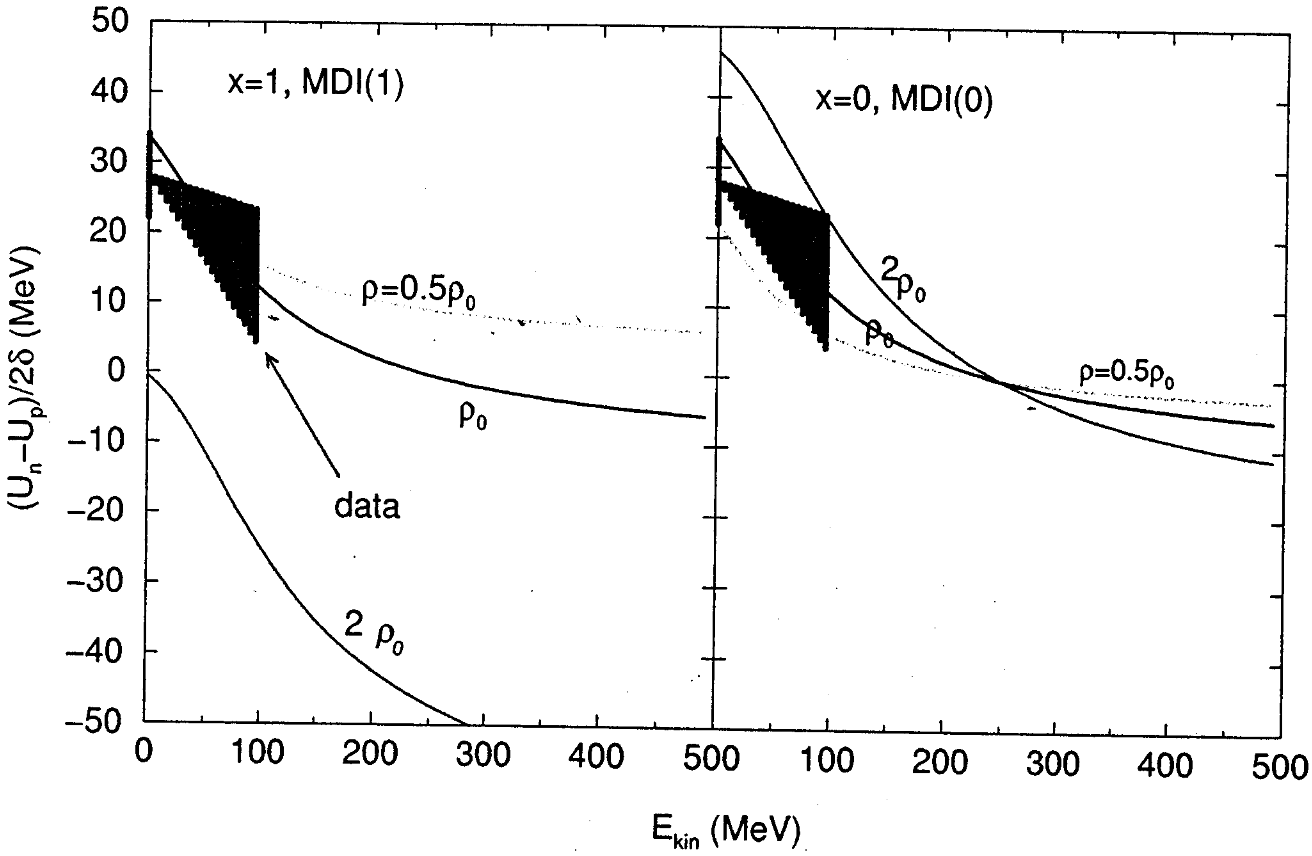
$$U(\rho,\delta,\vec{p},\tau) = A_u\frac{\rho_{\tau'}}{\rho_0} + A_l\frac{\rho_\tau}{\rho_0} + B(\frac{\rho}{\rho_0})^\sigma(1-x\delta^2) - 8\tau x\frac{B}{\sigma+1}\frac{\rho^{\sigma-1}}{\rho_0^\sigma}\delta\rho_{\tau'}$$
$$+\frac{2C_{\tau,\tau}}{\rho_0}\int d^3p'\frac{f_\tau(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2} + \frac{2C_{\tau,\tau'}}{\rho_0}\int d^3p'\frac{f_{\tau'}(\vec{r},\vec{p}')}{1+(\vec{p}-\vec{p}')^2/\Lambda^2}$$

$C_{unlike} = -103.4$ MeV, $C_{like} = -11.7$ MeV, $K_0 = 211$ MeV, $\tau = \pm 1/2$ for n/p

x is used to mimic different density-dependent symmetry energy predicted by various many-body theories, or to vary K_{asy}.

$$A_l(x) = -120.57 + x\frac{2B}{\sigma+1}, \quad A_u(x) = -95.98 - x\frac{2B}{\sigma+1}. \tag{1}$$

- **The formalism within Hartree-Fock approach using a Gogny force**
  C.B. Das, S. Das Gupta, C. Gale and B.A. Li, PRC 67, 034611 (2003).
- **Applications in heavy-ion collisions induced by neutron-rich nuclei:**
  (1) B. A. Li, C.B. Das, Subal Das Gupta and C. Gale, PRC **69**, 034614 (2004).
  (2) B. A. Li, C.B. Das, Subal Das Gupta and C. Gale, NPA 735, 563 (2004).



x=1, MDI(1)
x=0, MDI(0)
ρ=0.5ρ0
ρ0
2 ρ0
data
2ρ0
(Un−Up)/2δ (MeV)
Ekin (MeV)
50
40
30
20
10
0
−10
−20
−30
−40
−50
0
100
200
300
400
500

Evidence of the momentum-dependence of symmetry potential

from the difference of optical potentials for neutron and proton scatterings

(1) Neutron and proton scatterings on the same nucleus at the same energy

$$U_n - U_p = 2\delta\, U_{sym} + U_{Coulomb\ correction}$$

(2) Neutron or proton scatterings on a sequence of isotopes

$$U_p(\text{isotope 1}) - U_p(\text{isotope 2}) = 2(\delta_1 - \delta_2)\, U_{sym}$$

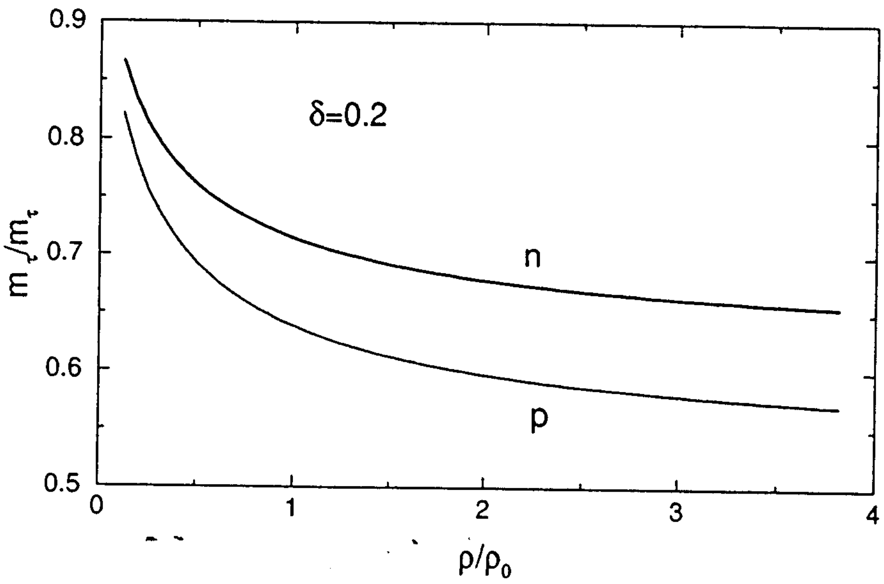
The finding: $U_{sym} = V_1 - \varepsilon_R E$, $V_1 \approx 28 \pm 6$ MeV, $\varepsilon_R \approx 0.1 - 0.2$

P.E. Hodgson, the Nucleon Optical Model, World Scientific, 1994

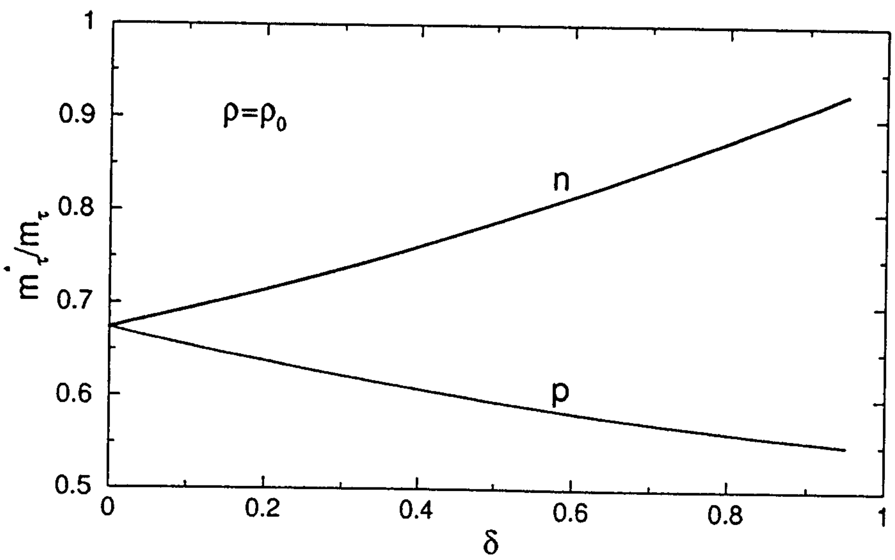
2.4.4. Energy dependence of isospin potential. **Some global analyses of neutron elastic scattering have been made that included a linear energy dependence of the asymmetry term. Values of the coefficients ϵ_R and ϵ_I in the expressions $(V_1 - \epsilon_R E)$ and $(W_1 - \epsilon_I E)$ are given in table 12.**

Table 12. Values of coefficient of linear energy dependence of isospin potential.

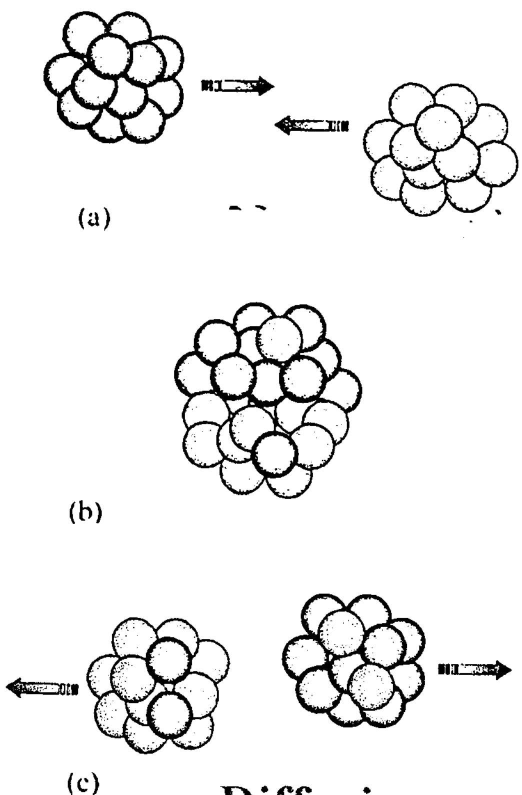
Reaction	E	Nucleus	ϵ_R	ϵ_I	Reference
Theoretical analyses			0.1	—	Rook (1973)
			0.17	—	Dabrowski and Haensel (1974)
			0.1	—	Jeukenne *et al* (1977b)
(n, n)	7–26	^{40}Ca, ^{208}Pb	0.19	—	Rapaport *et al* (1979a, b)
(p, n)	25–45	^{48}Ca–^{208}Pb	0.18	—	Patterson *et al* (1976)
(n, n), p, p)	10–50	^{208}Pb	0.183 ± 0.008	—	De Vito *et al* (1981)
(n, n), (p, p)	10–40	^{208}Pb	—	0.178 ± 0.052	De Leo and Micheletti (1981)
(p, p)	100	^{58}Ni–^{208}Pb	0.12 ± 0.06	—	Kwiatkowski and Wall (1978)



$$\frac{m^*_\tau}{m_\tau} = \left[1 + \frac{m_\tau}{p}\frac{dU}{dp}\right]^{-1}_{p_F^\tau}$$



Isospin diffusion



Diffusion

Particle Flux:

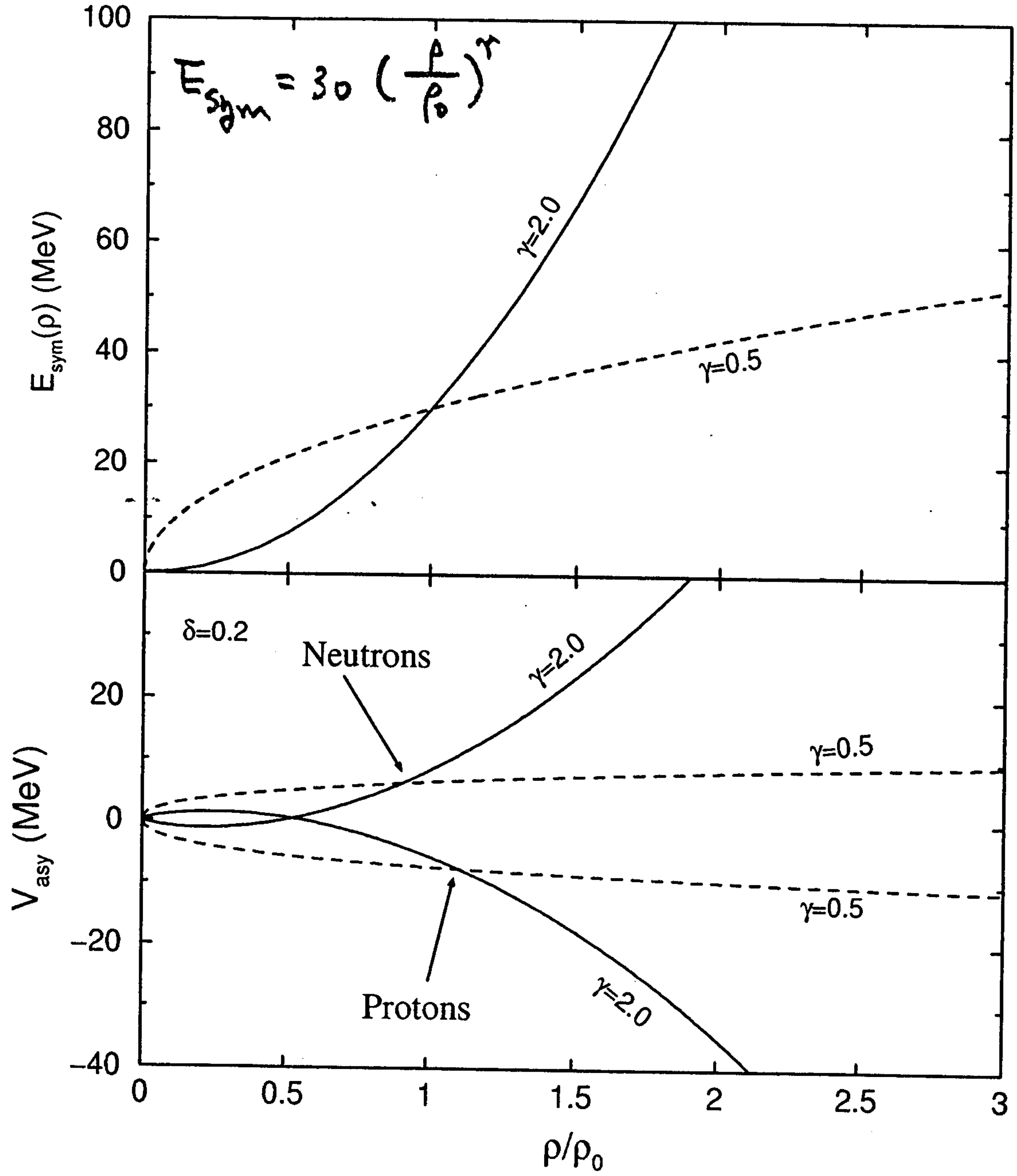
$$\mathbf{\Gamma}_i = n_i \left(\underline{\mathbf{v}}_i - \underline{\mathbf{v}} \right),$$

Isospin Flow:

$$\Gamma_I = \Gamma_n - \Gamma_p = -n D_I \frac{\partial \delta}{\partial \mathbf{r}}.$$

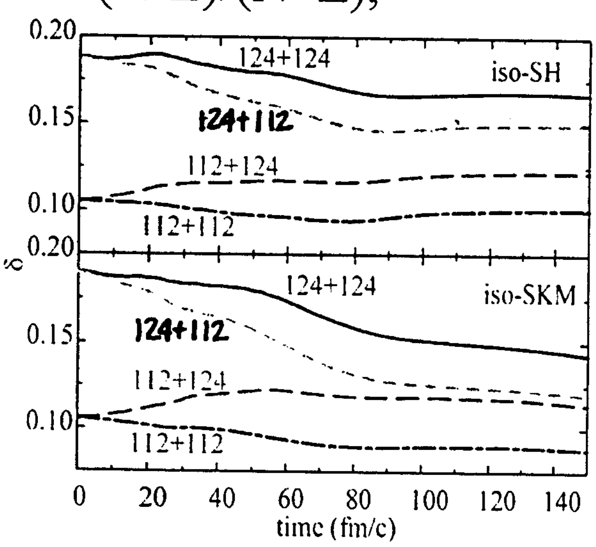
Isospin diffusion coef D_I

L. Shi
P. Danielewicz

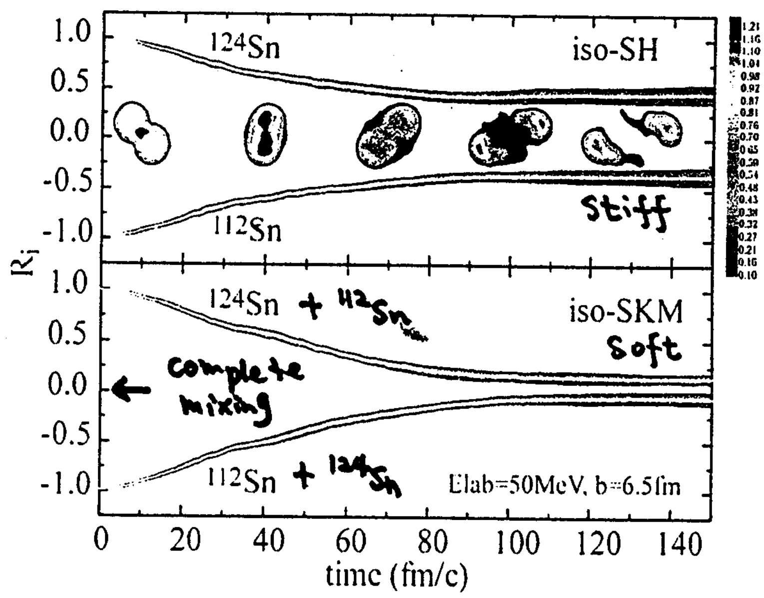

$E_{sym} = 30 \left(\frac{\rho}{\rho_0}\right)^{\gamma}$
$E_{sym}(\rho)$ (MeV)
γ=2.0
γ=0.5
δ=0.2
Neutrons
Protons
γ=2.0
γ=0.5
γ=0.5
γ=2.0
V_{asy} (MeV)
ρ/ρ_0

R_i from simulations by Shi and Danielewicz

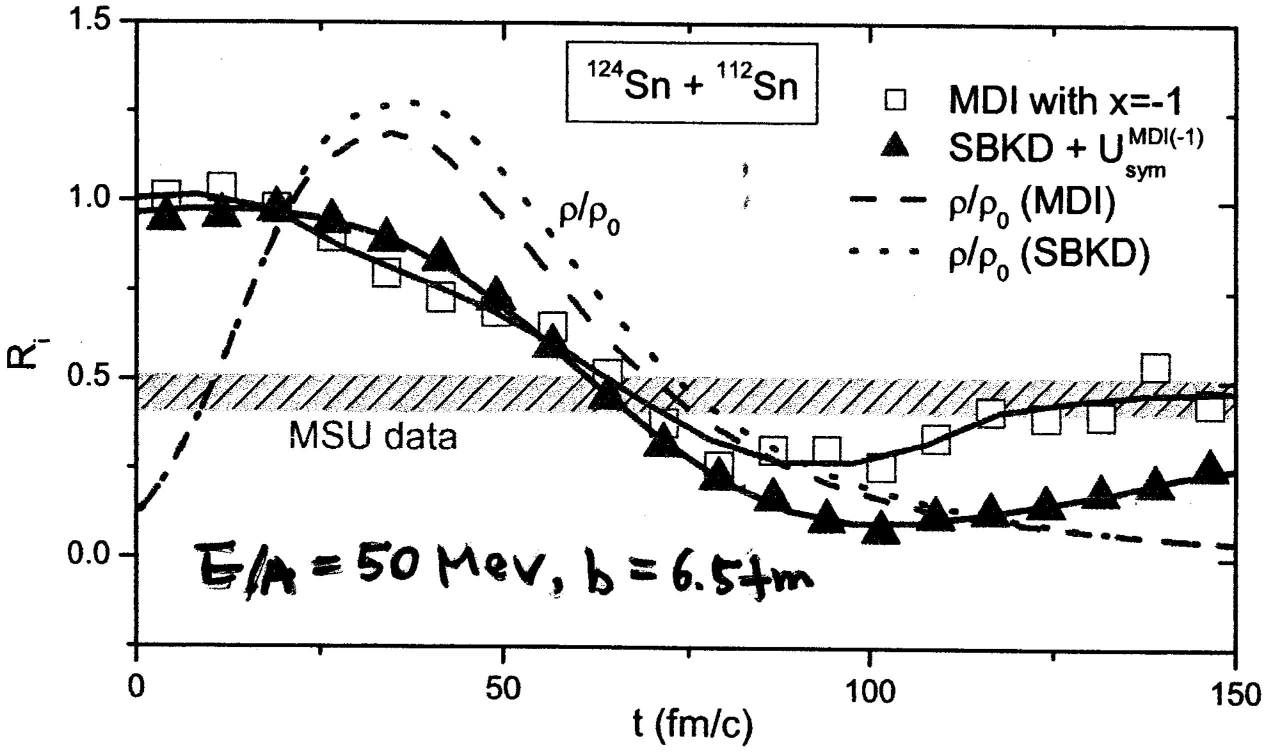
Projectile isospin asymmetry
δ from simulation
δ = (N-Z)/(N+Z),



$$R_i = \frac{(2\delta_i - \delta_{124+124} - \delta_{112+112})}{(\delta_{124+124} - \delta_{112+112})},$$

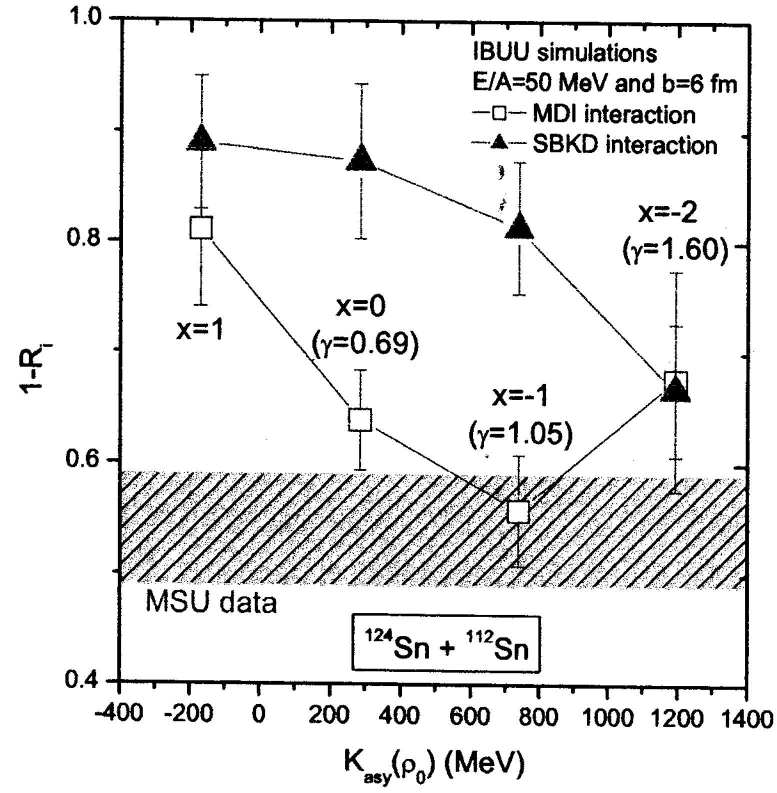


- R_i is a stable signal
- Non-diffusion effects: cancelled out
- R_i ~ IEOS ->Diffusion effect



- **Momentum-dependence is important for isospin diffusion**
- **For the Sn+Sn reactions at 50 MeV/A, the isospin diffusion happens in the density range of** $0.3 < \rho/\rho_0 < 1.2$

Strength of Isospin diffusion



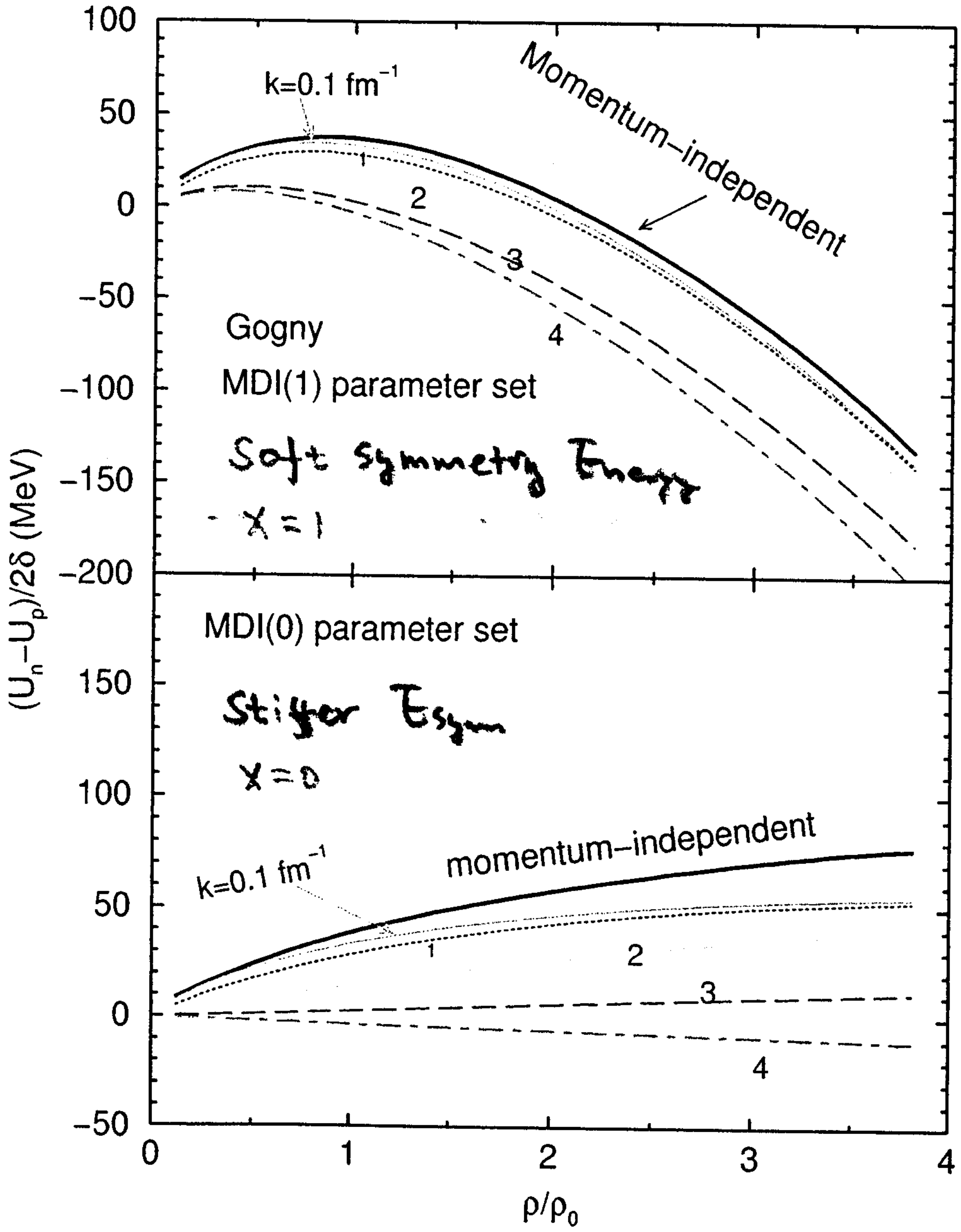
$$\mathbf{K(asymmetric\ \ matter) = K_0(symmetric\ \ matter) + K_{asy} \cdot \delta^2} \qquad (1)$$

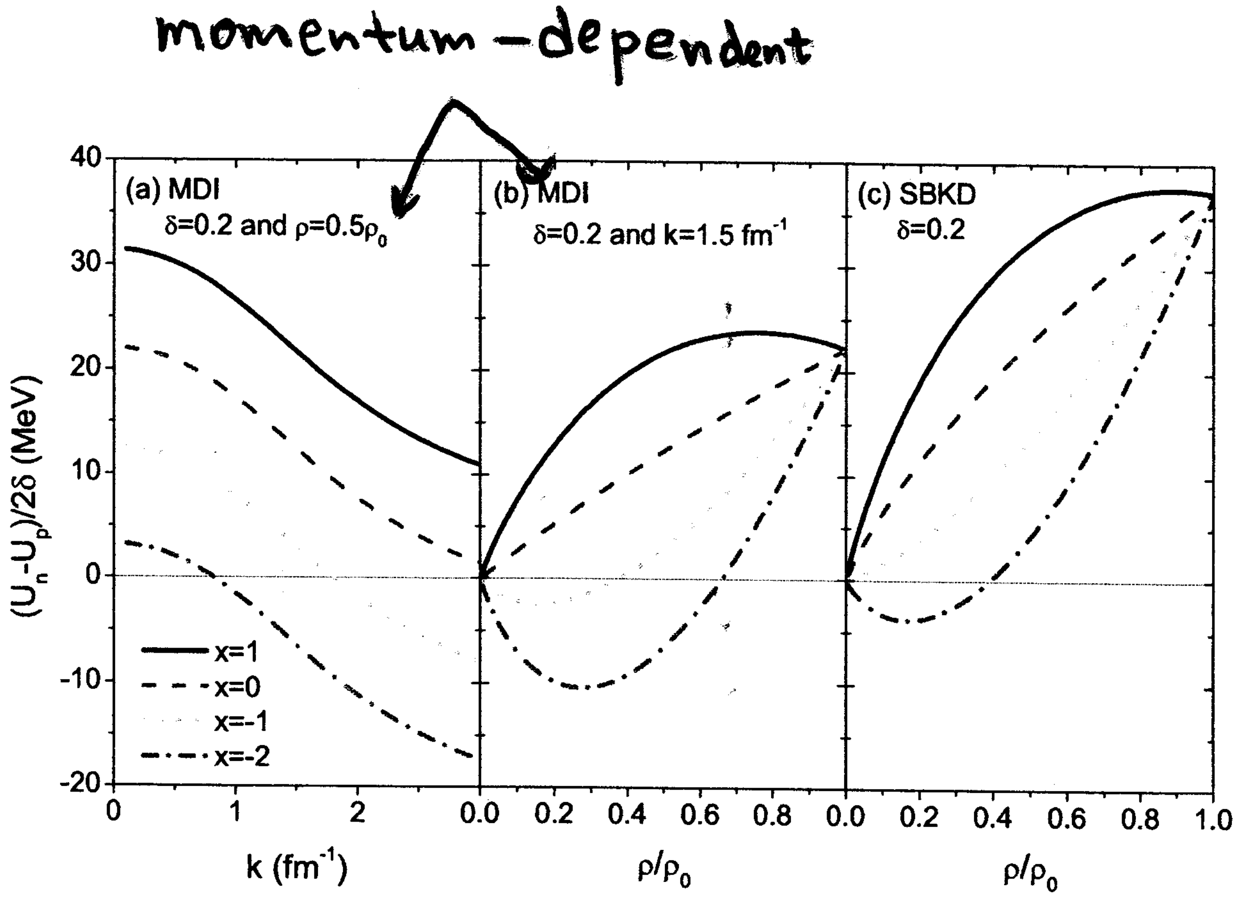
$$\mathbf{K_{asy}(\rho) = 18\rho\frac{dE_{sym}}{d\rho} + 9\rho^2\frac{d^2E_{sym}}{d\rho^2}} \qquad (2)$$

The data favors $E_{sym} = 31.6(\rho/\rho_0)^{1.05}$ and $K_{asy}(\rho_0) \approx 738 \pm 150$ MeV.

Giant Monopole resonance studies give:
-566 ± 1350 MeV $< K_{asy}(\rho_0) < 139 \pm 1617$ MeV by Shlomo and Youngblood in PRC 47, 529 (1993).

Strength of the symmetry potential





Strength of the symmetry potential

Pion probe of N/Z of high density participants

$N + N \rightarrow \Delta + N'$
$\quad\quad\quad\;\; \hookrightarrow \pi + N''$

Δ(1232) resonance model:

	π^+	π^0	π^-
nn	**0**	**1**	**5**
pp	**5**	**1**	**0**
np(pn)	**1**	**4**	**1**

Pion asymmetry: before re-absorption and re-scattering

$$\frac{\pi^-}{\pi^+} = \frac{5N^2 + NZ}{5Z^2 + NZ} \approx \left(\frac{N}{Z}\right)^2$$

Thermal model: (G.F. Bertsch, Nature 283 (1980) 281.)

$$\frac{\pi^-}{\pi^+} \propto \exp[(\mu_n - \mu_p)/kT] \text{ (the same factor for n/p ratio !)}$$

$$\mu_n - \mu_p = 2V_{sym}(\rho)\delta - V_{Coul} + kT\{\ln\frac{\rho_n}{\rho_p} + \sum_m \frac{m+1}{m} b_m (\frac{1}{2}\lambda_T^3)^m (\rho_n^m - \rho_p^m)\},$$ Henry Jaqaman, Aram Mekjian PRC (1984) ~.

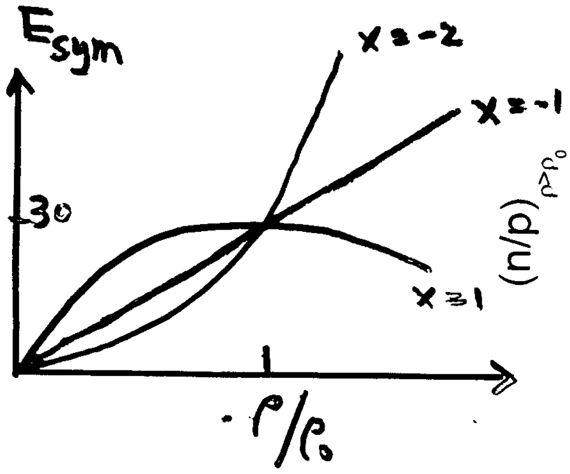
λ_T is the thermal wavelength and b_m are constants.

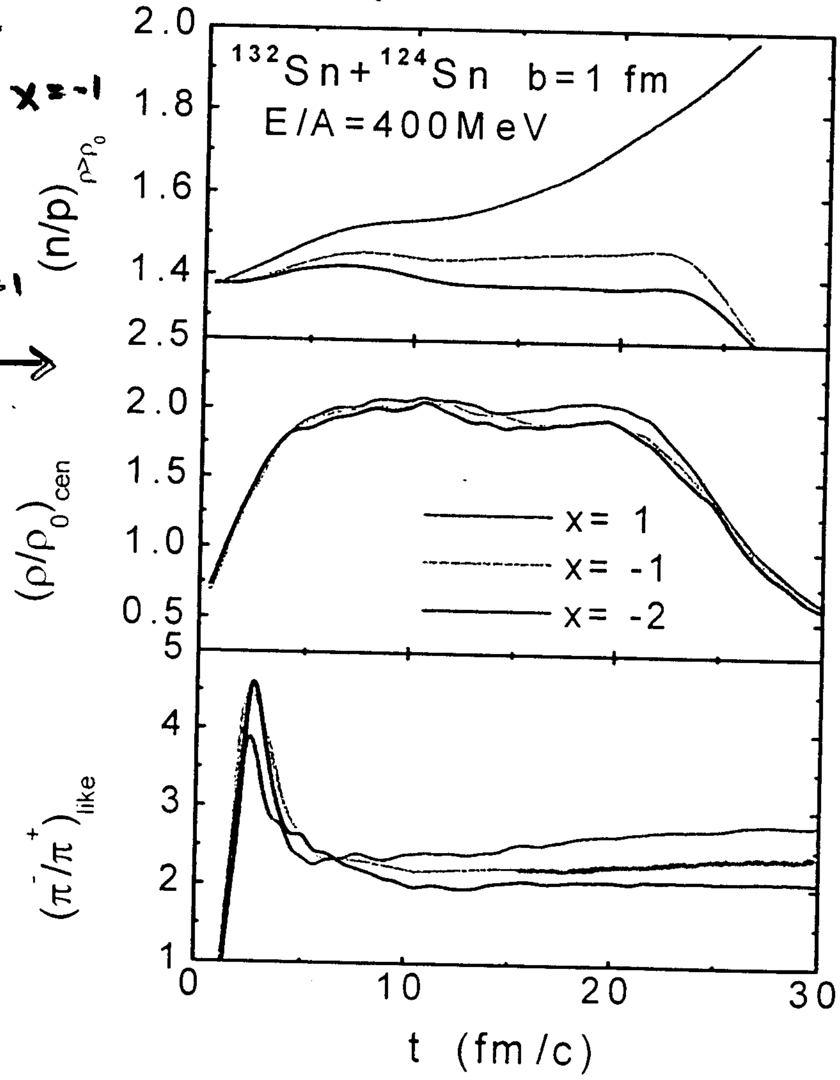
$$\rightarrow \frac{n}{p}, \frac{\pi^-}{\pi^+} \propto \exp[(2V_{sym}(\rho)\delta - V_{cou})/kT]$$

Pion-like ratio during the reaction:

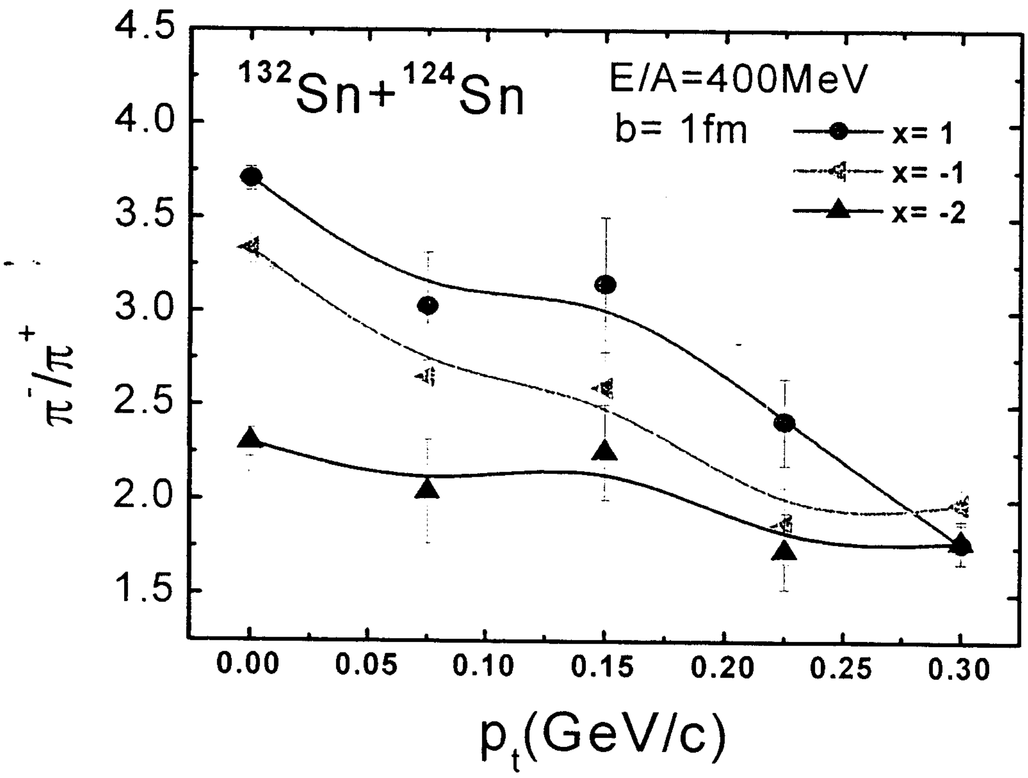
$$\left(\frac{\pi^-}{\pi^+}\right)_{like} = \frac{\pi^- + \Delta^- + \frac{1}{3}\Delta^0 + \frac{2}{3}N^{*0}}{\pi^+ + \Delta^{++} + \frac{1}{3}\Delta^+ + \frac{2}{3}N^{*+}} \xrightarrow{t\to\infty} \frac{\pi^-}{\pi^+}$$

Pion production with momentum-dependent interaction





$\pi^-/\ \pi^+$ ratio at low transverse momentum



Summary

- $E_{sym}(\rho)$ is important for both astrophysics and heavy-ion reactions
- Isospin diffusion data from NSCL/MSU favors

  $E_{sym}(\rho) = 31.6\ (\rho/\rho_0)^{1.05}$

  $Kasy(\rho_0) = 738 \pm 150$ MeV for $\rho/\rho_0 < 1.2$
- π^- / π^+ ratio at low transverse momentum in central heavy-ion collisions is promising for determining the symmetry energy at higher densities.