Remarks about weak-interaction processes

K. Langanke

GSI Darmstadt and TUD, Darmstadt

March 9, 2006

Nuclear beta decay, energetics

Q-value defined as the total kinetic energy released in the reaction

- β^- decay, $Q_{\beta^-} = M_i - M_f + E_i - E_f$

$$A(Z,N) \rightarrow A(Z+1,N-1) + e^- + \bar{\nu}_e$$

- β^+ decay, $Q_{\beta^+} = M_i - M_f + E_i - E_f - 2m_e$

$$A(Z,N) \rightarrow A(Z-1,N+1) + e^+ + \nu_e$$

- Electron capture, $Q_{\mathrm{EC}} = M_i - M_f + E_i - E_f$

$$A(Z,N) + e^- \rightarrow A(Z-1,N+1) + \nu_e$$

Transition rates for β decay

Fermi's golden rule:

$$\lambda = \frac{2\pi}{\hbar} \int |\mathcal{M}_{if}|^2 (2\pi\hbar)^3 \delta^{(4)}(p_f + p_e + p_\nu - p_i) \frac{d^3 p_f}{(2\pi\hbar)^3} \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 p_\nu}{(2\pi\hbar)^3}$$

$$|\mathcal{M}_{if}|^2 = \frac{1}{2J_i + 1} \sum_{\text{lepton spins}} \sum_{M_i, M_f} |\langle f | \boldsymbol{H}_w | i \rangle|^2$$

$$\lambda = \frac{1}{2\pi\hbar^7} \int |\mathcal{M}_{if}|^2 \delta(M_f^{\text{nuc}} + E_e + E_\nu - M_i^{\text{nuc}}) p_e^2 p_\nu^2 dp_e dp_\nu \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

Transition rates for β decay

$$W = E_e/(m_e c^2); \qquad W_0 = \frac{M_i^{\mathrm{nuc}} - M_f^{\mathrm{nuc}}}{m_e c^2} = \frac{Q}{m_e c^2} + 1$$

$$\lambda = \frac{m_e^5 c^4 G_V^2}{2\pi\hbar^7} \int_1^{W_0} C(W) F(Z,W) (W^2 - 1)^{1/2} W (W_0 - W)^2 dW$$

$$C(W) = \frac{1}{G_V^2} \int |\mathcal{M}_{if}|^2 \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

$F(Z,W)$ Fermi function, accounts for distortion of the electron (positron) wave function due to Coulomb effects. We need to compute shape factor,

$$C(W) = \frac{1}{G_V^2} \int \frac{1}{2J_i + 1} \sum_{\text{lepton spins}} \sum_{M_i, M_f} |\langle f | \boldsymbol{H}_w | i \rangle|^2 \frac{d\Omega_e}{4\pi} \frac{d\Omega_\nu}{4\pi}$$

between states: $|i\rangle = |J_i M_i; T_i T_{z_i}\rangle$; $\quad |f\rangle = |J_f M_f; T_f T_{z_f}; e^-; \bar{\nu}\rangle$

Weak Hamiltonian

Current-Current interaction:

$$\boldsymbol{H}_w = \frac{G_V}{\sqrt{2}} \int d^3 r \mathcal{J}^\mu(\boldsymbol{r}) j_\mu(\boldsymbol{r})$$

$$\langle f|\boldsymbol{H}_w|i\rangle = \frac{G_V}{\sqrt{2}} \int d^3 r \langle J_f M_f; T_f T_{z_f}; e, \nu | j_\mu \mathcal{J}^\mu | J_i M_i; T_i T_{z_i}\rangle$$

Assuming plane waves for electron and neutrino:

$$\langle e; \nu | j_\mu | 0\rangle = e^{-i(\boldsymbol{p}_e + \boldsymbol{p}_\nu)\cdot \boldsymbol{r}} \bar{u}\gamma_\mu (1-\gamma_5) v$$

$$\langle f|\boldsymbol{H}_w|i\rangle = \frac{G_V}{\sqrt{2}} l_\mu \int d^3 r e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \langle J_f M_f; T_f T_{z_f} | \mathcal{J}^\mu | J_i M_i; T_i T_{z_i}\rangle$$

$$l_\mu = \bar{u}\gamma_\mu (1-\gamma_5) v$$

Non-relativistic reduction

Assuming one nucleon participates in the decay and that we can use the free current (impulse approximation):

$$\langle f|\boldsymbol{H}_w|i\rangle = \frac{G_V}{\sqrt{2}} l_\mu \int d^3 r e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \bar{\psi}_f \gamma^\mu (1 + g_A \gamma_5) \boldsymbol{t}_\pm \psi_i$$

$$\psi = \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+M} \end{pmatrix} \phi \rightarrow \begin{pmatrix} \phi \\ 0 \end{pmatrix}$$

$$\langle f|\boldsymbol{H}_w|i\rangle = \frac{G_V}{\sqrt{2}} \int d^3 r e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \phi_f (l_0 \mathbf{1} + g_A \boldsymbol{l} \cdot \boldsymbol{\sigma}) \boldsymbol{t}_\pm \phi_i$$

Generalization to A particles:

$$\boldsymbol{H}_w = \frac{G_V}{\sqrt{2}} \sum_{k=1}^{A} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_k} (l_0 \mathbf{1}^k + g_A \boldsymbol{l} \cdot \boldsymbol{\sigma}^k) \boldsymbol{t}_\pm^k$$

Non-relativistic reduction

$$\boldsymbol{H}_w = \frac{G_V}{\sqrt{2}} \sum_{k=1}^{A} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}_k} (l_0 \mathbf{1}^k + g_A \boldsymbol{l} \cdot \boldsymbol{\sigma}^k) \boldsymbol{t}_{\pm}^k$$

$$e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} = \sum_l \sqrt{4\pi(2l+1)}(-i)^l j_l(qr) Y_{l0}(\theta, \varphi)$$

$$j_l(qr) \approx \frac{(qr)^l}{(2l+1)!!}$$

- Zero order: Allowed transitions (Fermi, Gamow-Teller)
- Higher orders: Forbidden transitions.

ft-value

$$\lambda = \frac{\ln 2}{K} \int_1^{W_0} C(W) F(Z,W) (W^2-1)^{1/2} W (W_0 - W)^2 dW$$

For allowed transitions: $C(W) = B(F) + B(GT)$,

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{K} [B(F) + B(GT)] f(Z, W_0)$$

$$ft_{1/2} = \frac{K}{B(F) + B(GT)}, \quad K = 6144.4 \pm 1.6 \text{ s}$$

Fermi Transitions

$$B(F) = \frac{1}{2J_i+1} \sum_{M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \boldsymbol{t}_{\pm}^{k} |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(F) = [T_i(T_i+1) - T_{z_i}(T_{z_i} \pm 1)]\delta_{J_i,J_f}\delta_{T_i,T_f}\delta_{T_{z_f},T_{z_i}\pm 1}$$

Energetics (Isobaric Analog State):

$$E_{\mathrm{IAS}} = Q_\beta + \mathrm{sign}(T_{z_i})[E_C(Z+1) - E_C(Z) - (m_n - m_H)]$$

Selection rule:

$$\Delta J = 0 \qquad \Delta T = 0 \qquad \pi_i = \pi_f$$

Sum rule (sum over all the final states):

$$S(F) = S_-(F) - S_+(F) = 2T_{z_i} = (N-Z)$$

Gamow-Teller Transitions

$$B(GT) = \frac{g_A^2}{2J_i+1} \sum_{m,M_i,M_f} |\langle J_f M_f; T_f T_{z_f}| \sum_{k=1}^{A} \sigma_m^k t_\pm^k |J_i M_i; T_i T_{z_i}\rangle|^2$$

$$B(GT) = \frac{g_A^2}{2J_i+1} |\langle J_f; T_f T_{z_f}|| \sum_{k=1}^{A} \sigma^k t_\pm^k ||J_i; T_i T_{z_i}\rangle|^2$$

$$g_A = -1.2720 \pm 0.0018$$

Selection rule:

$$\Delta J = 0,1 \ (\text{no } J_i = 0 \rightarrow J_f = 0) \qquad \Delta T = 0,1 \qquad \pi_i = \pi_f$$

Ikeda sum rule:

$$S(GT) = S_-(GT) - S_+(GT) = 3(N-Z)$$

Gamow-Teller transitions II

β^- decay $|J_i; T, T\rangle$

- Final state $|J_f; T-1, T-1\rangle$

$$B(GT) = \frac{2g_A^2}{2J_i+1} \frac{|\langle J_f; T-1 ||| \sum_{k=1}^{A} \sigma^k t^k ||| J_i; T\rangle|^2}{2T+1}$$

- Final state $|J_f; T, T-1\rangle$

$$B(GT) = \frac{2g_A^2}{2J_i+1} \frac{|\langle J_f; T ||| \sum_{k=1}^{A} \sigma^k t^k ||| J_i; T\rangle|^2}{(2T+1)(T+1)}$$

- Final state $|J_f; T+1, T-1\rangle$

$$B(GT) = \frac{2g_A^2}{2J_i+1} \frac{|\langle J_f; T+1 ||| \sum_{k=1}^{A} \sigma^k t^k ||| J_i; T\rangle|^2}{(2T+1)(T+1)(2T+3)}$$

Forbidden Transitions

Involve operators $r^l Y_{lm}$ and $r^l [Y_{lm} \otimes \boldsymbol{\sigma}]^K$
Selection rules

Decay type	ΔJ	ΔT	$\Delta\pi$	log ft
Superallowed	$0^+ \to 0^+$	0	no	3.1–3.6
Allowed	0,1	0,1	no	2.9–10
First forbidden	0,1,2	0,1	yes	5–19
Second forbidden	1,2,3	0,1	no	10–18
Third forbidden	2,3,4	0,1	yes	17–22
Fourth forbidden	3,4,5	0,1	no	22–24

Electron Capture

Fermi's golden rule:

$$\sigma = \frac{2\pi}{\hbar v_e} \int |\mathcal{M}_{if}|^2 (2\pi\hbar)^3 \delta^{(4)}(p_f + p_\nu - p_i - p_e) \frac{d^3 p_f}{(2\pi\hbar)^3} \frac{d^3 p_\nu}{(2\pi\hbar)^3}$$

$$\sigma_{i,f}(E_e) = \frac{G_V^2}{2\pi\hbar^4} F(Z, E_e)[B(F) + B(GT)] p_\nu^2$$

Neutrino cross sections

Charged current: $(Z, A) + \nu_e \rightarrow (Z+1, A) + e^-$

$$\sigma_{i,f}(E_\nu) = \frac{G_V^2}{\pi} p_e E_e F(Z+1, E_e)[B(F) + B(GT)]$$

Neutral current: $(Z, A) + \nu \rightarrow (Z, A)^* + \nu$

$$\sigma_{i,f}(E_\nu) = \frac{G_F}{\pi}(E_\nu - w)^2 B(GT_0)$$

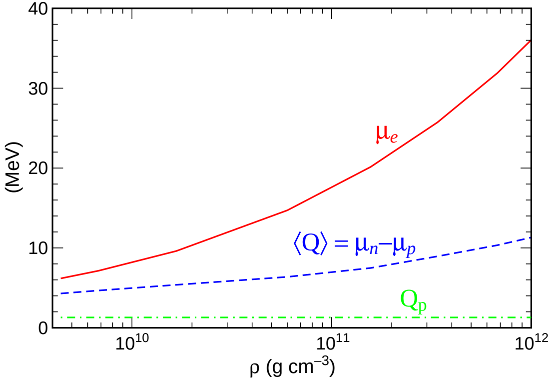
with $w = E_f - E_i$

In general, multipoles beyond allowed transitions are necessary. See Donelly and Peccei, Phys. Repts. **50**, 1 (1979).

General considerations: Which multipoles?

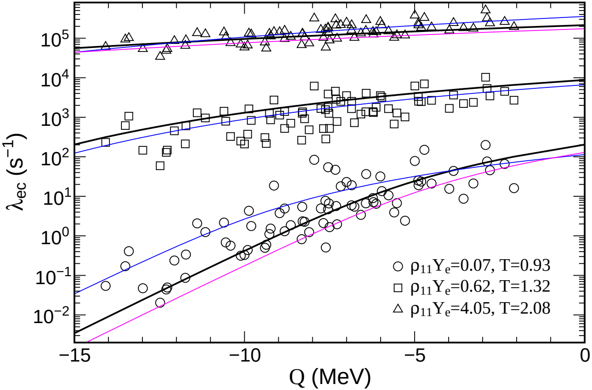
- Multipole operators O_λ
  - $\sim \left(\frac{qR}{\hbar c}\right)^\lambda \quad ; q \approx E_\nu$
  - successively higher rank with increasing E_ν
- Collective nuclear excitations:
  - $[H, O_\lambda] \neq 0 \rightarrow$ strength is fragmented
  - centroid $E^\lambda_{coll} \sim \lambda\hbar\omega \sim \lambda\frac{41}{A^{1/3}}\mathrm{MeV}$
- Phase space:
  - $\sim p_{lep}E_{lep} \rightarrow$ high E_{lep} preferred
  - average nuclear excitation $\bar{\omega}$ lags behind with increasing E_ν
  - if $E_\nu >> \bar{\omega}$, σ sensitive to total strength

Electron capture: energetics during collapse



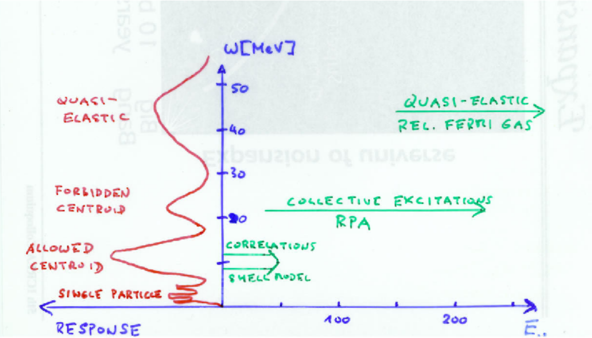
capture rate becomes less dependent on details of GT distribution with increasing density (chem. potential); for $\rho_{11} >\sim 1$, it depends essentially only on total strength and centroid

Example: electron capture at high electron energy

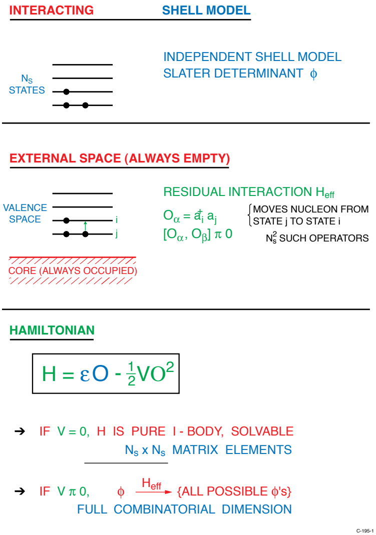


Assumption: capture proceeds by a single transition ($E_f - E_i = const$) with a constant strength

Remarks about response: Which models?



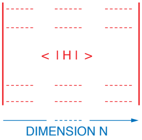
Shell Model.



INTERACTING SHELL MODEL
N_S STATES
INDEPENDENT SHELL MODEL
SLATER DETERMINANT ϕ
EXTERNAL SPACE (ALWAYS EMPTY)
VALENCE SPACE
i
j
CORE (ALWAYS OCCUPIED)
RESIDUAL INTERACTION H_eff
Oα = a†i aj
[Oα, Oβ] π 0
MOVES NUCLEON FROM STATE j TO STATE i
N_S^2 SUCH OPERATORS
HAMILTONIAN
H = εO - ½VO²
IF V = 0, H IS PURE I - BODY, SOLVABLE
N_S x N_S MATRIX ELEMENTS
IF V π 0, ϕ H_eff {ALL POSSIBLE ϕ's}
FULL COMBINATORIAL DIMENSION
C-195-1

Diagonalization shell model.

DIAGONALIZATION APPROACH



"GIANT" MATRIX

$$N = \binom{N_s}{N^P_{VAL}} \binom{N_s}{N^N_{VAL}}$$

$\approx 10^9$ FOR ^{60}Zn

REDUCTION OF SIZE DUE TO SYMMETRIES

MODERN ALGORITHM (STRASSBOURG - MADRID)

- LANCZOS ALGORITHM (FEW LOWEST EIGENSTATES)
- STORAGE OF ONLY H_{pp}, H_{nn}, H_{pn}
- EFFICIENT ALGORITHM TO CONSTRUCT <.......H......> FROM H_{pp}, H_{nn}, H_{pn}

ALL EVEN-EVEN NUCLEI IN PF-SHELL

C-195-2

SHELL MODEL MONTE CARLO

GOAL: DETERMINE NUCLEAR PROPERTIES, NOT ALL ~10^9 COMPONENTS OF W.F.

➔ CONSIDER THERMAL AVERAGE IN CANONICAL (FIXED NUMBER) EMSEMBLE

$$\langle A \rangle = \frac{\mathrm{Tr}\,(e^{-\beta H} A)}{\mathrm{Tr}\,(e^{-\beta H})} \qquad \beta = \frac{1}{T}$$

➔ HUBBARD - STRATONOVICH TRANSFORMATION

2-BODY ⇒ MANY 1-BODY EVOLUTIONS IN FLUCTUATING EXTERNAL FIELDS

SUPPOSE

$$e^{-\beta H} \longrightarrow e^{\frac{1}{2}\beta V O^2} = \int \frac{d\sigma}{\sqrt{2\pi/\beta V}}\, e^{\frac{1}{2}\beta V \sigma^2} \underbrace{e^{\beta \sigma V O}}_{\text{1-BODY PROPAGATOR}}$$

HOWEVER: MANY NON-COMMUTING O's

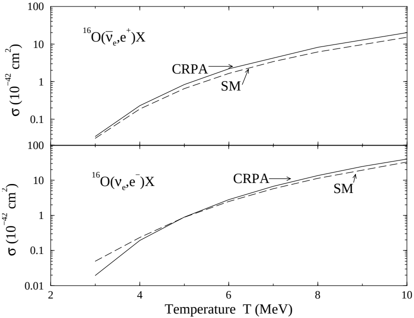
$$e^{-\beta H} = (e^{-\Delta\beta H})^{N_t} \quad ; \quad \Delta\beta = \frac{\beta}{N_t} \quad \text{“TIME SLICE”}$$

⇒ SEPARATE σ-FIELDS AT EACH TIME SLICE FOR EACH O

➔ MONTE-CARLO EVALUATION OF σ-INTEGRALS EMBARRASSINGLY PARALLEL

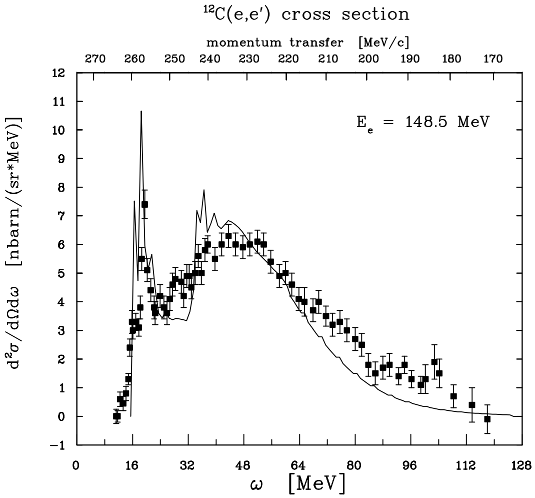
C-195-3

Shell Model versus RPA.

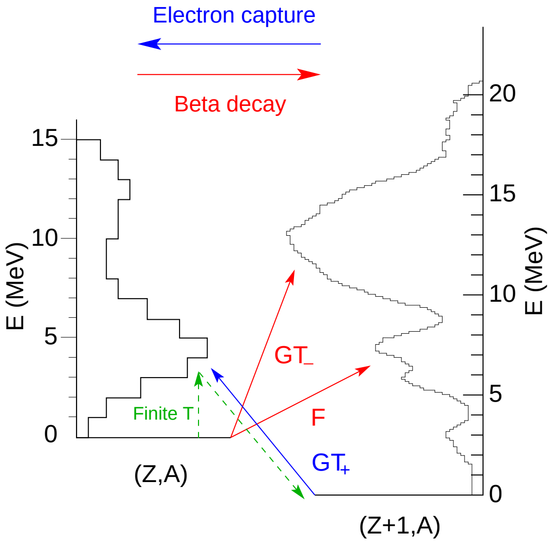


The RPA considers only (1p-1h) correlations; it is well suited to describe the centroid of giant resonances

Inelastic electron scattering: RPA description of response



Beta-decay, electron capture, charged-current



Systematics of Fermi and Gamow-Teller centroids

ESTIMATE OF CHARGED-CURRENT CROSS SECTION

$$\sigma(E_\nu) = \frac{G_F^2 \cos^2\Theta_c}{\pi} k_e E_e F(Z+1, E_e) \left(|M_F|^2 + \left(g_A^{\text{eff}}\right)^2 |M_{GT}|^2 \right)$$

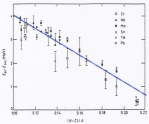
FERMI TRANSITIONS

- $|M_F|^2 = (N-Z)$ ISOBARIC ANALOG
- $E_{IAS} \approx \left(\frac{1.728\, Z}{1.12\, A^{1/3} + 0.78} - 1.293 \right)$ MeV

GAMOW-TELLER TRANSITIONS

- $|M_{GT}|^2 = 3(N-Z)$ VERY NEUTRON RICH
  IKEDA SUM RULE

- $\langle E_{GT} \rangle - \langle E_{IAS} \rangle \approx A + B(N-Z)$

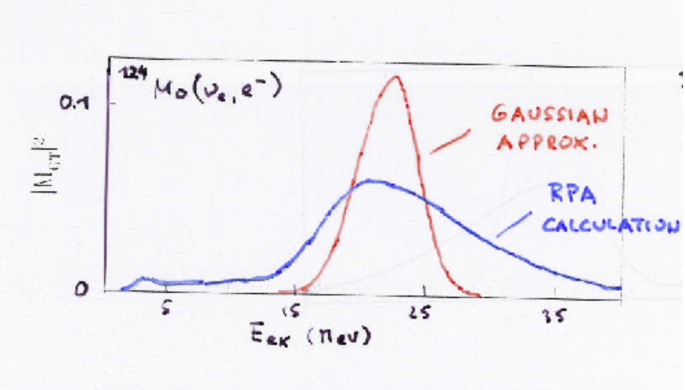


- DISTRIBUTION
  a) δ-FUNCTION
  b) GAUSSIAN AROUND $\langle E_{GT} \rangle$ WITH WIDTH ~ 5 MeV

THE OPEN QUESTIONS:

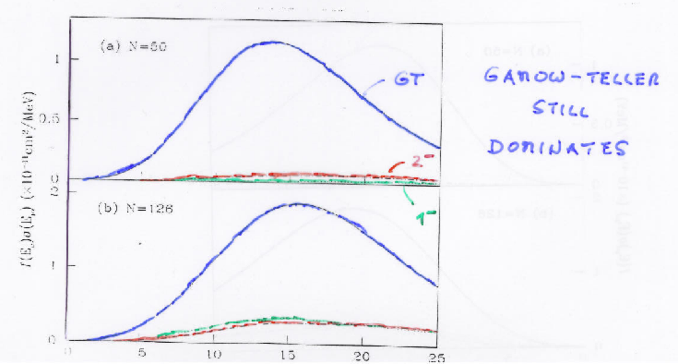
- LOW LYING GT-STRENGTH
- FORBIDDEN TRANSITIONS

How well do we know charged-current cross sections?



RPA vs simple Gaussian: $\sim$ factor 2 (Surmann+Engel)

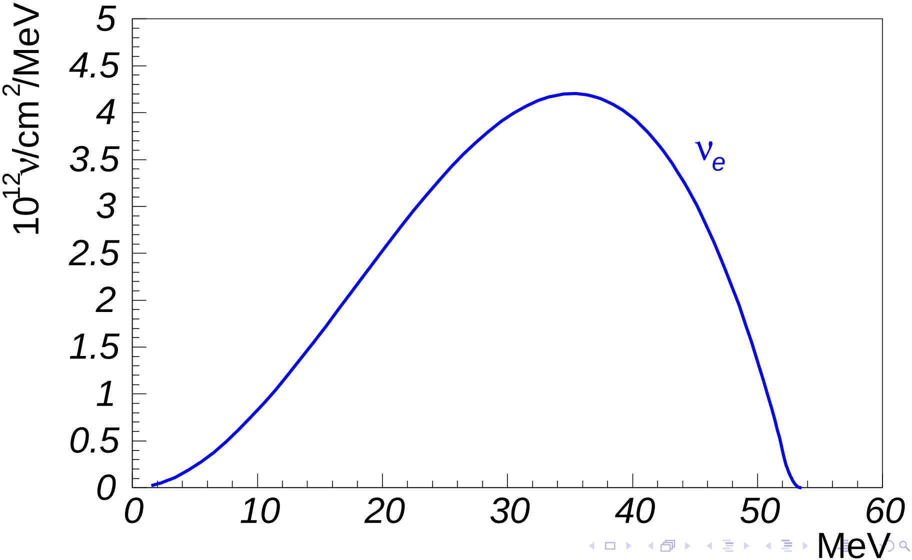
What about other multipoles?



for neutrino spectrum with $T = 4$ MeV

Influence of higher multipoles

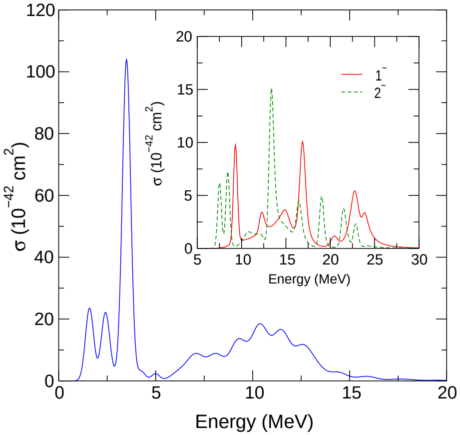
Spectrum for muon decay: $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$



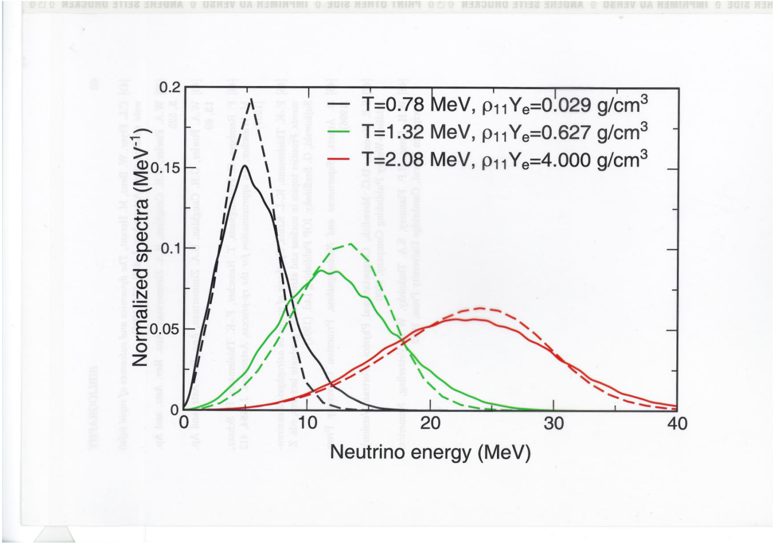
^{56}Fe$(\nu_e, e^-)^{56}$Co measured by KARMEN collaboration:

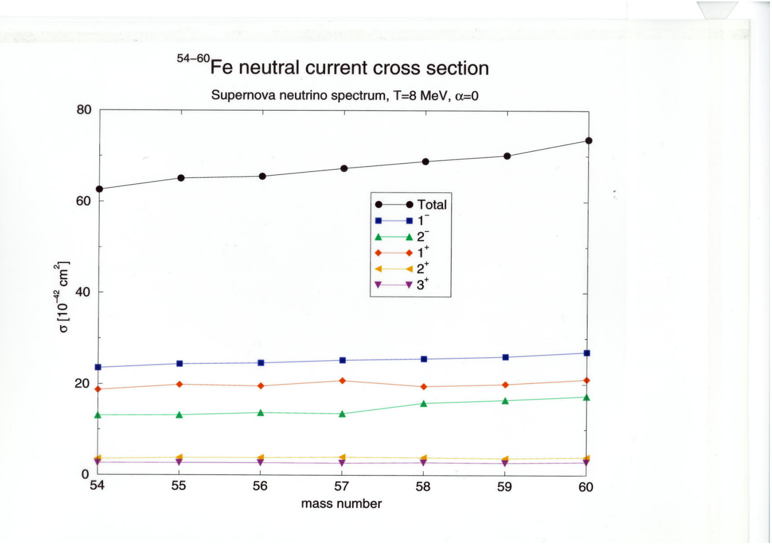
$\sigma_{exp} = 2.56 \pm 1.08(stat) \pm 0.43(syst) \times 10^{-40}\ cm^2$

$\sigma_{th} = 2.38 \times 10^{-40}\ cm^2$



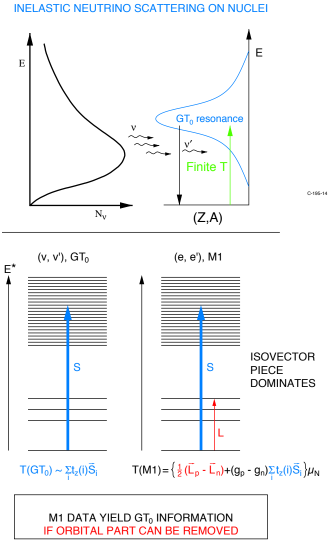
Typical supernova neutrino spectra



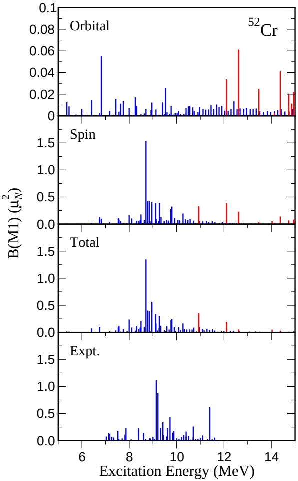


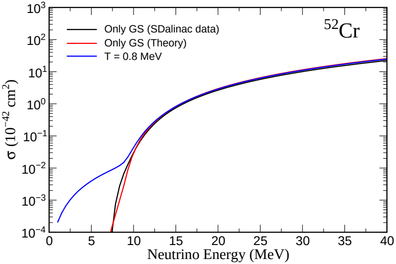
$^{54-60}$Fe neutral current cross section
Supernova neutrino spectrum, T=8 MeV, α=0
σ [10^{-42} cm^2]
80
60
40
20
0
54
55
56
57
58
59
60
mass number
Total
1$^-$
2$^-$
1$^+$
2$^+$
3$^+$

Determining inelastic neutrino-nucleus cross sections



Neutrino cross sections from electron scattering





- high-precision SDalinac data
- large-scale shell model

- neutrino cross sections from (e, e') data
- validation of shell model
- G.Martinez-Pinedo, P. v. Neumann-Cosel, A. Richter